1 Problem Proposal

Modelling Canadian Heavy Crude Congestion pricing using Capacitated Transport networks Problem Proposal:

Alberta produces ∼4MM bbls/d (barrels per day) of crude oil of which ∼3 MM bbls/d is considered "heavy oil". Majority or 2/3 of Canadian heavy oil is consumed in PADD II and 1/3 in USGC refining center. The PADD III (also known as USGC - United States Coast Guard, see Figure 1) is considered a marginal consumption point for Canadian crude where the "last barrel" or marginal barrel is consumed. Canadian crude can reach the USGC through number of pipe options: Enbridge mainline, Express, and Keystone, or rail from either Edmonton or Hardisty. The cost of transport of the last barrel from Alberta to USGC is going to set the price of Canadian heavy benchmark (WCS) in Alberta.

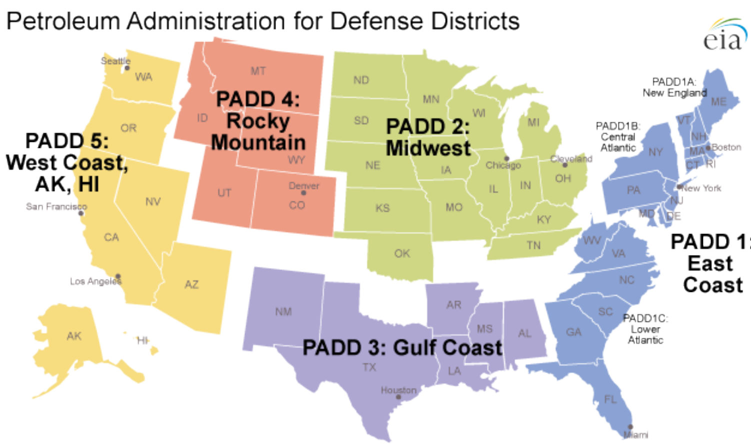

Figure 1: Strategic and Military Pipeline for Transmission of Petroleum map. Available at eia.gov

Pipe is considered a cheapest mode of transport of Canadian crude to the demand markets. The total offtake capacity from Alberta is on average less than the total available production. This leads often to the requirement of "call on rail" (the amount of rail capacity needed to clear an oversupply of barrels in the WCSB, see atbcapitalmarkets) or shut-ins (implementing a production cap lower than the available output of a specific site) to balance the market which in turn leads to significant price response. Prices are generally integrated if they respond to the same price shock. In reality looking at the USGC–Alberta spread pricing, we see a regular breakdown of the price relationship or "congestion pricing" (regulating demand by increasing prices without increasing supply). This is due to the fact that transient "sub-markets" can be form in a capacitated transport system.

The question is using stochastic transport optimization (following a similar method to [3] and [4]) can we model and answer the following questions:

- When there are documented disruptions in the transport system can we predict how large the congestion surcharge was and how did prices respond to the disruption?
- Can we predict the occurrence of congestion by perturbing input factors in the system?
- How does shape and connections in the transport network contribute to the propensity for frequency of the congestion and magnitude of congestion surcharge?

There is a fantastic work performed looking at the US gasoline market that can be used as a guide (where is the guide?) and starting point to look at the Canadian crude oil transport network. The majority of the data: (1) Pipe Tolls and Tariffs (2) Capacities (3) Supply and Demand factors are publicly available. (4) Pricing data and some of the historical flow data that is used for benchmarking and testing the models can be made available with some degree of modifications and can be anonymized.

