# MAST30025: Linear Statistical Models

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Assignment 2, 2019

May 6, 2019

*All R output is provided at the end of the assignment. Written using LaTeXon Overleaf.*

1. Prove Theorem 4.8: show that the maximum likelihood estimator of the error variance $\sigma^2$ is

$$\hat{\sigma^2} = \frac{SS_{Res}}{n}.$$

The likelihood function is given as

$$\begin{aligned}
L(\beta, \sigma^2) &= \prod_{i=1}^{n} \exp(-\epsilon_i^2/2\sigma^2) \\
&= \frac{1}{(2\sigma^2)^{n/2}} \exp(-\sum_{i=1}^{n} \epsilon_i^2/2\sigma^2) \\
&= \frac{1}{(2\sigma^2)^{n/2}} \exp(-(\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta)/\sigma^2)
\end{aligned}$$

We can then find the log-likelihood function

$$\begin{aligned}
\ell(\beta, \sigma^2) &= \ln(L(\beta, \sigma^2)) \\
&= \frac{-n}{2}\ln(2\pi\sigma^2) - \frac{1}{2} - (\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta).
\end{aligned}$$

Now, solve $\frac{\partial}{\partial\sigma^2}\ell(\beta, \sigma^2) = 0$ for the MLE:

$$\begin{aligned}
\frac{\partial}{\partial\sigma^2}\ell(\beta, \sigma^2) = \left(\frac{-n}{2}\right)\left(\frac{2}{\sigma}\right) - (\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta)\left(\frac{-1}{\sigma^3}\right) &= 0 \\
\left(\frac{n}{2}\right)\left(\frac{2}{\sigma}\right) &= (\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta)\left(\frac{1}{\sigma^3}\right) \\
\left(\frac{n}{\sigma}\right)\sigma^3 &= (\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta) \\
\sigma^2 &= \frac{(\mathbf{y} - X\beta)^T(\mathbf{y} - X\beta)}{n}
\end{aligned}$$

Sub in the estimator $SS_{Res} = (\mathbf{y} - X\mathbf{b})^T(\mathbf{y} - X\mathbf{b})$.
Therefore the MLE is given as $\hat{\sigma}^2 = \frac{SS_{Res}}{n}$.

2. An experiment is conducted to estimate the annual demand for cars, based on their cost, the current unemployment rate, and the current interest rate. A survey is conducted and the following measurements obtained:

(a) Fit a linear model to the data and estimate the parameters and variance.
The estimated parameters are:

$$\Rightarrow \mathbf{b} = (X^T X)^{-1} X^T \mathbf{y}$$
$$= \begin{bmatrix} -7.4044796 \\ 0.1207646 \\ 1.1174846 \\ 0.3861206 \end{bmatrix}, \text{ using R.}$$

The estimated variance is

$$\Rightarrow s^2 = \frac{SS_{Res}}{n-p}$$
$$= 0.3955368, \text{ using R.}$$

(b) Which two of the parameters have the highest (in magnitude) covariance in their estimators?
The covariance matrix of $\mathbf{b}$ given as $(X^T X)\sigma^2$ is

$$\Rightarrow (X^T X)^{-1}\sigma^2 = \begin{bmatrix} 13.49743324 & -0.054817613 & -0.69854293 & -1.029731987 \\ -0.05481761 & 0.024498395 & -0.01478859 & -0.001937333 \\ -0.69854293 & -0.014788594 & 0.06226378 & 0.031714790 \\ -1.02973199 & -0.001937333 & 0.03171479 & 0.135362495 \end{bmatrix} \sigma^2.$$

From the covariance matrix, parameters $\beta_0, \beta_3$ have the highest (in magnitude) covariance.

(c) Find a 99% confidence interval for the average number of \$8,000 cars sold in a year which has unemployment rate 9% and interest rate 5%.
Let $\alpha = 0.01$ which gives a $t$-quantile of 4.60409, and let $\mathbf{t} = \begin{bmatrix} 1 & 8 & 9 & 5 \end{bmatrix}^T$.
Then the 99% Confidence Interval for the Number of Cars Sold ($\times 10^3$) is

$$\Rightarrow \mathbf{t}^T \mathbf{b} \pm t_{\alpha/2} s \sqrt{\mathbf{t}^T (X^T X)^{-1} \mathbf{t}} = (3.926075, 7.173129), \text{ using R.}$$

(d) A prediction interval for the number of cars sold in such a year is calculated to be (4012,7087). Find the confidence level used.

**Algorithm 1** Simulate $\alpha$

```
for α ∈ range(0.01, 0.15) do
    t = t-quantile(α)
    Calculate PI given t
    if Lower Bound of PI == 4.012 & Upper Bound of PI == 7.087 then
        Return α
    end if
end for
```

From all generated Prediction Intervals, we find that $\alpha = 0.1$.

(e) Test for model relevance using a corrected sum of squares.
Let our null be $H_0 : \beta_1 = \beta_2 = \beta_3 = \mathbf{0}$ versus the alternative that some $\beta_i \neq \mathbf{0}$, $i \in \{1, 2, 3\}$.
We first need to find the $SS_{Res}$ and the Corrected $SS_{Reg}$:

$$\begin{aligned}\Rightarrow SS_{Res} &= (\mathbf{y} - X\mathbf{b})^T(\mathbf{y} - X\mathbf{b}) \\ &= 1.582147, \text{ using R.}\end{aligned}$$

$$\begin{aligned}\Rightarrow R(\gamma_1|\gamma_2) &= R(\beta) - R(\gamma_2) \\ &= \mathbf{y}^T X\mathbf{b} - \frac{(\sum_{i=1}^n y_i)^2}{n} \\ &= 27.85785, \text{ using R.}\end{aligned}$$

We then use an $F$ statistic to test for model relevance,

$$\begin{aligned}F &= \frac{MS_{Reg}}{MS_{Res}} \\ &= \frac{R(\gamma_1|\gamma_2)/k}{SS_{Res}/(n-k-1)}, \quad k = 3 \\ &= 23.47683, \text{ using R.}\end{aligned}$$

Since our $F$ statistic is significantly larger than $F_{3,4} = 6.591382$ at $\alpha = 0.05$, we reject the null hypothesis and conclude that there is evidence that at least one the parameters are not zero.

3. Consider two full rank linear models $y = X_1\gamma_1 + \epsilon_1$ and $y = X\beta + \epsilon_2$, where all predictors in the first model ($\gamma_1$) are also contained in the second model ($\beta$). Show that the $\text{SS}_{Res}$ for the first model is at least the $\text{SS}_{Res}$ for the second model.

   We are given that $\beta = \begin{bmatrix}\gamma_1 & | & \gamma_2\end{bmatrix}^T$.

