Interpolación lineal

(1950, 151326) $x_0 = 1950$ $f(x_0) = 151326$

(1990, 249633) $x_1 = 1990$ $f(x_1) = 249633$

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$f_1(x) = 151326 + \frac{249633 - 151326}{1990 - 1950}(x - 1950)$$

$$f_1(x) = 151326 + 2457.675(x - 1950)$$

$$f_1(x) = 151326 + 2457.675x - 4792466.25$$

$$\boxed{f_1(x) = 2457.675x - 4641140.25}$$

Polinomio de grado 1

x : años
$f_1(x)$: núm. de habitantes

$$f_1(1985) = 2457.675(1985) - 4641140.25$$
$$= 237344.625 \approx 237345$$

$$f_1(2024) = 2457.675(2024) - 4641140.25$$
$$= 333193.95 \approx 333194$$

Interpolación Cuadrática

$(\underset{x_0}{1950}, \underset{f(x_0)}{151326})$

$(\underset{x_1}{1970}, \underset{f(x_1)}{203302})$

$(\underset{x_2}{1990}, \underset{f(x_2)}{249633})$

$$f_2(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$b_0 = f(x_0) = 151326$$

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{203302-151326}{1970-1950} = 2598.8$$

Elcazar

Edgar

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} =$$

$$\frac{2316.55 - 2598.8}{1990 - 1950} = \frac{-282.25}{40} = -7.05625$$

Itzel

$$f[x_2, x_1] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{249633-203302}{1990-1970} = \frac{46331}{20}$$

$$= 2316.55$$

$$f_2(x) = 151326 + 2598.8(x-1950) + (-7.05625)(x-1950)(x-1970)$$

$$30259.3x - 7.05625x^2 - 32022918.375$$

$$-7.05625x^2 + 30259.3x - 32022918.375$$

La concentración de salida de un reactor se mide en distintos momentos durante un periodo de 24 horas:

t, h	0	1	5.5	6	10	12	14	16	18	20	24
$c, mg/L$	1	1.5	2.3	2.27	2.1	4	5	5.5	5	3	1.2

1.- Graficar los datos $\quad \overset{x_0}{(1,} \overset{f(x_0)}{1.5)}, \quad \overset{x_1}{(12,} \overset{f(x_1)}{4)}$

2.- Interpolación lineal

3.- Interpolación cuadrática $(1, 1.5), (12, 4), (24, 1.2)$

4.- Graficar ambas interpolaciones

Interpolación Lineal

$$b_0 = f(x_0) = 1.5$$

Daniel

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{4 - 1.5}{12 - 1} = \frac{5}{22} = 0.22\overline{72}$$

Edgar

$$f_1(x) = b_0 + b_1(x - x_0) = 1.5 + 0.2\overline{27}(x - 1)$$
$$= 1.5 + 0.2\overline{27}x - 0.2\overline{27} = 0.2\overline{27}x + 1.\overline{27}$$

Interpolación Cuadrática

$(1, 1.5) \quad (12, 4) \quad (24, 1.2)$

$$b_0 = f(x_0) = 1.5$$

$$b_1 = f[x_1, x_0] = 0.2272$$

Gleaza

$$b_2 = f[x_2, x_1, x_0] = \frac{-0.2333 - 0.2272}{24 - 1} = -0.02$$

Itzel

$$f[x_2, x_1] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1.2 - 4}{24 - 12} = -\frac{7}{30} = -0.2333$$

Edwin

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

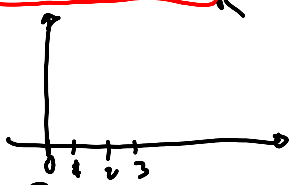
$$1.5 + 0.2272(x - 1) + (-0.02)(x - 1)(x - 12)$$

$$1.0698x - 0.383x^2 - 3.1252$$

$$f_2(x) = 0.2333x^2 - 2.8057x + 4.0724$$

$f_2(0) =$ ___

$f_2(1) =$ ___



Error

$$\sum_{i=0} |f_2(x_i) - c_i| = \boxed{}$$

Aproximado

$(0, f_2(0)), (1, f_2(1))$

Reales

t	0	1	5.5
C	1	1.5	2.3

$(0, 1) \quad (1, 1.5) \quad (5.5, 2.3)$

$(1, 1.5), (12, 4) \quad (16, 5.5)$

Error = 3. ___

$$b_3 = f[x_3, x_2, x_1, x_0]$$

$$f[x_3, x_2, x_1, x_0] = \frac{f[x_3, x_2, x_1] - \boxed{f[x_2, x_1, x_0]}}{x_3 - x_0}$$