2 I. Y. Zhu's paper notes

The competitive market is modeling with what we call the welfare-maximizing market allocation problem:

$$
\begin{aligned}
\max_{\vec{f},\vec{b}} \quad & \sum_{s\in\mathcal{S}} W_s(b_s) - \sum_{(i,j)\in\mathcal{E}} c_{ij} f_{ij} - \sum_{k\in\mathcal{K}} W_k(b_k) \\
\text{s.t.} \quad & -b_s + \sum_{i\in I(s)} f_{si} - \sum_{j\in O(s)} f_{sj} = 0, \qquad \forall s \in \mathcal{S}, \\
& b_k + \sum_{i\in I(k)} f_{ik} - \sum_{j\in O(k)} f_{kj} = 0, \qquad \forall k \in \mathcal{K}, \\
& 0 \le f_{ij} \le u_{ij}, \quad \forall (i,j) \in \mathcal{E}, \\
& b_s \ge 0, \quad \forall s \in \mathcal{S}, \\
& b_k \ge 0, \quad \forall k \in \mathcal{K}
\end{aligned}
$$

- $\mathcal{S}, \mathcal{K}$ denotes the nodes in a network $\mathcal{N}$ populated by consumers and producers, respectively.
- For a node $s \in \mathcal{S}$ ($k \in \mathcal{K}$), $W_s(b_s)$ ($W_k(b_k)$) is the welfare (production cost) for consuming (producing) b_s (b_k) units.
- Assume W_s is strictly concave, increasing, and differentiable. Assume W_k is convex, increasing and differentiable.
- $\mathcal{E}$ denotes the set of transportation links (i,j) from a node (i,j) (i.e., it is the edge set of a directed graph)
- f_{ij} denotes the flow from i to j on link $(i,j) \in \mathcal{E}$. $f_{i,j}$ is nonnegative, and bounded above by the capacity u_{ij}, with per-unit transportation cost $c_{i,j}$
- $I(j) = (n,j) \in \mathcal{E} | n \in \mathcal{N}$ The set of links that flow into the jth node from n.
- $O(i) = (i,n) \in \mathcal{E} | n \in \mathcal{N}$ The set of sets that flow out of the ith node to n.
- $\mathcal{P}(i,j)$ denotes the set of paths from i to j, and p_{ij}^q the cost of a path q which is the sum of costs of the links from i to j.

In words, we are maximizing the welfare of the consumers less the cost of transportation and less the cost of production subject to the flow balancing, i.e., all that is produced is consumed.

Definition 1 (Slater's condition)**.** For an optimization problem of the form

$$
\begin{aligned}
\min \quad & f_0(x) \\
\text{s.t.} \quad & f_i(x) \le 0, \qquad i = 1, \dots, m,, \\
& Ax = b
\end{aligned}
$$

If there exists an $x \in \operatorname{relint} \mathcal{D} = \cap_{i=0}^m \operatorname{dom}(f_i)$ such that

$$
f_i(x) < 0, \qquad i = 1, \dots, m, \qquad Ax = b,
$$

we say that the optimization problem satisfies *Slater's condition.* That is, the optimization problem has an interior point in the feasible region.

Theorem 1. *Given the following optimization problem*

$$
\begin{aligned}
& \textit{Optimize} \qquad f(\mathbf{x}) \\
& \textit{Subject to} \\
& \qquad g_i(\mathbf{x}) \le 0,\ h_j(\mathbf{x}) = 0.
\end{aligned}
$$

with associated Lagrangian

$$
L(\mathbf{x}, \mu, \lambda) = f(\mathbf{x}) + \mu^T \mathbf{g}(\mathbf{x}) + \lambda^T \mathbf{h}(\mathbf{x}).
$$

Let $x^\star$ and $(\lambda^\star, \mu^\star)$ be any primal and dual optimal points with zero duality gap. Then $x^\star$ is the minimizer of $L(x, \lambda^\star, \mu^\star)$ and hence its gradient vanishes at $x^\star$,

$$\nabla f_0(x^\star) + \sum_{i=1}^{m} \lambda_i^\star \nabla f_i(x^\star) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^\star) = 0.$$

In particular,

$$\begin{aligned} f_i(x^\star) &\leq 0, & i &= 1, \dots, m \\ h_i(x^\star) &= 0, & i &= 1 \dots, p \\ \lambda_i^\star &\geq 0, & i &= 1, \dots, m \\ \lambda_i^\star f_i(x^\star) &= 0, & i &= 1, \dots, m. \end{aligned}$$

