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Indice

1 Problemas

1.1 Algoritmo Hungaro

```
// Resolve o problema de assignment (matriz n x n)
// Colocar os valores da matriz em 'a' (pode < 0)
// assignment() retorna um par com o valor do
// assignment minimo, e a coluna escolhida por cada linha
//
// O(n^3)
// 64c53e

template<typename T> struct hungarian {
    int n;
    vector<vector<T>> a;
    vector<T> u, v;
    vector<int> p, way;
    T inf;

    hungarian(int n_) : n(n_), u(n+1), v(n+1), p(n+1),
       way(n+1) {
        a = vector<vector<T>>(n, vector<T>(n));
        inf = numeric_limits<T>::max();
    }
    pair<T, vector<int>> assignment() {
        for (int i = 1; i <= n; i++) {
            p[0] = i;
            int j0 = 0;
            vector<T> minv(n+1, inf);
            vector<int> used(n+1, 0);
            do {
                used[j0] = true;
                int i0 = p[j0], j1 = -1;
                T delta = inf;
                for (int j = 1; j <= n; j++) if (!used[j]) {
                    T cur = a[i0-1][j-1] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j]
                        = j0;
                    if (minv[j] < delta) delta = minv[j], j1
                        = j;
                }
                for (int j = 0; j <= n; j++)
                    if (used[j]) u[p[j]] += delta, v[j] -=
                        delta;
                    else minv[j] -= delta;
                j0 = j1;
            } while (p[j0] != 0);
            do {
                int j1 = way[j0];
                p[j0] = p[j1];
                j0 = j1;
            } while (j0);
        }
        vector<int> ans(n);
        for (int j = 1; j <= n; j++) ans[p[j]-1] = j-1;
        return make_pair(-v[0], ans);
    }
};
```

1.2 Algoritmo MO - queries em caminhos de arvore

```
// Problema que resolve: https://www.spoj.com/problems/COT2/
//
// Complexidade sendo c = O(update) e SQ = sqrt(n):
// O((n + q) * sqrt(n) * c)
// 395329
```

```
const int MAX = 40010, SQ = 400;

vector<int> g[MAX];

namespace LCA { ... }

int in[MAX], out[MAX], vtx[2 * MAX];
bool on[MAX];

int dif, freq[MAX];
vector<int> w;

void dfs(int v, int p, int &t) {
    vtx[t] = v, in[v] = t++;
    for (int u : g[v]) if (u != p) {
        dfs(u, v, t);
    }
    vtx[t] = v, out[v] = t++;
}

void update(int p) { // faca alteracoes aqui
    int v = vtx[p];
    if (not on[v]) { // insere vtx v
        dif += (freq[w[v]] == 0);
        freq[w[v]]++;
    }
    else { // retira o vertice v
        dif -= (freq[w[v]] == 1);
        freq[w[v]]--;
    }
    on[v] = not on[v];
}

vector<tuple<int, int, int>> build_queries(const
   vector<pair<int, int>>& q) {
    LCA::build(0);
    vector<tuple<int, int, int>> ret;
    for (auto [l, r] : q){
        if (in[r] < in[l]) swap(l, r);
        int p = LCA::lca(l, r);
        int init = (p == l) ? in[l] : out[l];
            ret.emplace_back(init, in[r], in[p]);
    }
    return ret;
}

vector<int> mo_tree(const vector<pair<int, int>>& vq){
    int t = 0;
    dfs(0, -1, t);

    auto q = build_queries(vq);

    vector<int> ord(q.size());
    iota(ord.begin(), ord.end(), 0);
    sort(ord.begin(), ord.end(), [&] (int l, int r) {
        int bl = get<0>(q[l]) / SQ, br = get<0>(q[r]) / SQ;
        if (bl != br) return bl < br;
        else if (bl % 2 == 1) return get<1>(q[l]) <
           get<1>(q[r]);
        else return get<1>(q[l]) > get<1>(q[r]);
    });

    memset(freq, 0, sizeof freq);
    dif = 0;

    vector<int> ret(q.size());
    int l = 0, r = -1;
    for (int i : ord) {
        auto [ql, qr, qp] = q[i];
        while (r < qr) update(++r);
        while (l > ql) update(--l);
        while (l < ql) update(l++);
        while (r > qr) update(r--);

        if (qp < l or qp > r) { // se LCA estah entre as
           pontas
            update(qp);
            ret[i] = dif;
            update(qp);
        }
        else ret[i] = dif;
    }
    return ret;
```

```
}
```

1.3 Angle Range Intersection

```
// Computa intersecao de angulos
// Os angulos (arcos) precisam ter comprimeiro < pi
// (caso contrario a intersecao eh estranha)
//
// Tudo O(1)
// 5e1c85

struct angle_range {
    static constexpr ld ALL = 1e9, NIL = -1e9;
    ld l, r;
    angle_range() : l(ALL), r(ALL) {}
    angle_range(ld l_, ld r_) : l(l_), r(r_) { fix(l),
       fix(r); }

    void fix(ld& theta) {
        if (theta == ALL or theta == NIL) return;
        if (theta > 2*pi) theta -= 2*pi;
        if (theta < 0) theta += 2*pi;
    }
    bool empty() { return l == NIL; }
    bool contains(ld q) {
        fix(q);
        if (l == ALL) return true;
        if (l == NIL) return false;
        if (l < r) return l < q and q < r;
        return q > l or q < r;
    }
    friend angle_range operator &(angle_range p, angle_range
       q) {
        if (p.l == ALL or q.l == NIL) return q;
        if (q.l == ALL or p.l == NIL) return p;
        if (p.l > p.r and q.l > q.r) return {max(p.l, q.l) ,
           min(p.r, q.r)};
        if (q.l > q.r) swap(p.l, q.l), swap(p.r, q.r);
        if (p.l > p.r) {
            if (q.r > p.l) return {max(q.l, p.l) , q.r};
            else if (q.l < p.r) return {q.l, min(q.r, p.r)};
            return {NIL, NIL};
        }
        if (max(p.l, q.l) > min(p.r, q.r)) return {NIL, NIL};
        return {max(p.l, q.l), min(p.r, q.r)};
    }
};
```

1.4 Area da Uniao de Retangulos

```
// O(n log(n))
// bea565

namespace seg {
    pair<int, ll> seg[4*MAX];
    ll lazy[4*MAX], *v;
    int n;

    pair<int, ll> merge(pair<int, ll> l, pair<int, ll> r){
        if (l.second == r.second) return {l.first+r.first,
           l.second};
        else if (l.second < r.second) return l;
        else return r;
    }

    pair<int, ll> build(int p=1, int l=0, int r=n-1) {
        lazy[p] = 0;
        if (l == r) return seg[p] = {1, v[l]};
        int m = (l+r)/2;
        return seg[p] = merge(build(2*p, l, m), build(2*p+1,
           m+1, r));
    }
    void build(int n2, ll* v2) {
        n = n2, v = v2;
        build();
    }
    void prop(int p, int l, int r) {
        seg[p].second += lazy[p];
        if (l != r) lazy[2*p] += lazy[p], lazy[2*p+1] +=
           lazy[p];
```

```
        lazy[p] = 0;
    }
    pair<int, ll> query(int a, int b, int p=1, int l=0, int
       r=n-1) {
        prop(p, l, r);
        if (a <= l and r <= b) return seg[p];
        if (b < l or r < a) return {0, LINF};
        int m = (l+r)/2;
        return merge(query(a, b, 2*p, l, m), query(a, b,
           2*p+1, m+1, r));
    }
    pair<int, ll> update(int a, int b, int x, int p=1, int
       l=0, int r=n-1) {
        prop(p, l, r);
        if (a <= l and r <= b) {
            lazy[p] += x;
            prop(p, l, r);
            return seg[p];
        }
        if (b < l or r < a) return seg[p];
        int m = (l+r)/2;
        return seg[p] = merge(update(a, b, x, 2*p, l, m),
                update(a, b, x, 2*p+1, m+1, r));
    }
};

ll seg_vec[MAX];

ll area_sq(vector<pair<pair<int, int>, pair<int, int>>> &sq){
    vector<pair<pair<int, int>, pair<int, int>>> up;
    for (auto it : sq){
        int x1, y1, x2, y2;
        tie(x1, y1) = it.first;
        tie(x2, y2) = it.second;
        up.push_back({{x1+1,  1}, {y1, y2}});
        up.push_back({{x2+1, -1}, {y1, y2}});
    }
    sort(up.begin(), up.end());
    memset(seg_vec, 0, sizeof seg_vec);
    ll H_MAX = MAX;
    seg::build(H_MAX-1, seg_vec);
    auto it = up.begin();
    ll ans = 0;
    while (it != up.end()){
        ll L = (*it).first.first;
        while (it != up.end() && (*it).first.first == L){
            int x, inc, y1, y2;
            tie(x, inc) = it->first;
            tie(y1, y2) = it->second;
            seg::update(y1+1, y2, inc);
            it++;
        }
        if (it == up.end()) break;
        ll R = (*it).first.first;

        ll W = R-L;
        auto jt = seg::query(0, H_MAX-1);
        ll H = H_MAX - 1;
        if (jt.second == 0) H -= jt.first;
        ans += W*H;
    }
    return ans;
}
```

1.5 Area Maxima de Histograma

```
// Assume que todas as barras tem largura 1,
// e altura dada no vetor v
//
// O(n)
// e43846

ll area(vector<int> v) {
    ll ret = 0;
    stack<int> s;
    // valores iniciais pra dar tudo certo
    v.insert(v.begin(), -1);
    v.insert(v.end(), -1);
    s.push(0);

    for(int i = 0; i < (int) v.size(); i++) {
        while (v[s.top()] > v[i]) {
```

```
            ll h = v[s.top()]; s.pop();
            ret = max(ret, h * (i - s.top() - 1));
        }
        s.push(i);
    }

    return ret;
}
```

1.6 Binomial modular

```
// Computa C(n, k) mod m em O(m + log(m) log(n))
// = O(rapido)
// ed4344

ll divi[MAX];

ll expo(ll a, ll b, ll m) {
    if (!b) return 1;
    ll ans = expo(a*a%m, b/2, m);
    if (b%2) ans *= a;
    return ans%m;
}

ll inv(ll a, ll b){
    return 1<a ? b - inv(b%a,a)*b/a : 1;
}

template<typename T> tuple<T, T, T> ext_gcd(T a, T b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b%a, a);
    return {g, y - b/a*x, x};
}

template<typename T = ll> struct crt {
    T a, m;

    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator * (crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m/g*C.m;
        T ans = a + (x*(C.a-a)/g % (C.m/g))*m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

pair<ll, ll> divide_show(ll n, int p, int k, int pak) {
    if (n == 0) return {0, 1};
    ll blocos = n/pak, falta = n%pak;
    ll periodo = divi[pak], resto = divi[falta];
    ll r = expo(periodo, blocos, pak)*resto%pak;

    auto rec = divide_show(n/p, p, k, pak);
    ll y = n/p + rec.first;
    r = r*rec.second % pak;

    return {y, r};
}

ll solve_pak(ll n, ll x, int p, int k, int pak) {
    divi[0] = 1;
    for (int i = 1; i <= pak; i++) {
        divi[i] = divi[i-1];
        if (i%p) divi[i] = divi[i] * i % pak;
    }

    auto dn = divide_show(n, p, k, pak), dx = divide_show(x,
       p, k, pak),
         dnx = divide_show(n-x, p, k, pak);
    ll y = dn.first-dx.first-dnx.first, r =
        (dn.second*inv(dx.second, pak)%pak)*inv(dnx.second,
           pak)%pak;
    return expo(p, y, pak) * r % pak;
}

ll solve(ll n, ll x, int mod) {
    vector<pair<int, int>> f;
    int mod2 = mod;
    for (int i = 2; i*i <= mod2; i++) if (mod2%i==0) {
```

```
        int c = 0;
        while (mod2%i==0) mod2 /= i, c++;
        f.push_back({i, c});
    }
    if (mod2 > 1) f.push_back({mod2, 1});
    crt ans(0, 1);
    for (int i = 0; i < f.size(); i++) {
        int pak = 1;
        for (int j = 0; j < f[i].second; j++) pak *=
           f[i].first;
        ans = ans * crt(solve_pak(n, x, f[i].first,
           f[i].second, pak), pak);
    }
    return ans.a;
}
```

1.7 Closest pair of points

```
// O(nlogn)
// f90265

pair<pt, pt> closest_pair_of_points(vector<pt> v) {
    int n = v.size();
    sort(v.begin(), v.end());
    for (int i = 1; i < n; i++) if (v[i] == v[i-1]) return
       {v[i-1], v[i]};
    auto cmp_y = [&](const pt &l, const pt &r) {
        if (l.y != r.y) return l.y < r.y;
        return l.x < r.x;
    };
    set<pt, decltype(cmp_y)> s(cmp_y);
    int l = 0, r = -1;
    ll d2_min = numeric_limits<ll>::max();
    pt pl, pr;
    const int magic = 5;
    while (r+1 < n) {
        auto it = s.insert(v[++r]).first;
        int cnt = magic/2;
        while (cnt-- and it != s.begin()) it--;
        cnt = 0;
        while (cnt++ < magic and it != s.end()) {
            if (!((*it) == v[r])) {
                ll d2 = dist2(*it, v[r]);
                if (d2_min > d2) {
                    d2_min = d2;
                    pl = *it;
                    pr = v[r];
                }
            }
            it++;
        }
        while (l < r and sq(v[l].x-v[r].x) > d2_min)
           s.erase(v[l++]);
    }
    return {pl, pr};
}
```

1.8 Coloracao de Grafo de Intervalo

```
// Colore os intervalos com o numero minimo
// de cores de tal forma que dois intervalos
// que se interceptam tem cores diferentes
// As cores vao de 1 ate n
//
// O(n log(n))
// 83a32d

vector<int> coloring(vector<pair<int, int>>& v) {
    int n = v.size();
    vector<pair<int, pair<int, int>>> ev;
    for (int i = 0; i < n; i++) {
        ev.push_back({v[i].first, {1, i}});
        ev.push_back({v[i].second, {0, i}});
    }
    sort(ev.begin(), ev.end());
    vector<int> ans(n), avl(n);
    for (int i = 0; i < n; i++) avl.push_back(n-i);
    for (auto i : ev) {
        if (i.second.first == 1) {
            ans[i.second.second] = avl.back();
```

```
            avl.pop_back();
        } else avl.push_back(ans[i.second.second]);
    }
    return ans;
}
```

1.9 Conectividade Dinamica

```
// Offline com Divide and Conquer e
// DSU com rollback
// O(n log^2(n))
// 043d93

typedef pair<int, int> T;

namespace data {
    int n, ans;
    int p[MAX], sz[MAX];
    stack<int> S;

    void build(int n2) {
        n = n2;
        for (int i = 0; i < n; i++) p[i] = i, sz[i] = 1;
        ans = n;
    }
    int find(int k) {
        while (p[k] != k) k = p[k];
        return k;
    }
    void add(T x) {
        int a = x.first, b = x.second;
        a = find(a), b = find(b);
        if (a == b) return S.push(-1);
        ans--;
        if (sz[a] > sz[b]) swap(a, b);
        S.push(a);
        sz[b] += sz[a];
        p[a] = b;
    }
    int query() {
        return ans;
    }
    void rollback() {
        int u = S.top(); S.pop();
        if (u == -1) return;
        sz[p[u]] -= sz[u];
        p[u] = u;
        ans++;
    }
};

int ponta[MAX]; // outra ponta do intervalo ou -1 se for
   query
int ans[MAX], n, q;
T qu[MAX];

void solve(int l = 0, int r = q-1) {
    if (l >= r) {
        ans[l] = data::query(); // agora a estrutura ta certa
        return;
    }
    int m = (l+r)/2, qnt = 1;
    for (int i = m+1; i <= r; i++) if (ponta[i]+1 and
       ponta[i] < l)
        data::add(qu[i]), qnt++;
    solve(l, m);
    while (--qnt) data::rollback();
    for (int i = l; i <= m; i++) if (ponta[i]+1 and ponta[i]
       > r)
        data::add(qu[i]), qnt++;
    solve(m+1, r);
    while (qnt--) data::rollback();
}
```

