Nome: Gabriel Hipólito Ferreira da Silva

1) a) $f(x) = -x, \; -L \leq x < L; \; f(x+2L) = f(x)$ (ímpar)

$\operatorname{sen}(m\pi) = 0$, $\forall$ para m inteiro

$a_m = 0$

$$b_m = \frac{2}{L}\int_0^L f(x)\operatorname{sen}\left(\frac{m\pi x}{L}\right)dx = \frac{2}{L}\int_0^L \overset{u}{-x} \cdot \overbrace{\operatorname{sen}\left(\frac{m\pi x}{L}\right)dx}^{dv}$$

Integrando por partes:

$u = -x \Rightarrow \frac{du}{dx} = -1 \Rightarrow -dx = du$

$u = \frac{m\pi x}{L}$

$\frac{du}{dx} = \frac{m\pi}{L} \Rightarrow du = \frac{m\pi}{L}dx \Rightarrow L.du = m\pi.dx$

$dx = \frac{L}{m\pi}du$

$\int dv = \int \operatorname{sen}\left(\frac{m\pi x}{L}\right)dx$

$v = \left[-\cos\left(\frac{m\pi x}{L}\right)\right] \cdot \frac{L}{m\pi}$

$v = -\frac{L}{m\pi}\cos\left(\frac{m\pi x}{L}\right)$

$\int u\,dv = uv - \int v\,du$

$$\int -x \cdot \operatorname{sen}\left(\frac{m\pi x}{L}\right) = \left[(-x)\left[-\frac{L}{m\pi}\cos\left(\frac{m\pi x}{L}\right)\right] - \int_0^L +\frac{L}{m\pi}\cos\left(\frac{m\pi x}{L}\right)dx\right]_0^L \frac{2}{L}$$

$$\left[\frac{xL}{m\pi}\cos\left(\frac{m\pi x}{L}\right) - \frac{L}{m\pi}\int_0^L \cos\left(\frac{m\pi x}{L}\right)dx\right]\frac{2}{L}$$

$$\frac{xL}{m\pi}\cdot\cos\left(\frac{m\pi x}{L}\right) - \frac{L}{m\pi}\left(\left[\operatorname{sen}\left(\frac{m\pi x}{L}\right)\right]\cdot\frac{L}{m\pi}\right)\Bigg|_0^L \frac{2}{L}$$

$$\left\{\frac{L.L}{m\pi}\cdot\cos\left(\frac{m\pi \cancel{L}}{\cancel{L}}\right) - \frac{L}{m\pi}\left[\operatorname{sen}\left(\frac{m\pi.L}{L}\right)\right]\frac{L}{m\pi}\right\}_0 \frac{2}{L}$$

$$\frac{L^2}{m\pi}\cdot\underbrace{\cancel{\cos(m\pi)}}_{(-1)^m} - \cancel{\left(\frac{L}{m\pi}\right)^2 \cdot \operatorname{sen}(m\pi)}^{0} - {}^{0}$$

$$\left[\frac{L^2}{m\pi}\cdot(-1)^m - \left(\cancel{\frac{0.L}{m\pi}}^{0}\cdot\cos\left(\frac{m\pi.0}{L}\right)\right) - \cancel{\frac{L}{m\pi}}^{0}\left(\operatorname{sen}\left(\cancel{\frac{m\pi.0}{L}}^{0}\right)\cdot\frac{L}{m\pi}\right)\right]\frac{2}{L}$$

$$\left[\frac{L^2}{m\pi}\cdot(-1)^m\right]\frac{2}{L} = \frac{2}{\cancel{L}}\cdot\frac{\cancel{L}.L}{m\pi}\cdot(-1)^m = \frac{2L}{m\pi}(-1)^m \begin{cases} m \text{ par: } \frac{2L}{m\pi} \\ m \text{ ímpar: } -\frac{2L}{m\pi} \end{cases}$$

$a_0 = 0.$

$$\sum_{m=1}^{\infty} b_m \operatorname{sen}\left(\frac{m\pi x}{L}\right) = \sum_{m=1}^{\infty}\left[\frac{2L}{m\pi}(-1)^m\right]\cdot \operatorname{sen}\left(\frac{m\pi x}{L}\right)$$

b) $f(x) = \begin{cases} 1, & -L \leq x < 0 \\ 0, & 0 \leq x < L \end{cases}$; $f(x+2L) = f(x)$ $T = 2L$

$a_n = 0$, pois $f(x)$ é ímpar.

