Tarefa Básica - Matriz Inversa

01 - $A = B^{-1}$ $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$

$B \cdot B^{-1} = I$

$$\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\begin{cases} 3x - 5 = 1 \\ xy + 10 = 0 \end{cases}$ $\begin{cases} 3 - 3 = 0 \\ y + 6 = 1 \end{cases}$

$3x - 5 = 1$
$3x = 6$
$\boxed{x = 2}$

$3x - 5 = 1$
$(3.2) - 5 = 1$
Verdadeiro

$2.y + 10 = 0$
$2y + 10 = 0$
$y = \frac{-10}{2}$
$y = -5$

$\boxed{\begin{matrix} x + y = ? \\ 2 + (-5) = -3 \end{matrix}}$ C

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02 -
$$A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix} = 0$$

Se Det A = 0, não tem inversa.
$3 + k^2 - (1 + 3k) = 0$
$k^2 - 3k + 2 = 0$
$\Delta = 9 - 8 = 1$
$k = \frac{3 \pm 1}{2}$ $k1 = 2$ $k2 = 1$

$\boxed{C: 1 \text{ e } 2}$

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03 -
$B = A^{-1}$
$B = ?$

$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

$B = A^{-1} = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \cdot \frac{1}{2}$

det $A = 12 - 10$
det $A = 2$

$B = \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$ C

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04 -
tornar inversível.
$\neq 0$

$$\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$$

$\begin{pmatrix} 1 & 2 \\ 1 & x \end{pmatrix} = x - 2 \neq 0$
$x \neq 2$

$\begin{pmatrix} x & 1 \\ 3 & 1 \end{pmatrix} = x - 3 \neq 0$
$x \neq 3$

$\{x \neq 2 \text{ e } x \neq 3\}$ A

05 - $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} x & y & z \\ a & b & c \\ d & E & F \end{bmatrix}$

$+\begin{cases} -x - a + 2d = 1 \\ 2x + 1a - 2d = 0 \\ x + a - d = 0 \end{cases}$ $\begin{cases} -y - b + 2c = 0 \\ 2y + b - 2c = 1 \\ y + b - c = 0 \end{cases}$ $\begin{cases} -z - c + 2F = 0 \\ 2z + c - 2F = 0 \\ z + c - F = 1 \end{cases}$

$\boxed{x = 1}$ $\boxed{y = 1}$ $\boxed{z = 0}$

$2 + a - 2d = 0$
$a = 2d - 2$
$\boxed{a = 0}$

$2 + b - 2c = 1$
$b = 2c - 1$
$\boxed{b = -1}$

$c - 2F = 0$
$c = 2F$
$\boxed{c = 2}$

$1 + 2d - 2 - d = 0$
$-1 + d = 0$
$\boxed{d = 1}$

$1 + 2c - 1 - c = 0$
$\boxed{c = 0}$

$2F - F = 1$
$\boxed{F = 1}$

$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ $A + A^{-1} = ?$

$A + A^{-1} = \begin{bmatrix} -1+1 & -1+1 & 2+0 \\ 2+0 & 1-1 & -2+2 \\ 1+1 & 1+0 & -1+1 \end{bmatrix}$ $A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

B

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06- $(x.A)^t = B$ $\quad ((x.A)^t)^t = B$ transpor em

$x.A = B^t$ ambos os lados

isolar x $\quad x.A = B^t$

$x.A.A^{-1} = B^t.A^{-1}$

$x.1 = B^t.A^{-1}$

$x = B^t.A^{-1}$

(B)

07- $A = ?$ $\quad A.B = C$ $\quad C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ $B = \begin{bmatrix} x \\ y \end{bmatrix}$

$A^{-1} = ?$ $\quad A = \dfrac{C}{B}$

$$\begin{bmatrix} \frac{4x}{x} + \frac{5y}{y} \\ \frac{5x}{x} + \frac{6y}{y} \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} = A \qquad A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$$

(D)

08- $A = \begin{pmatrix} 2 & k \\ -2 & 1 \end{pmatrix}$ $\quad A^{-1} = \begin{pmatrix} -1 & k \\ -2 & -2 \end{pmatrix}$ $\quad K = -2$

Se $K = -2$, $\det A = \det A^{-1}$

$\det A = 2 - 4$ $\quad \det A^{-1} = 2 - 4$

$\det A = -2$ $\quad \det A^{-1} = -2$

(B)

09-

A) $(a+b)x(a-b)$

$\boxed{a^2 - ab + ba - b^2}$

B) Se $A.B = B.A$ então:

$(A+B)^2 = A^2 + 2.A.B + B^2$

C) $\det(A^{-1}) = \dfrac{1}{\det A}$

$\dfrac{\det(A^{-1})}{1} = \det A$

$\boxed{1 = \dfrac{\det A}{\det(A^{-1})}}$

D) $B = A^{-1}$

$$\boxed{\det B = \frac{1}{\det A}}$$ ← relação