

app([],Y,Y).

app([H|T],Y,[H|Z]) :- app(T,Y,Z).

% ?- app([1,2], [3,4], Z). % mode: (+,+,-)

% ?- app([1,2], Y, [1,2,3,4]). % mode: (+,-,+)

% ?- app(X, [3,4], [1,2,3,4]). % mode: (-,+,+)

% ?- app(X, Y, [1,2,3]). % mode: (-,-,+)

% ?- cons(snoc(snoc(nil,3),4),5,Out)

mode:

(+,+,-)

% ?- cons(snoc(snoc(nil,3),4),Out,snoc(snoc(snoc(nil, 5), 3), 4)) mode: (+,-,+)

% ?- cons(Out,5,snoc(snoc(snoc(nil, 5), 3), 4))

mode: (-,+,+)

% ?- cons(Out,X,snoc(snoc(snoc(nil, 5), 3), 4))

mode: (-,-,+)

cons(nil,E,snoc(nil,E)).

cons(snoc(BL,N),E,snoc(BLN,N)) :- cons(BL,E,BLN).

% ?- toBl([1,3,7],Out) mode: (+,-)

toBl([],nil).

toBl([H|T],O) :- toBl(T,Bl),

cons(Bl,H,O).

% ?- snocF([1,2,3,4],5,Out). mode: (+,+,-)

% ?- snocF([1,2,3,4],Out,[1,2,3,4,5]). mode: (+,-,+)

% ?- snocF(Out,5,[1,2,3,4,5]). mode: (-,+,+)

% ?- snocF(X,Y,[1,2,3,4,5]). mode: (-,-,+)

snocF([],N,[N]).

snocF([H|T],N,[H|O]) :- snocF(T,N,O).

```
% ?- fromBl(snoc(snoc(snoc(nil,1),3),7),Out). mode: (+,-)
```

fromBl(nil,[]).
$$\text{fromBl}(\text{snoc}(\text{Bl}, \text{X}), \text{O}) :- (\text{fromBl}(\text{Bl}, \text{E})),$$

$$(\text{snocF}(\text{E}, \text{X}, \text{O})).$$

```
% ?- numMt(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
```

numMt(empty,1).

$$\begin{aligned} \text{numMt}(\text{node}(_, \mathbf{L}, \mathbf{R}), \mathbf{Es}) &:- \text{numMt}(\mathbf{L}, \mathbf{LEs}), \\ &\quad \text{numMt}(\mathbf{R}, \mathbf{REs}), \\ &\quad \mathbf{Es} \text{ is } \mathbf{LEs} + \mathbf{REs}. \end{aligned}$$

```
% ?- numN(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
```

numN(empty,0).

```

numN(node(_,L,R),Ns) :- numN(L,LNs),
                        numN(R,RNs),
                        Ns is 1+LNs+RNs.

```

```
% ?- insertL(5,node(8,empty,empty),Out). mode: (+,+,-)
```

```
% ?- insertL(Out,node(8,empty,empty),node(8,node(5,empty,empty),empty)). mode:(-,+,+)
```

```
% ?- insertL(5,Out,node(8,node(5,empty,empty),empty)). mode: (+,-,+)
```

```
% ?- insertL(X,Y,node(8,node(5,empty,empty),empty)). mode: (-,-,+)
```

```
insertL(X,empty,node(X,empty,empty)).
```

