Computer Graphics

Lecture 17

Curve Fitting

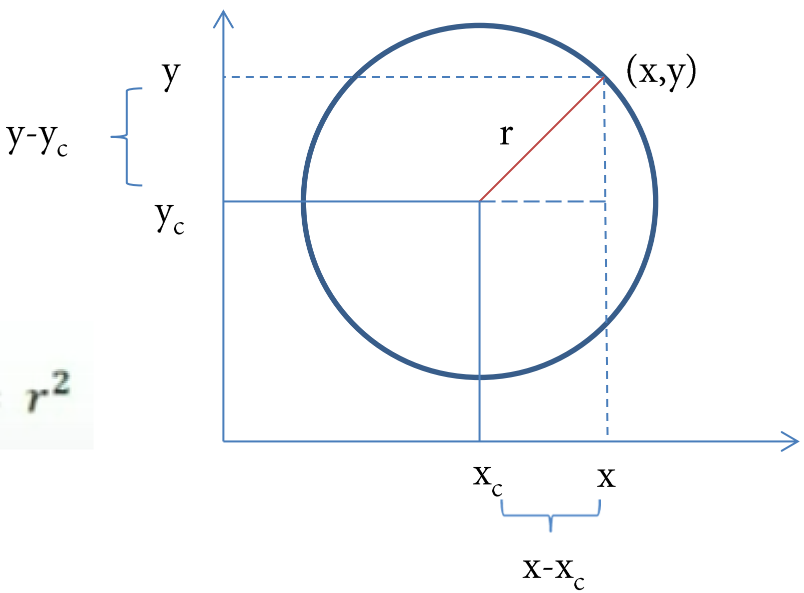
Circle Drawing

Properties of a circle

- Set of points that are distance r from given center point (x_c,y_c)
  - Center (x_c,y_c)
  - Distance r – radius

At any point (x,y)

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$



Circle Drawing

Solve for y

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

$$(y - y_c)^2 = r^2 - (x - x_c)^2$$

$$y - y_c = \sqrt{r^2 - (x - x_c)^2}$$

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$

Like wise solve for x

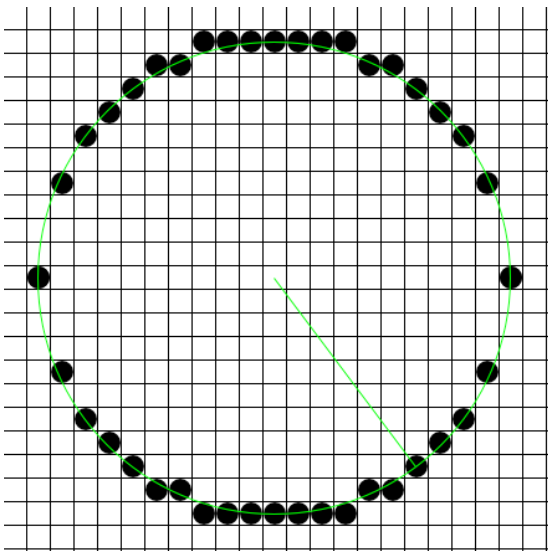
$$x = x_c \pm \sqrt{r^2 - (y - y_c)^2}$$

Drawing algorithm to plot a circle ?

Problems with Circle Drawing

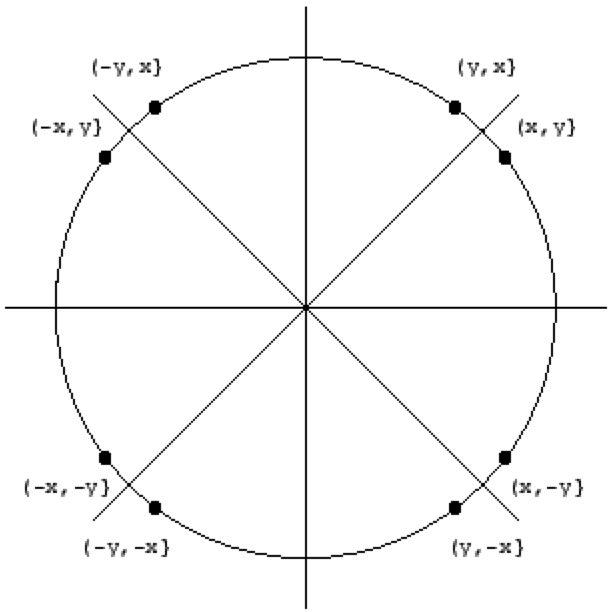
Number of computational operations

More spaces between plotted pixel position



Better solutions ?

