Measurements of Black Hole Properties

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January 2023

1 Introduction

According to the no-hair theorem, we only characterize black holes in three properties: mass, angular momentum, and electric charge. While charged black holes can easily be neutralized through discharge, we therefore consider mainly a black hole's mass and angular momentum, also known as spin.

2 Measurement of the Mass of Black Holes

From my understanding, I think measuring the mass of black holes should be considered separately for black holes binaries and galactic center black holes. In the following sections, a brief explanation on the methods on how to measure the mass of a black hole will be presented. Generally, observing the motion of gas in the accretion disk around a black hole and assuming that the gas motions in the innermost regions reflect keplerian rotation can give us a good estimation of the mass of the black hole [6].

2.1 Black Hole Binaries

2.1.1 Black Holes in Binary Systems

Measuring the masses of black holes in binary systems is quite similar to measuring the stellar masses in binary star systems. For black hole binaries, Kepler's Third Law of Motion should be considered. With the original equation of Kepler's Third Law of Motion,

$$P^2 \propto a^3 \tag{1}$$

where P is the orbital period, and a is the semi-major axis, we can transform this to fit binary systems of masses m_1, the mass of the star, and m_2, the mass of the black hole:

$$m_1 + m_2 = \frac{4\pi^2 a^3}{P^2} \tag{2}$$

and while considering orbital inclination i:

$$m_1 + m_2 = \frac{4\pi^2 {a_{obs}}^3}{P^2 \sin^3 i} \tag{3}$$

where a_{obs} is the observed semi-major axis. Another form of Kepler's Third Law can also be applied to the measurement of mass of a black hole.

$$\frac{{m_2}^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{P {v_{obs}}^3}{2\pi G} \tag{4}$$

where v_{obs} is the observed velocity. If we assume $m_1 = 0$ and $\sin i = 1$, we are able to calculate the minimum allowable mass of the compact object, M.

$$M = \frac{P {v_{obs}}^3}{2\pi G} \tag{5}$$

Thus, in order to obtain M, P and v_{obs} need to be measured through observables. v_{obs} can be derived through observing the spectrum of the star, which will be redshifted or blueshifted due to Doppler effect. By plotting a velocity versus time diagram through observational data from telescopes, we are able to determine P and thus obtain M. With P and v_{obs} we are able to determine a.

We can also calculate the sum of the two masses, $m_1 + m_2$ and subtract that of the star, according to Equation 3. m_1 can be estimated through various ways other than kinematics and is relatively easy. For example, we can estimate stellar mass through asteroseismic scaling relations [5], which only requires observables from telescopes. Mass-luminosity relation can also be employed in this scheme.

However, in order to obtain a more precise estimation of the mass of a black hole in Equation 4, i and the mass ratio $q = \frac{m_2}{m_1}$ should be considered, where

$$m_2 = \frac{P {v_{obs}}^3}{2\pi G} \frac{q(1+q)^2}{\sin^3 i} \tag{6}$$

I believe that in order to calculate i, one needs to employ X-ray intensity variations along with time. If, for example, X-ray emissions from the black hole is observed to be eclipsed periodically, we are able to estimate that the orbital inclination is high. We can also extract and analyze the motion and orbit of the companion star. With more detailed modeling, one can determined i. Another approach is to use ellipsoidal modulations, where due to the strong gravity of the black hole, the star can be tidally distorted, which can be measured through analyzing its light curve. Using the equation given by [7], we are able to constrain both q and i.

2.1.2 Binary Black Holes

Binary black holes are also another situation we need to consider separately. For binary black holes like the ones observed by LIGO in 2015 through gravitational waves, we can consider calculating the chirp mass of the system [1], which only requires the gravitational wave frequency from observations. The mass ratio of the two black holes is also required and can be determined in the early stage of the inspiral using the equations mentioned in [9] and [2]. The total mass can be determined in the final stage of the coalescence.

2.2 Galactic Center Black Holes

Lone black holes like the supermassive black hole existing in the center of Milky Way requires a slightly different version of Kepler's Third Law of Motion in the determination of their masses:

$$P^2 = \frac{4\pi^2}{GM}a^3 \tag{7}$$

and resulting in

$$M = \frac{4\pi^2 a^3}{GP^2} \tag{8}$$

where P and a of stellar orbits can be determined through observational data of stars orbiting around the black hole. Usually, with the observed motions of stars orbiting around a black hole, one can directly fit the modelled orbits of these stars and obtain mass estimates from models [4]. However, this method is only plausible for nearby black holes in which we are able to observe the orbits of stars orbiting around it.