   Now consider the parameters of a reduced model $\mathbf{y} = X\gamma + \epsilon$, which are

$$\gamma = \begin{bmatrix}\beta_0 & \dots & \beta_r & 0 & \dots & 0\end{bmatrix}^T,$$

   where $r$ is the number of parameters in $\gamma_1$ and the remaining $k - r$ parameters in $\beta$ are 0.

   Then the reduced model $y = X_1\gamma_1 + \epsilon_1$ minimises $SS_{Res(reduced)}$, the full model $y = X\beta + \epsilon_2$ must have $SS_{Res(full)}$ which is at most equal to the reduced model's $SS_{Res}$.

   This means that

$$SS_{Res(full)} - SS_{Res(reduced)} \geq 0,$$

   which is positive semi-definite.

   Hence, the $\text{SS}_{Res}$ for the reduced model is at least the $\text{SS}_{Res}$ for the the full model.

4. In this question, we study a data set of 50 US states. We wish to use a linear model to model the murder rate in terms of the other variables. *Refer to Assignment spec for information.*
   **Refer to R output below for plots**.

   (a) Plot the data and comment. Should we consider any variable transformations?

   - The pairs plot show us that there seems to be a possibility of heteroskedasticity in Murder with respect to Illiteracy and Area.

- After applying a *log* variable transformation to Illiteracy and Area, we see that the variable HS.Grad shares a positive linear correlation to *log*(Illiteracy) and may end up being removed during feature selection.
- Although Population looks like it may need a variable transformation, the background behind the data set tells us that Population and Murder are in the same units and is implied it should not require a transformation. (My original final model had used a transformation on Population and the diagnostic plots for those are also provided in the R output at the end.)

(b) Perform model selection using forward selection, using all variable transformations which maybe relevant.
At a significance level of $\alpha = 0.05$, we add (in order) to our base model: Life.Exp, Frost, Area, Population, Illiteracy

(c) Starting from the full model, perform model selection using step-wise selection with the AIC.
Our full model is reduced to only use: Life.Exp, Area, Illiteracy, Population, Frost

(d) Write down your final fitted model (including any variable transformations used).

$$\begin{aligned}\text{Area} = 110.42398 - 1.55019 \times \text{Life.Exp} + 0.69359 \times \log(\text{Area}) + 1.78533 \\ \times \log(\text{Illiteracy}) + 0.000142 \times \text{Population} - 0.01173 \times \text{Frost} + \epsilon\end{aligned} \tag{1}$$

(e) Produce diagnostic plots for your final model and comment.

i. **Residuals vs Fitted**: We have an even spread of residuals which have a 0 trend-line.
ii. **QQ-Plot**: The majority of points follow the line although there seems to be some over estimation in the tails. This is most likely caused due to the outliers within the Population and Area variables.
iii. **Scale-Location**: The spread is quite even and spread out. There seems to be no visible correlation or trend and the variance is also less than 1 which is good.
iv. **Residuals vs Leverage**: The majority of the points have small leverage with the exception of a few outliers which are impacting the line. The Cook's Distance are also all under 0.5 which is desirable since it means all points are relevant to the model.

Overall, the final linear model seems to be a good fit for the data given the graphs produced in the diagnostic plots. However, there are a few outliers which can be omitted and may improve the model.

5. For ridge regression, we choose parameter estimators which minimise

$$\sum_{i=1}^{n} e_i^2 + \lambda \sum_{j=0}^{k} b_j^2,$$

where $\lambda$ is a constant penalty parameter.

(a) Show that these estimators are given by

$$\mathbf{b} = (X^T X + \lambda I)^{-1} X^T \mathbf{y}.$$

An alternative form of $\sum_{i=1}^{n} e_i^2 + \lambda \sum_{j=0}^{k} b_j^2$, is $(\mathbf{y} - X\mathbf{b})^T(\mathbf{y} - X\mathbf{b}) + \lambda \mathbf{b}^T\mathbf{b}$.

We want a $\mathbf{b}$ that minimises the normal equations for the equation above, which can be found by solving $\frac{\partial}{\partial \mathbf{b}}((\mathbf{y} - X\mathbf{b})^T(\mathbf{y} - X\mathbf{b}) + \lambda \mathbf{b}^T\mathbf{b}) = 0$.

First, we expand the terms:

$$(\mathbf{y} - X\mathbf{b})^T(\mathbf{y} - X\mathbf{b}) + \lambda \mathbf{b}^T\mathbf{b} = \mathbf{y}^T\mathbf{y} - 2(X^TX)^T\mathbf{b} + \mathbf{b}^T(X^TX + \lambda I)\mathbf{b}.$$

Let $A = (X^TX - \lambda I)$. Since $A$ is symmetric,

$$\frac{\partial}{\partial \mathbf{b}}(\mathbf{y}^T\mathbf{y} - 2(X^TX)^T\mathbf{b} + \mathbf{b}^TA\mathbf{b}) = 2A\mathbf{b} - 2(X^T\mathbf{y}).$$

Now we can solve the normal equations:

$$\begin{aligned} 2(X^T\mathbf{y}) &= 2(X^TX + \lambda I)\mathbf{b} \\ \mathbf{b} &= (X^TX + \lambda I)^{-1}X^T\mathbf{y}, \text{ as required.} \end{aligned}$$

(b) Calculate the ridge regression estimates for the data from Q2 with penalty parameter $\lambda = 0.5$. In order to avoid penalising some parameters unfairly, we must first scale every predictor variable so that it is standardised (mean 0, variance 1), and centre the response variable (mean 0), in which case an intercept parameter is not used. (Hint: This can be done with the *scale* function).

$$\begin{aligned} \mathbf{b} &= (X^TX + \lambda I)^{-1}X^T\mathbf{y} \\ &= \begin{bmatrix} 0.3494789 \\ 1.7899861 \\ 0.3432961 \end{bmatrix}, \text{ using R.} \end{aligned}$$

(c) One way to calculate the optimal value for the penalty parameter is to minimise the AIC. Since the number of parameters $p$ does not change, we use a slightly modified version:

$$AIC = n\ln\left(\frac{SS_{Res}}{n}\right) + 2df,$$

where $df$ is the "effective degrees of freedom" defined by

$$df = tr(H) = tr(X(X^TX + \lambda I)^{-1}X^T).$$

For the data from Q2, construct a plot of $\lambda$ against AIC. Thereby find the optimal value for $\lambda$.

$$\lambda = 0.14, \text{ using R.}$$

# R output for A2

Question 2. a)

```r
y = c(5.5,5.9,6.5,5.9,8,9,10,10.8)
X = matrix(c(rep(1,8),7.2,10,9,5.5,9,9.8,14.5,8,8.7,9.4,
             10,9,12,11,12,13.7,5.5,4.4,4,7,5,6.2,5.8,3.9),8,4)
b = solve(t(X)%*%X,t(X)%*%y)
e = y - X%*%b
n = dim(X)[1]
p = dim(X)[2]
s2 = sum(e^2)/(n-p)

b
```

```
##            [,1]
## [1,] -7.4044796
## [2,]  0.1207646
## [3,]  1.1174846
## [4,]  0.3861206
```

```r
s2
```

```
## [1] 0.3955368
```