$$f[x_3, x_2, x_1] = \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}$$

$$f[x_3, x_2] = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

o

$$b_4 = f[x_4, x_3, x_2, x_1, x_0]$$

$$f[x_4, x_3, x_2, x_1, x_0] = \frac{f[x_4, x_3, x_2, x_1] - f[x_3, x_2, x_1, x_0]}{x_4 - x_0}$$

$$f[x_4, x_3, x_2, x_1] = \frac{f[x_4, x_3] - f[x_3 - x_2] - f[x_2 - x_1]}{x_4 - x_1}$$

$$f[x_4, x_3, x_2, x_1] = \frac{f[x_4, x_3, x_2] - f[x_3, x_2, x_1]}{x_4 - x_1}$$

$$f[x_4, x_3, x_2] = \frac{f[x_4, x_3] - f[x_3, x_2]}{x_4 - x_2}$$

$$f[x_3, x_2, x_1] = \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}$$

$$f[x_4, x_3, x_2, x_1] = \frac{\frac{f[x_4, x_3] - f[x_3, x_2]}{x_4 - x_2} - \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1}}{x_4 - x_1}$$

Para las funciones dadas $f(x)$, sean $x_0 = 0$, $x_1 = 0.6$ y $x_2 = 0.9$. Construya polinomios de interpolación de grados uno y dos a lo máximo para aproximar $f(0.45)$, y calcule el error real.

$f(x) = \sqrt{1+x}$

$f_1(0.45)$

$f_2(0.45)$

$f(x_0)$

$f(x_1)$

$f(x_2)$

$$f_n(\overset{0.45}{x}) = \underbrace{b_0 + b_1(\overset{0.45}{x} - x_0) + b_2(\overset{0.45}{x} - x_0)(\overset{0.45}{x} - x_1)}_{f}$$

propiedad de que

$$f(x_k) = P(x_k) \qquad \text{para cada } k = 0, 1, \ldots, n.$$

Este polinomio está dado por

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

donde para cada $k = 0, 1, \ldots, n$.

$$L_{n,k}(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)}{(x_k - x_0)(x_k - x_1)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)}$$

$$= \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

Grad 1: Interpolación Lineal de Lagrange

$$P(x) = f(x_0)L_{n,0}(x) + f(x_1)L_{n,1}(x)$$

$(x_0, f(x_0))$ y $(x_1, f(x_1))$ se conocen

$$L_{n,0}(x) = \prod_{\substack{i=0 \\ i\neq 0}}^{1} \frac{(x-x_i)}{(x_0-x_i)} = \frac{(x-x_1)}{(x_0-x_1)}$$

$$L_{n,1}(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{1} \frac{(x-x_i)}{(x_1-x_i)} = \frac{(x-x_0)}{(x_1-x_0)}$$

$$\boxed{P(x) = f(x_0)\frac{(x-x_1)}{(x_0-x_1)} + f(x_1)\frac{(x-x_0)}{(x_1-x_0)}}$$

$(\underset{x_0}{1}, \underset{f(x_0)}{1.5}), (\underset{x_1}{12}, \underset{f(x_1)}{4})$

$$P(x) = 1.5\frac{(x-12)}{(1-12)} + 4\frac{(x-1)}{(12-1)}$$

$$P(x) = \frac{1.5x-18}{-11} + \frac{4x-4}{11}$$

$$P(x) = \frac{1}{11}(-1.5x+18+4x-4) = \frac{1}{11}(2.5x+14)$$

$$P(x) = 0.227x + 1.27$$

Polinomio de Lagrange de grado 1.