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{L}\int_{-L}^{0} 1\,dx = \frac{1}{L}\left(x\Big|_{-L}^{0}\right) = \frac{1}{L}\left(0-(-L)\right)$$

$$\Rightarrow a_0 = \frac{1}{L}(L) \Rightarrow a_0 = 1$$

$$b_m = \frac{1}{L}\int_{-L}^{0} \operatorname{sen}\left(\frac{m\pi x}{L}\right)dx \qquad u = \frac{m\pi x}{L} \Rightarrow du = \frac{m\pi}{L}\,dx$$

$$dx = \frac{L}{m\pi}\,du$$

$$= \frac{L}{m\pi}\cos\left(\frac{m\pi x}{L}\right)\Big]_{-L}^{0} = -\frac{1}{m\pi}\left(1-\cos m\pi\right) \Rightarrow \text{se} \begin{cases} 0, & m \text{ par} \\ \frac{-2}{m\pi}, & m \text{ ímpar} \end{cases}$$

$$f(x) = \frac{1}{2} + \sum_{m=1}^{\infty}\left[\frac{1}{m\pi}\left(1-\cos m\pi\right)\operatorname{sen}\frac{m\pi x}{L}\right]$$

c) $f(x) = \begin{cases} x+1, & -1 \leq x < 0 \\ 1-x, & 0 \leq x < 1 \end{cases}$ $f(x+2) = f(x)$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{m\pi x}{L}\right)dx$$

$$a_n = 0 \; \frac{1}{L}\left[\int_{-1}^{0} x+1\,dx + \int_{0}^{1} 1-x\,dx\right] = \frac{1}{L}\left(\frac{x^2}{2}+x\right)\Big|_{-1}^{0} - \left(x-\frac{x^2}{2}\right]_{0}^{1}$$

$$\frac{1}{L}\left(\cancel{\frac{1}{2}}+\frac{1}{2}\right) = \frac{1}{L}$$

$a_n > 0$

$$\frac{1}{L}\left[\int_{-1}^{0} x+1 \cos\frac{m\pi x}{L}\,dx + \int_{0}^{1} 1-x\cos\frac{m\pi x}{L}\,dx\right.$$

$u = x+1 \Rightarrow du = 1$

$dv = \cos\frac{m\pi x}{L} \Rightarrow v = \frac{1}{m\pi}\operatorname{sen}\frac{m\pi x}{L}$

$u = 1-x \Rightarrow du = -1$

$dv = \cos\frac{m\pi x}{L}$

$v = \frac{1}{m\pi}\cdot\operatorname{sen}\left(\frac{m\pi x}{L}\right)$

$$\frac{1}{L}\Big[(x+1)\frac{L}{m\pi}\,sen\left(\frac{m\pi x}{L}\right)\Big|_{-1}^{0} - \int_{-1}^{0}\frac{L}{m\pi}\,sen\,\frac{m\pi x}{L}\,dx + \Big[(1-x)\frac{L}{m\pi}\,sen\left(\frac{m\pi x}{L}\right)\Big|_{0}^{1} + \int_0^1 \frac{L}{m\pi}\,sen\left(\frac{m\pi x}{L}\right)$$

$$\frac{1}{L}\Big[-\frac{L^2}{m\pi\; m\pi}\cos\frac{m\pi x}{L}\Big|_1^0 - \frac{L^2}{(m\pi)^2}\cos\frac{m\pi x}{L}\Big|_0^1\Big]$$

