Network Control Model

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Overview

Network Control Theory

- A first order difference equation to model the dynamics of neural processes
- $x(t+1) = \mathbf{A}x(t) + \underbrace{\mathbf{B}\mu(t)}_{\text{external input}}$

  where $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ describes the state of each node
  $\mathbf{A} \in \mathbb{R}^{n \times n}$ is a weighted adjacency matrix
  μ is a vector of external inputs (stimuli)
  $\mathbf{B}$ is a matrix that maps the stimuli to corresponding brain regions

Implementation

- neurolib: *Dataset("hcp").Cmat* $\leftarrow$ Structural Connectivity
- nctpy: *get_control_inputs*
  integrate_u

Implementation

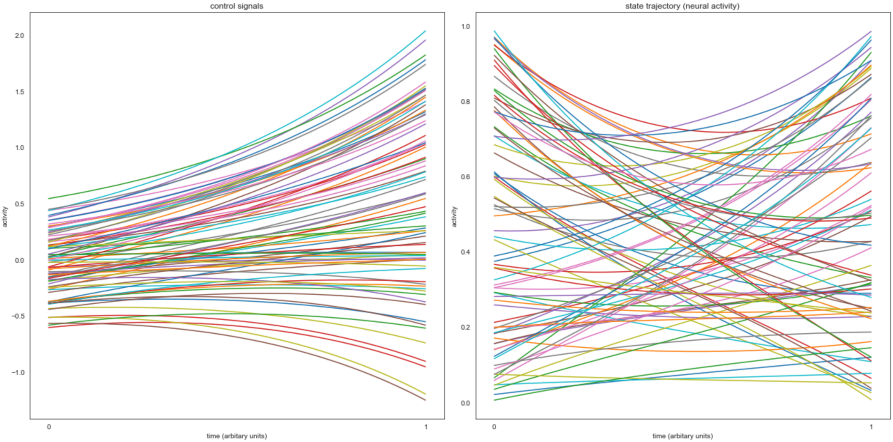
- *get_control_inputs*: function used to calculate the control signals required to transition a system from an initial state to a target state.
- **Inputs**:
  - `A_norm`: Normalised adjacency matrix of the network.
  - `T`: Time horizon for the control process.
  - `B`: Control input matrix (control set).
  - `x0`: Initial state vector.
  - `xf`: Target state vector.
  - `system`: Type of system (e.g., 'continuous' or 'discrete').
- **Outputs**:
  - `control_signals (u)`: Control signals required for the state transition.
  - `x`: State trajectory of the system.

Example

Parameter:

- x0 = np.random.rand(n, 1)
- xf = np.random.rand(n, 1)
- B (control_set) = uniform full control set ($n \times n$ identity matrix)
- system = continuous

Example

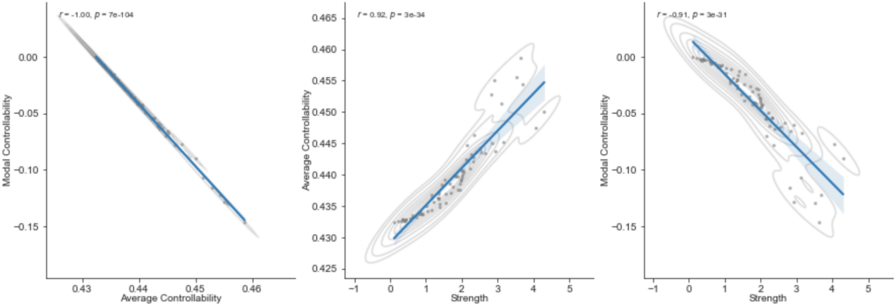


Energy: 24819.5

Controllability

- Average Controllability: Average Controllability indexes nodes' general capacity to control dynamics, a region with higher average controllability is better able to broadcast an impulse.
- Modal Controllability: Modal controllability identifies brain areas that can steer the system into difficult-to-reach states.

Controllability



Data-Driven Control Set Optimisation

ComputeOptimizedControlEnergy:

- Inputs:
  - A
  - T
  - system
  - ⋮
  - n_steps =5
  - learning_rate = 0.01
- Outputs:
  - optimised B
  - optimised energy

Example

$$B = \begin{pmatrix} 1.31 \\ 0.56 \\ 0.80 \\ \vdots \\ 0.72 \\ 1.33 \end{pmatrix} \left.\vphantom{\begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix}}\right\} 80$$

Energy: $24819 \to 16904$

Thank You!