1.10 Conectividade Dinamica 2

```
// Offline com link-cut trees
// O(n log(n))
// d38e4e
```

```
namespace lct {
    struct node {
        int p, ch[2];
        int val, sub;
        bool rev;
        node() {}
        node(int v) : p(-1), val(v), sub(v), rev(0) { ch[0]
           = ch[1] = -1; }
    };

    node t[2*MAX]; // MAXN + MAXQ
    map<pair<int, int>, int> aresta;
    int sz;

    void prop(int x) {
        if (t[x].rev) {
            swap(t[x].ch[0], t[x].ch[1]);
            if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
            if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
        }
        t[x].rev = 0;
    }
    void update(int x) {
        t[x].sub = t[x].val;
        for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
            prop(t[x].ch[i]);
            t[x].sub = min(t[x].sub, t[t[x].ch[i]].sub);
        }
    }
    bool is_root(int x) {
        return t[x].p == -1 or (t[t[x].p].ch[0] != x and
           t[t[x].p].ch[1] != x);
    }
    void rotate(int x) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
        bool d = t[p].ch[0] == x;
        t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
        if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
        t[x].p = pp, t[p].p = x;
        update(p), update(x);
    }
    int splay(int x) {
        while (!is_root(x)) {
            int p = t[x].p, pp = t[p].p;
            if (!is_root(p)) prop(pp);
            prop(p), prop(x);
            if (!is_root(p)) rotate((t[pp].ch[0] ==
               p)^(t[p].ch[0] == x) ? x : p);
            rotate(x);
        }
        return prop(x), x;
    }
    int access(int v) {
        int last = -1;
        for (int w = v; w+1; update(last = w), splay(v), w =
           t[v].p)
            splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
        return last;
    }
    void make_tree(int v, int w=INF) { t[v] = node(w); }
    bool conn(int v, int w) {
        access(v), access(w);
        return v == w ? true : t[v].p != -1;
    }
    void rootify(int v) {
        access(v);
        t[v].rev ^= 1;
    }
    int query(int v, int w) {
        rootify(w), access(v);
        return t[v].sub;
    }
    void link_(int v, int w) {
        rootify(w);
        t[w].p = v;
    }
    void link(int v, int w, int x) { // v--w com peso x
        int id = MAX + sz++;
        aresta[make_pair(v, w)] = id;
        make_tree(id, x);
        link_(v, id), link_(id, w);
    }
    void cut_(int v, int w) {
```

```
        rootify(w), access(v);
        t[v].ch[0] = t[t[v].ch[0]].p = -1;
    }
    void cut(int v, int w) {
        int id = aresta[make_pair(v, w)];
        cut_(v, id), cut_(id, w);
    }
}

void dyn_conn() {
    int n, q; cin >> n >> q;
    vector<int> p(2*q, -1); // outra ponta do intervalo
    for (int i = 0; i < n; i++) lct::make_tree(i);
    vector<pair<int, int>> qu(q);
    map<pair<int, int>, int> m;
    for (int i = 0; i < q; i++) {
        char c; cin >> c;
        if (c == '?') continue;
        int a, b; cin >> a >> b; a--, b--;
        if (a > b) swap(a, b);
        qu[i] = {a, b};
        if (c == '+') {
            p[i] = i+q, p[i+q] = i;
            m[make_pair(a, b)] = i;
        } else {
            int j = m[make_pair(a, b)];
            p[i] = j, p[j] = i;
        }
    }
    int ans = n;
    for (int i = 0; i < q; i++) {
        if (p[i] == -1) {
            cout << ans << endl; // numero de comp conexos
            continue;
        }
        int a = qu[i].first, b = qu[i].second;
        if (p[i] > i) { // +
            if (lct::conn(a, b)) {
                int mi = lct::query(a, b);
                if (p[i] < mi) {
                    p[p[i]] = p[i];
                    continue;
                }
                lct::cut(qu[p[mi]].first, qu[p[mi]].second),
                    ans++;
                p[mi] = mi;
            }
            lct::link(a, b, p[i]), ans--;
        } else if (p[i] != i) lct::cut(a, b), ans++; // -
    }
}
```

1.11 Conj. Indep. Maximo com Peso em Grafo de Intervalo

```
// Retorna os indices ordenados dos intervalos selecionados
// Se tiver empate, retorna o que minimiza o comprimento
//   total
//
// O(n log(n))
// c4dbe2

vector<int> ind_set(vector<tuple<int, int, int>>& v) {
    vector<tuple<int, int, int>> w;
    for (int i = 0; i < v.size(); i++) {
        w.push_back(tuple(get<0>(v[i]), 0, i));
        w.push_back(tuple(get<1>(v[i]), 1, i));
    }
    sort(w.begin(), w.end());

    vector<int> nxt(v.size());
    vector<pair<ll, int>> dp(v.size());
    int last = -1;
    for (auto [fim, t, i] : w) {
        if (t == 0) {
            nxt[i] = last;
            continue;
        }
        dp[i] = {0, 0};
        if (last != -1) dp[i] = max(dp[i], dp[last]);
        pair<ll, int> pega = {get<2>(v[i]), -(get<1>(v[i]) -
            get<0>(v[i]) + 1)};
```

```
        if (nxt[i] != -1) pega.first += dp[nxt[i]].first,
            pega.second += dp[nxt[i]].second;
        if (pega > dp[i]) dp[i] = pega;
        else nxt[i] = last;
        last = i;
    }
    pair<ll, int> ans = {0, 0};
    int idx = -1;
    for (int i = 0; i < v.size(); i++) if (dp[i] > ans) ans
        = dp[i], idx = i;
    vector<int> ret;
    while (idx != -1) {
        if (get<2>(v[idx]) > 0 and
            (nxt[idx] == -1 or get<1>(v[nxt[idx]]) <
                get<0>(v[idx]))) ret.push_back(idx);
        idx = nxt[idx];
    }
    sort(ret.begin(), ret.end());
    return ret;
}
```

1.12 Distancia maxima entre dois pontos

```
// max_dist2(v) - O(n log(n))
// max_dist_manhattan - O(n)

// Quadrado da Distancia Euclidiana (precisa copiar
    convex_hull, ccw e pt)
// bdace4
ll max_dist2(vector<pt> v) {
    v = convex_hull(v);
    if (v.size() <= 2) return dist2(v[0], v[1%v.size()]);
    ll ans = 0;
    int n = v.size(), j = 0;
    for (int i = 0; i < n; i++) {
        while (!ccw(v[(i+1)%n]-v[i], pt(0, 0),
            v[(j+1)%n]-v[j])) j = (j+1)%n;
        ans = max({ans, dist2(v[i], v[j]), dist2(v[(i+1)%n],
            v[j])});
    }
    return ans;
}

// Distancia de Manhattan
// 4e96f0
template<typename T> T max_dist_manhattan(vector<pair<T, T>>
    v) {
    T min_sum, max_sum, min_dif, max_dif;
    min_sum = max_sum = v[0].first + v[0].second;
    min_dif = max_dif = v[0].first - v[0].second;
    for (auto [x, y] : v) {
        min_sum = min(min_sum, x+y);
        max_sum = max(max_sum, x+y);
        min_dif = min(min_dif, x-y);
        max_dif = max(max_dif, x-y);
    }
    return max(max_sum - min_sum, max_dif - min_dif);
}
```

1.13 Distinct Range Query

```
// build - O(n (log n + log(sigma)))
// query - O(log(sigma))
// 5c7aa1

namespace perseg { };

int qt[MAX];

void build(vector<int>& v) {
    int n = v.size();
    perseg::build(n);
    map<int, int> last;
    int at = 0;
    for (int i = 0; i < n; i++) {
        if (last.count(v[i])) {
            perseg::update(last[v[i]], -1);
            at++;
        }
        perseg::update(i, 1);
```

```
        qt[i] = ++at;
        last[v[i]] = i;
    }
}

int query(int l, int r) {
    return perseg::query(l, r, qt[r]);
}
```

1.14 Distinct Range Query com Update

```
// build - O(n log(n))
// query - O(log^2(n))
// update - O(log^2(n))
// 2306f3

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <class T>
    using ord_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

int v[MAX], n, nxt[MAX], prv[MAX];
map<int, set<int> > ocor;

namespace bit {
    ord_set<pair<int, int>> bit[MAX];

    void build() {
        for (int i = 1; i <= n; i++)
            bit[i].insert({nxt[i-1], i-1});
        for (int i = 1; i <= n; i++) {
            int j = i + (i&-i);
            if (j <= n) for (auto x : bit[i])
                bit[j].insert(x);
        }
    }
    int pref(int p, int x) {
        int ret = 0;
        for (; p; p -= p&-p) ret += bit[p].order_of_key({x,
            -INF});
        return ret;
    }
    int query(int l, int r, int x) {
        return pref(r+1, x) - pref(l, x);
    }
    void update(int p, int x) {
        int p2 = p;
        for (p++; p <= n; p += p&-p) {
            bit[p].erase({nxt[p2], p2});
            bit[p].insert({x, p2});
        }
    }
}

void build() {
    for (int i = 0; i < n; i++) nxt[i] = INF;
    for (int i = 0; i < n; i++) prv[i] = -INF;
    vector<pair<int, int>> t;
    for (int i = 0; i < n; i++) t.push_back({v[i], i});
    sort(t.begin(), t.end());
    for (int i = 0; i < n; i++) {
        if (i and t[i].first == t[i-1].first)
            prv[t[i].second] = t[i-1].second;
        if (i+1 < n and t[i].first == t[i+1].first)
            nxt[t[i].second] = t[i+1].second;
    }

    for (int i = 0; i < n; i++) ocor[v[i]].insert(i);

    bit::build();
}

void muda(int p, int x) {
    bit::update(p, x);
    nxt[p] = x;
}

int query(int a, int b) {
    return b-a+1 - bit::query(a, b, b+1);
}
```

```
void update(int p, int x) { // mudar valor na pos. p para x
    if (prv[p] > -INF) muda(prv[p], nxt[p]);
    if (nxt[p] < INF) prv[nxt[p]] = prv[p];

    ocor[v[p]].erase(p);
    if (!ocor[x].size()) {
        muda(p, INF);
        prv[p] = -INF;
    } else if (*ocor[x].rbegin() < p) {
        int i = *ocor[x].rbegin();
        prv[p] = i;
        muda(p, INF);
        muda(i, p);
    } else {
        int i = *ocor[x].lower_bound(p);
        if (prv[i] > -INF) {
            muda(prv[i], p);
            prv[p] = prv[i];
        } else prv[p] = -INF;
        prv[i] = p;
        muda(p, i);
    }
    v[p] = x; ocor[x].insert(p);
}
```

1.15 Dominator Points

```
// Se um ponto A tem ambas as coordenadas >= B, dizemos
// que A domina B
// is_dominated(p) fala se existe algum ponto no conjunto
// que domina p
// insert(p) insere p no conjunto
// (se p for dominado por alguem, nao vai inserir)
// o multiset 'quina' guarda informacao sobre os pontos
// nao dominados por um elemento do conjunto que nao dominam
// outro ponto nao dominado por um elemento do conjunto
// No caso, armazena os valores de x+y esses pontos
//
// Complexidades:
// is_dominated - O(log(n))
// insert - O(log(n)) amortizado
// query - O(1)
// 09ffdc

struct dominator_points {
    set<pair<int, int>> se;
    multiset<int> quina;

    bool is_dominated(pair<int, int> p) {
        auto it = se.lower_bound(p);
        if (it == se.end()) return 0;
        return it->second >= p.second;
    }
    void mid(pair<int, int> a, pair<int, int> b, bool rem) {
        pair<int, int> m = {a.first+1, b.second+1};
        int val = m.first + m.second;
        if (!rem) quina.insert(val);
        else quina.erase(quina.find(val));
    }
    bool insert(pair<int, int> p) {
        if (is_dominated(p)) return 0;
        auto it = se.lower_bound(p);
        if (it != se.begin() and it != se.end())
            mid(*prev(it), *it, 1);
        while (it != se.begin()) {
            it--;
            if (it->second > p.second) break;
            if (it != se.begin()) mid(*prev(it), *it, 1);
            it = se.erase(it);
        }
        it = se.insert(p).first;
        if (it != se.begin()) mid(*prev(it), *it, 0);
        if (next(it) != se.end()) mid(*it, *next(it), 0);
        return 1;
    }
    int query() {
        if (!quina.size()) return INF;
        return *quina.begin();
    }
};
```

1.16 DP de Dominacao 3D

```
// Computa para todo ponto i,
// dp[i] = 1 + max_{j dominado por i} dp[j]
// em que ser dominado eh ter as 3 coordenadas menores
// Da pra adaptar facil para outras dps
//
// O(n log^2 n), O(n) de memoria
// 7c8896

void lis2d(vector<vector<tuple<int, int, int>>>& v,
    vector<int>& dp, int l, int r) {
    if (l == r) {
        for (int i = 0; i < v[l].size(); i++) {
            int ii = get<2>(v[l][i]);
            dp[ii] = max(dp[ii], 1);
        }
        return;
    }
    int m = (l+r)/2;
    lis2d(v, dp, l, m);

    vector<tuple<int, int, int>> vv[2];
    vector<int> Z;
    for (int i = l; i <= r; i++) for (auto it : v[i]) {
        vv[i > m].push_back(it);
        Z.push_back(get<1>(it));
    }
    sort(vv[0].begin(), vv[0].end());
    sort(vv[1].begin(), vv[1].end());
    sort(Z.begin(), Z.end());
    auto get_z = [&](int z) { return lower_bound(Z.begin(),
        Z.end(), z) - Z.begin(); };
    vector<int> bit(Z.size());

    int i = 0;
    for (auto [y, z, id] : vv[1]) {
        while (i < vv[0].size() and get<0>(vv[0][i]) < y) {
            auto [y2, z2, id2] = vv[0][i++];
            for (int p = get_z(z2)+1; p <= Z.size(); p +=
                p&-p)
                bit[p-1] = max(bit[p-1], dp[id2]);
        }
        int q = 0;
        for (int p = get_z(z); p; p -= p&-p) q = max(q,
            bit[p-1]);
        dp[id] = max(dp[id], q + 1);
    }
    lis2d(v, dp, m+1, r);
}

vector<int> solve(vector<tuple<int, int, int>> v) {
    int n = v.size();
    vector<tuple<int, int, int, int>> vv;
    for (int i = 0; i < n; i++) {
        auto [x, y, z] = v[i];
        vv.emplace_back(x, y, z, i);
    }
    sort(vv.begin(), vv.end());

    vector<vector<tuple<int, int, int>>> V;
    for (int i = 0; i < n; i++) {
        int j = i;
        V.emplace_back();
        while (j < n and get<0>(vv[j]) == get<0>(vv[i])) {
            auto [x, y, z, id] = vv[j++];
            V.back().emplace_back(y, z, id);
        }
        i = j-1;
    }
    vector<int> dp(n);
    lis2d(V, dp, 0, V.size()-1);
    return dp;
}
```

1.17 Gray Code

```
// Gera uma permutacao de 0 a 2^n-1, de forma que
// duas posicoes adjacentes diferem em exatamente 1 bit
//
// O(2^n)
// 840df4
```

```
vector<int> gray_code(int n) {
    vector<int> ret(1<<n);
    for (int i = 0; i < (1<<n); i++) ret[i] = i^(i>>1);
    return ret;
}
```