Question 2. b)

```r
c = solve(t(X)%*%X)
c
```

```
##             [,1]         [,2]        [,3]         [,4]
## [1,] 13.49743324 -0.054817613 -0.69854293 -1.029731987
## [2,] -0.05481761  0.024498395 -0.01478859 -0.001937333
## [3,] -0.69854293 -0.014788594  0.06226378  0.031714790
## [4,] -1.02973199 -0.001937333  0.03171479  0.135362495
```

Question 2. c)

```r
s = sqrt(sum(e^2)/(n-p))
alpha = 0.01
ta = qt(1-alpha/2, df=(n-p))
t = c(1,8,9,5)
ttb = t(t)%*%b
CI = c(ttb) + c(-1,1)*c(ta*s*sqrt(t(t)%*%solve(t(X)%*%X)%*%t))

CI
```

```
## [1] 3.926075 7.173129
```

Question 2. d)

```r
for (alpha in seq(0.01, 0.15, by = 0.0005)) {
  # t_alpha given an alpha value
  ta = qt(1-alpha/2, df=(n-p))
  # Generate Prediction Interval given alpha
  PI = c(ttb) + c(-1,1)*c(ta*s*sqrt(1+t(t)%*%solve(t(X)%*%X)%*%t))
  if (round(PI[1],3) == 4.012 && round(PI[2],3) == 7.087) {
    print(alpha)
```

```
    print(round(PI,3))
  }
}
```

```
## [1] 0.1
## [1] 4.012 7.087
```

Question 2. e)

```
SSRes = t(y-X%*%b)%*%(y-X%*%b)
CorrectedSSReg = t(y)%*%X%*%b - sum(y)^2/n
k = 3 # num parameters
Fstat = (CorrectedSSReg/k)/(SSRes/(n-k-1))
Fval = qf(0.95,k,n-k-1)

Fstat
```

```
##          [,1]
## [1,] 23.47683
```

```
Fval
```

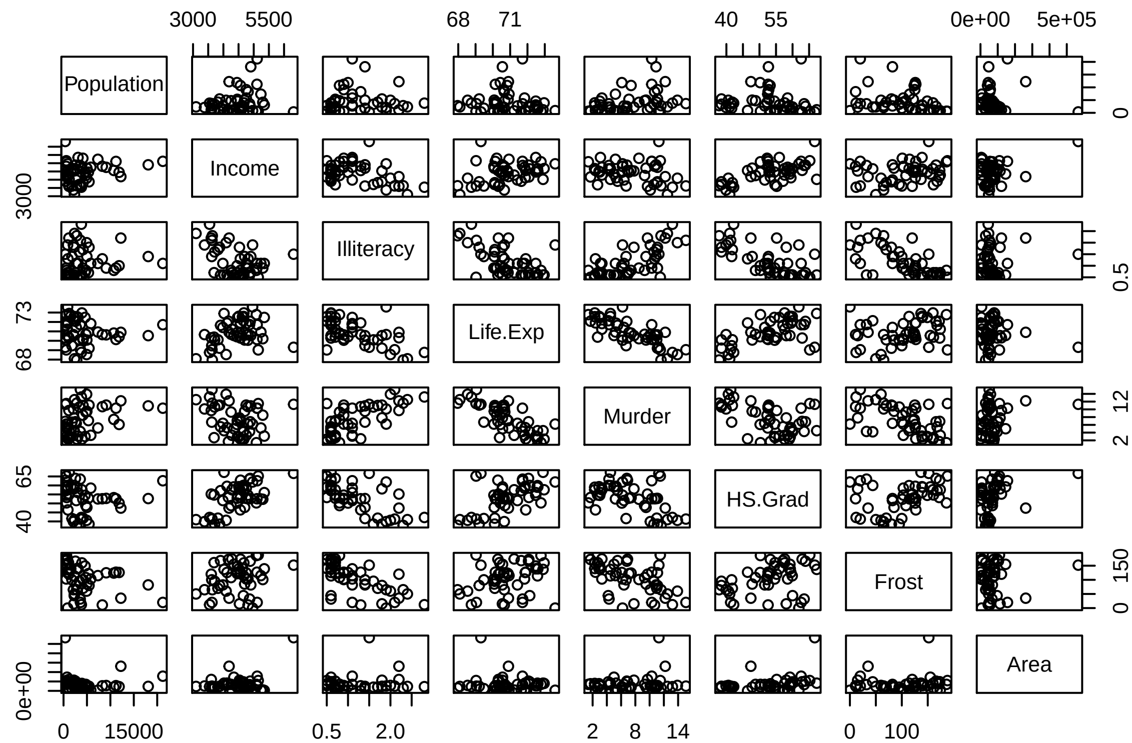
```
## [1] 6.591382
```

```
Fstat > Fval
```

```
##      [,1]
## [1,] TRUE
```

Question 4. a)

```
data(state)
statedata <- data.frame(state.x77, row.names=state.abb, check.names=TRUE)

pairs(statedata)
```




```r
statedata$Area = log(statedata$Area) # log Area
statedata$Illiteracy = log(statedata$Illiteracy) # log Illiteracy
```

