Linear Regression

Introduction

In this exercise, you will implement linear regression and get to see it work on data.

Files included in this exercise

`ex1.m` - Octave script that will help step you through the exercise
`ex1 multi.m` - Octave script for the later parts of the exercise
`ex1data1.txt` - Dataset for linear regression with one variable
`ex1data2.txt` - Dataset for linear regression with multiple variables
`warmUpExercise.m` - Simple example function in Octave
`plotData.m` - Function to display the dataset
`computeCost.m` - Function to compute the cost of linear regression
`gradientDescent.m` - Function to run gradient descent
`computeCostMulti.m` - Cost function for multiple variables
`gradientDescentMulti.m` - Gradient descent for multiple variables
`featureNormalize.m` - Function to normalize features
`normalEqn.m` - Function to compute the normal equations

2 Linear regression with one variable

In this part of this exercise, you will implement linear regression with one variable to predict profits for a food truck. Suppose you are the CEO of a restaurant franchise and are considering different cities for opening a new outlet. The chain already has trucks in various cities and you have data for profits and populations from the cities.

You would like to use this data to help you select which city to expand to next.

The file `ex1data1.txt` contains the dataset for our linear regression problem. The first column is the population of a city and the second column is the profit of a food truck in that city. A negative value for profit indicates a loss.

The `ex1.m` script has already been set up to load this data for you.

2.1 Plotting the Data

Before starting on any task, it is often useful to understand the data by visualizing it. For this dataset, you can use a scatter plot to visualize the data, since it has only two properties to plot (profit and population). (Many other problems that you will encounter in real life are multi-dimensional and can't be plotted on a 2-d plot.)

In `ex1.m`, the dataset is loaded from the data file into the variables X and y:

```
data = load('ex1data1.txt');          % read comma separated data
X = data(:, 1); y = data(:, 2);
m = length(y);                        % number of training examples
```

Next, the script calls the `plotData` function to create a scatter plot of the data. Your job is to complete `plotData.m` to draw the plot; modify the file and fill in the following code:

```
plot(x, y, 'rx', 'MarkerSize', 10);        % Plot the data
ylabel('Profit in $10,000s');              % Set the y−axis label
xlabel('Population of City in 10,000s');   % Set the x−axis label
```

Now, when you continue to run `ex1.m`, our end result should look like Figure 1, with the same red "x" markers and axis labels.

To learn more about the plot command, you can type `help plot` at the Octave command prompt or to search online for plotting documentation. (To change the markers to red "x", we used the option 'rx' together with the plot command, i.e., `plot(..,[your options here],.., 'rx');`)

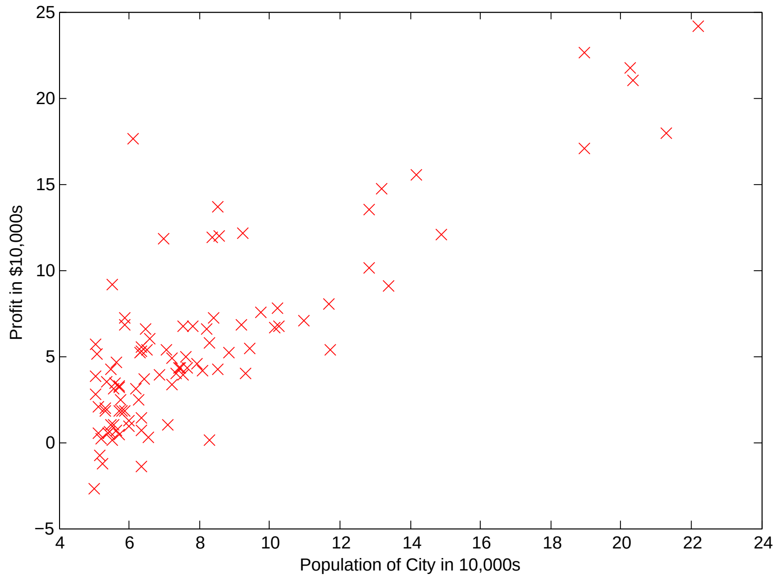


Figure 1: Scatter plot of training data

2.2 Gradient Descent

In this part, you will fit the linear regression parameters θ to our dataset using gradient descent.