1.18 Half-plane intersection

```
// Cada half-plane eh identificado por uma reta e a regiao
   ccw a ela
//
// O(n log n)
// f56e1c

vector<pt> hp_intersection(vector<line> &v) {
    deque<pt> dq = {{INF, INF}, {-INF, INF}, {-INF, -INF},
       {INF, -INF}};

#warning considerar trocar por compare_angle
    sort(v.begin(), v.end(), [&](line r, line s) { return
       angle(r.q-r.p) < angle(s.q-s.p); });

    for(int i = 0; i < v.size() and dq.size() > 1; i++) {
        pt p1 = dq.front(), p2 = dq.back();
        while (dq.size() and !ccw(v[i].p, v[i].q, dq.back()))
            p1 = dq.back(), dq.pop_back();
        while (dq.size() and !ccw(v[i].p, v[i].q,
           dq.front()))
            p2 = dq.front(), dq.pop_front();

        if (!dq.size()) break;
        if (p1 == dq.front() and p2 == dq.back()) continue;
        dq.push_back(inter(v[i], line(dq.back(), p1)));
        dq.push_front(inter(v[i], line(dq.front(), p2)));

        if (dq.size() > 1 and dq.back() == dq.front())
           dq.pop_back();
    }
    return vector<pt>(dq.begin(), dq.end());
}
```

1.19 Heap Sort

```
// O(n log n)
// 385e91

void down(vector<int>& v, int n, int i) {
    while ((i = 2*i+1) < n) {
        if (i+1 < n and v[i] < v[i+1]) i++;
        if (v[i] < v[(i-1)/2]) break;
        swap(v[i], v[(i-1)/2]);
    }
}
void heap_sort(vector<int>& v) {
    int n = v.size();
    for (int i = n/2-1; i >= 0; i--) down(v, n, i);
    for (int i = n-1; i > 0; i--)
        swap(v[0], v[i]), down(v, i, 0);
}
```

1.20 Inversion Count

```
// Computa o numero de inversoes para transformar
// l em r (se nao tem como, retorna -1)
//
// O(n log(n))
// eef01f

template<typename T> ll inv_count(vector<T> l, vector<T> r =
   {}) {
    if (!r.size()) {
        r = l;
        sort(r.begin(), r.end());
    }
    int n = l.size();
    vector<int> v(n), bit(n);
    vector<pair<T, int>> w;
```

```
    for (int i = 0; i < n; i++) w.push_back({r[i], i+1});
    sort(w.begin(), w.end());
    for (int i = 0; i < n; i++) {
        auto it = lower_bound(w.begin(), w.end(),
            make_pair(l[i], 0));
        if (it == w.end() or it->first != l[i]) return -1;
            // nao da
        v[i] = it->second;
        it->second = -1;
    }

    ll ans = 0;
    for (int i = n-1; i >= 0; i--) {
        for (int j = v[i]-1; j; j -= j&-j) ans += bit[j];
        for (int j = v[i]; j < n; j += j&-j) bit[j]++;
    }
    return ans;
}
```

1.21 LIS - Longest Increasing Subsequence

```
// Calcula e retorna uma LIS
//
// O(n.log(n))
// 4749e8

template<typename T> vector<T> lis(vector<T>& v) {
    int n = v.size(), m = -1;
    vector<T> d(n+1, INF);
    vector<int> l(n);
    d[0] = -INF;

    for (int i = 0; i < n; i++) {
        // Para non-decreasing use upper_bound()
        int t = lower_bound(d.begin(), d.end(), v[i]) -
            d.begin();
        d[t] = v[i], l[i] = t, m = max(m, t);
    }

    int p = n;
    vector<T> ret;
    while (p--) if (l[p] == m) {
        ret.push_back(v[p]);
        m--;
    }
    reverse(ret.begin(),ret.end());

    return ret;
}
```

1.22 LIS2 - Longest Increasing Subsequence

```
// Calcula o tamanho da LIS
//
// O(n log(n))
// 402def

template<typename T> int lis(vector<T> &v){
    vector<T> ans;
    for (T t : v){
        // Para non-decreasing use upper_bound()
        auto it = lower_bound(ans.begin(), ans.end(), t);
        if (it == ans.end()) ans.push_back(t);
        else *it = t;
    }
    return ans.size();
}
```

1.23 Mininum Enclosing Circle

```
// O(n) com alta probabilidade
// b0a6ba

const double EPS = 1e-12;
mt19937 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

struct pt {
```

```
    double x, y;
    pt(double x_ = 0, double y_ = 0) : x(x_), y(y_) {}
    pt operator + (const pt& p) const { return pt(x+p.x,
       y+p.y); }
    pt operator - (const pt& p) const { return pt(x-p.x,
       y-p.y); }
    pt operator * (double c) const { return pt(x*c, y*c); }
    pt operator / (double c) const { return pt(x/c, y/c); }
};

double dot(pt p, pt q) { return p.x*q.x+p.y*q.y; }
double cross(pt p, pt q) { return p.x*q.y-p.y*q.x; }
double dist(pt p, pt q) { return sqrt(dot(p-q, p-q)); }

pt center(pt p, pt q, pt r) {
    pt a = p-r, b = q-r;
    pt c = pt(dot(a, p+r)/2, dot(b, q+r)/2);
    return pt(cross(c, pt(a.y, b.y)), cross(pt(a.x, b.x),
       c)) / cross(a, b);
}

struct circle {
    pt cen;
    double r;
    circle(pt cen_, double r_) : cen(cen_), r(r_) {}
    circle(pt a, pt b, pt c) {
        cen = center(a, b, c);
        r = dist(cen, a);
    }
    bool inside(pt p) { return dist(p, cen) < r+EPS; }
};

circle minCirc(vector<pt> v) {
    shuffle(v.begin(), v.end(), rng);
    circle ret = circle(pt(0, 0), 0);
    for (int i = 0; i < v.size(); i++) if
       (!ret.inside(v[i])) {
        ret = circle(v[i], 0);
        for (int j = 0; j < i; j++) if (!ret.inside(v[j])) {
            ret = circle((v[i]+v[j])/2, dist(v[i], v[j])/2);
            for (int k = 0; k < j; k++) if
                (!ret.inside(v[k]))
                    ret = circle(v[i], v[j], v[k]);
        }
    }
    return ret;
}
```

1.24 Minkowski Sum

```
// Computa A+B = {a+b : a \in A, b \in B}, em que
// A e B sao poligonos convexos
// A+B eh um poligono convexo com no max |A|+|B| pontos
//
// O(|A|+|B|)

// d7cca8
vector<pt> minkowski(vector<pt> p, vector<pt> q) {
    auto fix = [](vector<pt>& P) {
        rotate(P.begin(), min_element(P.begin(), P.end()),
           P.end());
        P.push_back(P[0]), P.push_back(P[1]);
    };
    fix(p), fix(q);
    vector<pt> ret;
    int i = 0, j = 0;
    while (i < p.size()-2 or j < q.size()-2) {
        ret.push_back(p[i] + q[j]);
        auto c = ((p[i+1] - p[i]) ^ (q[j+1] - q[j]));
        if (c >= 0) i = min<int>(i+1, p.size()-2);
        if (c <= 0) j = min<int>(j+1, q.size()-2);
    }
    return ret;
}

// 2f5dd2
ld dist_convex(vector<pt> p, vector<pt> q) {
    for (pt& i : p) i = i * -1;
    auto s = minkowski(p, q);
    if (inpol(s, pt(0, 0))) return 0;
    return 1;
    ld ans = DINF;
```

```
    for (int i = 0; i < s.size(); i++) ans = min(ans,
            disttoseg(pt(0, 0), line(s[(i+1)%s.size()],
               s[i])));
    return ans;
}
```

1.25 MO - DSU

```
// Dado uma lista de arestas de um grafo, responde
// para cada query(l, r), quantos componentes conexos
// o grafo tem se soh considerar as arestas l, l+1, ..., r
// Da pra adaptar pra usar MO com qualquer estrutura
   rollbackavel
//
// O(m sqrt(q) log(n))
// f98540

struct dsu {
    int n, ans;
    vector<int> p, sz;
    stack<int> S;

    dsu(int n_) : n(n_), ans(n), p(n), sz(n) {
        for (int i = 0; i < n; i++) p[i] = i, sz[i] = 1;
    }
    int find(int k) {
        while (p[k] != k) k = p[k];
        return k;
    }
    void add(pair<int, int> x) {
        int a = x.first, b = x.second;
        a = find(a), b = find(b);
        if (a == b) return S.push(-1);
        ans--;
        if (sz[a] > sz[b]) swap(a, b);
        S.push(a);
        sz[b] += sz[a];
        p[a] = b;
    }
    int query() { return ans; }
    void rollback() {
        int u = S.top(); S.pop();
        if (u == -1) return;
        sz[p[u]] -= sz[u];
        p[u] = u;
        ans++;
    }
};

int n;
vector<pair<int, int>> ar; // vetor com as arestas

vector<int> MO(vector<pair<int, int>> &q) {
    int SQ = ar.size() / sqrt(q.size()) + 1;
    int m = q.size();
    vector<int> ord(m);
    iota(ord.begin(), ord.end(), 0);
    sort(ord.begin(), ord.end(), [&](int l, int r) {
        if (q[l].first / SQ != q[r].first / SQ) return
           q[l].first < q[r].first;
        return q[l].second < q[r].second;
    });
    vector<int> ret(m);

    for (int i = 0; i < m; i++) {
        dsu D(n);
        int fim = q[ord[i]].first/SQ*SQ + SQ - 1;
        int last_r = fim;
        int j = i-1;
        while (j+1 < m and q[ord[j+1]].first / SQ ==
           q[ord[i]].first / SQ) {
            auto [l, r] = q[ord[++j]];

            if (l / SQ == r / SQ) {
                dsu D2(n);
                for (int k = l; k <= r; k++) D2.add(ar[k]);
                ret[ord[j]] = D2.query();
                continue;
            }

            while (last_r < r) D.add(ar[++last_r]);
            for (int k = l; k <= fim; k++) D.add(ar[k]);
```

```
            ret[ord[j]] = D.query();

            for (int k = 1; k <= fim; k++) D.rollback();
        }
        i = j;
    }
    return ret;
}
```

1.26 Mo - numero de distintos em range

```
// Para ter o bound abaixo, escolher
// SQ = n / sqrt(q)
//
// O(n * sqrt(q))
// e94f60

const int MAX = 1e5+10;
const int SQ = sqrt(MAX);
int v[MAX];

int ans, freq[MAX];

inline void insert(int p) {
    int o = v[p];
    freq[o]++;
    ans += (freq[o] == 1);
}

inline void erase(int p) {
    int o = v[p];
    ans -= (freq[o] == 1);
    freq[o]--;
}

inline ll hilbert(int x, int y) {
    static int N = 1 << (__builtin_clz(0) -
        __builtin_clz(MAX));
    int rx, ry, s;
    ll d = 0;
    for (s = N/2; s > 0; s /= 2) {
        rx = (x & s) > 0, ry = (y & s) > 0;
        d += s * ll(s) * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = N-1 - x, y = N-1 - y;
            swap(x, y);
        }
    }
    return d;
}

#define HILBERT true
vector<int> MO(vector<pair<int, int>> &q) {
    ans = 0;
    int m = q.size();
    vector<int> ord(m);
    iota(ord.begin(), ord.end(), 0);
#if HILBERT
    vector<ll> h(m);
    for (int i = 0; i < m; i++) h[i] = hilbert(q[i].first,
        q[i].second);
    sort(ord.begin(), ord.end(), [&](int l, int r) { return
        h[l] < h[r]; });
#else
    sort(ord.begin(), ord.end(), [&](int l, int r) {
        if (q[l].first / SQ != q[r].first / SQ) return
            q[l].first < q[r].first;
        if ((q[l].first / SQ) % 2) return q[l].second >
            q[r].second;
        return q[l].second < q[r].second;
    });
#endif
    vector<int> ret(m);
    int l = 0, r = -1;

    for (int i : ord) {
        int ql, qr;
        tie(ql, qr) = q[i];
        while (r < qr) insert(++r);
        while (l > ql) insert(--l);
        while (l < ql) erase(l++);
```

```
        while (r > qr) erase(r--);
        ret[i] = ans;
    }
    return ret;
}
```

1.27 Palindromic Factorization

```
// Precisa da eertree
// Computa o numero de formas de particionar cada
// prefixo da string em strings palindromicas
//
// O(n log n), considerando alfabeto O(1)
// 9e6e22

struct eertree { ... };

ll factorization(string s) {
    int n = s.size(), sz = 2;
    eertree PT(n);
    vector<int> diff(n+2), slink(n+2), sans(n+2), dp(n+1);
    dp[0] = 1;
    for (int i = 1; i <= n; i++) {
        PT.add(s[i-1]);
        if (PT.size()+2 > sz) {
            diff[sz] = PT.len[sz] - PT.len[PT.link[sz]];
            if (diff[sz] == diff[PT.link[sz]])
                slink[sz] = slink[PT.link[sz]];
            else slink[sz] = PT.link[sz];
            sz++;
        }
        for (int v = PT.last; PT.len[v] > 0; v = slink[v]) {
            sans[v] = dp[i - (PT.len[slink[v]] + diff[v])];
            if (diff[v] == diff[PT.link[v]])
                sans[v] = (sans[v] + sans[PT.link[v]]) % MOD;
            dp[i] = (dp[i] + sans[v]) % MOD;
        }
    }
    return dp[n];
}
```

1.28 Parsing de Expressao

```
// Operacoes associativas a esquerda por default
// Para mudar isso, colocar em r_assoc
// Operacoes com maior prioridade sao feitas primeiro
//
// 68921b

bool blank(char c) {
    return c == ' ';
}

bool is_unary(char c) {
    return c == '+' or c == '-';
}

bool is_op(char c) {
    if (is_unary(c)) return true;
    return c == '*' or c == '/' or c == '+' or c == '-';
}

bool r_assoc(char op) {
    // operator unario - deve ser assoc. a direita
    return op < 0;
}

int priority(char op) {
    // operator unario - deve ter precedencia maior
    if (op < 0) return INF;

    if (op == '*' or op == '/') return 2;
    if (op == '+' or op == '-') return 1;
    return -1;
}

void process_op(stack<int>& st, stack<int>& op) {
    char o = op.top(); op.pop();
    if (o < 0) {
        o *= -1;
        int l = st.top(); st.pop();
        if (o == '+') st.push(l);
        if (o == '-') st.push(-l);
```

```
    } else {
        int r = st.top(); st.pop();
        int l = st.top(); st.pop();
        if (o == '*') st.push(l * r);
        if (o == '/') st.push(l / r);
        if (o == '+') st.push(l + r);
        if (o == '-') st.push(l - r);
    }
}

int eval(string& s) {
    stack<int> st, op;
    bool un = true;
    for (int i = 0; i < s.size(); i++) {
        if (blank(s[i])) continue;

        if (s[i] == '(') {
            op.push('(');
            un = true;
        } else if (s[i] == ')') {
            while (op.top() != '(') process_op(st, op);
            op.pop();
            un = false;
        } else if (is_op(s[i])) {
            char o = s[i];
            if (un and is_unary(o)) o *= -1;
            while (op.size() and (
                        (!r_assoc(o) and priority(op.top())
                            >= priority(o)) or
                        (r_assoc(o) and priority(op.top()) >
                            priority(o))))
                process_op(st, op);
            op.push(o);
            un = true;
        } else {
            int val = 0;
            while (i < s.size() and isalnum(s[i]))
                val = val * 10 + s[i++] - '0';
            i--;
            st.push(val);
            un = false;
        }
    }

    while (op.size()) process_op(st, op);
    return st.top();
}
```