Question 4. b)

```r
basemodel = lm(Murder ~ 1, data=statedata)

add1(basemodel, scope= ~ . + Population + Income + Illiteracy + Life.Exp + HS.Grad + Frost + Area, data=
```

```
## Single term additions
##
## Model:
## Murder ~ 1
##            Df Sum of Sq    RSS    AIC F value    Pr(>F)
## <none>                  667.75 131.59
## Population  1     78.85 588.89 127.31  6.4273 0.0145504 *
## Income      1     35.35 632.40 130.88  2.6829 0.1079683
## Illiteracy  1    322.29 345.46 100.64 44.7810 2.183e-08 ***
## Life.Exp    1    407.14 260.61  86.55 74.9887 2.260e-11 ***
## HS.Grad     1    159.00 508.75 120.00 15.0017 0.0003248 ***
## Frost       1    193.91 473.84 116.44 19.6433 5.405e-05 ***
## Area        1     58.63 609.12 129.00  4.6201 0.0366687 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# we add Life.Exp
model1 = lm(Murder ~ Life.Exp, data=statedata)
add1(model1, scope= ~ . + Population + Income + Illiteracy + HS.Grad + Frost + Area, data=statedata, te
```

```
## Single term additions
##
## Model:
## Murder ~ Life.Exp
##            Df Sum of Sq    RSS    AIC F value    Pr(>F)
## <none>                  260.61 86.550
## Population  1    56.615 203.99 76.303 13.0442 0.0007374 ***
## Income      1     0.958 259.65 88.366  0.1733 0.6790605
## Illiteracy  1    61.648 198.96 75.054 14.5629 0.0003952 ***
## HS.Grad     1     1.124 259.48 88.334  0.2035 0.6539823
## Frost       1    80.104 180.50 70.187 20.8575 3.576e-05 ***
## Area        1    30.223 230.38 82.386  6.1656 0.0166517 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# we add Frost
model2 = lm(Murder ~ Life.Exp + Frost, data=statedata)
add1(model2, scope= ~ . + Population + Income + Illiteracy + HS.Grad +  Area, data=statedata, test="F")
```

```
## Single term additions
##
## Model:
## Murder ~ Life.Exp + Frost
##            Df Sum of Sq    RSS    AIC F value   Pr(>F)
## <none>                  180.50 70.187
## Population  1   23.7098 156.79 65.146  6.9559 0.011358 *
## Income      1    5.5598 174.94 70.622  1.4619 0.232807
## Illiteracy  1    6.4775 174.03 70.359  1.7122 0.197204
## HS.Grad     1    2.0679 178.44 71.610  0.5331 0.469015
## Area        1   30.9733 149.53 62.774  9.5283 0.003422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# we add Area
model3 = lm(Murder ~ Life.Exp + Frost + Area, data=statedata)
add1(model3, scope= ~ . + Population + Income + Illiteracy + HS.Grad, data=statedata, test="F")
```

```
## Single term additions
##
## Model:
## Murder ~ Life.Exp + Frost + Area
##            Df Sum of Sq    RSS    AIC F value  Pr(>F)
## <none>                  149.53 62.774
## Population  1    16.347 133.18 58.985  5.5235 0.02321 *
## Income      1     4.786 144.75 63.147  1.4879 0.22889
## Illiteracy  1    13.479 136.05 60.050  4.4584 0.04032 *
## HS.Grad     1     0.190 149.34 64.710  0.0572 0.81200
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# we add Population
model4 = lm(Murder ~ Life.Exp + Frost + Area + Population, data=statedata)
add1(model4, scope= ~ . + Income + Illiteracy + HS.Grad, data=statedata, test="F")
```

```
## Single term additions
```

```
##
## Model:
## Murder ~ Life.Exp + Frost + Area + Population
##            Df Sum of Sq    RSS    AIC F value  Pr(>F)
## <none>                  133.18 58.985
## Income      1    0.9201 132.26 60.639  0.3061 0.58289
## Illiteracy  1   14.2593 118.92 55.323  5.2757 0.02644 *
## HS.Grad     1    0.0829 133.10 60.954  0.0274 0.86929
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# we add Illiteracy
model5 = lm(Murder ~ Life.Exp + Frost + Area + Population + Illiteracy, data=statedata)
add1(model5, scope= ~ . + Income + HS.Grad, data=statedata, test="F")
```

```
## Single term additions
##
## Model:
## Murder ~ Life.Exp + Frost + Area + Population + Illiteracy
##         Df Sum of Sq    RSS    AIC F value Pr(>F)
## <none>               118.92 55.323
## Income   1    2.2064 116.72 56.387  0.8129 0.3723
## HS.Grad  1    2.0227 116.90 56.465  0.7440 0.3932
```

```
# Our final model keeps Life.Exp, Frost, Area, Population, Illiteracy

model5
```

```
##
## Call:
## lm(formula = Murder ~ Life.Exp + Frost + Area + Population +
##     Illiteracy, data = statedata)
##
## Coefficients:
## (Intercept)     Life.Exp        Frost          Area   Population
##   1.104e+02   -1.550e+00   -1.173e-02    6.936e-01    1.422e-04
##  Illiteracy
##   1.785e+00
```

Question 4. c)

```
fullmodel = lm(Murder ~ Population + Income + Illiteracy + Life.Exp + HS.Grad + Frost + Area, data=stat
model = step(fullmodel, scope= ~ . + Population + Income + Illiteracy + Life.Exp + HS.Grad + Frost + Ar
```

```
## Start:  AIC=58.2
## Murder ~ Population + Income + Illiteracy + Life.Exp + HS.Grad +
##     Frost + Area
##
##              Df Sum of Sq    RSS    AIC
## - HS.Grad     1     0.432 116.72 56.387
## - Income      1     0.616 116.90 56.465
## <none>                    116.29 58.201
## - Frost       1     8.555 124.84 59.751
## - Population  1    12.255 128.54 61.211
## - Illiteracy  1    14.806 131.09 62.194
## - Area        1    23.755 140.04 65.496
## - Life.Exp    1   124.645 240.93 92.624
```

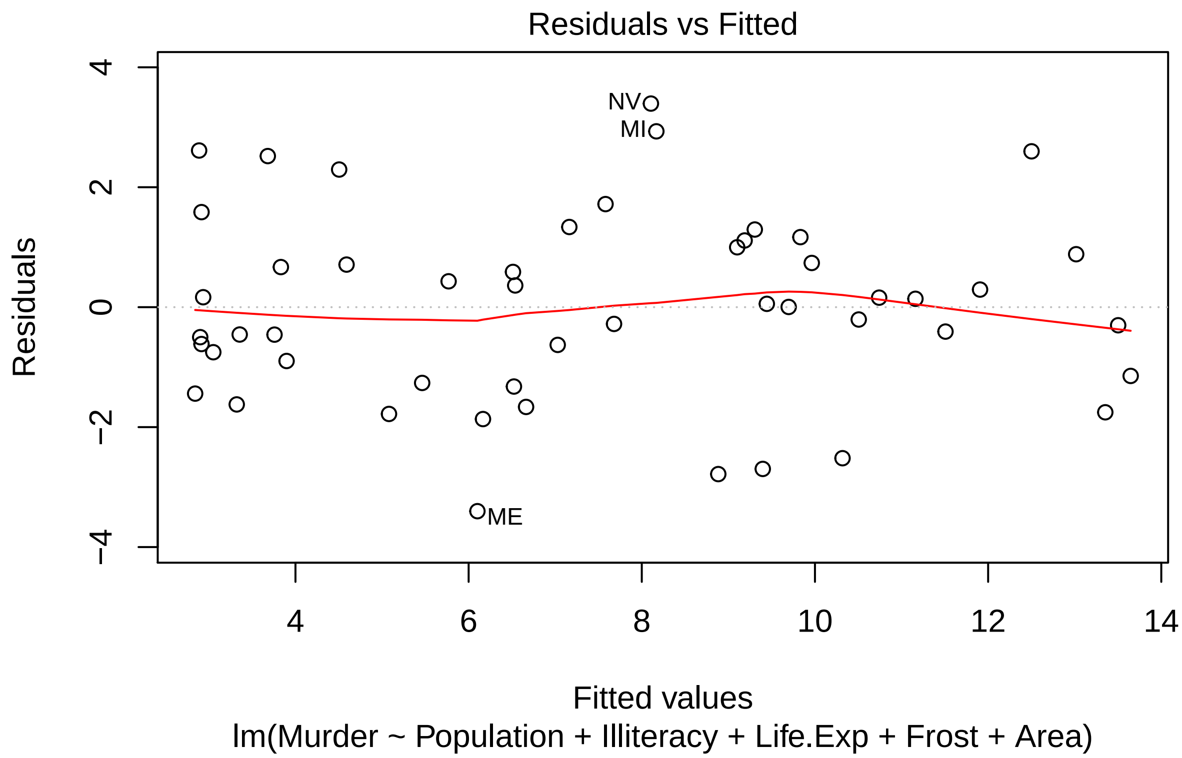
```
##
## Step:  AIC=56.39
## Murder ~ Population + Income + Illiteracy + Life.Exp + Frost +
##     Area
##
##              Df Sum of Sq    RSS    AIC
## - Income      1     2.206 118.92 55.323
## <none>                    116.72 56.387
## + HS.Grad     1     0.432 116.29 58.201
## - Frost       1     9.542 126.26 58.316
## - Population  1    11.960 128.68 59.264
## - Illiteracy  1    15.546 132.26 60.639
## - Area        1    30.621 147.34 66.035
## - Life.Exp    1   133.825 250.54 92.580
##
## Step:  AIC=55.32
## Murder ~ Population + Illiteracy + Life.Exp + Frost + Area
##
##              Df Sum of Sq    RSS    AIC
## <none>                    118.92 55.323
## + Income      1     2.206 116.72 56.387
## + HS.Grad     1     2.023 116.90 56.465
## - Frost       1     8.663 127.59 56.839
## - Illiteracy  1    14.259 133.18 58.985
## - Population  1    17.127 136.05 60.050
## - Area        1    29.940 148.86 64.551
## - Life.Exp    1   132.043 250.97 90.665
```