Midpoint Circle Drawing Algorithm

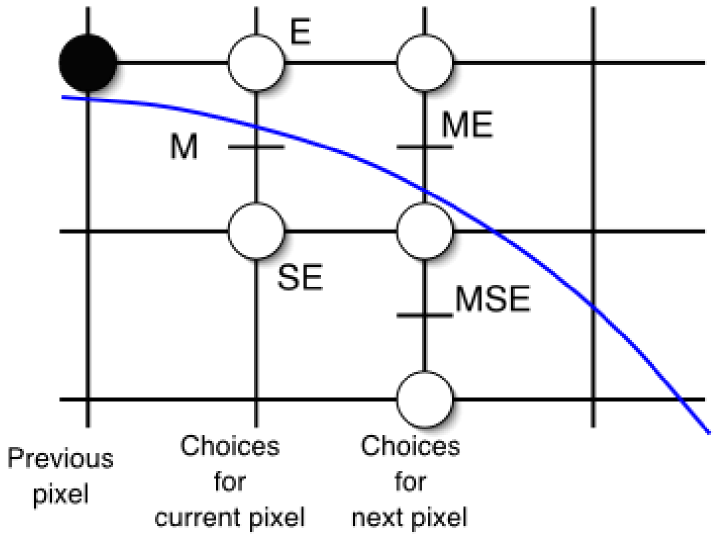


Consider the center of the circle is (0,0)

Using Symmetry property of the circle

- Calculate values for the one octant
- Generate the rest of the 7 octant

Midpoint Circle Drawing Algorithm



Let's start in first octant starting value (0,r)

Let the current position is (x_k, y_k)

Then

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k \text{ or } y_k - 1$$

When x move from x_k to x_{k+1} , y_{k+1} have two choices (y_k or y_k -1)

Midpoint Circle Drawing Algorithm

Consider

$$p_k = f(x, y) = x^2 + y^2 - r^2$$

$= \langle < 0\ (x, y)\ inside\ the\ circle \mid = 0\ (x, y)\ on\ the\ circle \mid > 0\ (x, y) outside\ the\ circle \rangle$

$p_k \rightarrow (x_k + 1, y_k - {}^1/_2)$ Taking midpoint in y axis

If $p_k < 0$ $the\ next\ point\ (x_k + 1, y_k)$

$else\ (p_k > 0)\ the\ next\ point\ (x_k + 1, y_k - 1)$

Mid point $y_k - (y_k - y_k - 1)/2$
$\rightarrow y_k - 1/2$

$$p_k = f(x_k + 1, y_k - {}^1/_2)$$

Midpoint Circle Drawing Algorithm

$$p_k = f(x_k + 1, y_k - {}^1\!/_2)$$

$$p_k = (x_k + 1)^2 + (y_k - {}^1\!/_2)^2 - r^2 \rightarrow (2)$$

Since

$$p_k = f(x, y) = x^2 + y^2 - r^2$$

Midpoint Circle Drawing Algorithm

in k+1 step

$$p_{k+1} = (x_{k+1} + 1)^2 + (y_{k+1} - {}^1\!/_2)^2 - r^2$$

Replace $x_{k+1} = x_k + 1$

$$p_{k+1} = ((x_k + 1) + 1)^2 + (y_{k+1} - {}^1\!/_2)^2 - r^2$$

$$p_{k+1} = (x_k + 1)^2 + 1 + 2(x_k + 1) + (y_{k+1} - {}^1\!/_2)^2 - r^2 \rightarrow (3)$$

(3)-(2)

$$p_{k+1} - p_k = (x_k + 1)^2 + 1 + 2(x_k + 1) + \left(y_{k+1} - {}^1\!/_2\right)^2 - r^2 - (x_k + 1)^2 - \left(y_k - {}^1\!/_2\right)^2 + r^2$$

$$p_{k+1} - p_k = 1 + 2(x_k + 1) + \left(y_{k+1} - {}^1\!/_2\right)^2 - \left(y_k - {}^1\!/_2\right)^2$$

$$p_{k+1} = p_k + 2(x_k + 1) + \left(y_{k+1} - {}^1\!/_2\right)^2 - \left(y_k - {}^1\!/_2\right)^2 + 1 \rightarrow (4)$$

Midpoint Circle Drawing Algorithm

If $p_k < 0$ *the next point* $(x_k + 1, y_k)$

$$p_{k+1} = p_k + 2(x_k + 1) + \left(y_k - {}^1/_2\right)^2 - \left(y_k - {}^1/_2\right)^2 + 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + 1$$

Since $x_{k+1} = (x_k+1)$

$$p_{k+1} = p_k + 2x_{k+1} + 1 \rightarrow (5)$$

else $(p_k > 0)$ *the next point* $(x_k + 1, y_k - 1)$

$$p_{k+1} = p_k + 2(x_k + 1) + \left((y_k - 1) - {}^1/_2\right)^2 - \left(y_k - {}^1/_2\right)^2 + 1$$