$$\frac{1}{L}\Big[-\frac{L^2}{(m\pi)^2}\left(1-\cos\frac{m\pi}{L}\right) - \frac{L^2}{(n\pi)^2}\left(1-\cos\frac{m\pi}{L}\right)\Big]$$

$$\frac{1}{L}\Big[-\frac{2L^2}{(m\pi)^2}\left(1-\cos\left(\frac{m\pi}{L}\right)\right)\Big] = -\frac{2L}{(m\pi)^2}\left(1-\cos\frac{m\pi}{L}\right)$$

$b_m > 0$

$$b_m = \frac{1}{L}\int_{-L}^{L} f(x)\,sen\left(\frac{m\pi x}{L}\right)dx = \frac{1}{L}\Big[\int_{-L}^{0}(x+L)\,sen\,\frac{m\pi x}{L} + \int_0^1 (1-x)\,sen\,\frac{m\pi x}{L}\,dx\Big]$$

$u = x+1 \Rightarrow du = 1$ | $u = 1-x \Rightarrow du = -1$

$dv = sen\,\frac{m\pi x}{L} \Rightarrow v = -\frac{L}{m\pi}\cos\frac{m\pi x}{L}$ | $dv = -1 \Rightarrow v = -\frac{L}{m\pi}\cos m\pi x$

$$= \frac{1}{L}\Big[-(x+1)\frac{L}{m\pi}\cdot\cos\frac{m\pi x}{L}\Big|_{-1}^{0} + \int_{-1}^{0}\cos\frac{m\pi x}{L}\cdot\frac{L}{m\pi}\,dx + \Big|(1-x)\frac{-L}{m\pi}\cos\left(\frac{m\pi}{L}\right)\Big|_0^1 - \int_0^1\frac{1}{m\pi}\cos(m\pi)\,dx$$

$$= \frac{1}{L}\Big[-\frac{L}{m\pi} + \frac{L^2}{(m\pi)^2}\cdot sen\,\frac{m\pi x}{L}\Big|_{-1}^{0} + \frac{L}{m\pi} - \frac{L^2}{(m\pi)^2}\cdot sen\left(\frac{m\pi x}{L}\right)\Big|_0^1\Big]$$

$$= \frac{1}{L}\Big[-\frac{L}{m\pi} + \frac{L^2}{(m\pi)^2}\,sen\,\frac{m\pi}{L} + \frac{L}{m\pi} - \frac{L^2}{(m\pi)^2}\,sen\,\frac{m\pi}{L}\Big]$$

$$= \frac{1}{L}\Big[-\frac{2L^2}{(m\pi)^2}\cdot sen\,\frac{m\pi}{L}\Big] = \frac{2L}{(m\pi)^2}\,sen\left(\frac{m\pi}{L}\right)$$

$$f(x) = \frac{1}{2} + \sum_{m=1}^{\infty}\left(-\frac{2L}{(m\pi)^2}\left(1-\cos\frac{m\pi}{L}\right)\cos\frac{m\pi x}{L} + \frac{2L}{(m\pi)^2}\,sen\,\frac{m\pi}{L}\cdot sen\,\frac{m\pi x}{L}\right)$$

2) $f(x) = \begin{cases} -1, & -1 \le x < 0 \\ 1, & 0 \le x < 1 \end{cases}$

a) $[-1,1]$ f é contínua $\Rightarrow f(\bar{x}) = f(\bar{x}) = f(x)\ \forall x \in$