These four equations together with

$$\nabla f_0(x^\star) + \sum_{i=1}^{m} \lambda_i^\star \nabla f_i(x^\star) + \sum_{i=1}^{p} \mu_i \nabla h_i(x^\star) = 0$$

are known as the Karush–Kuhn–Tucker *(KKT) conditions. For any convex optimization problem with differentiable objective and constraint functions, any points that satisfy the KKT conditions are primal and have zero duality gap. Further, if Slater's condition is satisfied, these conditions are necessary and sufficient for optimality. See [2, Ch. 5]*

3 Discussion with Nima

3.1 Data sources

There are several sources of data that we can use for this project:

- The US Energy Information Administration, `https://www.eia.gov/maps/`, contains maps that can be filtered to show the crude pipelines and an interactive map displaying all the refineries and other data.
- The Alberta Energy Regulator, here provides an annual energy report and includes the data sets used to generate the reports known as the ST98. Also see the statistical reports available here.
- Oil Sands Magazine `https://www.oilsandsmagazine.com/projects/crude-oil-liquids-pipelines` has a lot of data including maps that show the toll rates for oil and the production capacity at various sites.

3.2 Market characteristics

There are many miscellaneous market characteristics of note that are written here:

- Rail is the most expensive method of transport and costs \$17 per barrel. The total rail capacity is 300,000 barrels per day.
- WCS trades as a basis to WTI. That is, if WCS is \$40 and WTI \$50, it is often said that WCS is -\$10. That is, WCS is worth \$10 less than WTI. This basis is taken to include the difference in transportation costs from Alberta to Cushing, and a grade adjustment. The grade adjustment is said to be negligible. Further, note that the differential between Heavy and Light oil is about \$5 per barrel.
- Note that as a result of COVID-19, the demand for US oil dropped from 17mm barrels to 13mm barrels per day. Given that Canada only produces 4mm barrels per day, this was a substantial drop in demand.

- There are two notable sources of pipeline disruption. The most notable source of disruption are related to the keystone pipeline. In particular, Nima recalls a disruption in 2019 in possibly November or December, and a disruption in 2017 in January.
- Traders use what is known as a calendar month average. I don't think this will be too important but remember this when considering how traders might work.
- The oil price spreads generally follow a mean reverting process, so an OU process is going to be a good candidate.

4 SEM with constraints

$$
\begin{aligned}
z(\beta) := \text{minimize} \quad & \sum_{s\in\mathcal{S}} \alpha_s \\
\text{subject to} \quad & \lambda_s^t = \eta^t + \rho_s + \epsilon_s^t + w_s^t, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \epsilon_s^t \geq -\alpha_s, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T} \\
& \epsilon_s^t \leq \alpha_s, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T} \\
& w_s^t \leq \psi^t M, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \sum_{t\in\mathcal{T}} \psi^t \leq \lfloor \beta T \rfloor \\
& w_s^t \leq \pi_s^t M, \quad \forall s\in\mathcal{S}, t\in\mathcal{T} \\
& \epsilon_s^t + (1-\pi_s^t)M \geq \alpha_s, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \sum_{s} \gamma_s^t \geq \psi^t, \forall t\in\mathcal{T} \\
& \epsilon_s^t \leq -\alpha_s + (1-\gamma_s^t)M, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T} \\
& \gamma_s^t, \psi^t, \pi_s^t \in \{0,1\}, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T} \\
& w_s^t \geq 0 \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}
\end{aligned}
$$