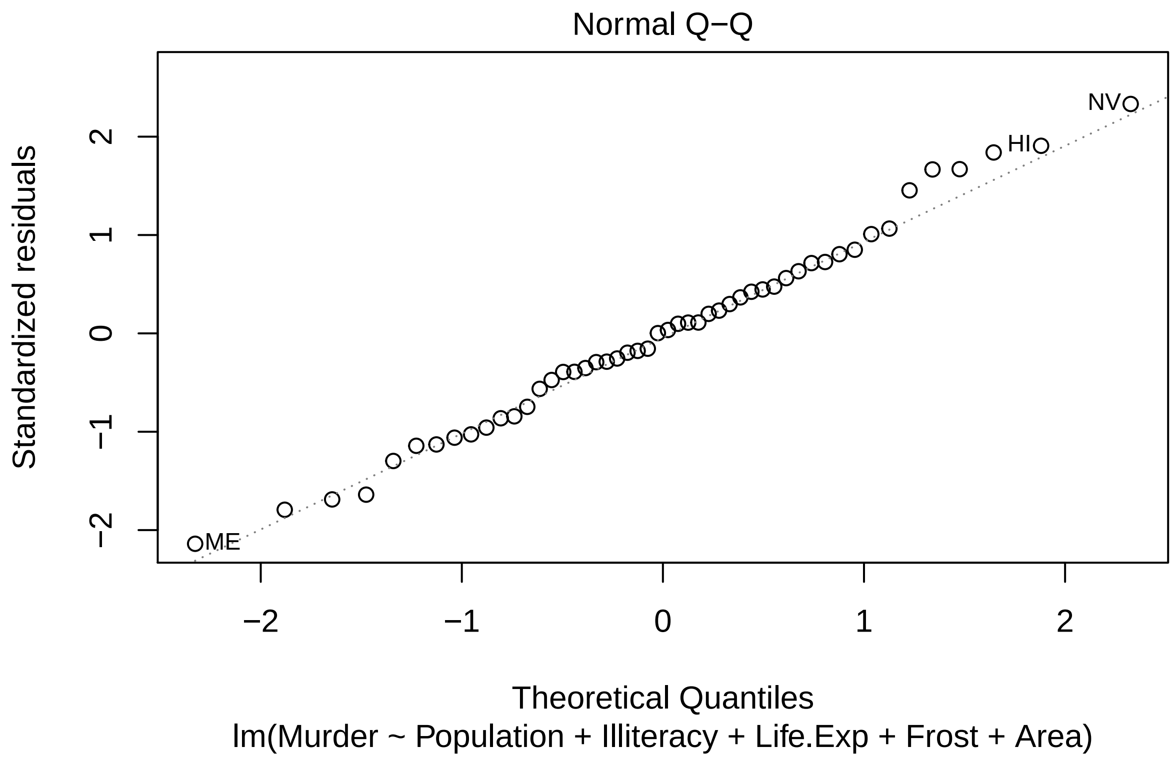
```r
# Our final model keeps Life.Exp, Area, Illiteracy, Population, Frost at a significance level of alpha=

model
```

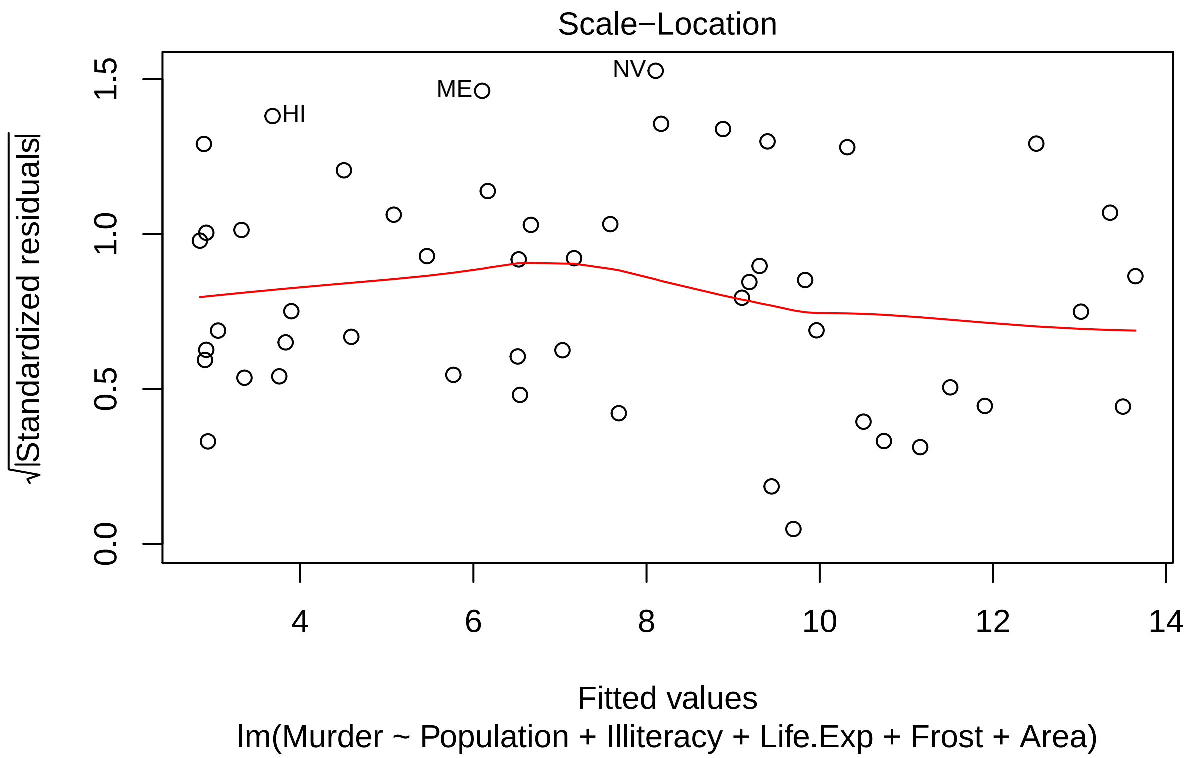
```
##
## Call:
## lm(formula = Murder ~ Population + Illiteracy + Life.Exp + Frost +
##     Area, data = statedata)
##
## Coefficients:
## (Intercept)   Population   Illiteracy     Life.Exp        Frost
##   1.104e+02    1.422e-04    1.785e+00   -1.550e+00   -1.173e-02
##        Area
##   6.936e-01
```

