

app([],Y,Y).

app([H|T],Y,[H|Z]) :- app(T,Y,Z).

% ?- app([1,2], [3,4], Z). % mode: (+,+,-)

% ?- app([1,2], Y, [1,2,3,4]). % mode: (+,-,+)

% ?- app(X, [3,4], [1,2,3,4]). % mode: (-,+,+)

% ?- app(X, Y, [1,2,3]). % mode: (-,-,+)

% ?- deal([1,2,3,4,5,6,7],Out). mode: (+,-)

% ?- deal(Out,([1,3,5,7],[2,4,6])). mode(-,+,+)

deal([],([],[])).

deal([X],([X],[])).

deal([X,Y|XYs],[X|Xs],[Y|Ys]) :- deal(XYs,(Xs,Ys)).

% ?- merge([1,2,7,9],[3,4,5,8],Out). mode: (+,+,-).

merge([],Ys,Ys).

merge(Xs,[],Xs).

merge([X|Xs],[Y|Ys],[X|O]) :- X<Y,merge(Xs,[Y|Ys],O).

merge([X|Xs],[Y|Ys],[Y|O]) :- X>=Y,merge([X|Xs],Ys,O).

% ?- ms([8,2,7,1,5,6],X). mode: (+,-)

ms([],[]).

ms([X],[X]).