1.29 RMQ com Divide and Conquer

```
// Responde todas as queries em
// O(n log(n))
// 5a6ebd

typedef pair<pair<int, int>, int> iii;
#define f first
#define s second

int n, q, v[MAX];
iii qu[MAX];
int ans[MAX], pref[MAX], sulf[MAX];

void solve(int l=0, int r=n-1, int ql=0, int qr=q-1) {
    if (l > r or ql > qr) return;
    int m = (l+r)/2;
    int qL = partition(qu+ql, qu+qr+1, [=](iii x){return
       x.f.s < m;}) - qu;
    int qR = partition(qu+qL, qu+qr+1, [=](iii x){return
       x.f.f <=m;}) - qu;

    pref[m] = sulf[m] = v[m];
    for (int i = m-1; i >= l; i--) pref[i] = min(v[i],
       pref[i+1]);
    for (int i = m+1; i <= r; i++) sulf[i] = min(v[i],
       sulf[i-1]);

    for (int i = qL; i < qR; i++)
        ans[qu[i].s] = min(pref[qu[i].f.f], sulf[qu[i].f.s]);

    solve(l, m-1, ql, qL-1), solve(m+1, r, qR, qr);
}
```

1.30 Segment Intersection

```
// Verifica, dado n segmentos, se existe algum par de
    segmentos
// que se intersecta
//
// O(n log n)
// 3957d8

bool operator < (const line& a, const line& b) { //
    comparador pro sweepline
    if (a.p == b.p) return ccw(a.p, a.q, b.q);
    if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps
        < b.p.x))
        return ccw(a.p, a.q, b.p);
    return ccw(a.p, b.q, b.p);
}

bool has_intersection(vector<line> v) {
    auto intersects = [&](pair<line, int> a, pair<line, int>
        b) {
        return interseg(a.first, b.first);
    };
    vector<pair<pt, pair<int, int>>> w;
    for (int i = 0; i < v.size(); i++) {
        if (v[i].q < v[i].p) swap(v[i].p, v[i].q);
        w.push_back({v[i].p, {0, i}});
        w.push_back({v[i].q, {1, i}});
    }
    sort(w.begin(), w.end());
    set<pair<line, int>> se;
    for (auto i : w) {
        line at = v[i.second.second];
        if (i.second.first == 0) {
            auto nxt = se.lower_bound({at, i.second.second});
            if (nxt != se.end() and intersects(*nxt, {at,
                i.second.second})) return 1;
            if (nxt != se.begin() and intersects(*(--nxt),
                {at, i.second.second})) return 1;
            se.insert({at, i.second.second});
        } else {
            auto nxt = se.upper_bound({at,
                i.second.second}), cur = nxt, prev = --cur;
            if (nxt != se.end() and prev != se.begin()
                and intersects(*nxt, *(--prev))) return 1;
            se.erase(cur);
        }
    }
    return 0;
}
```

1.31 Sequencia de de Brujin

```
// Se passar sem o terceiro parametro, gera um vetor com
    valores
// em [0, k) de tamanho k^n de forma que todos os subarrays
    ciclicos
// de tamanho n ocorrem exatamente uma vez
// Se passar com um limite lim, gera o menor vetor com
    valores
// em [0, k) que possui lim subarrays de tamanho n distintos
// (assume que lim <= k^n)
//
// Linear no tamanho da resposta
// 19720c

vector<int> de_brujin(int n, int k, int lim = INF) {
    if (k == 1) return vector<int>(lim == INF ? 1 : n, 0);
    vector<int> l = {0}, ret; // l eh lyndon word
    while (true) {
        if (l.size() == 0) {
            if (lim == INF) break;
            l.push_back(0);
        }
        if (n % l.size() == 0) for (int i : l) {
            ret.push_back(i);
            if (ret.size() == n+lim-1) return ret;
        }
        int p = l.size();
        while (l.size() < n) l.push_back(l[l.size()%p]);
        while (l.size() and l.back() == k-1) l.pop_back();
        if (l.size()) l.back()++;
```

```
    }
    return ret;
}
```

1.32 Shortest Addition Chain

```
// Computa o menor numero de adicoes para construir
// cada valor, comecando com 1 (e podendo salvar variaveis)
// Retorna um par com a dp e o pai na arvore
// A arvore eh tao que o taminho da raiz (1) ate x
// contem os valores que devem ser criados para gerar x
// A profundidade de x na arvore eh dp[x]
// DP funciona para ateh 300, mas a arvore soh funciona
// para ateh 148
//
// 84fcff

// recuperacao certa soh ateh 148 (erra para 149, 233, 298)
pair<vector<int>, vector<int>> addition_chain() {
    int MAX = 301;
    vector<int> dp(MAX), p(MAX);
    for (int n = 2; n < MAX; n++) {
        pair<int, int> val = {INF, -1};
        for (int i = 1; i < n; i++) for (int j = i; j; j =
            p[j])
             if (j == n-i) val = min(val, pair(dp[i]+1, i));
        tie(dp[n], p[n]) = val;
        if (n == 9) p[n] = 8;
        if (n == 149 or n == 233) dp[n]--;
    }
    return {dp, p};
}
```

1.33 Simple Polygon

```
// Verifica se um poligono com n pontos eh simples
//
// O(n log n)
// c724a4

bool operator < (const line& a, const line& b) { //
    comparador pro sweepline
     if (a.p == b.p) return ccw(a.p, a.q, b.q);
     if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or a.p.x+eps
         < b.p.x))
          return ccw(a.p, a.q, b.p);
     return ccw(a.p, b.q, b.p);
}

bool simple(vector<pt> v) {
     auto intersects = [&](pair<line, int> a, pair<line, int>
         b) {
         if ((a.second+1)%v.size() == b.second or
             (b.second+1)%v.size() == a.second) return false;
         return interseg(a.first, b.first);
     };
     vector<line> seg;
     vector<pair<pt, pair<int, int>>> w;
     for (int i = 0; i < v.size(); i++) {
         pt at = v[i], nxt = v[(i+1)%v.size()];
         if (nxt < at) swap(at, nxt);
         seg.push_back(line(at, nxt));
         w.push_back({at, {0, i}});
         w.push_back({nxt, {1, i}});
         // casos degenerados estranhos
         if (isinseg(v[(i+2)%v.size()], line(at, nxt)))
             return 0;
         if (isinseg(v[(i+v.size()-1)%v.size()], line(at,
             nxt))) return 0;
     }
     sort(w.begin(), w.end());
     set<pair<line, int>> se;
     for (auto i : w) {
         line at = seg[i.second.second];
         if (i.second.first == 0) {
             auto nxt = se.lower_bound({at, i.second.second});
             if (nxt != se.end() and intersects(*nxt, {at,
                 i.second.second})) return 0;
             if (nxt != se.begin() and intersects(*(--nxt),
                 {at, i.second.second})) return 0;
```

```
            se.insert({at, i.second.second});
        } else {
            auto nxt = se.upper_bound({at,
                i.second.second}), cur = nxt, prev = --cur;
            if (nxt != se.end() and prev != se.begin()
                and intersects(*nxt, *(--prev))) return 0;
            se.erase(cur);
        }
    }
    return 1;
}
```

1.34 Sweep Direction

```
// Passa por todas as ordenacoes dos pontos definitas por
   "direcoes"
// Assume que nao existem pontos coincidentes
//
// O(n^2 log n)
// 6bb68d

void sweep_direction(vector<pt> v) {
    int n = v.size();
    sort(v.begin(), v.end(), [](pt a, pt b) {
        if (a.x != b.x) return a.x < b.x;
        return a.y > b.y;
    });
    vector<int> at(n);
    iota(at.begin(), at.end(), 0);
    vector<pair<int, int>> swapp;
    for (int i = 0; i < n; i++) for (int j = i+1; j < n; j++)
        swapp.push_back({i, j}), swapp.push_back({j, i});

    sort(swapp.begin(), swapp.end(), [&](auto a, auto b) {
        pt A = rotate90(v[a.first] - v[a.second]);
        pt B = rotate90(v[b.first] - v[b.second]);
        if (quad(A) == quad(B) and !sarea2(pt(0, 0), A, B))
            return a < b;
        return compare_angle(A, B);
    });
    for (auto par : swapp) {
        assert(abs(at[par.first] - at[par.second]) == 1);
        int l = min(at[par.first], at[par.second]),
            r = n-1 - max(at[par.first], at[par.second]);
        // l e r sao quantos caras tem de cada lado do par
           de pontos
        // (cada par eh visitado duas vezes)
        swap(v[at[par.first]], v[at[par.second]]);
        swap(at[par.first], at[par.second]);
    }
}
```

1.35 Triangulacao de Delaunay

```
// Computa a triangulacao de Delaunay, o dual
// do diagrama de Voronoi (a menos de casos degenerados)
// Retorna um grafo indexado pelos indices dos pontos, e as
   arestas
// sao as arestas da triangulacao
// As arestas partindo de um vertice ja vem ordenadas por
   angulo,
// ou seja, se o vertice v nao esta no convex hull, (v, v_i,
   v_{i+1})
// eh um triangulo da triangulacao, em que v_i eh o i-esimo
   vizinho
// Usa o alg d&c, precisa representar MAX_COOR^4, por isso
   __int128
// pra aguentar valores ateh 1e9
//
// Propriedades:
// 1 - O grafo tem no max 3n-6 arestas
// 2 - Para todo triangulo, a circunf. que passa pelos 3
   pontos
//     nao contem estritamente nenhum ponto
// 3 - A MST euclidiana eh subgrafo desse grafo
// 4 - Cada ponto eh vizinho do ponto mais proximo dele
//
// O(n log n)
// 83ebab
```

```cpp
typedef struct QuadEdge* Q;
struct QuadEdge {
    int id;
    pt o;
    Q rot, nxt;
    bool used;

    QuadEdge(int id_ = -1, pt o_ = pt(INF, INF)) :
        id(id_), o(o_), rot(nullptr), nxt(nullptr),
            used(false) {}

    Q rev() const { return rot->rot; }
    Q next() const { return nxt; }
    Q prev() const { return rot->next()->rot; }
    pt dest() const { return rev()->o; }
};

Q edge(pt from, pt to, int id_from, int id_to) {
    Q e1 = new QuadEdge(id_from, from);
    Q e2 = new QuadEdge(id_to, to);
    Q e3 = new QuadEdge;
    Q e4 = new QuadEdge;
    tie(e1->rot, e2->rot, e3->rot, e4->rot) = {e3, e4, e2,
        e1};
    tie(e1->nxt, e2->nxt, e3->nxt, e4->nxt) = {e1, e2, e4,
        e3};
    return e1;
}

void splice(Q a, Q b) {
    swap(a->nxt->rot->nxt, b->nxt->rot->nxt);
    swap(a->nxt, b->nxt);
}

void del_edge(Q& e, Q ne) { // delete e and assign e <- ne
    splice(e, e->prev());
    splice(e->rev(), e->rev()->prev());
    delete e->rev()->rot, delete e->rev();
    delete e->rot; delete e;
    e = ne;
}
```

```cpp
Q conn(Q a, Q b) {
    Q e = edge(a->dest(), b->o, a->rev()->id, b->id);
    splice(e, a->rev()->prev());
    splice(e->rev(), b);
    return e;
}

bool in_c(pt a, pt b, pt c, pt p) { // p ta na circunf. (a,
    b, c) ?
    __int128 p2 = p*p, A = a*a - p2, B = b*b - p2, C = c*c -
        p2;
    return sarea2(p, a, b) * C + sarea2(p, b, c) * A +
        sarea2(p, c, a) * B > 0;
}

pair<Q, Q> build_tr(vector<pt>& p, int l, int r) {
    if (r-l+1 <= 3) {
        Q a = edge(p[l], p[l+1], l, l+1), b = edge(p[l+1],
            p[r], l+1, r);
        if (r-l+1 == 2) return {a, a->rev()};
        splice(a->rev(), b);
        ll ar = sarea2(p[l], p[l+1], p[r]);
        Q c = ar ? conn(b, a) : 0;
        if (ar >= 0) return {a, b->rev()};
        return {c->rev(), c};
    }
    int m = (l+r)/2;
    auto [la, ra] = build_tr(p, l, m);
    auto [lb, rb] = build_tr(p, m+1, r);
    while (true) {
        if (ccw(lb->o, ra->o, ra->dest())) ra =
            ra->rev()->prev();
        else if (ccw(lb->o, ra->o, lb->dest())) lb =
            lb->rev()->next();
        else break;
    }
    Q b = conn(lb->rev(), ra);
    auto valid = [&](Q e) { return ccw(e->dest(), b->dest(),
        b->o); };
    if (ra->o == la->o) la = b->rev();
    if (lb->o == rb->o) rb = b;
    while (true) {
```

```
        Q L = b->rev()->next();
        if (valid(L)) while (in_c(b->dest(), b->o,
            L->dest(), L->next()->dest()))
            del_edge(L, L->next());
        Q R = b->prev();
        if (valid(R)) while (in_c(b->dest(), b->o,
            R->dest(), R->prev()->dest()))
            del_edge(R, R->prev());
        if (!valid(L) and !valid(R)) break;
        if (!valid(L) or (valid(R) and in_c(L->dest(), L->o,
            R->o, R->dest())))
            b = conn(R, b->rev());
        else b = conn(b->rev(), L->rev());
    }
    return {la, rb};
}

vector<vector<int>> delaunay(vector<pt> v) {
    int n = v.size();
    auto tmp = v;
    vector<int> idx(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&](int l, int r) { return
        v[l] < v[r]; });
    for (int i = 0; i < n; i++) v[i] = tmp[idx[i]];
    assert(unique(v.begin(), v.end()) == v.end());
    vector<vector<int>> g(n);
    bool col = true;
    for (int i = 2; i < n; i++) if (sarea2(v[i], v[i-1],
        v[i-2])) col = false;
    if (col) {
        for (int i = 1; i < n; i++)
            g[idx[i-1]].push_back(idx[i]),
                g[idx[i]].push_back(idx[i-1]);
        return g;
    }
    Q e = build_tr(v, 0, n-1).first;
    vector<Q> edg = {e};
    for (int i = 0; i < edg.size(); e = edg[i++]) {
        for (Q at = e; !at->used; at = at->next()) {
            at->used = true;
            g[idx[at->id]].push_back(idx[at->rev()->id]);
            edg.push_back(at->rev());
        }
    }
    return g;
}
```

1.36 Triangulos em Grafos

```
// get_triangles(i) encontra todos os triangulos ijk no grafo
// Custo nas arestas
// retorna {custo do triangulo, {j, k}}
//
// O(m sqrt(m) log(n)) se chamar para todos os vertices
// f1adbc

vector<pair<int, int>> g[MAX]; // {para, peso}

#warning o 'g' deve estar ordenado
vector<pair<int, pair<int, int>>> get_triangles(int i) {
    vector<pair<int, pair<int, int>>> tri;
    for (pair<int, int> j : g[i]) {
        int a = i, b = j.first;
        if (g[a].size() > g[b].size()) swap(a, b);
        for (pair<int, int> c : g[a]) if (c.first != b and
            c.first > j.first) {
            auto it = lower_bound(g[b].begin(), g[b].end(),
                make_pair(c.first, -INF));
            if (it == g[b].end() or it->first != c.first)
                continue;
            tri.push_back({j.second+c.second+it->second, {a
                == i ? b : a, c.first}});
        }
    }
    return tri;
}
```