2.2.1 Update Equations

The objective of linear regression is to minimize the cost function

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

where the hypothesis $h_\theta(x)$ is given by the linear model

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1$$

Recall that the parameters of your model are the θ_j values. These are the values you will adjust to minimize cost $J(\theta)$. One way to do this is to use the batch gradient descent algorithm. In batch gradient descent, each iteration performs the update

$$\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} \qquad \text{(simultaneously update } \theta_j \text{ for all } j).$$

With each step of gradient descent, your parameters θ_j come closer to the optimal values that will achieve the lowest cost $J(\theta)$.

Implementation Note: We store each example as a row in the the `X` matrix in Octave. To take into account the intercept term (θ_0), we add an additional first column to `X` and set it to all ones. This allows us to treat θ_0 as simply another 'feature'.

2.2.2 Implementation

In `ex1.m`, we have already already set up the data for linear regression. In the following lines, we add another dimension to our data to accommodate the θ_0 intercept term. We also initialize the initial parameters to 0 and the learning rate `alpha` to 0.01.

```
X = [ones(m, 1), data(:,1)]; % Add a column of ones to x
theta = zeros(2, 1); % initialize fitting parameters

iterations = 1500;
alpha = 0.01;
```

2.2.3 Computing the cost $J(\theta)$

As you perform gradient descent to learn minimize the cost function $J(\theta)$, it is helpful to monitor the convergence by computing the cost. In this section, you will implement a function to calculate $J(\theta)$ so you can check the convergence of your gradient descent implementation.

Your next task is to complete the code in the file `computeCost.m`, which is a function that computes $J(\theta)$. As you are doing this, remember that the variables X and y are not scalar values, but matrices whose rows represent the examples from the training set.

Once you have completed the function, the next step in `ex1.m` will run `computeCost` once using θ initialized to zeros, and you will see the cost printed to the screen.

You should expect to see a cost of `32.07`.

2.2.4 Gradient descent

Next, you will implement gradient descent in the file `gradientDescent.m`. The loop structure has been written for you, and you only need to supply the updates to θ within each iteration.

As you program, make sure you understand what you are trying to optimize and what is being updated. Keep in mind that the cost $J(\theta)$ is parameterized by the vector θ, not X and y. That is, we minimize the value of $J(\theta)$ by changing the values of the vector θ, not by changing X or y. Refer to the equations in this handout and to the video lectures if you are uncertain.

A good way to verify that gradient descent is working correctly is to look at the value of $J(\theta)$ and check that it is decreasing with each step. The starter code for `gradientDescent.m` calls `computeCost` on every iteration and prints the cost. Assuming you have implemented gradient descent and `computeCost` correctly, your value of $J(\theta)$ should never increase, and should converge to a steady value by the end of the algorithm.

After you are finished, `ex1.m` will use your final parameters to plot the linear fit. The result should look something like Figure 2:

Your final values for θ will also be used to make predictions on profits in areas of 35,000 and 70,000 people. Note the way that the following lines in `ex1.m` uses matrix multiplication, rather than explicit summation or looping, to calculate the predictions. This is an example of code vectorization in Octave.

```
predict1 = [1, 3.5] * theta;
predict2 = [1, 7] * theta;
```

2.3 Debugging

Here are some things to keep in mind as you implement gradient descent:

- Octave array indices start from one, not zero. If you're storing θ_0 and θ_1 in a vector called `theta`, the values will be `theta(1)` and `theta(2)`.