Another method to measure the mass of the central black hole is by considering the active galactic nucleus, whose properties depend on the black hole's mass. Since clouds are orbiting around the supermassive black hole in the broad line region, we can measure the velocity and radius of the clouds to obtain the mass of the black hole with the following equation derived from circular motion:

$$M_{\mathrm{BH}} = \frac{fRv^2}{G} \tag{9}$$

where f is a factor that accounts for parameters like i that might not be available for every source. f can be determined and averaged with nearby black holes with known masses. R is the radius of the clouds, and v is the velocity of the clouds in the broad line region. To obtain R, we can use reverberation mapping to measure time delays of the flares emitted from the accretion disk of the black hole while employing cross correlation functions. To obtain v, we can observe the red-shifted or blue-shifted emission lines. However, this reverberation mapping method might produce large uncertainties as we might not be able to know the structure of the broad line region.

3 Measurement of the Spin of Black Holes

Measuring the spin of a black hole is more complicated than measuring the mass of a black hole. We commonly use the spin parameter $a = cJ/GM^2$ (where J is the angular momentum, M is the mass of the black hole) to measure the spin of the black hole. There are several commonly used methods to measure the spin of a black hole. These methods can mainly be separated into two categories: methods using electromagnetic waves and methods using gravitational waves.

3.1 X-Ray Reflection

The first method is through X-ray reflection. When we want to measure the spin of the black hole, we consider the accretion disk of the black hole because its structure is influenced by frame-dragging – rotating spacetime dragging nearby matter into the black hole, which is related to the black hole's spin. As the gas gets nearer to the black hole, it will eventually be plunged into the black hole's event horizon, according to relativity. The innermost stable circular orbit (ISCO) denotes this transition and is dependent on the spin of the black hole (the location of ISCO varies monotonically with the spin parameter): the higher the spin, the closer ISCO is to the black hole. In geometrically-thin accretion disks, ISCO can be shown in the X-ray emissions from the inner disk, and we can observe the redshifts in the X-ray spectrum, which is caused by (i) Doppler effect, (ii) time-dilation of Special Relativity, and (iii) gravitational redshifts of General Relativity. As ISCO gets closer to the event horizon, the more redshifted the X-rays reflected are. These emission lines will have a blueshifted peak and an extended redshifted wing. Usually, we can construct an accurate model of the accretion disk reflection spectrum from the X-ray reflection spectra, and the model is characterized by several parameters [10]. There are also constraints in the X-ray reflection technique: parameters like the ionization state of the accretion disk affects the disk spectrum in complicated ways; assumptions made about the accretion disk can deviate from the real scenario; X-ray spectrum observed can be affected by line of sight absorption or other more distant matter.

3.2 Gravitational Waves

For the merging black holes binary systems like the one detected by aLIGO, we can analyze the gravitational waves and obtain their spins. The inspiral stage of the merging black holes will contain information about the magnitude and orientation of the spin because it is affected by these factors. Thus, the gravitational wave data that we mainly use to determine black hole spin comes from the inspiral stage. The gravitational wave signals will be affected by the effective spin and the precession spin, which are correlated to the masses, spin parameters, and the direction of spin. Through the analyses of gravitational wave signals in the inspiral stage, one can obtain individual spins of the two black holes before merging. In order to obtain the spin of the merged black hole in ringdown, though we can theoretically measure the gravitational wave signals in the ringdown stage, these signals are hard to detect, and we would rather estimate the spin through constructing a model with mass ratios and effective spins obtained from the inspiral stage. Compared to methods using electromagnetic waves to obtain black hole spin, the gravitational wave analyses can be less complicated due to the fact that gravitational wave signals do not depend on accretion. However, this method is also disadvantageous because of its low signal to noise ratios and limited frequency range of gravitational waves able to be detected.

4 Detailed Review of Thermal Continuum Fitting

Thermal continuum fitting is another technique for measuring black hole spin. As mentioned in Section 3.1, the location of ISCO depends on the black hole's spin. Since the spin of the black hole will also affect the temperature of the inner disk, the temperature will also be correlated to ISCO such that for higher spins, ISCO is smaller, leading to a higher temperature. This effect can be shown using the Newtonian disk model:

$$T_{\text{eff}} = 3.3 \times 10^7 \eta^{-1/4} \left(\frac{M}{10\mathrm{M}_\odot}\right)^{-1/4} \left(\frac{L}{L_{\text{Edd}}}\right)^{1/4} \left(\frac{r_g}{r}\right)^{3/4} \left(1 - \sqrt{\frac{r_{\text{isco}}}{r}}\right)^{1/4}$$

(10)

where η is the radiative efficiency defined as $\eta = r_g/2r_{\text{isco}}$, L_{Edd} is the standard Eddington luminosity defined as $L_{\text{Edd}} = 4\pi GMm_pc/\sigma_{\text{T}}$ (Eddington luminosity is the maximum luminosity when the outward force of radiation is balanced with the inward force of gravitation), σ_{T} is the effective area of an electron when it is illuminated by radiation, and r_g is the gravitational radius defined as $r_g = GM/c^2$. Thus, as spin increases, which will result in r_{isco} decreasing, T_{eff} will increase. This is because as spin increases, radiation is emitted from a smaller effective area, thus the temperature of the radiation emitted will increase.