2 Primitivas

2.1 Aritmetica Modular

```
// O mod tem q ser primo
// d99334

template<int p> struct mod_int {
    ll pow(ll b, ll e) {
        if (e == 0) return 1;
        ll r = pow(b*b%p, e/2);
        if (e%2 == 1) r = (r*b)%p;
        return r;
    }
    ll inv(ll b) { return pow(b, p-2); }

    using m = mod_int;
    int v;
    mod_int() : v(0) {}
    mod_int(ll v_) {
        if (v_ >= p or v_ <= -p) v_ %= p;
        if (v_ < 0) v_ += p;
        v = v_;
    }
    m& operator+=(const m &a) {
        v += a.v;
        if (v >= p) v -= p;
        return *this;
    }
    m& operator-=(const m &a) {
        v -= a.v;
        if (v < 0) v += p;
        return *this;
    }
    m& operator*=(const m &a) {
        v = v * ll(a.v) % p;
        return *this;
    }
    m& operator/=(const m &a) {
        v = v* inv(a.v) % p;
        return *this;
    }
    m operator-(){ return m(-v); }
    m& operator^=(ll e) {
        if (e < 0){
            v = inv(v);
            e = -e;
        }
        v = pow(v, e%(p-1));
        return *this;
    }
    bool operator==(const m &a) { return v == a.v; }
    bool operator!=(const m &a) { return v != a.v; }

    friend istream &operator>>(istream &in, m& a) {
        ll val; in >> val;
        a = m(val);
        return in;
    }
    friend ostream &operator<<(ostream &out, m a) {
        return out << a.v;
    }
    friend m operator+(m a, m b) { return a+=b; }
    friend m operator-(m a, m b) { return a-=b; }
    friend m operator*(m a, m b) { return a*=b; }
    friend m operator/(m a, m b) { return a/=b; }
    friend m operator^(m a, ll e) { return a^=e; }
};

typedef mod_int<(int)1e9+7> mint;
```

2.2 Big Integer

```
// Complexidades: (para n digitos)
// Soma, subtracao, comparacao - O(n)
// Multiplicacao - O(n log(n))
// Divisao, resto - O(n^2)

struct bint {
    static const int BASE = 1e9;
    vector<int> v;
```

```
bool neg;

bint() : neg(0) {}
bint(int val) : bint() { *this = val; }
bint(long long val) : bint() { *this = val; }

void trim() {
    while (v.size() and v.back() == 0) v.pop_back();
    if (!v.size()) neg = 0;
}

// converter de/para string | cin/cout
bint(const char* s) : bint() { from_string(string(s)); }
bint(const string& s) : bint() { from_string(s); }
void from_string(const string& s) {
    v.clear(), neg = 0;
    int ini = 0;
    while (ini < s.size() and (s[ini] == '-' or s[ini]
        == '+' or s[ini] == '0'))
        if (s[ini++] == '-') neg = 1;
    for (int i = s.size()-1; i >= ini; i -= 9) {
        int at = 0;
        for (int j = max(ini, i - 8); j <= i; j++) at =
            10*at + (s[j]-'0');
        v.push_back(at);
    }
    if (!v.size()) neg = 0;
}
string to_string() const {
    if (!v.size()) return "0";
    string ret;
    if (neg) ret += '-';
    for (int i = v.size()-1; i >= 0; i--) {
        string at = ::to_string(v[i]);
        int add = 9 - at.size();
        if (i+1 < v.size()) for (int j = 0; j < add;
            j++) ret += '0';
        ret += at;
    }
    return ret;
}
friend istream& operator>>(istream& in, bint& val) {
    string s; in >> s;
    val = s;
    return in;
}
friend ostream& operator<<(ostream& out, const bint&
    val) {
    string s = val.to_string();
    out << s;
    return out;
}

// operators
friend bint abs(bint val) {
    val.neg = 0;
    return val;
}
friend bint operator-(bint val) {
    if (val != 0) val.neg ^= 1;
    return val;
}
bint& operator=(const bint& val) { v = val.v, neg =
    val.neg; return *this; }
bint& operator=(long long val) {
    v.clear(), neg = 0;
    if (val < 0) neg = 1, val *= -1;
    for (; val; val /= BASE) v.push_back(val % BASE);
    return *this;
}
int cmp(const bint& r) const { // menor: -1 | igual: 0 |
    maior: 1
    if (neg != r.neg) return neg ? -1 : 1;
    if (v.size() != r.v.size()) {
        int ret = v.size() < r.v.size() ? -1 : 1;
        return neg ? -ret : ret;
    }
    for (int i = int(v.size())-1; i >= 0; i--) {
        if (v[i] != r.v[i]) {
            int ret = v[i] < r.v[i] ? -1 : 1;
            return neg ? -ret : ret;
        }
    }
    return 0;
```

```
}
friend bool operator<(const bint& l, const bint& r) {
    return l.cmp(r) == -1; }
friend bool operator>(const bint& l, const bint& r) {
    return l.cmp(r) == 1; }
friend bool operator<=(const bint& l, const bint& r) {
    return l.cmp(r) <= 0; }
friend bool operator>=(const bint& l, const bint& r) {
    return l.cmp(r) >= 0; }
friend bool operator==(const bint& l, const bint& r) {
    return l.cmp(r) == 0; }
friend bool operator!=(const bint& l, const bint& r) {
    return l.cmp(r) != 0; }

bint& operator +=(const bint& r) {
    if (!r.v.size()) return *this;
    if (neg != r.neg) return *this -= -r;
    for (int i = 0, c = 0; i < r.v.size() or c; i++) {
        if (i == v.size()) v.push_back(0);
        v[i] += c + (i < r.v.size() ? r.v[i] : 0);
        if ((c = v[i] >= BASE)) v[i] -= BASE;
    }
    return *this;
}
friend bint operator+(bint a, const bint& b) { return a
    += b; }
bint& operator -=(const bint& r) {
    if (!r.v.size()) return *this;
    if (neg != r.neg) return *this += -r;
    if ((!neg and *this < r) or (neg and r < *this)) {
        *this = r - *this;
        neg ^= 1;
        return *this;
    }
    for (int i = 0, c = 0; i < r.v.size() or c; i++) {
        v[i] -= c + (i < r.v.size() ? r.v[i] : 0);
        if ((c = v[i] < 0)) v[i] += BASE;
    }
    trim();
    return *this;
}
friend bint operator-(bint a, const bint& b) { return a
    -= b; }

// operators de * / %
bint& operator *=(int val) {
    if (val < 0) val *= -1, neg ^= 1;
    for (int i = 0, c = 0; i < v.size() or c; i++) {
        if (i == v.size()) v.push_back(0);
        long long at = (long long) v[i] * val + c;
        v[i] = at % BASE;
        c = at / BASE;
    }
    trim();
    return *this;
}
friend bint operator *(bint a, int b) { return a *= b; }
friend bint operator *(int a, bint b) { return b *= a; }
using cplx = complex<double>;
void fft(vector<cplx>& a, bool f, int N, vector<int>&
    rev) const {
    for (int i = 0; i < N; i++) if (i < rev[i])
        swap(a[i], a[rev[i]]);
    vector<cplx> roots(N);
    for (int n = 2; n <= N; n *= 2) {
        const static double PI = acos(-1);
        for (int i = 0; i < n/2; i++) {
            double alpha = (2*PI*i)/n;
            if (f) alpha = -alpha;
            roots[i] = cplx(cos(alpha), sin(alpha));
        }
        for (int pos = 0; pos < N; pos += n)
            for (int l = pos, r = pos+n/2, m = 0; m <
                n/2; l++, r++, m++) {
                auto t = roots[m]*a[r];
                a[r] = a[l] - t;
                a[l] = a[l] + t;
            }
    }
    if (!f) return;
    auto invN = cplx(1)/cplx(N);
    for (int i = 0; i < N; i++) a[i] *= invN;
}
vector<long long> convolution(const vector<int>& a,
```

```
    const vector<int>& b) const {
    vector<cplx> l(a.begin(), a.end()), r(b.begin(),
        b.end());
    int ln = l.size(), rn = r.size(), N = ln+rn+1, n =
        1, log_n = 0;
    while (n <= N) n <<= 1, log_n++;
    vector<int> rev(n);
    for (int i = 0; i < n; i++) {
        rev[i] = 0;
        for (int j = 0; j < log_n; j++) if (i>>j&1)
            rev[i] |= 1 << (log_n-1-j);
    }
    l.resize(n), r.resize(n);
    fft(l, false, n, rev), fft(r, false, n, rev);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    fft(l, true, n, rev);
    vector<long long> ret;
    for (auto& i : l) ret.push_back(round(i.real()));
    return ret;
}
vector<int> convert_base(const vector<int>& a, int from,
    int to) const {
    static vector<long long> pot(10, 1);
    if (pot[1] == 1) for (int i = 1; i < 10; i++) pot[i]
        = 10*pot[i-1];
    vector<int> ret;
    long long at = 0;
    int digits = 0;
    for (int i : a) {
        at += i * pot[digits];
        digits += from;
        while (digits >= to) {
            ret.push_back(at % pot[to]);
            at /= pot[to];
            digits -= to;
        }
    }
    ret.push_back(at);
    while (ret.size() and ret.back() == 0)
        ret.pop_back();
    return ret;
}
bint operator*(const bint& r) const { // O(n log(n))
    bint ret;
    ret.neg = neg ^ r.neg;
    auto conv = convolution(convert_base(v, 9, 4),
        convert_base(r.v, 9, 4));
    long long c = 0;
    for (auto i : conv) {
        long long at = i+c;
        ret.v.push_back(at % 10000);
        c = at / 10000;
    }
    for (; c; c /= 10000) ret.v.push_back(c%10000);
    ret.v = convert_base(ret.v, 4, 9);
    if (!ret.v.size()) ret.neg = 0;
    return ret;
}
bint& operator*=(const bint& r) { return *this = *this *
    r; };
bint& operator/=(int val) {
    if (val < 0) neg ^= 1, val *= -1;
    for (int i = int(v.size())-1, c = 0; i >= 0; i--) {
        long long at = v[i] + c * (long long) BASE;
        v[i] = at / val;
        c = at % val;
    }
    trim();
    return *this;
}
friend bint operator/(bint a, int b) { return a /= b; }
int operator %=(int val) {
    if (val < 0) val *= -1;
    long long at = 0;
    for (int i = int(v.size())-1; i >= 0; i--)
        at = (BASE * at + v[i]) % val;
    if (neg) at *= -1;
    return at;
}
friend int operator%(bint a, int b) { return a %= b; }
friend pair<bint, bint> divmod(const bint& a_, const
    bint& b_) { // O(n^2)
    if (a_ == 0) return {0, 0};
    int norm = BASE / (b_.v.back() + 1);
```

```
        bint a = abs(a_) * norm;
        bint b = abs(b_) * norm;
        bint q, r;
        for (int i = a.v.size() - 1; i >= 0; i--) {
            r *= BASE, r += a.v[i];
            long long upper = b.v.size() < r.v.size() ?
               r.v[b.v.size()] : 0;
            int lower = b.v.size() - 1 < r.v.size() ?
               r.v[b.v.size() - 1] : 0;
            int d = (upper * BASE + lower) / b.v.back();
            r -= b*d;
            while (r < 0) r += b, d--; // roda O(1) vezes
            q.v.push_back(d);
        }
        reverse(q.v.begin(), q.v.end());
        q.neg = a_.neg ^ b_.neg;
        r.neg = a_.neg;
        q.trim(), r.trim();
        return {q, r / norm};
    }
    bint operator/(const bint& val) { return divmod(*this,
       val).first; }
    bint& operator/=(const bint& val) { return *this = *this
       / val; }
    bint operator%(const bint& val) { return divmod(*this,
       val).second; }
    bint& operator%=(const bint& val) { return *this = *this
       % val; }
};
```

2.3 Matroid

```
// Matroids de Grafo e Particao
// De modo geral, toda Matroid contem um build() linear
// e uma funcao constante oracle()
// oracle(i) responde se o conjunto continua independente
// apos adicao do elemento i
// oracle(i, j) responde se o conjunto continua indepente
// apos trocar o elemento i pelo elemento j
//
// Intersecao sem peso O(r^2 n)
// em que n eh o tamanho do conjunto e r eh o tamanho da
   resposta

// Matroid Grafica
// Matroid das florestas de um grafo
// Um conjunto de arestas eh independente se formam uma
   floresta
//
// build() : O(n)
// oracle() : O(1)
// 691847

struct graphic_matroid {
    int n, m, t;
    vector<array<int, 2>> edges;
    vector<vector<int>> g;
    vector<int> comp, in, out;
    graphic_matroid(int n_, vector<array<int, 2>> edges_)
        : n(n_), m(edges_.size()), edges(edges_), g(n),
          comp(n), in(n), out(n) {}
    void dfs(int u) {
        in[u] = t++;
        for (auto v : g[u]) if (in[v] == -1)
            comp[v] = comp[u], dfs(v);
        out[u] = t;
    }
    void build(vector<int> I) {
        t = 0;
        for (int u = 0; u < n; u++) g[u].clear(), in[u] = -1;
        for (int e : I) {
            auto [u, v] = edges[e];
            g[u].push_back(v), g[v].push_back(u);
        }
        for (int u = 0; u < n; u++) if (in[u] == -1)
            comp[u] = u, dfs(u);
    }
    bool is_ancestor(int u, int v) {
        return in[u] <= in[v] and in[v] < out[u];
    }
    bool oracle(int e) {
        return comp[edges[e][0]] != comp[edges[e][1]];
```

```
    }
    bool oracle(int e, int f) {
        if (oracle(f)) return true;
        int u = edges[e][in[edges[e][0]] < in[edges[e][1]]];
        return is_ancestor(u, edges[f][0]) != is_ancestor(u,
            edges[f][1]);
    }
};

// Matroid de particao ou cores
// Um conjunto eh independente se a quantidade de elementos
// de cada cor nao excede a capacidade da cor
// Quando todas as capacidades sao 1, um conjunto eh
   independente
// se todas as suas cores sao distintas
//
// build() : O(n)
// oracle() : O(1)
// caa72a

struct partition_matroid {
    vector<int> cap, color, d;
    partition_matroid(vector<int> cap_, vector<int> color_)
        : cap(cap_), color(color_), d(cap.size()) {}
    void build(vector<int> I) {
        fill(d.begin(), d.end(), 0);
        for (int u : I) d[color[u]]++;
    }
    bool oracle(int u) {
        return d[color[u]] < cap[color[u]];
    }
    bool oracle(int u, int v) {
        return color[u] == color[v] or oracle(v);
    }
};

// Intersecao de matroid sem pesos
// Dadas duas matroids M1 e M2 definidas sobre o mesmo
// conjunto I, retorna o maior subconjunto de I
// que eh independente tanto para M1 quanto para M2
//
// O(r^2*n)
```

```
// 899f94

// Matroid "pesada" deve ser a M2
template<typename Matroid1, typename Matroid2>
vector<int> matroid_intersection(int n, Matroid1 M1,
    Matroid2 M2) {
    vector<bool> b(n);
    vector<int> I[2];
    bool converged = false;
    while (!converged) {
        I[0].clear(), I[1].clear();
        for (int u = 0; u < n; u++) I[b[u]].push_back(u);

        M1.build(I[1]), M2.build(I[1]);
        vector<bool> target(n), pushed(n);
        queue<int> q;
        for (int u : I[0]) {
            target[u] = M2.oracle(u);
            if (M1.oracle(u)) pushed[u] = true, q.push(u);
        }
        vector<int> p(n, -1);
        converged = true;
        while (q.size()) {
            int u = q.front(); q.pop();
            if (target[u]) {
                converged = false;
                for (int v = u; v != -1; v = p[v]) b[v] =
                    !b[v];
                break;
            }
            for (int v : I[!b[u]]) if (!pushed[v]) {
                if ((b[u] and M1.oracle(u, v)) or (b[v] and
                    M2.oracle(v, u)))
                    p[v] = u, pushed[v] = true, q.push(v);
            }
        }
    }
    return I[1];
}

// Intersecao de matroid com pesos
// Dadas duas matroids M1 e M2 e uma funcao de pesos w,
```

```
    todas definidas sobre
// um conjunto I retorna o maior subconjunto de I
    (desempatado pelo menor peso)
// que eh independente tanto para M1 quanto para M2
// A resposta eh construida incrementando o tamanho conjunto
    I de 1 em 1
// Se nao tiver custo negativo, nao precisa de SPFA
//
// O(r^3*n) com SPFA
// O(r^2*n*log(n)) com Dijkstra e potencial
// 3a09d1

template<typename T, typename Matroid1, typename Matroid2>
vector<int> weighted_matroid_intersection(int n, vector<T>
    w, Matroid1 M1, Matroid2 M2) {
     vector<bool> b(n), target(n), is_inside(n);
     vector<int> I[2], from(n);
     vector<pair<T, int>> d(n);
     auto check_edge = [&](int u, int v) {
          return (b[u] and M1.oracle(u, v)) or (b[v] and
              M2.oracle(v, u));
     };
     while (true) {
          I[0].clear(), I[1].clear();
          for (int u = 0; u < n; u++) I[b[u]].push_back(u);
          // I[1] contem o conjunto de tamanho I[1].size() de
              menor peso
          M1.build(I[1]), M2.build(I[1]);
          for (int u = 0; u < n; u++) {
               target[u] = false, is_inside[u] = false, from[u]
                   = -1;
               d[u] = {numeric_limits<T>::max(), INF};
          }
          deque<T> q;
          sort(I[0].begin(), I[0].end(), [&](int i, int j){
              return w[i] < w[j]; });
          for (int u : I[0]) {
               target[u] = M2.oracle(u);
               if (M1.oracle(u)) {
                    if (is_inside[u]) continue;
                    d[u] = {w[u], 0};
                    if (!q.empty() and d[u] > d[q.front()])
                        q.push_back(u);
                    else q.push_front(u);
                    is_inside[u] = true;
               }
          }
          while (q.size()) {
               int u = q.front(); q.pop_front();
               is_inside[u] = false;
               for (int v : I[!b[u]]) if (check_edge(u, v)) {
                    pair<T, int> nd(d[u].first + w[v],
                        d[u].second + 1);
                    if (nd < d[v]) {
                         from[v] = u, d[v] = nd;
                         if (is_inside[v]) continue;
                         if (q.size() and d[v] > d[q.front()])
                             q.push_back(v);
                         else q.push_front(v);
                         is_inside[v] = true;
                    }
               }
          }
          pair<T, int> mini = pair(numeric_limits<T>::max(),
              INF);
          int targ = -1;
          for (int u : I[0]) if (target[u] and d[u] < mini)
               mini = d[u], targ = u;
          if (targ != -1) for (int u = targ; u != -1; u =
              from[u])
               b[u] = !b[u], w[u] *= -1;
          else break;
     }
     return I[1];
}
```