Note that under the change of variables $\overline{w}_s^t := w_s^t + \epsilon_s^t - \alpha_s$, this can be simplified to

$$
\begin{aligned}
z(\beta) := \text{minimize} \quad & \sum_{s\in\mathcal{S}} \alpha_s \\
\text{subject to} \quad & \lambda_s^t = \eta^t + \rho_s + \alpha_s + \overline{w}_s^t, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \overline{w}_s^t \geq -2\alpha_s, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \overline{w}_s^t \leq \psi^t M, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \sum_{t\in\mathcal{T}} \psi^t \leq \lfloor \beta T \rfloor, \\
& \overline{w}_s^t \leq -2\alpha_s + (1-\gamma_s^t)M, \qquad \forall s\in\mathcal{S}, t\in\mathcal{T}, \\
& \sum_{s} \gamma_s^t \geq \psi^t, \qquad \forall t\in\mathcal{T}, \\
& \gamma_s^t, \psi^t \in \{0,1\}, \qquad \forall t\in\mathcal{T}.
\end{aligned}
$$

Given optimal values of $\overline{w}_s^t$ we can return to the original w_s^t and ϵ_s^t as follows: If $\overline{w}_s^t \geq 0$, then $w_s^t = \overline{w}_s^t$ and $\epsilon_s^t = \alpha_s$. If $\overline{w}_s^t < 0$, then $w_s^t = 0$ and $\epsilon_s^t = \overline{w}_s^t + \alpha_s$. Note that for larger data sets, we may need to use an additional set of constraints to reduce the scope of the problem. See [1, pp. 23–24] for details.

4.1 SEM notes

This model takes as an input a set of spatial prices $\boldsymbol{\lambda} = \{\lambda_s^t\}_{s\in\mathcal{S},t\in\mathcal{T}}$ and returns an estimate of congestion over a given time horizon. The variable η^t captures a node-invariant underlying trend, ρ_s, ϵ_s^t, and α_s capture time-invariant neutral bands. All the remaining price variation is captured by the variables w_s^t representing surcharges. We represent the constraints on $w_s^t by \mathcal{W} \subseteq \mathbb{R}^+$. That is, $\mathcal{W}$ restricts the domain of w_s^t.

If prices differences are perfectly constant , theoptimal objective value will be zero and the prices λ_s^t can be explained entirely by the node-invariant η^t and the time-invariant ρ_s. Consider perturbing the λ_s^t with Ornstein–Uhlenbeck stochastic processes to avoid this. Similarly, we can use this as a test that the optimization routine is working.

The parameters

$$w_s^t \leq \psi^t M, \sum_{t\in\mathcal{T}} \psi^t \leq \lfloor \beta T \rfloor, \psi^t \in \{0,1\}$$

represent time constraints. The parameter ψ^t is an indicator variable for when the congestion surcharge is free ($\psi^t = 1$) or fixed to zero ($\psi^t = 0$). The parameter M represents a large value such that w_s^t will never reach its upper bound when $\psi^t = 1$. Finally β is a fraction of time periods for which the underlying network is congested. The remaining $\lceil (1-\beta)T \rceil$ periods with no congestion (and hence $\psi^t = 0$) are used to fit ρ_s so that we can precisely estimate w_s^t when $\psi^t = 1$. The goal of these is to ensure that η^t, (resp. w_s^t) are the largest (resp. smallest) possible value out of the set of optimal solutions.

References

[1] John R. Birge, Timothy C. Y. Chan, J. Michael Pavlin, and Ian Yihang Zhu, *Spatial price integration in commodity markets with capacitated transportation networks*, Available at SSRN: https://ssrn.com/abstract=3544530. 4

[2] Stephen P. Boyd, *Convex optimization*, Cambridge University Press, 2009, available online at: https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf. 3

[3] Michael Pavlin, Ian Yihang Zhu, Timothy C. Y. Chan, and John R. Birge, *Price integration in competitive markets with capacitated transporation networks*, Commodity and Energy Markets Association Annual Meeting, 2019, https://www.dropbox.com/s/afjccbi7827mxe1/Pavlin2019.pdf?dl=0. 1

[4] Ian Yihang Zhu, *Spatial price integration in competitive markets with capacitated transporatation networks*, mathesis, University of Toronto, 2018, https://tspace.library.utoronto.ca/bitstream/1807/91504/3/Zhu_Ian_Yihang_201811_MAS_thesis.pdf. 1