ms(Xs,M) :- deal(Xs,(As,Bs)),

ms(As,Ms1),

ms(Bs,Ms2),

merge(Ms1,Ms2,M),!.

```
% ?- cons(snoc(snoc(nil,3),4),5,Out)
(+,+, -)
```

```
% ?- cons(snoc(snoc(nil,3),4),Out,snoc(snoc(snoc(nil, 5), 3), 4)) mode: (+,-,+)
```

```
% ?- cons(Out,5,snoc(snoc(snoc(nil, 5), 3), 4)) mode: (-,+,+)
```

```
% ?- cons(Out,X,snoc(snoc(snoc(nil, 5), 3), 4)) mode: (-,-,+)
```

```
cons(nil,E,snoc(nil,E)).
```

```
cons(snoc(BL,N),E,snoc(BLN,N)) :- cons(BL,E,BLN).
```

```
% ?- toBl([1,3,7],Out) mode: (+,-)
```

```
toBl([],nil).
```

```
toBl([H|T],O) :- toBl(T,Bl),
```

```
cons(Bl,H,O).
```

```
% ?- snocF([1,2,3,4],5,Out). mode: (+,+, -)
```

```
% ?- snocF([1,2,3,4],Out,[1,2,3,4,5]). mode: (+,-,+)
```

```
% ?- snocF(Out,5,[1,2,3,4,5]). mode: (-,+,+)
```

```
% ?- snocF(X,Y,[1,2,3,4,5]). mode: (-,-,+)
```

```
snocF([],N,[N]).
```

```
snocF([H|T],N,[H|O]) :- snocF(T,N,O).
```

```
% ?- fromBl(snoc(snoc(snoc(nil,1),3),7),Out). mode: (+,-)
```

```
fromBl(nil,[]).
```

```
fromBl(snoc(BL,X),O) :- (fromBl(Bl,E),
```

```
(snocF(E,X,O)).
```

```
% ?- numMt(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
```

```
numMt(empty,1).
```

```
numMt(node(_,L,R),Es) :- numMt(L,LEs),
```

```
numMt(R,REs),
```

```
Es is LEs+REs.
```

```
% ?- numN(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
```

```
numN(empty,0).
```

```
numN(node(_,L,R),Ns) :- numN(L,LNs),
```

```
numN(R,RNs),
```

```
Ns is 1+LNs+RN.
```

```
% ?- insertL(5,node(8,empty,empty),Out). mode: (+,+,-)
```

```
% ?- insertL(Out,node(8,empty,empty),node(8,node(5,empty,empty),empty)). mode:(-,+,+)
```

```
% ?- insertL(5,Out,node(8,node(5,empty,empty),empty)). mode: (+,-,+)
```

```
% ?- insertL(X,Y,node(8,node(5,empty,empty),empty)). mode: (-,-,+)
```

```
insertL(X,empty,node(X,empty,empty)).
```

```
insertL(X,node(N,L,R),node(N,New,R)) :- insertL(X,L,New).
```

```
% ?- insertR(X,Y,node(8,empty,node(5,empty,empty))). mode: (-,-,+)
```

```
% ?- insertR(5,Out,node(8,empty,node(5,empty,empty))). mode: (+,-,+)
```

```
% ?- insertR(Out,node(8,empty,empty),node(8,empty,node(5,empty,empty))). mode: (-,+,+)
```

```
% ?- insertR(5,node(8,empty,empty),Out). mode: (+,+,-)
```

```
insertR(X,empty,node(X,empty,empty)).
```

```
insertR(X,node(N,L,R),node(N,L,New)) :- insertR(X,R,New).
```

```
% ?- sumN(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
```

```
sumN(empty,0).
```

```
sumN(node(N,R,L),O) :- sumN(R,SumR),
```

```
    sumN(L,SumL),
```

```
    O is N+SumL+SumR.
```

```
% ?- inOrd(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
inOrd(empty,[]).
```

```
inOrd(node(N,L,R),O) :- inOrd(L,GoL),
```

```
    inOrd(R,GoR),
```

```
    app(GoL,[N],O1),
```

```
    app(O1,GoR,O).
```

```
% ?- numEs(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
numEs(leaf(_),1).
```

```
numEs(node2(_,L,R),O) :- numEs(L,LEs),
```

```
    numEs(R,REs),
```

```
    O is 1+LEs+REs.
```

```
% ?- sumN2(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
sumN2(leaf(N),N).
```

```
sumN2(node2(N,R,L),O) :- sumN2(R,SumR),
```

```
    sumN2(L,SumL),
```

```
    O is N+SumL+SumR.
```

```
% ?- inOrd2(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
inOrd2(leaf(N),[N]).
```

```
inOrd2(node2(N,L,R),O) :- inOrd2(L,GoL),
```

```
    inOrd2(R,GoR),
```

```
    app(GoL,[N],O1),
```

```
    app(O1,GoR,O).
```

```
% conv21(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
% the mode: (-,+) below only works when there are 2 children
```

```
% conv21(Out,node(3,node(1,empty,empty),node(2,empty,empty))).
```

```
conv21(leaf(N),node(N,empty,empty)).
```

```
conv21(node2(N,L,R),node(N,LO,RO)) :- conv21(L,LO),
```

```
    conv21(R,RO).
```

```
% ?- toBl_It([1,2,3,4],X). mode: (+,-)
```

```
toBl_It(L,O) :- toBl_H(L,nil,O).
```

```
toBl_H([],A,A).
```

```
toBl_H([H|T],A,BI) :- toBl_H(T,snoc(A,H),BI).
```

```
% ?- fromBl_It(snoc(snoc(snoc(nil,1),2),3),O). mode: (+,-)
```

```
fromBl_It(BI,O) :- fromBl_H(BI,[],O).
```

```
fromBl_H(nil,A,A).
```

```
fromBl_H(snoc(L,X),A,RL) :- fromBl_H(L,[X|A],RL).
```

```
% ?- sumN_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
sumN_It(T,O) :- sumN_H([T],0,O).
```

```
sumN_H([],A,A).
```

```
sumN_H([empty|Ts],A,Sn) :- sumN_H(Ts,A,Sn).
```

```
sumN_H([node(N,L,R)|Ts],A,Sn) :- AN is A+N,  
                                sumN_H([L,R|Ts],AN,Sn).
```

```
% ?- numE_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
numE_It(T,O) :- numE_H([T],0,O).
```

```
numE_H([],A,A).
```

```
numE_H([empty|Ts],A,En) :- A1 is A+1, numE_H(Ts,A1,En).
```

```
numE_H([node(_,L,R)|Ts],A,En) :- numE_H([L,R|Ts],A,En).
```

```
% ?- numN_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
numN_It(T,O) :- numN_H([T],0,O).
```

```
numN_H([],A,A).
```

```
numN_H([empty|Ts],A,Nn) :- numN_H(Ts,A,Nn).
```

```
numN_H([node(_,L,R)|Ts],A,Nn) :- A1 is A+1, numN_H([L,R|Ts],A1,Nn).
```

```
% ?- sumN2_It(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
sumN2_It(T,O) :- sumN2_H([T],0,O).
```

```
sumN2_H([],A,A).
```

```
sumN2_H([leaf(N)|Ts],A,Sn) :- AN is A+N,
```

```
sumN2_H(Ts,AN,Sn).
```

```
sumN2_H([node2(N,L,R)|Ts],A,Sn) :- AN is A+N,
```

```
sumN2_H([L,R|Ts],AN,Sn).
```

```
% ?- inOrd2_It(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
inOrd2_It(T2,O) :- inOrd2_H([T2],[],O).
```

```
inOrd2_H([],A,A).
```

```
inOrd2_H([leaf(N)|Ts],A,IO) :- inOrd2_H(Ts,[N|A],IO).
```

```
inOrd2_H([node2(N,L,R)|Ts],A,IO) :- inOrd2_H([R,leaf(N),L|Ts],A,IO).
```

```
% less than function with infinity
```

```
% ?- isA_lt_B(fin(1),fin(2)). mode: (+,+) (I included this cause it is the only one.)
```

```
isA_lt_B(neginf,fin(_)).
```

```
isA_lt_B(fin(N),fin(M)) :- N<M.
```

```
isA_lt_B(fin(_),posinf).
```

```
% ?-
```

```
bst_H(neginf,posinf,node(10,node(5,node(3,empty,empty),node(8,empty,empty)),node(15,empty,empty)),Out). mode: (+,+,+)
```

```
bst_H(_,_,empty).
```

```
bst_H(Lo,Hi,node(N,L,R)) :- isA_lt_B(Lo,fin(N)),
```

```
isA_lt_B(fin(N),Hi),
```

```
bst_H(Lo,fin(N),L),
```

```
bst_H(fin(N),Hi,R).
```

```
% ?-
```

```
bst(node(10,node(5,node(3,empty,empty),node(8,empty,empty)),node(15,empty,empty)),Out).  
mode: (+,-)
```

```
bst(node(N,L,R),true) :- bst_H(neginf,posinf,node(N,L,R)),!.
```

```
bst(_,false).
```

```
% ?- bst2_H(neginf,posinf,node2(15,node2(10,leaf(5),leaf(12)),node2(20,leaf(18),leaf(25)))).  
mode: (+,+,+)
```

```
bst2_H(Lo,Hi,leaf(N)) :- isA_lt_B(Lo,fin(N)),  
                           isA_lt_B(fin(N),Hi).  
  
bst2_H(Lo,Hi,node2(N,L,R)) :- isA_lt_B(Lo,fin(N)),  
                               isA_lt_B(fin(N),Hi),  
                               bst2_H(Lo,fin(N),L),  
                               bst2_H(fin(N),Hi,R).
```

```
% ?- bst2(node2(15,node2(10,leaf(5),leaf(12)),node2(20,leaf(18),leaf(25))),Out). mode: (+,-)  
bst2(node2(N,L,R),true) :- bst2_H(neginf,posinf,node2(N,L,R)),!.  
bst2(_,false).
```