Regra de Cramer e Escalonamento - Tarefa Bónus 1

1) a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$ $D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - (1) = 5$

$D_X = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - (3) = \boxed{3}$ $\quad x = \frac{D_X}{D} = \frac{3}{5}$

$D_Y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = \boxed{-4}$ $\quad y = \frac{D_Y}{D} = -\frac{4}{5}$

$V = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = -10 - (13) = \boxed{-23}$

$0 + 9 + 4$

$0 - 12 + 2$

$D_X = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} \begin{matrix} 1 & -1 \\ -1 & 0 \\ 7 & 1 \end{matrix} = -22 - (1) = \boxed{-23}$

$0 + 3 - 2$

$-0 - 21 - 1$

$x = \frac{D_X}{D} = \frac{-23}{-23} = 1$

$D_Y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} \begin{matrix} 3 & 1 \\ 2 & -1 \\ 4 & 7 \end{matrix} = 32 - (55) = \boxed{-23}$

$-4 + 63 - 4$

$6 + 12 + 14$

$y = \frac{D_Y}{D} = \frac{-23}{-23} = 1$

$D_Z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = 6 - (-17) = \boxed{23}$

$0 - 3 - 14$

$0 + 4 + 2$

$z = \frac{D_Z}{D} = \frac{23}{-23} = -1$

$V = \{ (1, 1, -1) \}$

2) $\begin{cases} 3x+4y-z=1 \\ 4x+5y+2z=12 \\ x-2y+3z=8 \end{cases}$ $\cdot(-1)$

$3x+4y-z=1$
$x-2y+3z=8$ +

$4x+2y+2z=9$

$4x+2y+2z=9$
$-4x-5y-2z=-12$

$-3y=-3\ (-1)$
$3y=3$
$y=\frac{3}{3}$
$y=1$

(A)

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3) $\begin{cases} x+2y+z=1 \\ 3x+y-11z=-2 \\ 2x+3y-z=1 \end{cases}$

$\begin{pmatrix} 1 & 2 & 1 & | & 1 \\ 3 & 1 & -11 & | & -2 \\ 2 & 3 & -1 & | & 1 \end{pmatrix}$ $\begin{matrix} \cdot(-3)+ \\ \\ \cdot(-2)+ \end{matrix}$ $\sim$ $\begin{pmatrix} 0 & -5 & -14 & | & -5 \\ 0 & -1 & -3 & | & -1 \end{pmatrix}$ $\cdot(-5)+$

$-y=-1\ (-1)$
$y=1$

$\begin{pmatrix} 0 & 0 & 1 & | & 0 \end{pmatrix}$ $z=0$

$x+2=1$
$x=1-2$
$x=-1$

$x=A$
$y=B$
$z=C$

$A+B+C$
$-1+1+0=0$

(C)

4) $\begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases}$

$$\begin{pmatrix} 1 & 2 & -3 & 29 \\ 1 & 2 & 2 & 4 \\ 1 & -1 & -2 & 8 \end{pmatrix} \sim \begin{pmatrix} 0 & 3 & -1 & 21 \\ 0 & 3 & 4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 15 & 0 & 80 \end{pmatrix}$$

$15y = 80$

$y = \frac{80}{15}$

$y \cong 5$

$15 - 1z = 21$

$-1z = 21 - 15$

$z = \frac{6}{-1}$

$z = -6$

$x + y + z$

$1 + 5 + (-6) = 0$ (A)

$x - 5 - 2.(-6) = 8$

$x - 5 + 12 = 8$

$x = 8 + 5 - 12$

$x = 1$

5) $\begin{cases} 2x+y=5 \\ 2y+z=3 \\ 3x+2y+z=7 \end{cases}$

$3x+2y-2y+z-z=7-3$

$3x=4$

$\boxed{x=\frac{4}{3}}$

$2x+y=5$

$y=5-\frac{8}{3}$

$\boxed{y=\frac{7}{3}}$

(D) $4:\frac{7}{3}$ e $\frac{5}{3}$

Conclusão por eliminatória.

6) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$ $\quad D = \begin{vmatrix} 1x & 0y & 0z \\ 2x & 1y & 0z \\ -1x & 2y & 2z \end{vmatrix} = \begin{vmatrix} 3 \\ 7 \\ -1 \end{vmatrix}$

$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} = 1.1.2 = 2$

$x = \frac{DX}{D} = \frac{6}{2} = 3$

$DX = \begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} = 3.1.2 = 6$

$y = \frac{DY}{D} = \frac{2}{2} = 1$

$0+0+12$

$DY = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & 7 \\ -1 & -1 \end{matrix} = 14-12 = 2$

$z = \frac{DZ}{D} = \frac{0}{2} = \boxed{0}$

$14+0+0$

$-3+14+0$

$DZ = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & -1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{matrix} = 11-11 = 0$

$z = 0$

(E)

$-1+0+12$

Regra de Cramer e Escalonamento - Tarefa Básica 2

1) $\begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$

$$D = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 3 & -1 \\ 1 & 0 & -5 \end{vmatrix} \quad \begin{matrix} 2 & -1 \\ 1 & 3 \\ 1 & 0 \end{matrix}$$