$[-1,1) = [-1,0) \cup [0,1)$

$\lim_{x\to -1} -1 = -1 = f(x)$ $\quad f'(x) = 0$

$\lim_{x\to 0^+} -1 = -1 = f(x)$ $\quad \lim_{x\to 1^+} 0 = 0$

$\lim_{x\to 0^-} 1 = 1 = f(x)$ $\quad \lim_{x\to 0} 0 = 0$

$\lim_{x\to 1^+} 1 = 1 = f(x)$ $\quad \lim_{x\to 0^-} 0 = 0$

$\lim_{x\to 0^-} 0 = 0$

Scanned with CamScanner

$\therefore$ f e f' de x são seccionalmente contínuas.

b) $T=2$

$a_n = 0$: $a_n = \frac{1}{L}\left[\int_{-1}^{0} -1\cos 0\, dx + \int_{0}^{1} 1\cos 0\, dx\right]$

$a_n = \frac{1}{L}\left[x\Big|_{-1}^{0} + x\Big|_{0}^{1}\right] = \frac{1}{L}\cancel{(1-1)}^{0} = 0$

$\ast$ $a_n > 0$

$a_n = \frac{1}{L}\left[\int_{-1}^{0} -1\cos\frac{n\pi x}{L}\, dx + \int_{0}^{1} 1\cos\frac{n\pi x}{L}\, dx\right]$

$a_n = \frac{1}{L}\left[-\frac{L}{n\pi}\,\text{sen}\,\frac{n\pi x}{L}\Big|_{-1}^{0} + \frac{L}{n\pi}\cdot\text{sen}\,\frac{n\pi x}{L}\Big|_{0}^{1}\right.$

$a_n = \frac{1}{L}\left[+\frac{L}{n\pi}\cdot\text{sen}\,\frac{n\pi x}{L} + \frac{L}{n\pi}\cdot\text{sen}\left(\frac{n\pi}{L}\right)\right) = \frac{1}{L}\cdot(0) \Rightarrow a_n = 0$

$\cdot$ $b_n > 0$

$b_n = \frac{1}{L}\left[\int_{-1}^{0} -1\,\text{sen}\,\frac{n\pi x}{L}\, dx + \int_{0}^{1} 1\,\text{sen}\,\frac{n\pi x}{L}\, dx\right.$

$b_n = \frac{1}{L}\left[+\cos\frac{n\pi x}{L}\Big|_{-1}^{0}\cdot\frac{L}{n\pi} - \frac{L}{n\pi}\cos\frac{n\pi x}{L}\Big|_{0}^{1}\right]$

$b_n = \frac{1}{L}\left[\left(\cos 0 - \cos\frac{n\pi}{L}\right)\cdot\frac{L}{n\pi} - \frac{L}{n\pi}\left|\cos\frac{n\pi}{L} - \cos 0\right]\right]$

$b_n = \frac{1}{L}\left(1 - \cos\frac{n\pi}{L} + \frac{L}{n\pi}\right(\cos\frac{n\pi}{L} - 1$

$b_n = \frac{1}{\cancel{L}}\cdot\frac{2\cancel{L}}{n\pi}\left(1 - \cos\frac{n\pi}{L}\right) = \frac{2}{n\pi}\left(1 - \cos\frac{n\pi}{L}\right)$

$f(x) \text{ "="} \sum_{n=1}^{2} \frac{2}{n\pi}\left(\cos\frac{n\pi}{L} + 1\right)\text{sen}\,\frac{n\pi x}{L}$

3) a) $f(x) = \begin{cases} 0, & -\pi \leq x < 0 \text{ ou } x = \pi \\ \pi, & 0 < x < \pi \end{cases}$

f' e f são contínuas em $[-\pi, \pi]$

$[-\pi, \pi] = [-\pi, 0] \cup [0, \pi]$

* f e f' são contínuas A e B.
* $f(\pi), f(0), f^+(0), f^-(\pi), f^+(\pi), f^-(0), f'(0)$ e $f'(\pi)$

$\therefore$, são seccionalmente contínuas.

b) $a_0 = \frac{1}{\cancel{\pi}} \int_0^{\pi} \cancel{\pi} \cos 0 \, dx = x \Big|_0^{\pi} = \pi$