Calculations

$$p_{k+1} = p_k + 2(x_k + 1) + \left((y_k - 1) - {}^{1}\!/_{2}\right)^2 - \left(y_k - {}^{1}\!/_{2}\right)^2 + 1$$

$$= p_k + 2(x_k + 1) + \left((y_k - 1)^2 + {}^{1}\!/_{4} - (y_k - 1)\right) - \left(y_k - {}^{1}\!/_{2}\right)^2 + 1$$

$$= p_k + 2(x_k + 1) + \left((y_k - 1)^2 + {}^{1}\!/_{4} - (y_k - 1)\right) - (y_k{}^2 + {}^{1}\!/_{4} - y_k) + 1$$

$$= p_k + 2(x_k + 1) + \left(y_k{}^2 + 1 - 2y_k + {}^{1}\!/_{4} - (y_k - 1)\right) - (y_k{}^2 + {}^{1}\!/_{4} - y_k) + 1$$

$$= p_k + 2(x_k + 1) + y_k{}^2 + 1 - 2y_k + {}^{1}\!/_{4} - y_k + 1 - y_k{}^2 - {}^{1}\!/_{4} + y_k + 1$$

$$= p_k + 2(x_k + 1) + 1 - 2y_k + 1 + 1$$

$$= p_k + 2(x_k + 1) + 2 - 2y_k + 1$$

$$= p_k + 2(x_k + 1) - 2(y_k - 1) + 1$$

Since

$$x_{k+1} = x_k + 1 \;\&\; y_{k+1} = y_k - 1$$

$$p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1 \rightarrow (6)$$

Initial point

Initial point (0,r)

$$x_{k+1} = x_k + 1 = 1$$

$$y_{k+1} = y_k - {}^1\!/_2 = r - {}^1\!/_2$$

$$p_1 = f(1, r - {}^1\!/_2)$$

$$p_1 = 1^2 + (r - {}^1\!/_2)^2 - r^2$$

$$= 1 + r^2 + {}^1\!/_4 - r - r^2$$

$$= {}^5\!/_4 - r$$

Since we calculating in pixel level $p_1 = 1 - r$

Algorithm with example

Midpoint Circle Drawing Algorithm

Step 1: Input Radius r and circle centre (x_c,y_c) and plot the first point (0,r)

Step 2: Calculate the Initial value of the Decision Parameter

$$p_1 = 1 - r$$

Step 3: At Each x point perform the following test

If $p_k < 0$ *the next point is* $(x_k + 1, y_k)$ and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise $(p_k > 0)$ *the next point is* $(x_k + 1, y_k - 1)$ and

$$p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1$$

Step 4: Repeat step 3 until x>=y

Step 5: Calculate other octant values by using symmetry property of the circle

Step 6: Add Centre value with the calculated x,y value

$$x = x + x_c$$

$$y = y + y_c$$

Problem: Draw a circle with radius 5 with centre (2,2) by using Midpoint Circle Drawing Algorithm

Solution

r=5 centre=(2,2)

To use the Symmetry property of the circle consider the centre as (0,0)

To Find

$$p_1 = 1 - r$$

$$=1\text{-}5 = -4$$

$p_1 < 0$ *the next point is* $(x_k + 1, y_k) \dashrightarrow (1,5)$

$$\mathbf{p_{k+1} = p_k + 2x_{k+1} + 1}$$

x	y	p_k
0	5	-4
1	5	-1
2	5	4
3	4	3
4	3	

Let k=1

$\mathbf{p_2 = p_1 + 2x_2 + 1}$

$\mathbf{= -4 + 2.1 + 1 = -1}$

$p_2 < 0$ *the next point is* $(x_k + 1, y_k) \dashrightarrow (2,5)$

$$\mathbf{p_{k+1} = p_k + 2x_{k+1} + 1}$$

Let k=2

$\mathbf{p_3 = p_2 + 2x_3 + 1}$ $\quad\quad\mathbf{= -1 + 2.2 + 1 = 4}$

$p_3 > 0$ *the next point is* $(x_k + 1, y_k - 1) \dashrightarrow (3,4)$

$$\mathbf{p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1}$$

Let k=3

$\mathbf{p_4 = p_3 + 2x_4 - 2y_4 + 1}$

$\mathbf{= 4 + 2.3 - 2.4 + 1 = 3}$

$p_4 > 0$ *the next point is* $(x_k + 1, y_k - 1) \dashrightarrow (4,3)$

Now x>y. it's the end of 1st octant

1st Octant	
x	y
0	5
1	5
2	5
3	4

2nd Octant	
x	y
4	3
5	2
5	1
5	0

2nd Quadrant	
x	y
5	0
5	-1
5	-2
4	-3
3	-4
2	-5
1	-5
0	-5

3rd Quadrant	
x	y
0	-5
-1	-5
-2	-5
-3	-4
-4	-3
-5	-2
-5	-1
-5	0

4th Quadrant	
x	y
-5	0
-5	1
-5	2
-4	3
-3	4
-2	5
-1	5
0	5

Linier translation

Add the centre value (2,2)

1st Quadrant		2nd Quadrant		3rd Quadrant		4th Quadrant	
2	7	7	2	2	-3	-3	2
3	7	7	1	1	-3	-3	3
4	7	7	0	0	-3	-3	4
5	6	6	-1	-1	-2	-2	5
6	5	5	-2	-2	-1	-1	6
7	4	4	-3	-3	0	0	7
7	3	3	-3	-3	1	1	7
7	2	2	-3	-3	2	2	7