Polinomio de Lagrange de 2° grado

Formula:

$$L_{n,0}(x) = \prod_{\substack{i=0 \\ i\neq 0}}^{2} \frac{(x-x_i)}{(x_0-x_i)} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

Antonio

$$L_{n,1}(x) = \prod_{\substack{i=0 \\ i\neq 1}}^{2} \frac{(x-x_i)}{(x_k-x_i)} = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

Daniel

$$L_{n,2}(x) = \prod_{\substack{i=0 \\ i\neq 2}}^{2} \frac{(x-x_i)}{x_k-x_i} = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P(x) = f(x_0)L_{n,0}(x) + f(x_1)L_{n,1}(x) + f(x_2)L_{n,2}(x)$$

Puntos: $(1, 1.5), (12, 4), (24, 1.2)$

Edgar

$$P(x) = 1.5\frac{(x-12)(x-24)}{(1-12)(1-24)} + 4\frac{(x-1)(x-24)}{(12-1)(12-24)} + 1.2\frac{(x-1)(x-12)}{(24-1)(24-12)}$$

$$= -\frac{152x^2 - 3701x - 7836}{7590}$$

$$= -0.02x^2 + 0.487x + 1.032$$

Use los polinomios interpolantes de Lagrange de grado tres para aproximar lo siguiente:
f(8.4) si
f(8.1) = 16.944,
f(8.3) = 17.56492,
f(8.6) = 18.505,
f(8.7) = 18.8209

Uziel

$$L_{3,0}(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$-\frac{1412}{5}x^3 - \frac{180736}{25}x^2 - 61678.984x + 175372.0933$$

$$-16.\bar{6}x^3 + 426.\bar{1}x^2 - 3640.17x + 10350.1$$

Eleazar

$$L_{3,1}(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = 41.666x^3 - 1058.333x^2 + 8956.25x - 25251.75$$

$$= \prod_{\substack{i=0 \\ i\neq 1}}^{3} \frac{(x-x_i)}{(x_1-x_i)} =$$

Edwin

$$L_{3,2}(x) = \prod_{\substack{i=0 \\ i\neq 2}}^{3} \frac{(x-x_i)}{(x_2-x_i)} = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$= -66.666x^3 + 1673.33x^2 + 13994x + 38993.4$$

Edgar

$$L_{3,3}(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = 41.6x^3 - 1041.6x^2 + 8677.916x - 24090.75$$

Daniel

Polinomio:

$$-12591x^3 - 14457.87x^2 + 4.9899x - 649009.32$$

Rendimiento de un proceso productivo en función de la temperatura

En una planta química se sintetiza un producto que es utilizado posteriormente como conservante de productos enlatados. El rendimiento del proceso depende de la temperatura.