2.4 Primitivas de fracao

```
// Funciona com o Big Int
// cdb445

template<typename T = int> struct frac {
```

```
    T num, den;
    template<class U, class V>
    frac(U num_ = 0, V den_ = 1) : num(num_), den(den_) {
        assert(den != 0);
        if (den < 0) num *= -1, den *= -1;
        T g = gcd(abs(num), den);
        num /= g, den /= g;
    }

    friend bool operator<(const frac& l, const frac& r) {
        return l.num * r.den < r.num * l.den;
    }
    friend frac operator+(const frac& l, const frac& r) {
        return {l.num*r.den + l.den*r.num, l.den*r.den};
    }
    friend frac operator-(const frac& l, const frac& r) {
        return {l.num*r.den - l.den*r.num, l.den*r.den};
    }
    friend frac operator*(const frac& l, const frac& r) {
        return {l.num*r.num, l.den*r.den};
    }
    friend frac operator/(const frac& l, const frac& r) {
        return {l.num*r.den, l.den*r.num};
    }
    friend ostream& operator<<(ostream& out, frac f) {
        out << f.num << '/' << f.den;
        return out;
    }
};
```

2.5 Primitivas de matriz - exponenciacao

```
// d05c24

#define MODULAR false
template<typename T> struct matrix : vector<vector<T>> {
    int n, m;

    void print() {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) cout <<
                (*this)[i][j] << " ";
            cout << endl;
        }
    }

    matrix(int n_, int m_, bool ident = false) :
            vector<vector<T>>(n_, vector<T>(m_, 0)), n(n_),
                m(m_) {
        if (ident) {
            assert(n == m);
            for (int i = 0; i < n; i++) (*this)[i][i] = 1;
        }
    }
    matrix(const vector<vector<T>>& c) :
        vector<vector<T>>(c),
        n(c.size()), m(c[0].size()) {}
    matrix(const initializer_list<initializer_list<T>>& c) {
        vector<vector<T>> val;
        for (auto& i : c) val.push_back(i);
        *this = matrix(val);
    }

    matrix<T> operator*(matrix<T>& r) {
        assert(m == r.n);
        matrix<T> M(n, r.m);
        for (int i = 0; i < n; i++) for (int k = 0; k < m;
            k++)
            for (int j = 0; j < r.m; j++) {
                T add = (*this)[i][k] * r[k][j];
#if MODULAR
#warning Usar matrix<ll> e soh colocar valores em [0, MOD)
    na matriz!
                M[i][j] += add%MOD;
                if (M[i][j] >= MOD) M[i][j] -= MOD;
#else
                M[i][j] += add;
#endif
            }
        return M;
    }
    matrix<T> operator^(ll e){
```

```
        matrix<T> M(n, n, true), at = *this;
        while (e) {
            if (e&1) M = M*at;
            e >>= 1;
            at = at*at;
        }
        return M;
    }
    void apply_transform(matrix M, ll e){
        auto& v = *this;
        while (e) {
            if (e&1) v = M*v;
            e >>= 1;
            M = M*M;
        }
    }
};
```

2.6 Primitivas Geometricas

```
typedef double ld;
const ld DINF = 1e18;
const ld pi = acos(-1.0);
const ld eps = 1e-9;

#define sq(x) ((x)*(x))

bool eq(ld a, ld b) {
    return abs(a - b) <= eps;
}

// a8b7d6
struct pt { // ponto
    ld x, y;
    pt(ld x_ = 0, ld y_ = 0) : x(x_), y(y_) {}
    bool operator < (const pt p) const {
        if (!eq(x, p.x)) return x < p.x;
        if (!eq(y, p.y)) return y < p.y;
        return 0;
    }
    bool operator == (const pt p) const {
        return eq(x, p.x) and eq(y, p.y);
    }
    pt operator + (const pt p) const { return pt(x+p.x,
       y+p.y); }
    pt operator - (const pt p) const { return pt(x-p.x,
       y-p.y); }
    pt operator * (const ld c) const { return pt(x*c  , y*c
       ); }
    pt operator / (const ld c) const { return pt(x/c  , y/c
       ); }
    ld operator * (const pt p) const { return x*p.x + y*p.y;
       }
    ld operator ^ (const pt p) const { return x*p.y - y*p.x;
       }
    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y;
    }
};

// 7ab617
struct line { // reta
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// PONTO & VETOR

// c684fb
ld dist(pt p, pt q) { // distancia
    return hypot(p.y - q.y, p.x - q.x);
}

// 80f2b6
ld dist2(pt p, pt q) { // quadrado da distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}
```

```cpp
// cf7f33
ld norm(pt v) { // norma do vetor
    return dist(pt(0, 0), v);
}

// 404df7
ld angle(pt v) { // angulo do vetor com o eixo x
    ld ang = atan2(v.y, v.x);
    if (ang < 0) ang += 2*pi;
    return ang;
}

// 1b1d4a
ld sarea(pt p, pt q, pt r) { // area com sinal
    return ((q-p)^(r-q))/2;
}

// 98c42f
bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
    return eq(sarea(p, q, r), 0);
}

// 85d09d
bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
    return sarea(p, q, r) > eps;
}

// 41a7b4
pt rotate(pt p, ld th) { // rotaciona o ponto th radianos
    return pt(p.x * cos(th) - p.y * sin(th),
            p.x * sin(th) + p.y * cos(th));
}

// e4ad5e
pt rotate90(pt p) { // rotaciona 90 graus
    return pt(-p.y, p.x);
}

// RETA

// 0fb984
bool isvert(line r) { // se r eh vertical
    return eq(r.p.x, r.q.x);
}

// 726d68
bool isinseg(pt p, line r) { // se p pertence ao seg de r
    pt a = r.p - p, b = r.q - p;
    return eq((a ^ b), 0) and (a * b) < eps;
}

// a0a30b
ld get_t(pt v, line r) { // retorna t tal que t*v pertence a
   reta r
    return (r.p^r.q) / ((r.p-r.q)^v);
}

// 2329fe
pt proj(pt p, line r) { // projecao do ponto p na reta r
    if (r.p == r.q) return r.p;
    r.q = r.q - r.p; p = p - r.p;
    pt proj = r.q * ((p*r.q) / (r.q*r.q));
    return proj + r.p;
}

// 111fd2
pt inter(line r, line s) { // r inter s
    if (eq((r.p - r.q) ^ (s.p - s.q), 0)) return pt(DINF,
        DINF);
    r.q = r.q - r.p, s.p = s.p - r.p, s.q = s.q - r.p;
    return r.q * get_t(r.q, s) + r.p;
}

// 35998c
bool interseg(line r, line s) { // se o seg de r intersecta
   o seg de s
    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
            ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

// 1b72e1
```

```
ld disttoline(pt p, line r) { // distancia do ponto a reta
    return 2 * abs(sarea(p, r.p, r.q)) / dist(r.p, r.q);
}

// 3679c0
ld disttoseg(pt p, line r) { // distancia do ponto ao seg
    if ((r.q - r.p)*(p - r.p) < 0) return dist(r.p, p);
    if ((r.p - r.q)*(p - r.q) < 0) return dist(r.q, p);
    return disttoline(p, r);
}

// 222358
ld distseg(line a, line b) { // distancia entre seg
    if (interseg(a, b)) return 0;

    ld ret = DINF;
    ret = min(ret, disttoseg(a.p, b));
    ret = min(ret, disttoseg(a.q, b));
    ret = min(ret, disttoseg(b.p, a));
    ret = min(ret, disttoseg(b.q, a));

    return ret;
}

// POLIGONO

// corta poligono com a reta r deixando os pontos p tal que
// ccw(r.p, r.q, p)
// 2538f9
vector<pt> cut_polygon(vector<pt> v, line r) { // O(n)
    vector<pt> ret;
    for (int j = 0; j < v.size(); j++) {
        if (ccw(r.p, r.q, v[j])) ret.push_back(v[j]);
        if (v.size() == 1) continue;
        line s(v[j], v[(j+1)%v.size()]);
        pt p = inter(r, s);
        if (isinseg(p, s)) ret.push_back(p);
    }
    ret.erase(unique(ret.begin(), ret.end()), ret.end());
    if (ret.size() > 1 and ret.back() == ret[0])
        ret.pop_back();
    return ret;
}

// distancia entre os retangulos a e b (lados paralelos aos
    eixos)
// assume que ta representado (inferior esquerdo, superior
    direito)
// 630253
ld dist_rect(pair<pt, pt> a, pair<pt, pt> b) {
    ld hor = 0, vert = 0;
    if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x -
        b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y -
        a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y -
        b.second.y;
    return dist(pt(0, 0), pt(hor, vert));
}

// 5df9cf
ld polarea(vector<pt> v) { // area do poligono
    ld ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea(pt(0, 0), v[i], v[(i + 1) % v.size()]);
    return abs(ret);
}

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
// a6423f
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i+1)%v.size();
        if (eq(p.y, v[i].y) and eq(p.y, v[j].y)) {
            if ((v[i]-p)*(v[j]-p) < eps) return 2;
            continue;
        }
        bool baixo = v[i].y+eps < p.y;
        if (baixo == (v[j].y+eps < p.y)) continue;
        auto t = (p-v[i])^(v[j]-v[i]);
```

```
        if (eq(t, 0)) return 2;
        if (baixo == (t > eps)) qt += baixo ? 1 : -1;
    }
    return qt != 0;
}

// c58350
bool interpol(vector<pt> v1, vector<pt> v2) { // se dois
   poligonos se intersectam - O(n*m)
    int n = v1.size(), m = v2.size();
    for (int i = 0; i < n; i++) if (inpol(v2, v1[i])) return
       1;
    for (int i = 0; i < n; i++) if (inpol(v1, v2[i])) return
       1;
    for (int i = 0; i < n; i++) for (int j = 0; j < m; j++)
        if (interseg(line(v1[i], v1[(i+1)%n]), line(v2[j],
           v2[(j+1)%m]))) return 1;
    return 0;
}

// 12559f
ld distpol(vector<pt> v1, vector<pt> v2) { // distancia
   entre poligonos
    if (interpol(v1, v2)) return 0;

    ld ret = DINF;

    for (int i = 0; i < v1.size(); i++) for (int j = 0; j <
       v2.size(); j++)
        ret = min(ret, distseg(line(v1[i], v1[(i + 1) %
           v1.size()]),
                    line(v2[j], v2[(j + 1) % v2.size()])));
    return ret;
}

// 32623c
vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n
   log(n))
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    sort(v.begin(), v.end());
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l[l.size()-2],
           l.back(), v[i]))
            l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u[u.size()-2],
           u.back(), v[i]))
            u.pop_back();
        u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

struct convex_pol {
    vector<pt> pol;

    // nao pode ter ponto colinear no convex hull
    convex_pol() {}
    convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

    // se o ponto ta dentro do hull - O(log(n))
    // 800813
    bool is_inside(pt p) {
        if (pol.size() == 1) return p == pol[0];
        int l = 1, r = pol.size();
        while (l < r) {
            int m = (l+r)/2;
            if (ccw(p, pol[0], pol[m])) l = m+1;
            else r = m;
        }
        if (l == 1) return isinseg(p, line(pol[0], pol[1]));
        if (l == pol.size()) return false;
        return !ccw(p, pol[l], pol[l-1]);
    }
    // ponto extremo em relacao a cmp(p, q) = p mais extremo
       q
    // (copiado de
       https://github.com/gustavoM32/caderno-zika)
    // 56ccd2
```

```
    int extreme(const function<bool(pt, pt)>& cmp) {
        int n = pol.size();
        auto extr = [&](int i, bool& cur_dir) {
            cur_dir = cmp(pol[(i+1)%n], pol[i]);
            return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
        };
        bool last_dir, cur_dir;
        if (extr(0, last_dir)) return 0;
        int l = 0, r = n;
        while (l+1 < r) {
            int m = (l+r)/2;
            if (extr(m, cur_dir)) return m;
            bool rel_dir = cmp(pol[m], pol[l]);
            if ((!last_dir and cur_dir) or
                    (last_dir == cur_dir and rel_dir ==
                       cur_dir)) {
                l = m;
                last_dir = cur_dir;
            } else r = m;
        }
        return l;
    }
    int max_dot(pt v) {
        return extreme([&](pt p, pt q) { return p*v > q*v;
           });
    }
    pair<int, int> tangents(pt p) {
        auto L = [&](pt q, pt r) { return ccw(p, q, r); };
        auto R = [&](pt q, pt r) { return ccw(p, r, q); };
        return {extreme(L), extreme(R)};
    }
};

// CIRCUNFERENCIA

// a125e4
pt getcenter(pt a, pt b, pt c) { // centro da circunf dado 3
    pontos
    b = (a + b) / 2;
    c = (a + c) / 2;
    return inter(line(b, b + rotate90(a - b)),
            line(c, c + rotate90(a - c)));
}

// cd80c0
vector<pt> circ_line_inter(pt a, pt b, pt c, ld r) { //
    intersecao da circunf (c, r) e reta ab
    vector<pt> ret;
    b = b-a, a = a-c;
    ld A = b*b;
    ld B = a*b;
    ld C = a*a - r*r;
    ld D = B*B - A*C;
    if (D < -eps) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+eps))/A);
    if (D > eps) ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

// fb11d8
vector<pt> circ_inter(pt a, pt b, ld r, ld R) { //
    intersecao da circunf (a, r) e (b, R)
    vector<pt> ret;
    ld d = dist(a, b);
    if (d > r+R or d+min(r, R) < max(r, R)) return ret;
    ld x = (d*d-R*R+r*r)/(2*d);
    ld y = sqrt(r*r-x*x);
    pt v = (b-a)/d;
    ret.push_back(a+v*x + rotate90(v)*y);
    if (y > 0) ret.push_back(a+v*x - rotate90(v)*y);
    return ret;
}

// 3a44fb
bool operator <(const line& a, const line& b) { //
    comparador pra reta
    // assume que as retas tem p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    if (!eq(angle(v1), angle(v2))) return angle(v1) <
       angle(v2);
    return ccw(a.p, a.q, b.p); // mesmo angulo
}
bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
```

```
}

// comparador pro set pra fazer sweep line com segmentos
// 36729f
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
        // assume que os segmentos tem p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or
            a.p.x+eps < b.p.x))
             return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador pro set pra fazer sweep angle com segmentos
// f778aa
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) + eps < get_t(dir, b);
    }
};
```