To measure the spin of black holes in practice, we need to fit the thermal spectrum into a model developed by Novikov and Thorne (NT model) [8] by inputting external parameters distance D, mass M, and inclination i. The need for D is due to the required conversion from thermal spectrum to luminosity values. i is also required because the deviation in inclination will result in the deviation in the thermal disk spectra. This model will output the spin parameter a and mass accretion rate $\dot{M}$. We should also wait for black hole binaries to enter a "thermal-dominant state" to obtain its thermal spectrum because it will become closer to a pure thermal disk.

However, there are also limitations in the continuum fitting approach. Due to the difficulty and lack of precision in the determination of M and i for black holes in the active galactic nucleus, the continuum fitting method will be more applicable to black hole X-ray binaries. During the model fitting process, we have assumed that the spin of the black hole and its orbit are in the same plane. If, however, these two factors are not in the same plane, the disk will be warped rather be a thin disk [3], which will affect the estimation of spin. There are also assumptions in the NT model that might not correspond to the real scenario. The NT model assumes that there is no torque at ISCO, which is not true because the magnetic fields can connect the matter at ISCO with the rotating matter in the disk.

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Black Holes

Mass and spin measurement approaches

Zihan Guo 1/25/2023

Black Hole Properties

- No-hair theorem: mass, spin, charge
- Mass
  - required for measuring spin and other phenomenon
- Spin
  - accreting matter converting into electromagnetic radiation
  - formation and growth

Mass Measurement

Black Holes in Binary Systems

- $M = \dfrac{m_{BH}{}^3 \sin^3 i}{(m_\star + m_{BH})^2} = \dfrac{P v_{obs}{}^3}{2\pi G}$
- v_{obs} : redshifted / blueshifted spectrum; plot observables
- P : obtained through plot

Mass Measurement

Black Holes in Binary Systems

- $m_{BH} = \frac{P{v_{obs}}^3}{2\pi G} \frac{q(1+q)^2}{\sin^3 i}$
- $q = \frac{m_{BH}}{m_\star}$
- i : inclination
    - modeling with X-ray emissions and motion of the companion star
    - ellipsoidal modulations (Masuda & Hotokezaka 2019)
        - determine q and i

Mass Measurement
Black Holes in Binary Systems

- Kepler's Third Law of Motion: $P^2 \propto a^3$
- $m_\star + m_{BH} = \dfrac{4\pi^2 {a_{obs}}^3}{P^2 \sin^3 i}$
- $m_\star$
  - asteroseismic scaling relations (T_{eff}, ν_{max}, $\Delta\nu$) (Kjeldsen & Bedding 1995)
  - mass-luminosity relation

Mass Measurement
Binary Black Holes

- chirp mass (Abbot et al. 2016)
  - gravitational wave frequency
  - mass ratio: inspiral
  - total mass: final stage of coalescence

Mass Measurement

Lone Black Holes

- $M = \dfrac{4\pi^2 a^3}{GP^2}$
- observational data of orbiting stars
- fitting models
- only plausible for nearby black holes

Mass Measurement

Lone Black Holes

- $M_{\rm BH} = \dfrac{fRv^2}{G}$
- f : factor
- R : reverberation mapping
- v : redshifted / blueshifted emission lines
- uncertainties: do not know the structure of the broad line region

Spin Measurement

X-Ray Reflection

- spin parameter: $a = \dfrac{cJ}{GM^2}$
- accretion disk influenced by frame-dragging
- inner stable circular orbit (ISCO): higher spin, closer ISCO, more redshifted
- construct accretion disk models
- constraints

Spin Measurement
Gravitational Waves

- inspiral: magnitude and orientation of spin
- effective spin and precession spin - masses, spin parameters, direction of spin
- ring-down: estimate with mass ratios and effective spins from inspire
- advantage: “cleaner”
- constraints: low signal-to-noise ratios, limited frequency

Spin Measurement

Thermal Continuum Fitting

- temperature
- $T_{\text{eff}} = 3.3 \times 10^7 \eta^{-1/4} (\frac{M}{10M_\odot})^{-1/4} (\frac{L}{L_{Edd}})^{1/4} (\frac{r_g}{r})^{3/4} (1 - \sqrt{\frac{r_{isco}}{r}})^{1/4}$
- (Reynolds 2021)
- spin increases → ISCO decreases → effective temperature increases

Spin Measurement
Thermal Continuum Fitting

- Novikov and Thorne model (Novikov & Thorne 1973) with D, M, and i
  - D : thermal spectrum to luminosity values
  - i : affect thermal disk spectra
- output a and $\dot{M}$
- thermal-dominant state

Spin Measurement

Thermal Continuum Fitting

- constraints
  - M and D for AGN and X-ray binaries
  - spin and orbit in the same plane (if not, disk would be warped)
  - no torque at ISCO

Future

- gravitational waves
- primordial black holes?
- higher precision
- new techniques