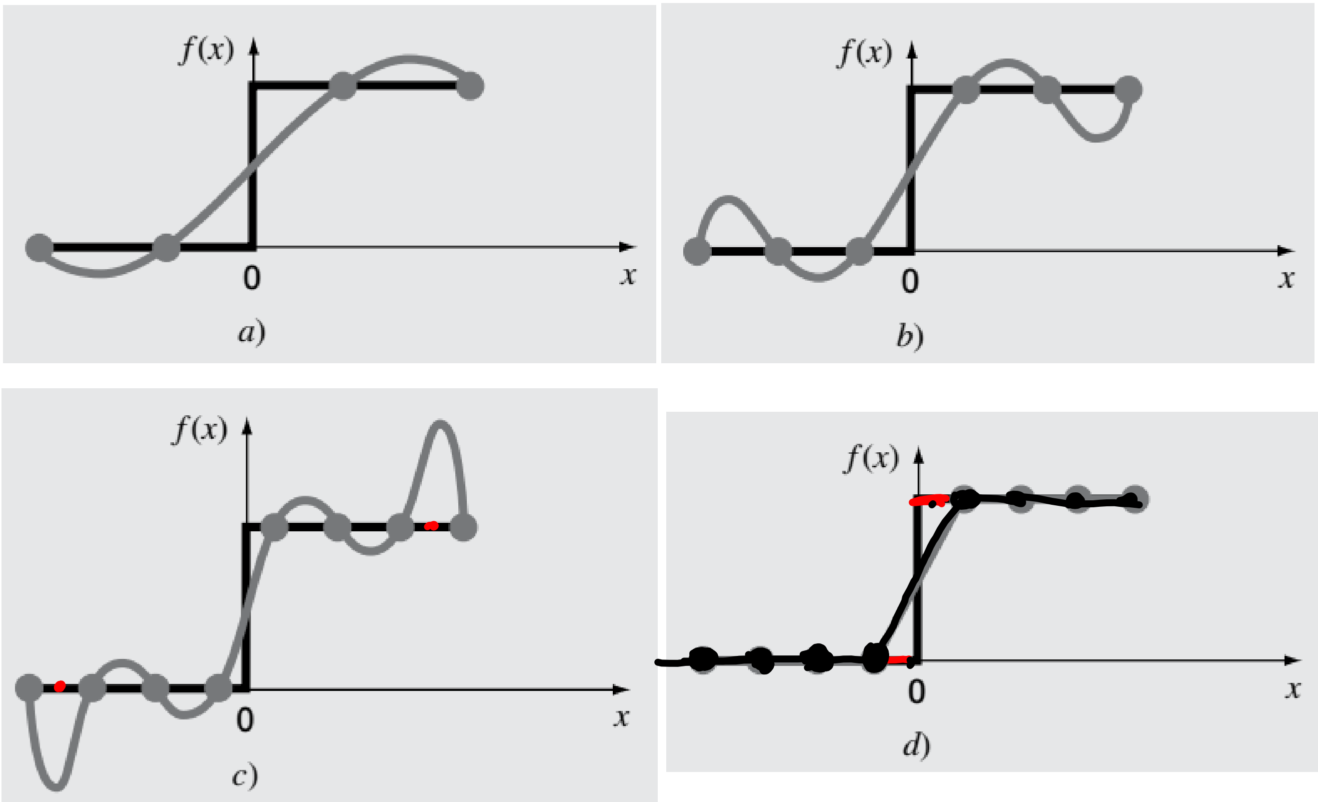
Se dispone de los siguientes datos

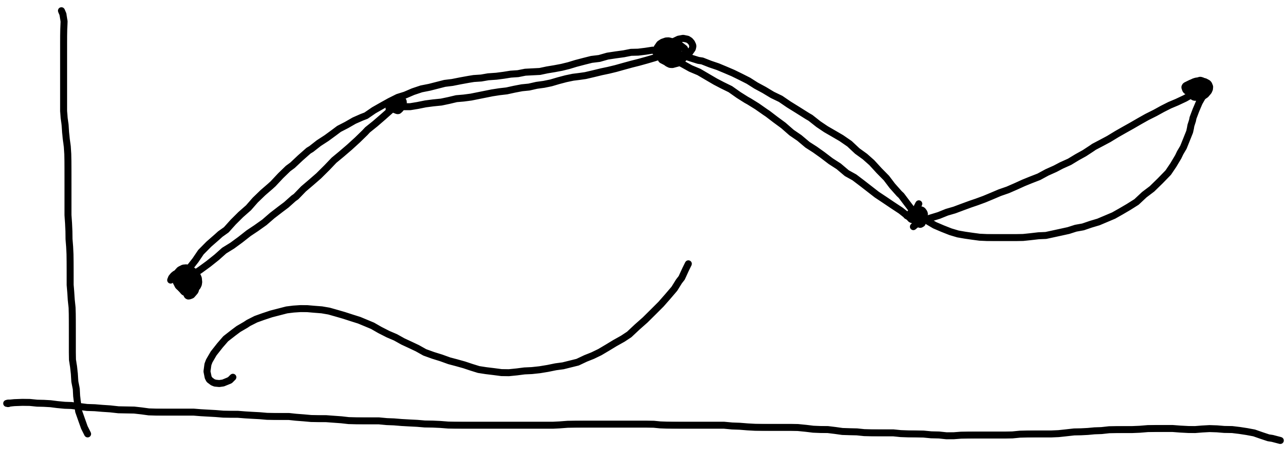
0 1 2 3 4 5 6

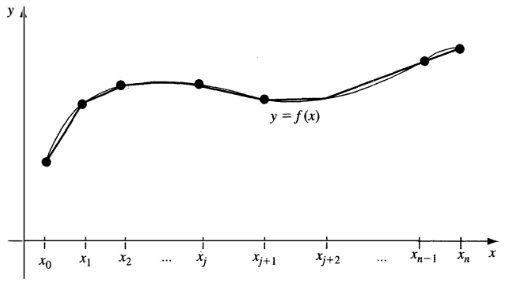
T (°C)	150	160	170	180	190	200	210
R (%)	35.5	37.8	43.6	45.7	47.3	50.1	51.2

Se considera un rendimiento óptimo el que va de 38.5 a 45, por lo que la planta trabaja a 175 °C. Si la temperatura de trabajo cae a 162 °C por una avería, ¿será el proceso satisfactorio hasta que sea reparada?