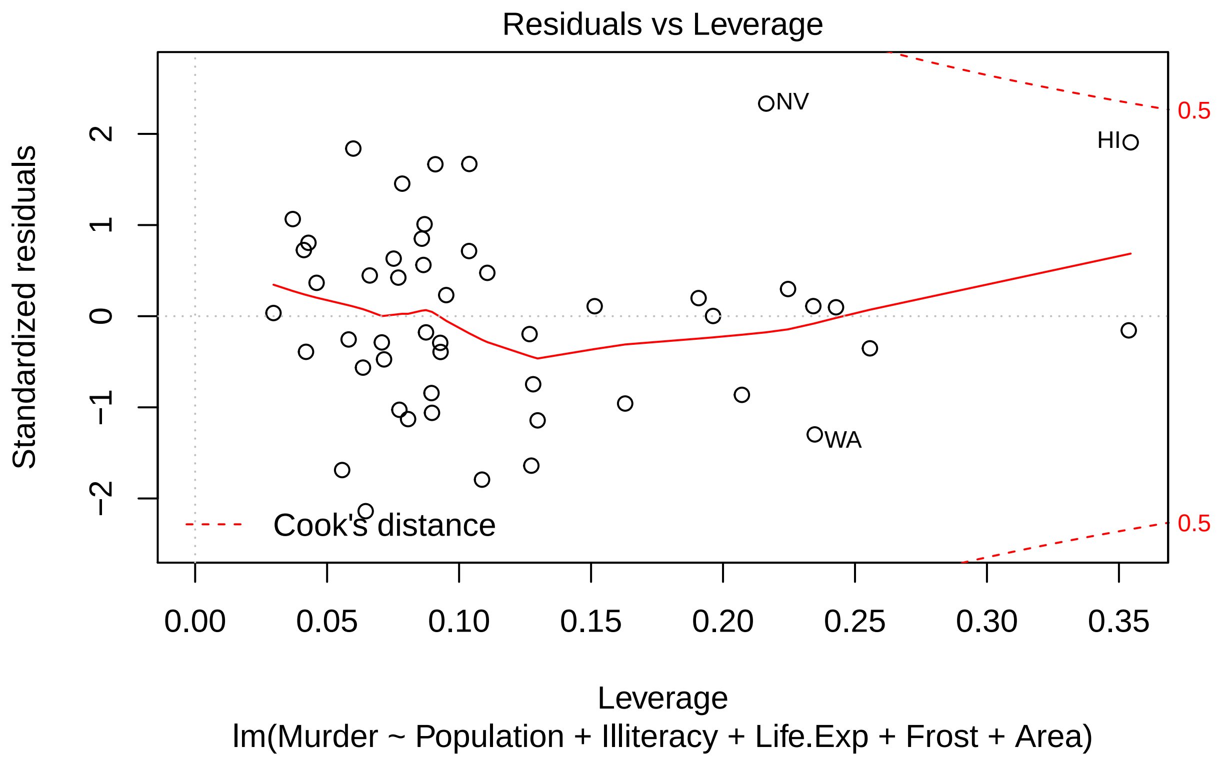
Question 4. e)

```r
plot(model)
```


Residuals vs Fitted
Residuals
4
2
0
−2
−4
NV
MI
ME
4
6
8
10
12
14
Fitted values
lm(Murder ~ Population + Illiteracy + Life.Exp + Frost + Area)


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(Murder ~ Population + Illiteracy + Life.Exp + Frost + Area)
NV
HI
ME
−2
−1
0
1
2


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
HI
ME
NV
4
6
8
10
12
14
Fitted values
lm(Murder ~ Population + Illiteracy + Life.Exp + Frost + Area)