$$\text{insertL}(X, \text{node}(N, L, R), \text{node}(N, \text{New}, R)) \text{ :- insertL}(X, L, \text{New}).$$

```
% ?- insertR(X,Y,node(8,empty,node(5,empty,empty))). mode: (-,-,+)
% ?- insertR(5,Out,node(8,empty,node(5,empty,empty))). mode: (+,-,+)
% ?- insertR(Out,node(8,empty,empty),node(8,empty,node(5,empty,empty))). mode: (-,+,+)
% ?- insertR(5,node(8,empty,empty),Out). mode: (+,+,-)
insertR(X,empty,node(X,empty,empty)).
insertR(X,node(N,L,R),node(N,L,New)) :- insertR(X,R,New).
```

```
% ?- sumN(node(5,node(3,empty,empty),empty),Out). mode: (+,-)
sumN(empty,0).
sumN(node(N,R,L),O) :- sumN(R,SumR),
                  sumN(L,SumL),
          O is N+SumL+SumR.
```

```
% ?- inOrd(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
inOrd(empty,[]).
inOrd(node(N,L,R),O) :- inOrd(L,GoL),
                  inOrd(R,GoR),
          app(GoL,[N],O1),
          app(O1,GoR,O).
```

```
% ?- numEs(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
numEs(leaf(_),1).
numEs(node2(_,L,R),O) :- numEs(L,LEs),
                  numEs(R,REs),
          O is 1+LEs+REs.
```

```
% ?- sumN2(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
sumN2(leaf(N),N).
```

```
sumN2(node2(N,R,L),O) :- sumN2(R,SumR),  
                           sumN2(L,SumL),  
                           O is N+SumL+SumR.
```

```
% ?- inOrd2(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
inOrd2(leaf(N),[N]).
```

```
inOrd2(node2(N,L,R),O) :- inOrd2(L,GoL),  
                           inOrd2(R,GoR),  
                           app(GoL,[N],O1),  
                           app(O1,GoR,O).
```

```
% conv21(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
% the mode: (-,+) below only works when there are 2 children
```

```
% conv21(Out,node(3,node(1,empty,empty),node(2,empty,empty))).
```

```
conv21(leaf(N),node(N,empty,empty)).
```

```
conv21(node2(N,L,R),node(N,LO,RO)) :- conv21(L,LO),  
                                       conv21(R,RO).
```

```
% ?- toBl_It([1,2,3,4],X). mode: (+,-)
```

```
toBl_It(L,O) :- toBl_H(L,nil,O).
```

```
toBl_H([],A,A).
```

```
toBl_H([H|T],A,BI) :- toBl_H(T,snoc(A,H),BI).
```

```
% ?- fromBl_It(snoc(snoc(snoc(nil,1),2),3),O). mode: (+,-)
```

```
fromBl_It(Bl,O) :- fromBl_H(Bl,[],O).
```

```
fromBl_H(nil,A,A).
```

```
fromBl_H(snoc(L,X),A,RL) :- fromBl_H(L,[X|A],RL).
```

```
% ?- sumN_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
sumN_It(T,O) :- sumN_H([T],0,O).
```

```
sumN_H([],A,A).
```

```
sumN_H([empty|Ts],A,Sn) :- sumN_H(Ts,A,Sn).
```

```
sumN_H([node(N,L,R)|Ts],A,Sn) :- AN is A+N,  
                                sumN_H([L,R|Ts],AN,Sn).
```

```
% ?- numE_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
numE_It(T,O) :- numE_H([T],0,O).
```

```
numE_H([],A,A).
```

```
numE_H([empty|Ts],A,En) :- A1 is A+1, numE_H(Ts,A1,En).
```

```
numE_H([node(_,L,R)|Ts],A,En) :- numE_H([L,R|Ts],A,En).
```

```
% ?- numN_It(node(2,node(1,empty,empty),node(3,empty,empty)),Out). mode: (+,-)
```

```
numN_It(T,O) :- numN_H([T],0,O).
```

```
numN_H([],A,A).
```

```
numN_H([empty|Ts],A,Nn) :- numN_H(Ts,A,Nn).
```

```
numN_H([node(_,L,R)|Ts],A,Nn) :- A1 is A+1, numN_H([L,R|Ts],A1,Nn).
```

```
% ?- sumN2_It(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
sumN2_It(T,O) :- sumN2_H([T],0,O).
```

```
sumN2_H([],A,A).
```

```
sumN2_H([leaf(N)|Ts],A,Sn) :- AN is A+N,
```

```
    sumN2_H(Ts,AN,Sn).
```

```
sumN2_H([node2(N,L,R)|Ts],A,Sn) :- AN is A+N,
```

```
    sumN2_H([L,R|Ts],AN,Sn).
```

```
% ?- inOrd2_It(node2(7,node2(3,leaf(1),leaf(2)),node2(3,leaf(1),leaf(2))),Out). mode: (+,-)
```

```
inOrd2_It(T2,O) :- inOrd2_H([T2],[],O).
```

```
inOrd2_H([],A,A).
```

```
inOrd2_H([leaf(N)|Ts],A,IO) :- inOrd2_H(Ts,[N|A],IO).
```

```
inOrd2_H([node2(N,L,R)|Ts],A,IO) :- inOrd2_H([R,leaf(N),L|Ts],A,IO).
```

```
% less than function with infinity
```

```
% ?- isA_lt_B(fin(1),fin(2)). mode: (+,+) (I included this cause it is the only one.)
```

```
isA_lt_B(neginf,fin(_)).
```

```
isA_lt_B(fin(N),fin(M)) :- N<M.
```

```
isA_lt_B(fin(_),posinf).
```

```
% ?-  
bst_H(neginf, posinf, node(10, node(5, node(3, empty, empty), node(8, empty, empty)), node(15, empty, empty)), Out). mode: (+,+,+,-)
```

```
bst_H(_,_,empty,true).
```

```
bst_H(Lo,Hi,node(N,L,R),B) :- isA_lt_B(Lo,fin(N)),  
                                isA_lt_B(fin(N),Hi),  
                                bst_H(Lo,fin(N),L,B),  
                                bst_H(fin(N),Hi,R,B).
```

```
% ?-  
bst(node(10,node(5,node(3,empty,empty),node(8,empty,empty)),node(15,empty,empty)),Out).  
mode: (+,-)
```

```
bst(empty,false).
```

```
bst(node(N,L,R),B) :- bst_H(neginf, posinf, node(N,L,R),B), !.
```

```
bst(_,false).
```

```
% ?-  
bst2_H(neginf, posinf, node2(15, node2(10, leaf(5), leaf(12)), node2(20, leaf(18), leaf(25))), Out).  
mode: (+,+,+,-)
```

```
bst2_H(Lo,Hi,leaf(N),true) :- isA_lt_B(Lo,fin(N)),  
                               isA_lt_B(fin(N),Hi).
```

```
bst2_H(Lo,Hi,node2(N,L,R),B) :- isA_lt_B(Lo,fin(N)),  
                                isA_lt_B(fin(N),Hi),  
                                bst2_H(Lo,fin(N),L,B),  
                                bst2_H(fin(N),Hi,R,B).
```

```
% ?- bst2(node2(15,node2(10,leaf(5),leaf(12)),node2(20,leaf(18),leaf(25))),Out). mode: (+,-)  
bst2(leaf(_),false).
```

```
bst2(node2(N,L,R),B) :- bst2_H(neginf, posinf, node2(N,L,R),B), !.
```

```
bst2(_,false).
```