* $a_n > 0$

$$\frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} \, dx = \frac{1}{\pi} \int_0^{\pi} \pi \cos \frac{n\cancel{\pi}x}{\cancel{\pi}} = \int_0^{\pi} \cos nx = \frac{1}{n} \operatorname{sen} nx \Big|_0^{\pi} = 0$$

* $b_n \ (n > 0)$

$$b_n = \frac{1}{\pi} \int_0^{\pi} \operatorname{sen} \frac{n\cancel{\pi}x}{\cancel{\pi}} \, dx = \int_0^{\pi} \operatorname{sen} x \, dx = -\frac{1}{nx} \cos nx \Big|_0^{\pi} = \frac{1}{nx} (\cos n\pi - 1)$$

$\begin{cases} \frac{2}{n}, & n \text{ ímpar} \\ 0, & n \text{ par} \end{cases}$

$$f(x) = \frac{\pi}{2} + \sum_{n=1,3,5}^{\infty} \frac{2}{n} \operatorname{sen}(nx)$$

$$f(x) = \frac{\pi}{2} + 2\left[\sum_{k=0}^{\infty} \frac{1}{(2k+1)} \operatorname{sen}(2k+1)x\right]$$

c) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$

$\operatorname{sen}(2k+1)x \quad (-1)^k$

$x = \frac{\pi}{2} \Rightarrow \operatorname{sen}(2k+1)\frac{\pi}{2} = (-1)^k$

$\frac{\pi}{2}$ é ponto de continuidade de f. Logo:

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)} \Rightarrow 2\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)} = \frac{\pi}{4}$$

4) a) x^3-2x

$(-x)^3-2(-x) \Rightarrow -x^3+2x \Rightarrow -(x^3-2x)$ $\boxed{f(x)=-f(x), \therefore \text{é ímpar.}}$

b) x^3-2x+1

$(-x)^3-2(-x)+1 \Rightarrow -x^3+2x+1 \Rightarrow -(x^3-2x-1)$ $\boxed{\begin{array}{l}\therefore \text{indeterminada.}\\ f(x) \neq f(-x)\\ f(x) \neq -f(x)\end{array}}$

c) $tg(2x) = \dfrac{sen(2x)}{cos(2x)} \Rightarrow \dfrac{sen(-2x)}{cos(-2x)} = \dfrac{-sen(2x)}{cos(2x)}$ $\boxed{\begin{array}{l}\text{é ímpar.}\\ f(x)=-f(x)\end{array}}$

d) $sec\, x = \dfrac{1}{cos\, x} \Rightarrow \dfrac{1}{cos(-x)} \Rightarrow \dfrac{1}{cos\, x}$, $\therefore f(x)=f(-x)$. par.

e) $|x|^3$ $\quad |-x|^3 = |x|^3 \Rightarrow f(x)=f(-x)$, $\therefore$ par

f) e^{-x}

$e^{-(-x)} = e^{x}$, $\therefore$ $f(x) \neq f(-x)$, $f(x) \neq -f(x)$ $\therefore$ é indeterminada.

5) $f(x) = \begin{cases} 1-x, & 0<x\leq 1 \\ 0, & 1<x\leq 2 \end{cases}$

$\begin{cases} 1-x, & 0<x\leq 1 \\ 0, & 1<x\leq 2 \\ (1-(-x)), & 0<-x\leq 1 \\ (-0), & 1<-x\leq 2 \end{cases} = \begin{cases} 1-x, & 0<x\leq 1 \\ 0, & 1<x\leq 2 \\ 1+x & -1\leq x\leq 0 \\ 0 & -2\leq x<-1 \end{cases}$

$a_n = n>0$

$a_n = \frac{2}{L}\int_0^2 f(x) \cos\frac{n\pi x}{L}\, dx = \frac{2}{2}\int_0^1 1-x \cos\frac{n\pi x}{2}\, dx$