$-9+0+5$

$-30+1+0$

$D = -32 + 4 = -28$

$$D_x = \begin{vmatrix} -5 & -1 & -3 \\ 11 & 3 & -1 \\ 3 & 0 & -5 \end{vmatrix} \quad \begin{matrix} -5 & -1 \\ 11 & 3 \\ 3 & 0 \end{matrix}$$

$-27+0+55$

$75+3+0$

$D_x = 78 - 28 = \frac{50}{-25} = -2 = x$

$$D_y = \begin{vmatrix} 2 & -5 & -3 \\ 1 & 11 & -1 \\ 1 & 3 & -5 \end{vmatrix} \quad \begin{matrix} 2 & -5 \\ 1 & 11 \\ 1 & 3 \end{matrix}$$

$-33-6+25$

$-110+5-9$

$D_y = -114 - (-14) = \frac{-100}{-25} = 4 = y$

$$D_z = \begin{vmatrix} 2 & -1 & -5 \\ 1 & 3 & 11 \\ 1 & 0 & 3 \end{vmatrix} \quad \begin{matrix} 2 & -1 \\ 1 & 3 \\ 1 & 0 \end{matrix}$$

$-15+0-3$

$18-11+0$

$D_z = 7 + 18 = \frac{25}{-25} = -1 = z$

$$V = \{(-2, 4, -1)\}$$

2) $\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$

$3z + 1{,}5z + z = 11$

$5{,}5z = 11$

$z = 2$

$2y = 3z$

$2y = 3 \cdot 2$

$2y = 6$

$y = \frac{6}{2}$

$y = 3$

$x = 2y$

$x = 2 \cdot 3$

$x = 6$

$x + 2y + 3z$

$6 + 6 + 6 = 18$ (B)

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3) $\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ 0 & 6 & 3 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & -1 \\ 0 & 6 \end{matrix}$

$0 - 12 + 6$

$2 - 1 = 9 + 6 = 15$

$0 \quad 6$

$-3 + 0 + 12$

$0 + 6 - 24$

$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 6 & -12 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & -1 \\ 0 & 6 \end{matrix}$

$12 + 0 + 0$

$= 12 + 18 = \frac{30}{15} = 2 = z$

$z = 2$ (D)

11

4) $A+B+C=68$ | $B+\frac{C}{5}=A$ | $C+\frac{A}{5}=3B$

$5A-5B-C=0$ | $A-15B+5C=0$

$-5+15+25$

$$\begin{cases} A+B+C=68 \\ 5A-5B-C=0 \\ A-15B+5C=0 \end{cases}$$

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 5 & -5 & -1 \\ 1 & -15 & 5 \end{vmatrix} \quad \begin{matrix} 1 & 1 \\ 5 & -5 \\ 1 & -15 \end{matrix}$$

$-25-1-75$

$A=ALi$

$B=BiA$

$C=CACO$

$D=-25-1-75+5-15-25$

$D=-136$

$0+1020+0$

$$DA=\begin{vmatrix} 68 & 1 & 1 \\ 0 & -5 & -1 \\ 0 & -15 & 5 \end{vmatrix} \quad \begin{matrix} 68 & 1 \\ 0 & -5 \\ 0 & -15 \end{matrix}$$

$DA=-1700-1020=\frac{-2720}{-136}=20=A$

$-1700+0+0$

$-340+0+0$

$$DC=\begin{vmatrix} 1 & 1 & 68 \\ 5 & -5 & 0 \\ 1 & -15 & 0 \end{vmatrix} \quad \begin{matrix} 1 & 1 \\ 5 & -5 \\ 1 & -15 \end{matrix}$$

$DC=-5100+340=\frac{-4760}{-136}=35=C$

$0+0-5100$

ALi tem 15R$ o menor que CACO

$35-20=15$ A

5) $A = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix}$ $X = \begin{vmatrix} x \\ y \\ z \end{vmatrix}$ $D = \begin{vmatrix} 0x & 3y & 4z \\ 1x & 0y & 5z \\ 2x & 1y & 0z \end{vmatrix}$

$$D = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 3 & \\ 0 & \\ 1 & \\ 2 & 1 \end{matrix} = 34 - 0 = 34$$

$0 + 30 + 4$

$$DX = \begin{vmatrix} 134 & 3 & 4 \\ 115 & 0 & 5 \\ 48 & 1 & 0 \end{vmatrix} \begin{matrix} 134 & 3 \\ 115 & 0 \\ 48 & 1 \end{matrix} = 1180 - 670 = \frac{510}{34} = 15 = X$$

$0 + 670 + 0$

$0 + 720 + 460$

$$DY = \begin{vmatrix} 0 & 134 & 4 \\ 1 & 115 & 5 \\ 2 & 48 & 0 \end{vmatrix} \begin{matrix} 0 & 134 \\ 1 & 115 \\ 2 & 48 \end{matrix} = 1532 - 920 = \frac{612}{34} = 18 = Y$$

$920 + 0 + 0$

$0 + 1340 + 192$

$$DZ = \begin{vmatrix} 0 & 3 & 134 \\ 1 & 0 & 115 \\ 2 & 1 & 48 \end{vmatrix} \begin{matrix} 0 & 3 \\ 1 & 0 \\ 2 & 1 \end{matrix} = 824 - 144 = \frac{680}{34} = 20 = Z$$

$0 + 0 + 144$

$0 + 690 + 134$

total a ser Pago $= X + Y + Z$

$15 + 18 + 20 = 53$

(A)