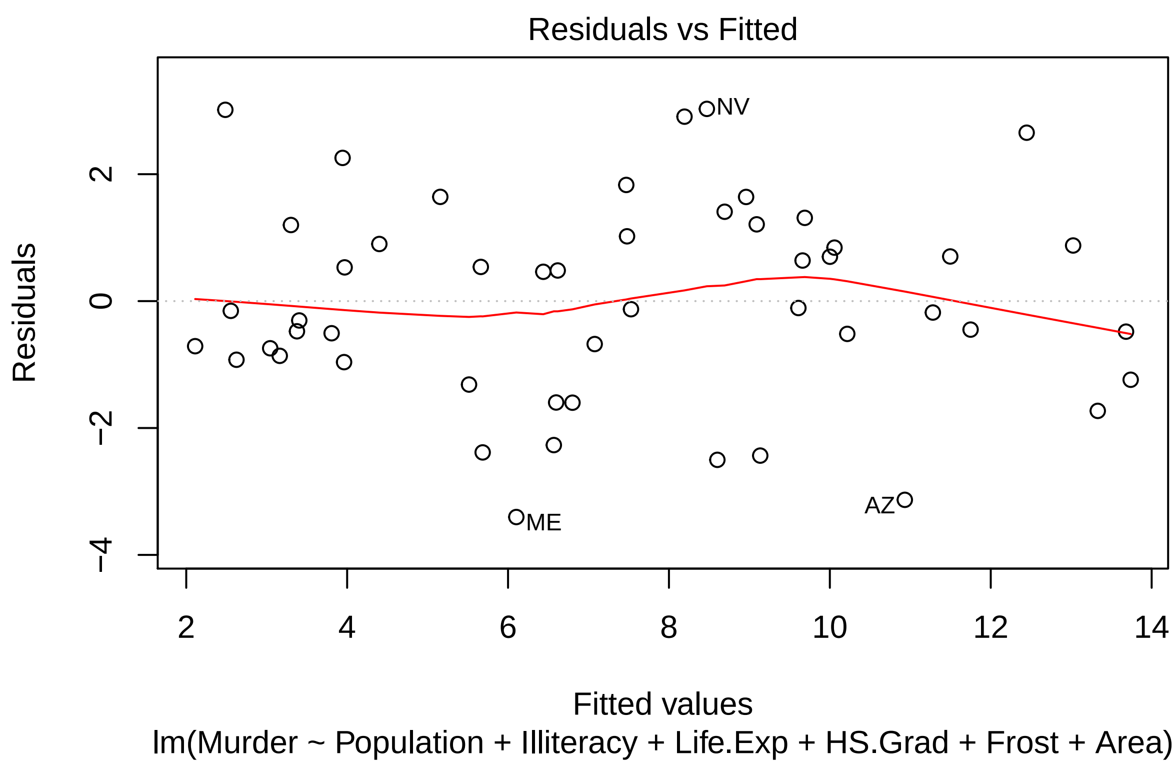

Compare this to another final model that also used a log transformation on Population.

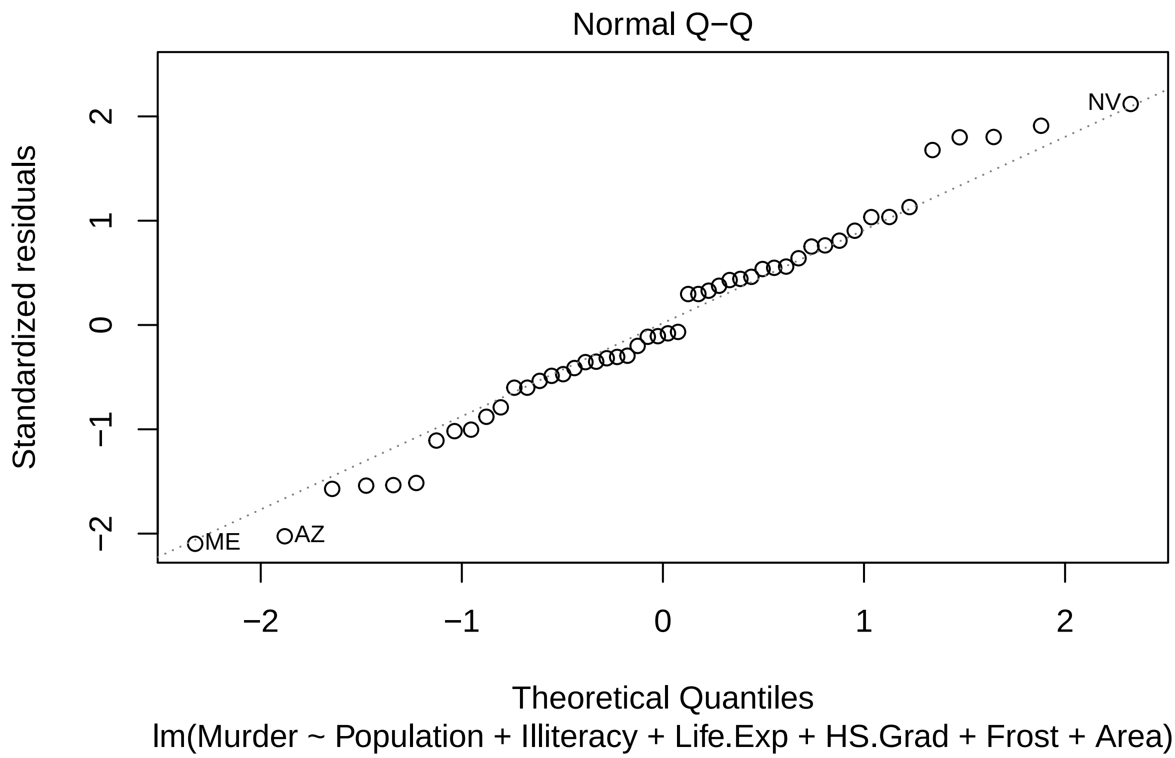
```
statedata$Population = log(statedata$Population)

fullmodel = lm(Murder ~ Population + Income + Illiteracy + Life.Exp + HS.Grad + Frost + Area, data=state
model = step(fullmodel, scope= ~ . + Population + Income + Illiteracy + Life.Exp + HS.Grad + Frost + Ar
```

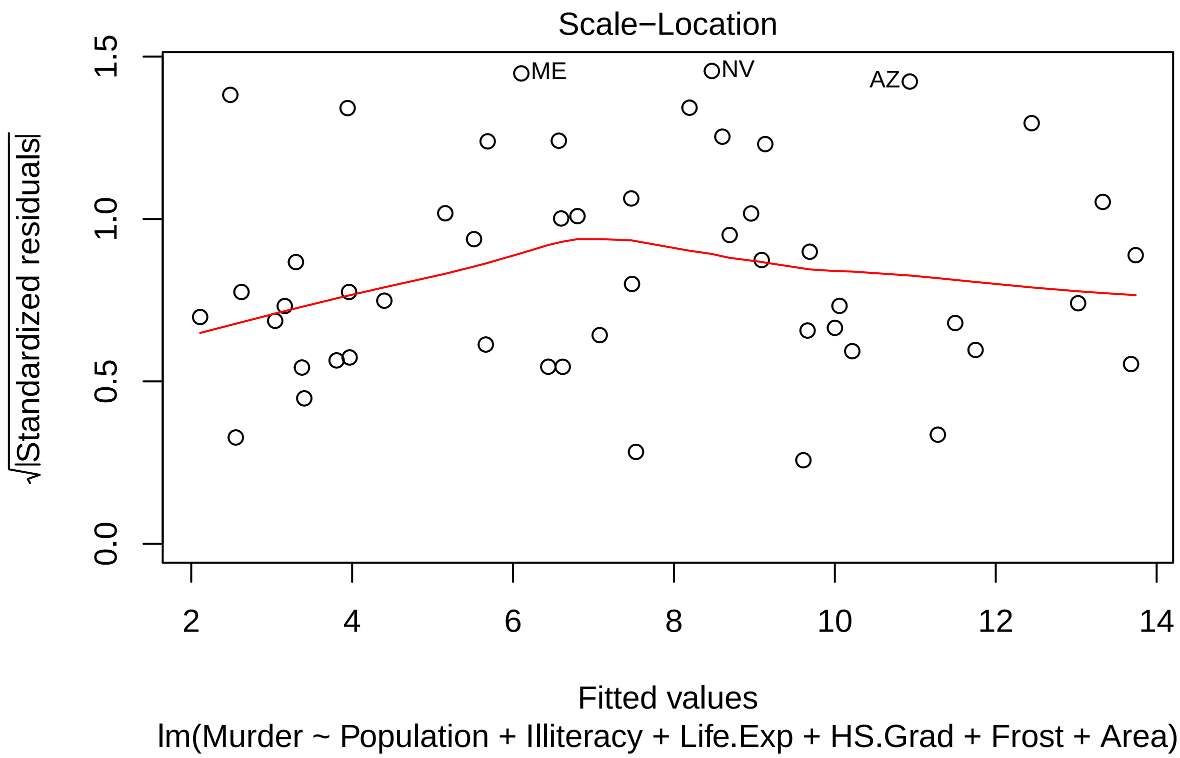
```
## Start:  AIC=59.82
## Murder ~ Population + Income + Illiteracy + Life.Exp + HS.Grad +
##     Frost + Area
##
##              Df Sum of Sq    RSS    AIC
## - Income      1     0.991 121.11 58.233
## - HS.Grad     1     1.219 121.34 58.327
## <none>                    120.12 59.822
## - Frost       1     6.267 126.38 60.365
## - Population  1     8.424 128.54 61.211
## - Illiteracy  1    16.539 136.66 64.272
## - Area        1    24.459 144.57 67.089
## - Life.Exp    1   127.765 247.88 94.046
##
## Step:  AIC=58.23
## Murder ~ Population + Illiteracy + Life.Exp + HS.Grad + Frost +
##     Area
##
##              Df Sum of Sq    RSS    AIC
## <none>                    121.11 58.233
```

```
## - HS.Grad     1      5.275 126.38 58.364
## - Frost       1      5.426 126.53 58.424
## + Income      1      0.991 120.12 59.822
## - Population  1     13.493 134.60 61.514
## - Illiteracy  1     19.123 140.23 63.563
## - Area        1     23.594 144.70 65.132
## - Life.Exp    1    131.223 252.33 92.936
```

```r
plot(model)
```