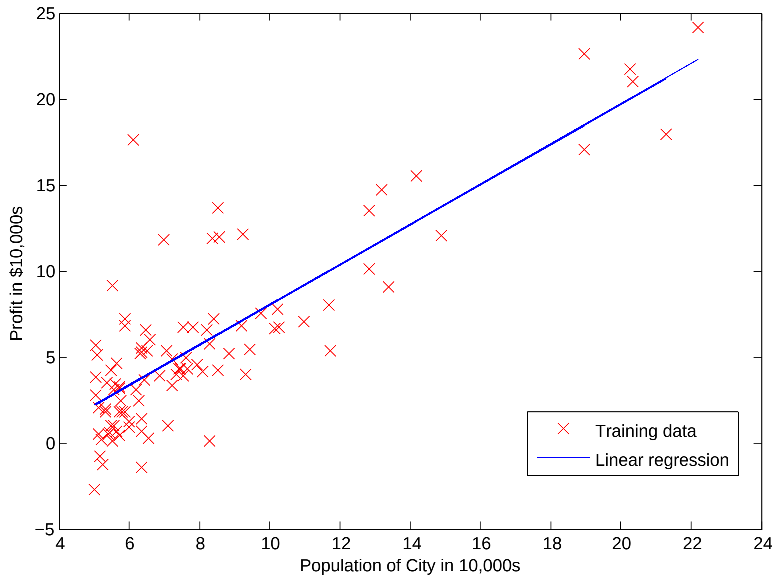


Figure 2: Training data with linear regression fit

- If you are seeing many errors at runtime, inspect your matrix operations to make sure that you're adding and multiplying matrices of compatible dimensions. Printing the dimensions of variables with the `size` command will help you debug.
- By default, Octave interprets math operators to be matrix operators. This is a common source of size incompatibility errors. If you don't want matrix multiplication, you need to add the "dot" notation to specify this to Octave. For example, `A*B` does a matrix multiply, while `A.*B` does an element-wise multiplication.

2.4 Visualizing $J(\theta)$

To understand the cost function $J(\theta)$ better, you will now plot the cost over a 2-dimensional grid of θ_0 and θ_1 values. You will not need to code anything new for this part, but you should understand how the code you have written already is creating these images.

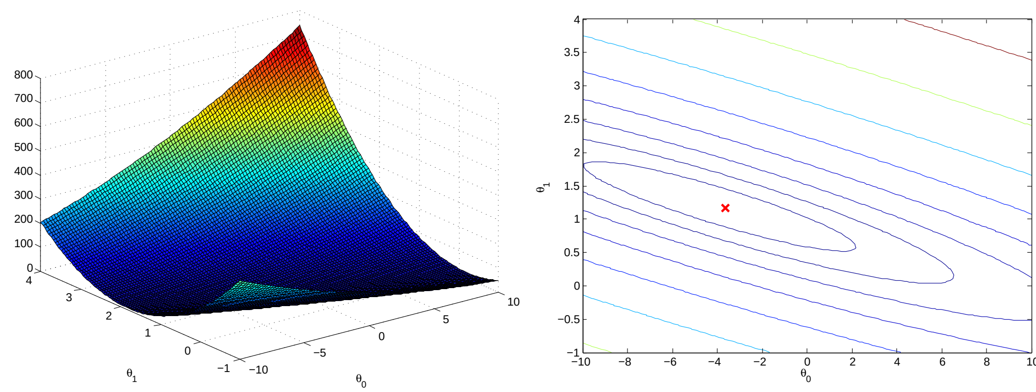
In the next step of `ex1.m`, there is code set up to calculate $J(\theta)$ over a grid of values using the `computeCost` function that you wrote.

```
% initialize J_vals to a matrix of 0's
```

```
J_vals = zeros(length(theta0_vals), length(theta1_vals));

% Fill out J_vals
for i = 1:length(theta0_vals)
    for j = 1:length(theta1_vals)
       t = [theta0_vals(i); theta1_vals(j)];
       J_vals(i,j) = computeCost(x, y, t);
    end
end
```

After these lines are executed, you will have a 2-D array of $J(\theta)$ values. The script **ex1.m** will then use these values to produce surface and contour plots of $J(\theta)$ using the **surf** and **contour** commands. The plots should look something like Figure 3:



(a) Surface

(b) Contour, showing minimum

Figure 3: Cost function $J(\theta)$

The purpose of these graphs is to show you that how $J(\theta)$ varies with changes in θ_0 and θ_1. The cost function $J(\theta)$ is bowl-shaped and has a global mininum. (This is easier to see in the contour plot than in the 3D surface plot). This minimum is the optimal point for θ_0 and θ_1, and each step of gradient descent moves closer to this point.

References:
Programming exercises by Andrew N.g., Machine Learning.