$= (1-x)\frac{2}{n\pi} sen\frac{n\pi x}{2}\Big|_0^1 - \int_0^1 \frac{2}{n\pi} sen\frac{n\pi x}{2}\, dx$

$= \frac{2}{n\pi}\cdot cos\frac{n\pi x}{2}\left(\frac{-2}{n\pi}\right)\Big|_0^1 = \frac{-4}{(n\pi)^2} cos\frac{n\pi x}{2}\Big|_0^1 \frac{-4}{(n\pi)^2}\left(cos\frac{n\pi}{2}-1\right)$

$a_n\ (n=0)$

$a_n = \frac{2}{L}\int_0^L f(x)\cdot cos\frac{n\pi x}{L}\, dx - \frac{2}{2}\int_0^1 (1-x)\cdot 1\, dx = x-\frac{x^2}{2}\Big|_0^1$

$= \left[1-\frac{1}{2}\right] = \frac{1}{2}$

$$f(x) "=" \frac{1}{2} + \sum_{n=1}^{\infty} -\frac{4}{(n\pi)^2}\left(\cos\frac{n\pi}{2} - 1\right)\cos\frac{n\pi x}{2}$$

$$f(x) = \frac{1}{4} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\left(\cos\frac{n\pi}{2} - 1\right)\cdot\cos\frac{n\pi x}{2}$$

$$\begin{cases} f(x), 0 < x < 2 \\ -f(-x)\ 0 < -x < 2 \end{cases} = \begin{cases} 1-x, 0 < x \leq 1 \\ 0 \quad 1 < x \leq 2 \\ (1-x)\ 0 \geq x \geq 1 \\ 0 \quad -1 > x \geq -2 \end{cases} = \begin{cases} 1-x, 0 < x \leq 1 \\ 0, 1 < x \leq 2 \\ -1-x, 0 \geq x \geq -1 \\ 0, -1 > x \geq -2 \end{cases}$$

$a_n = 0$

$$b_n = \frac{2}{2}\int_0^2 f(x)\,\text{sen}\left(\frac{n\pi x}{L}\right)dx = \frac{2}{2}\int_0^1 (1-x)\,\text{sen}\,\frac{n\pi x}{2}\,dx$$

$$= \left[-(1-x)\cdot\frac{2}{n\pi}\cdot\cos\frac{n\pi x}{2}\right|_0^1 + \int_0^1 \frac{2}{n\pi}\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{2}{n\pi} + \left[\frac{4}{(n\pi)^2}\cdot\text{sen}\left(\frac{n\pi x}{2}\right)\right|_0^1 = \frac{2}{n\pi} + \frac{4}{(n\pi)^2}\cdot\text{sen}\,\frac{n\pi}{2}$$

$$f(x) "=" \sum_{n=}^{\infty} b_n\,\text{sen}\,\frac{n\pi x}{2}$$

$$f(x) "=" \sum_{n}^{\infty} \frac{2}{n\pi} + \frac{4}{(n\pi)^2}\,\text{sen}\,\frac{n\pi}{2}\ \text{sen}\,\frac{n\pi x}{2}$$

$$f(x) = \frac{2}{\pi}\sum \frac{1}{n} + \frac{4}{(n\pi)^2}\cdot\text{sen}\,\frac{\pi n}{2}\cdot\text{sen}\,\frac{n\pi x}{2} \Rightarrow$$ série dos senos

6) a) $f(x) = 3 - x,\ 0 < x < 3$

$$f(x) = \begin{cases} f(x) =, 0 < x < 3 \\ f(-x), 0 < -x < 3 \end{cases} \qquad \begin{cases} f(x) \neq 0 < x < 3 \\ f(-x), -3 < x < 0 \end{cases}$$

$$= \begin{cases} 3-x, 0 < x < 3 \\ 3+x, -3 < x < 0 \end{cases}$$

$a_n,\ n=0$