2.7 Primitivas Geometricas 3D

```
typedef double ld;
const ld DINF = 1e18;
const ld eps = 1e-9;

#define sq(x) ((x)*(x))

bool eq(ld a, ld b) {
        return abs(a - b) <= eps;
}

struct pt { // ponto
        ld x, y, z;
        pt(ld x_ = 0, ld y_ = 0, ld z_ = 0) : x(x_), y(y_),
            z(z_) {}
        bool operator < (const pt p) const {
                if (!eq(x, p.x)) return x < p.x;
                if (!eq(y, p.y)) return y < p.y;
                if (!eq(z, p.z)) return z < p.z;
                return 0;
        }
        bool operator == (const pt p) const {
                return eq(x, p.x) and eq(y, p.y) and eq(z,
                    p.z);
        }
        pt operator + (const pt p) const { return pt(x+p.x,
            y+p.y, z+p.z); }
        pt operator - (const pt p) const { return pt(x-p.x,
            y-p.y, z-p.z); }
        pt operator * (const ld c) const { return pt(x*c  ,
            y*c  , z*c  ); }
        pt operator / (const ld c) const { return pt(x/c  ,
            y/c  , z/c  ); }
        ld operator * (const pt p) const { return x*p.x +
            y*p.y + z*p.z; }
        pt operator ^ (const pt p) const { return pt(y*p.z -
            z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); }
        friend istream& operator >> (istream& in, pt& p) {
                return in >> p.x >> p.y >> p.z;
        }
};

struct line { // reta
        pt p, q;
        line() {}
        line(pt p_, pt q_) : p(p_), q(q_) {}
        friend istream& operator >> (istream& in, line& r) {
                return in >> r.p >> r.q;
        }
};

struct plane { // plano
        array<pt, 3> p;  // pontos que definem o plano
        array<ld, 4> eq; // equacao do plano
        plane() {}
        plane(pt p_, pt q_, pt r_) : p({p_, q_, r_}) {
            build(); }
```

```
	friend istream& operator >> (istream& in, plane& P) {
		return in >> P.p[0] >> P.p[1] >> P.p[2];
		P.build();
	}
	void build() {
		pt dir = (p[1] - p[0]) ^ (p[2] - p[0]);
		eq = {dir.x, dir.y, dir.z, dir*p[0]*(-1)};
	}
};

// converte de coordenadas polares para cartesianas
// (angulos devem estar em radianos)
// phi eh o angulo com o eixo z (cima) theta eh o angulo de
   rotacao ao redor de z
pt convert(ld rho, ld th, ld phi) {
	return pt(sin(phi) * cos(th), sin(phi) * sin(th),
	   cos(phi)) * rho;
}

// projecao do ponto p na reta r
pt proj(pt p, line r) {
	if (r.p == r.q) return r.p;
	r.q = r.q - r.p; p = p - r.p;
	pt proj = r.q * ((p*r.q) / (r.q*r.q));
	return proj + r.p;
}

// projecao do ponto p no plano P
pt proj(pt p, plane P) {
	p = p - P.p[0], P.p[1] = P.p[1] - P.p[0], P.p[2] =
	   P.p[2] - P.p[0];
	pt norm = P.p[1] ^ P.p[2];
	pt proj = p - (norm * (norm * p) / (norm*norm));
	return proj + P.p[0];
}

// distancia
ld dist(pt a, pt b) {
	return sqrt(sq(a.x-b.x) + sq(a.y-b.y) + sq(a.z-b.z));
}

// distancia ponto reta
ld distline(pt p, line r) {
	return dist(p, proj(p, r));
}

// distancia de ponto para segmento
ld distseg(pt p, line r) {
	if ((r.q - r.p)*(p - r.p) < 0) return dist(r.p, p);
	if ((r.p - r.q)*(p - r.q) < 0) return dist(r.q, p);
	return distline(p, r);
}

// distancia de ponto a plano com sinal
ld sdist(pt p, plane P) {
	return P.eq[0]*p.x + P.eq[1]*p.y + P.eq[2]*p.z +
	   P.eq[3];
}

// distancia de ponto a plano
ld distplane(pt p, plane P) {
	return abs(sdist(p, P));
}

// se ponto pertence a reta
bool isinseg(pt p, line r) {
	return eq(distseg(p, r), 0);
}

// se ponto pertence ao triangulo definido por P.p
bool isinpol(pt p, vector<pt> v) {
	assert(v.size() >= 3);
	pt norm = (v[1]-v[0]) ^ (v[2]-v[1]);
	bool inside = true;
	int sign = -1;
	for (int i = 0; i < v.size(); i++) {
		line r(v[(i+1)%3], v[i]);
		if (isinseg(p, r)) return true;

		pt ar = v[(i+1)%3] - v[i];
		if (sign == -1) sign = ((ar^(p-v[i]))*norm >
		   0);
		else if (((ar^(p-v[i]))*norm > 0) != sign)
```

```
                inside = false;
        }
        return inside;
}

// distancia de ponto ate poligono
ld distpol(pt p, vector<pt> v) {
        pt p2 = proj(p, plane(v[0], v[1], v[2]));
        if (isinpol(p2, v)) return dist(p, p2);
        ld ret = DINF;
        for (int i = 0; i < v.size(); i++) {
                int j = (i+1)%v.size();
                ret = min(ret, distseg(p, line(v[i], v[j])));
        }
        return ret;
}

// intersecao de plano e segmento
// BOTH = o segmento esta no plano
// ONE = um dos pontos do segmento esta no plano
// PARAL = segmento paralelo ao plano
// CONCOR = segmento concorrente ao plano
enum RETCODE {BOTH, ONE, PARAL, CONCOR};
pair<RETCODE, pt> intersect(plane P, line r) {
    ld d1 = sdist(r.p, P);
    ld d2 = sdist(r.q, P);
    if (eq(d1, 0) and eq(d2, 0))
            return pair(BOTH, r.p);
    if (eq(d1, 0))
            return pair(ONE, r.p);
    if (eq(d2, 0))
            return pair(ONE, r.q);
    if ((d1 > 0 and d2 > 0) or (d1 < 0 and d2 < 0)) {
        if (eq(d1-d2, 0)) return pair(PARAL, pt());
        return pair(CONCOR, pt());
    }
    ld frac = d1 / (d1 - d2);
    pt res = r.p + ((r.q - r.p) * frac);
    return pair(ONE, res);
}

// rotaciona p ao redor do eixo u por um angulo a
pt rotate(pt p, pt u, ld a) {
        u = u / dist(u, pt());
        return u * (u * p) + (u ^ p ^ u) * cos(a) + (u ^ p)
            * sin(a);
}
```

2.8 Primitivas Geometricas Inteiras

```
#define sq(x) ((x)*(ll)(x))

// 840720
struct pt { // ponto
    int x, y;
    pt(int x_ = 0, int y_ = 0) : x(x_), y(y_) {}
    bool operator < (const pt p) const {
        if (x != p.x) return x < p.x;
        return y < p.y;
    }
    bool operator == (const pt p) const {
        return x == p.x and y == p.y;
    }
    pt operator + (const pt p) const { return pt(x+p.x,
        y+p.y); }
    pt operator - (const pt p) const { return pt(x-p.x,
        y-p.y); }
    pt operator * (const int c) const { return pt(x*c, y*c);
        }
    ll operator * (const pt p) const { return x*(ll)p.x +
        y*(ll)p.y; }
    ll operator ^ (const pt p) const { return x*(ll)p.y -
        y*(ll)p.x; }
    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y;
    }
};

// 7ab617
struct line { // reta
    pt p, q;
    line() {}
```

```
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// PONTO & VETOR

// 51563e
ll dist2(pt p, pt q) { // quadrado da distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}

// bf431d
ll sarea2(pt p, pt q, pt r) { // 2 * area com sinal
    return (q-p)^(r-q);
}

// a082d3
bool col(pt p, pt q, pt r) { // se p, q e r sao colin.
    return sarea2(p, q, r) == 0;
}

// 42bb09
bool ccw(pt p, pt q, pt r) { // se p, q, r sao ccw
    return sarea2(p, q, r) > 0;
}

// fcf924
int quad(pt p) { // quadrante de um ponto
    return (p.x<0)^3*(p.y<0);
}

// 77187b
bool compare_angle(pt p, pt q) { // retorna se ang(p) <
   ang(q)
    if (quad(p) != quad(q)) return quad(p) < quad(q);
    return ccw(q, pt(0, 0), p);
}

// e4ad5e
pt rotate90(pt p) { // rotaciona 90 graus
    return pt(-p.y, p.x);
}

// RETA

// c9f07f
bool isinseg(pt p, line r) { // se p pertence ao seg de r
    pt a = r.p - p, b = r.q - p;
    return (a ^ b) == 0 and (a * b) <= 0;
}

// 35998c
bool interseg(line r, line s) { // se o seg de r intersecta
   o seg de s
    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
            ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

// dd8702
int segpoints(line r) { // numero de pontos inteiros no
   segmento
    return 1 + __gcd(abs(r.p.x - r.q.x), abs(r.p.y - r.q.y));
}

// d273be
double get_t(pt v, line r) { // retorna t tal que t*v
   pertence a reta r
    return (r.p^r.q) / (double) ((r.p-r.q)^v);
}

// POLIGONO

// quadrado da distancia entre os retangulos a e b (lados
   paralelos aos eixos)
// assume que ta representado (inferior esquerdo, superior
   direito)
// e13018
ll dist2_rect(pair<pt, pt> a, pair<pt, pt> b) {
    int hor = 0, vert = 0;
```

```
    if (a.second.x < b.first.x) hor = b.first.x - a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x -
        b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y -
        a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y -
        b.second.y;
    return sq(hor) + sq(vert);
}

// d5f693
ll polarea2(vector<pt> v) { // 2 * area do poligono
    ll ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea2(pt(0, 0), v[i], v[(i + 1) % v.size()]);
    return abs(ret);
}

// se o ponto ta dentro do poligono: retorna 0 se ta fora,
// 1 se ta no interior e 2 se ta na borda
// afd587
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i+1)%v.size();
        if (p.y == v[i].y and p.y == v[j].y) {
            if ((v[i]-p)*(v[j]-p) <= 0) return 2;
            continue;
        }
        bool baixo = v[i].y < p.y;
        if (baixo == (v[j].y < p.y)) continue;
        auto t = (p-v[i])^(v[j]-v[i]);
        if (!t) return 2;
        if (baixo == (t > 0)) qt += baixo ? 1 : -1;
    }
    return qt != 0;
}

// 32623c
vector<pt> convex_hull(vector<pt> v) { // convex hull - O(n
    log(n))
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    sort(v.begin(), v.end());
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l[l.size()-2],
            l.back(), v[i]))
             l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u[u.size()-2],
            u.back(), v[i]))
             u.pop_back();
        u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

// af2d96
ll interior_points(vector<pt> v) { // pontos inteiros dentro
    de um poligono simples
    ll b = 0;
    for (int i = 0; i < v.size(); i++)
        b += segpoints(line(v[i], v[(i+1)%v.size()])) - 1;
    return (polarea2(v) - b) / 2 + 1;
}

struct convex_pol {
    vector<pt> pol;

    // nao pode ter ponto colinear no convex hull
    convex_pol() {}
    convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

    // se o ponto ta dentro do hull - O(log(n))
    // 800813
    bool is_inside(pt p) {
        if (pol.size() == 1) return p == pol[0];
        int l = 1, r = pol.size();
        while (l < r) {
```

```
        int m = (l+r)/2;
        if (ccw(p, pol[0], pol[m])) l = m+1;
        else r = m;
    }
    if (l == 1) return isinseg(p, line(pol[0], pol[1]));
    if (l == pol.size()) return false;
    return !ccw(p, pol[l], pol[l-1]);
}
// ponto extremo em relacao a cmp(p, q) = p mais extremo
    q
// (copiado de
    https://github.com/gustavoM32/caderno-zika)
// 56ccd2
int extreme(const function<bool(pt, pt)>& cmp) {
    int n = pol.size();
    auto extr = [&](int i, bool& cur_dir) {
        cur_dir = cmp(pol[(i+1)%n], pol[i]);
        return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
    };
    bool last_dir, cur_dir;
    if (extr(0, last_dir)) return 0;
    int l = 0, r = n;
    while (l+1 < r) {
        int m = (l+r)/2;
        if (extr(m, cur_dir)) return m;
        bool rel_dir = cmp(pol[m], pol[l]);
        if ((!last_dir and cur_dir) or
                (last_dir == cur_dir and rel_dir ==
                    cur_dir)) {
            l = m;
            last_dir = cur_dir;
        } else r = m;
    }
    return l;
}
int max_dot(pt v) {
    return extreme([&](pt p, pt q) { return p*v > q*v;
        });
}
pair<int, int> tangents(pt p) {
    auto L = [&](pt q, pt r) { return ccw(p, q, r); };
    auto R = [&](pt q, pt r) { return ccw(p, r, q); };
        return {extreme(L), extreme(R)};
    }
};

// dca598
bool operator <(const line& a, const line& b) { //
    comparador pra reta
    // assume que as retas tem p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    bool b1 = compare_angle(v1, v2), b2 = compare_angle(v2,
        v1);
    if (b1 or b2) return b1;
    return ccw(a.p, a.q, b.p); // mesmo angulo
}
bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
}

// comparador pro set pra fazer sweep line com segmentos
// 6774df
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
        // assume que os segmentos tem p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (a.p.x != a.q.x and (b.p.x == b.q.x or a.p.x <
            b.p.x))
             return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador pro set pra fazer sweep angle com segmentos
// 1ee7f5
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) < get_t(dir, b);
    }
};
```