Normal Q−Q
Standardized residuals
2
1
0
−1
−2
NV
ME
AZ
−2
−1
0
1
2
Theoretical Quantiles
lm(Murder ~ Population + Illiteracy + Life.Exp + HS.Grad + Frost + Area)


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
ME
NV
AZ
2
4
6
8
10
12
14
Fitted values
lm(Murder ~ Population + Illiteracy + Life.Exp + HS.Grad + Frost + Area)


Not taking a transformation is arguably better even though it looks like it should have. Residuals vs Fitted are not as spread, QQ-Plot suggests that the tails follow another distribution, Scale-Location seems to have a negative quadratic trend and the Residuals vs Leverage has points with much larger leverage compared to the previous final model.

Question 5. b)

```
Xscaled = scale(X[,-1]) # No intercept parameter (Piazza)
yscaled = scale(y, scale=FALSE) # Only centering, no scale (Piazza)
r = dim(t(Xscaled)%*%Xscaled)
lambda = diag(0.5, r)
b = solve(t(Xscaled)%*%Xscaled + lambda, t(Xscaled)%*%yscaled)

b
```

```
##            [,1]
## [1,] 0.3494789
## [2,] 1.7899861
## [3,] 0.3432961
```

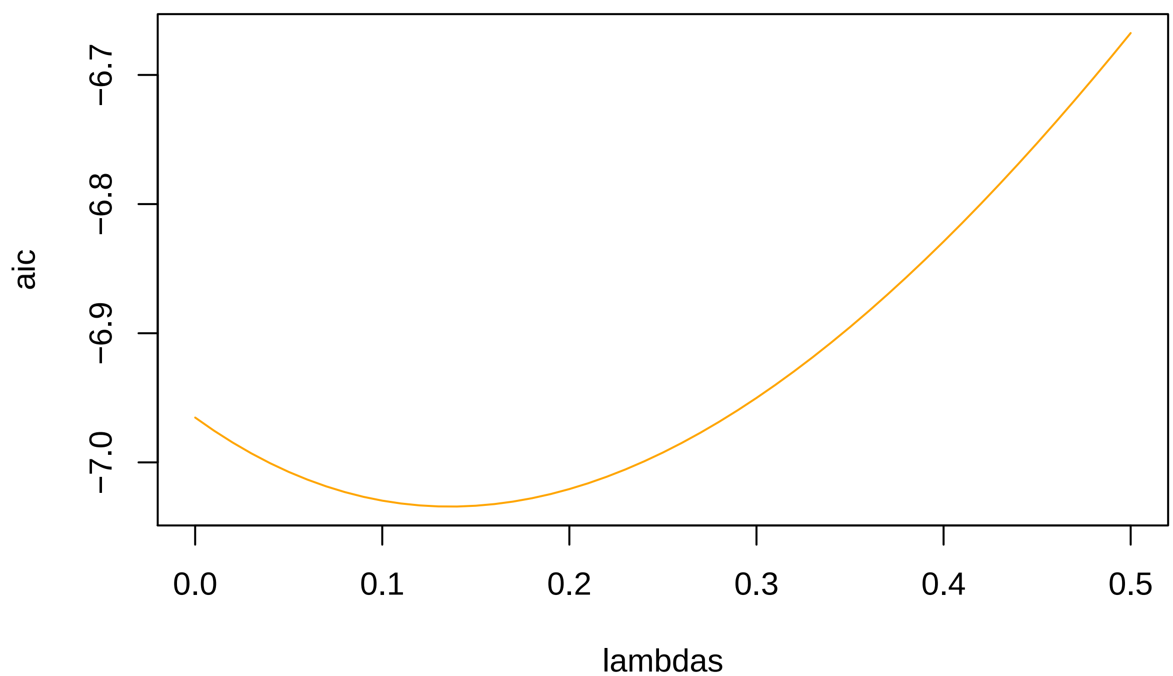
Question 5. c)

```
library(matrixcalc)
```

```
## Warning: package 'matrixcalc' was built under R version 3.5.2
```

```
aic = c()
lambdas = seq(0, 0.5, by=0.01)

for (i in lambdas){
```

```
  lambda = diag(i, r)

  ridgeb = solve(t(Xscaled)%*%Xscaled + lambda, t(Xscaled)%*%yscaled)
  SSRes = t(yscaled-Xscaled%*%ridgeb)%*%(yscaled-Xscaled%*%ridgeb)

  H = Xscaled%*%solve(t(Xscaled)%*%Xscaled + lambda)%*%t(Xscaled)

  aic = c(aic, n*log(SSRes/n) + 2*matrix.trace(H))
}

plot(lambdas, aic, col='orange', type='l')
```




```
lambda_aic = lambdas[which.min(aic)]

lambda_aic
```

```
## [1] 0.14
```