$162 \to 39.2011$ $x[6]$


f(x)
0
x
a)
b)
c)
d)





Dada una función f definida en $[a, b]$ y un conjunto de nodos $a = x_0 < x_1 < \cdots < x_n = b$ un **interpolante de trazador cúbico** S para f es una función que cumple con las condiciones siguientes:

a. $S(x)$ es un polinomio cúbico, denotado $S_j(x)$, en el subintervalo $[x_j, x_{j+1}]$ para cada $j = 0, 1, \ldots, n-1$;

b. $S(x_j) = f(x_j)$ para cada $j = 0, 1, \ldots, n$;

c. $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$ para cada $j = 0, 1, \ldots, n-2$;

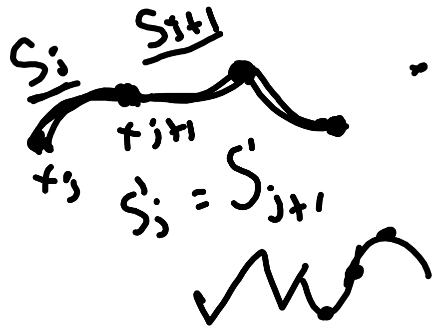
d. $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$ para cada $j = 0, 1, \ldots, n-2$;

e. $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$ para cada $j = 0, 1, \ldots, n-2$;

f. Una de las siguientes condiciones de frontera se satisface:

(i) $S''(x_0) = S''(x_n) = 0$ **(frontera libre o natural)**;

(ii) $S'(x_0) = f'(x_0)$ y $S'(x_n) = f'(x_n)$ **(frontera sujeta)**.



$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \ddots & & \vdots \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ \frac{3}{h_1}(a_2 - a_1) - \frac{3}{h_0}(a_1 - a_0) \\ \vdots \\ \frac{3}{h_{n-1}}(a_n - a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1} - a_{n-2}) \\ 0 \end{bmatrix} \quad \text{y} \quad \mathbf{x} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix}.$$

Trazador cúbico natural

Para construir el interpolante de trazador cúbico S de la función f, que se define en los números $x_0 < x_1 < \cdots < x_n$ y que satisface $S''(x_0) = S''(x_n) = 0$:

ENTRADA n; $x_0, x_1, \ldots, x_n$; $a_0 = f(x_0)$, $a_1 = f(x_1), \ldots, a_n = f(x_n)$.

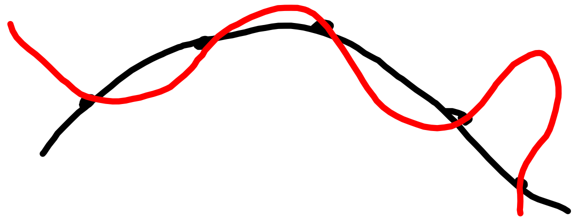
SALIDA a_j, b_j, c_j, d_j, para $j = 0, 1, \ldots, n-1$.
(*Nota*: $S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$ *para* $x_j \le x \le x_{j+1}$.)

Paso 1 Para $i = 0, 1, \ldots, n-1$ tome $h_i = x_{i+1} - x_i$.

Paso 2 Para $i = 1, 2, \ldots, n-1$ tome

$$\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1}).$$

Paso 3 Tome $l_0 = 1$;
$\mu_0 = 0$;
$z_0 = 0$.



Paso 4 Para $i = 1, 2, \ldots, n-1$
tome $l_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1}$;
$\mu_i = h_i / l_i$;
$z_i = (\alpha_i - h_{i-1}z_{i-1})/l_i$.

Paso 5 Tome $l_n = 1$;
$z_n = 0$;
$c_n = 0$.

Paso 6 Para $j = n-1, n-2, \ldots, 0$
tome $c_j = z_j - \mu_j c_{j+1}$;
$b_j = (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3$;
$d_j = (c_{j+1} - c_j)/(3h_j)$.

Paso 7 SALIDA $(a_j, b_j, c_j, d_j$, para $j = 0, 1, \ldots, n-1)$;
PARAR.

	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20
x	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	10.5	11.3	11.6	12.0	12.6	13.0	13.3
$f(x)$	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	1.4	0.9	0.7	0.6	0.5	0.4	0.25