3 Grafos

3.1 AGM Direcionada

```
// Fala o menor custo para selecionar arestas tal que
// o vertice 'r' alcance todos
// Se nao tem como, retorna LINF
//
// O(m log(n))
// dc345b

struct node {
    pair<ll, int> val;
    ll lazy;
    node *l, *r;
    node() {}
    node(pair<int, int> v) : val(v), lazy(0), l(NULL),
       r(NULL) {}

    void prop() {
        val.first += lazy;
        if (l) l->lazy += lazy;
        if (r) r->lazy += lazy;
        lazy = 0;
    }
};
void merge(node*& a, node* b) {
    if (!a) swap(a, b);
    if (!b) return;
    a->prop(), b->prop();
    if (a->val > b->val) swap(a, b);
    merge(rand()%2 ? a->l : a->r, b);
}
pair<ll, int> pop(node*& R) {
    R->prop();
    auto ret = R->val;
    node* tmp = R;
    merge(R->l, R->r);
    R = R->l;
    if (R) R->lazy -= ret.first;
    delete tmp;
    return ret;
}
void apaga(node* R) { if (R) apaga(R->l), apaga(R->r),
   delete R; }

ll dmst(int n, int r, vector<pair<pair<int, int>, int>>& ar)
   {
    vector<int> p(n); iota(p.begin(), p.end(), 0);
    function<int(int)> find = [&](int k) { return
       p[k]==k?k:p[k]=find(p[k]); };
    vector<node*> h(n);
    for (auto e : ar) merge(h[e.first.second], new
       node({e.second, e.first.first}));
    vector<int> pai(n, -1), path(n);
    pai[r] = r;
    ll ans = 0;

    for (int i = 0; i < n; i++) { // vai conectando todo
       mundo
        int u = i, at = 0;
        while (pai[u] == -1) {
            if (!h[u]) { // nao tem
                for (auto i : h) apaga(i);
                return LINF;
            }
            path[at++] = u, pai[u] = i;
            auto [mi, v] = pop(h[u]);
            ans += mi;

            if (pai[u = find(v)] == i) { // ciclo
                while (find(v = path[--at]) != u)
                    merge(h[u], h[v]), h[v] = NULL,
                       p[find(v)] = u;
                pai[u] = -1;
            }
        }
    }
    for (auto i : h) apaga(i);
    return ans;
}
```

3.2 Bellman-Ford

```
// Calcula a menor distancia
// entre a e todos os vertices e
// detecta ciclo negativo
// Retorna 1 se ha ciclo negativo
// Nao precisa representar o grafo,
// soh armazenar as arestas
//
// O(nm)
// 03059b

int n, m;
int d[MAX];
vector<pair<int, int>> ar; // vetor de arestas
vector<int> w;              // peso das arestas

bool bellman_ford(int a) {
    for (int i = 0; i < n; i++) d[i] = INF;
    d[a] = 0;

    for (int i = 0; i <= n; i++)
        for (int j = 0; j < m; j++) {
            if (d[ar[j].second] > d[ar[j].first] + w[j]) {
                if (i == n) return 1;

                d[ar[j].second] = d[ar[j].first] + w[j];
            }
        }

    return 0;
}
```

3.3 Block-Cut Tree

```
// Cria a block-cut tree, uma arvore com os blocos
// e os pontos de articulacao
// Blocos sao componentes 2-vertice-conexos maximais
// Uma 2-coloracao da arvore eh tal que uma cor sao
// os blocos, e a outra cor sao os pontos de art.
// Funciona para grafo nao conexo
//
// art[i] responde o numero de novas componentes conexas
// criadas apos a remocao de i do grafo g
// Se art[i] >= 1, i eh ponto de articulacao
//
// Para todo i <= blocks.size()
// blocks[i] eh uma componente 2-vertce-conexa maximal
// edgblocks[i] sao as arestas do bloco i
// tree[i] eh um vertice da arvore que corresponde ao bloco i
//
// pos[i] responde a qual vertice da arvore vertice i
   pertence
// Arvore tem no maximo 2n vertices
//
// O(n+m)
// 056fa2

struct block_cut_tree {
    vector<vector<int>> g, blocks, tree;
    vector<vector<pair<int, int>>> edgblocks;
    stack<int> s;
    stack<pair<int, int>> s2;
    vector<int> id, art, pos;

    block_cut_tree(vector<vector<int>> g_) : g(g_) {
        int n = g.size();
        id.resize(n, -1), art.resize(n), pos.resize(n);
        build();
    }

    int dfs(int i, int& t, int p = -1) {
        int lo = id[i] = t++;
        s.push(i);

        if (p != -1) s2.emplace(i, p);
        for (int j : g[i]) if (j != p and id[j] != -1)
            s2.emplace(i, j);

        for (int j : g[i]) if (j != p) {
            if (id[j] == -1) {
                int val = dfs(j, t, i);
```

```
                lo = min(lo, val);

                if (val >= id[i]) {
                    art[i]++;
                    blocks.emplace_back(1, i);
                    while (blocks.back().back() != j)
                        blocks.back().push_back(s.top()),
                            s.pop();

                    edgblocks.emplace_back(1, s2.top()),
                        s2.pop();
                    while (edgblocks.back().back() !=
                        pair(j, i))
                        edgblocks.back().push_back(s2.top()),
                            s2.pop();
                }
                // if (val > id[i]) aresta i-j eh ponte
            }
            else lo = min(lo, id[j]);
        }

        if (p == -1 and art[i]) art[i]--;
        return lo;
    }

    void build() {
        int t = 0;
        for (int i = 0; i < g.size(); i++) if (id[i] == -1)
            dfs(i, t, -1);

        tree.resize(blocks.size());
        for (int i = 0; i < g.size(); i++) if (art[i])
            pos[i] = tree.size(), tree.emplace_back();

        for (int i = 0; i < blocks.size(); i++) for (int j :
            blocks[i]) {
            if (!art[j]) pos[j] = i;
            else tree[i].push_back(pos[j]),
                tree[pos[j]].push_back(i);
        }
    }
};
```

3.4 Blossom - matching maximo em grafo geral

```
// O(n^3)
// Se for bipartido, nao precisa da funcao
// 'contract', e roda em O(nm)
// 4426a4

vector<int> g[MAX];
int match[MAX]; // match[i] = com quem i esta matchzado ou -1
int n, pai[MAX], base[MAX], vis[MAX];
queue<int> q;

void contract(int u, int v, bool first = 1) {
    static vector<bool> bloss;
    static int l;
    if (first) {
        bloss = vector<bool>(n, 0);
        vector<bool> teve(n, 0);
        int k = u; l = v;
        while (1) {
            teve[k = base[k]] = 1;
            if (match[k] == -1) break;
            k = pai[match[k]];
        }
        while (!teve[l = base[l]]) l = pai[match[l]];
    }
    while (base[u] != l) {
        bloss[base[u]] = bloss[base[match[u]]] = 1;
        pai[u] = v;
        v = match[u];
        u = pai[match[u]];
    }
    if (!first) return;
    contract(v, u, 0);
    for (int i = 0; i < n; i++) if (bloss[base[i]]) {
        base[i] = l;
        if (!vis[i]) q.push(i);
        vis[i] = 1;
    }
}

int getpath(int s) {
```

```
    for (int i = 0; i < n; i++) base[i] = i, pai[i] = -1,
        vis[i] = 0;
    vis[s] = 1; q = queue<int>(); q.push(s);
    while (q.size()) {
        int u = q.front(); q.pop();
        for (int i : g[u]) {
            if (base[i] == base[u] or match[u] == i)
                continue;
            if (i == s or (match[i] != -1 and pai[match[i]]
                != -1))
                contract(u, i);
            else if (pai[i] == -1) {
                pai[i] = u;
                if (match[i] == -1) return i;
                i = match[i];
                vis[i] = 1; q.push(i);
            }
        }
    }
    return -1;
}

int blossom() {
    int ans = 0;
    memset(match, -1, sizeof(match));
    for (int i = 0; i < n; i++) if (match[i] == -1)
        for (int j : g[i]) if (match[j] == -1) {
            match[i] = j;
            match[j] = i;
            ans++;
            break;
        }
    for (int i = 0; i < n; i++) if (match[i] == -1) {
        int j = getpath(i);
        if (j == -1) continue;
        ans++;
        while (j != -1) {
            int p = pai[j], pp = match[p];
            match[p] = j;
            match[j] = p;
            j = pp;
        }
    }
    return ans;
}
```

3.5 Centro de arvore

```
// Retorna o diametro e o(s) centro(s) da arvore
// Uma arvore tem sempre um ou dois centros e estes estao no
    meio do diametro
//
// O(n)
// c1adeb

vector<int> g[MAX];
int d[MAX], par[MAX];

pair<int, vector<int>> center() {
    int f, df;
    function<void(int)> dfs = [&] (int v) {
        if (d[v] > df) f = v, df = d[v];
        for (int u : g[v]) if (u != par[v])
            d[u] = d[v] + 1, par[u] = v, dfs(u);
    };

    f = df = par[0] = -1, d[0] = 0;
    dfs(0);
    int root = f;
    f = df = par[root] = -1, d[root] = 0;
    dfs(root);

    vector<int> c;
    while (f != -1) {
        if (d[f] == df/2 or d[f] == (df+1)/2) c.push_back(f);
        f = par[f];
    }

    return {df, c};
}
```

3.6 Centroid

```
// Computa os 2 centroids da arvore
//
// O(n)
// e16075

int n, subsize[MAX];
vector<int> g[MAX];

void dfs(int k, int p=-1) {
	subsize[k] = 1;
	for (int i : g[k]) if (i != p) {
		dfs(i, k);
		subsize[k] += subsize[i];
	}
}

int centroid(int k, int p=-1, int size=-1) {
	if (size == -1) size = subsize[k];
	for (int i : g[k]) if (i != p) if (subsize[i] > size/2)
		return centroid(i, k, size);
	return k;
}

pair<int, int> centroids(int k=0) {
	dfs(k);
	int i = centroid(k), i2 = i;
	for (int j : g[i]) if (2*subsize[j] == subsize[k]) i2 =
	    j;
	return {i, i2};
}
```

3.7 Centroid decomposition

```
// decomp(0, k) computa numero de caminhos com 'k' arestas
// Mudar depois do comentario
//
// O(n log(n))
// fe2541

vector<int> g[MAX];
int sz[MAX], rem[MAX];

void dfs(vector<int>& path, int i, int l=-1, int d=0) {
	path.push_back(d);
	for (int j : g[i]) if (j != l and !rem[j]) dfs(path, j,
	    i, d+1);
}

int dfs_sz(int i, int l=-1) {
	sz[i] = 1;
	for (int j : g[i]) if (j != l and !rem[j]) sz[i] +=
	    dfs_sz(j, i);
	return sz[i];
}

int centroid(int i, int l, int size) {
	for (int j : g[i]) if (j != l and !rem[j] and sz[j] >
	    size / 2)
		return centroid(j, i, size);
	return i;
}

ll decomp(int i, int k) {
	int c = centroid(i, i, dfs_sz(i));
	rem[c] = 1;

	// gasta O(n) aqui - dfs sem ir pros caras removidos
	ll ans = 0;
	vector<int> cnt(sz[i]);
	cnt[0] = 1;
	for (int j : g[c]) if (!rem[j]) {
		vector<int> path;
		dfs(path, j);
		for (int d : path) if (0 <= k-d-1 and k-d-1 < sz[i])
			ans += cnt[k-d-1];
		for (int d : path) cnt[d+1]++;
	}

	for (int j : g[c]) if (!rem[j]) ans += decomp(j, k);
	rem[c] = 0;
```

```
    return ans;
}
```

3.8 Centroid Tree

```
// Constroi a centroid tree
// p[i] eh o pai de i na centroid-tree
// dist[i][k] = distancia na arvore original entre i
// e o k-esimo ancestral na arvore da centroid
//
// O(n log(n)) de tempo e memoria
// a0e7c7

vector<int> g[MAX], dist[MAX];
int sz[MAX], rem[MAX], p[MAX];

int dfs_sz(int i, int l=-1) {
    sz[i] = 1;
    for (int j : g[i]) if (j != l and !rem[j]) sz[i] +=
        dfs_sz(j, i);
    return sz[i];
}

int centroid(int i, int l, int size) {
    for (int j : g[i]) if (j != l and !rem[j] and sz[j] >
        size / 2)
        return centroid(j, i, size);
    return i;
}

void dfs_dist(int i, int l, int d=0) {
    dist[i].push_back(d);
    for (int j : g[i]) if (j != l and !rem[j])
        dfs_dist(j, i, d+1);
}

void decomp(int i, int l = -1) {
    int c = centroid(i, i, dfs_sz(i));
    rem[c] = 1, p[c] = l;
    dfs_dist(c, c);
    for (int j : g[c]) if (!rem[j]) decomp(j, c);
}

void build(int n) {
    for (int i = 0; i < n; i++) rem[i] = 0, dist[i].clear();
    decomp(0);
    for (int i = 0; i < n; i++) reverse(dist[i].begin(),
        dist[i].end());
}
```

3.9 Dijkstra

```
// encontra menor distancia de x
// para todos os vertices
// se ao final do algoritmo d[i] = LINF,
// entao x nao alcanca i
//
// O(m log(n))
// 695ac4

ll d[MAX];
vector<pair<int, int>> g[MAX]; // {vizinho, peso}

int n;

void dijkstra(int v) {
    for (int i = 0; i < n; i++) d[i] = LINF;
    d[v] = 0;
    priority_queue<pair<ll, int>> pq;
    pq.emplace(0, v);

    while (pq.size()) {
        auto [ndist, u] = pq.top(); pq.pop();
        if (-ndist > d[u]) continue;

        for (auto [idx, w] : g[u]) if (d[idx] > d[u] + w) {
            d[idx] = d[u] + w;
            pq.emplace(-d[idx], idx);
        }
    }
```

```
}
```

3.10 Dinic

```
// O(min(m * max_flow, n^2 m))
// Grafo com capacidades 1 -> O(sqrt(n)*m)
// 67ce89

struct dinic {
    const bool scaling = false; // com scaling -> O(nm
        log(MAXCAP)),
    int lim;                    // com constante alta
    struct edge {
        int to, cap, rev, flow;
        bool res;
        edge(int to_, int cap_, int rev_, bool res_)
            : to(to_), cap(cap_), rev(rev_), flow(0),
                res(res_) {}
    };

    vector<vector<edge>> g;
    vector<int> lev, beg;
    ll F;
    dinic(int n) : g(n), F(0) {}

    void add(int a, int b, int c) {
        g[a].emplace_back(b, c, g[b].size(), false);
        g[b].emplace_back(a, 0, g[a].size()-1, true);
    }
    bool bfs(int s, int t) {
        lev = vector<int>(g.size(), -1); lev[s] = 0;
        beg = vector<int>(g.size(), 0);
        queue<int> q; q.push(s);
        while (q.size()) {
            int u = q.front(); q.pop();
            for (auto& i : g[u]) {
                if (lev[i.to] != -1 or (i.flow == i.cap))
                    continue;
                if (scaling and i.cap - i.flow < lim)
                    continue;
                lev[i.to] = lev[u] + 1;
                q.push(i.to);
            }
        }
        return lev[t] != -1;
    }
    int dfs(int v, int s, int f = INF) {
        if (!f or v == s) return f;
        for (int& i = beg[v]; i < g[v].size(); i++) {
            auto& e = g[v][i];
            if (lev[e.to] != lev[v] + 1) continue;
            int foi = dfs(e.to, s, min(f, e.cap - e.flow));
            if (!foi) continue;
            e.flow += foi, g[e.to][e.rev].flow -= foi;
            return foi;
        }
        return 0;
    }
    ll max_flow(int s, int t) {
        for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
            while (bfs(s, t)) while (int ff = dfs(s, t)) F
                += ff;
        return F;
    }
};

// Recupera as arestas do corte s-t
// d23977
vector<pair<int, int>> get_cut(dinic& g, int s, int t) {
    g.max_flow(s, t);
    vector<pair<int, int>> cut;
    vector<int> vis(g.g.size(), 0), st = {s};
    vis[s] = 1;
    while (st.size()) {
        int u = st.back(); st.pop_back();
        for (auto e : g.g[u]) if (!vis[e.to] and e.flow <
            e.cap)
            vis[e.to] = 1, st.push_back(e.to);
    }
    for (int i = 0; i < g.g.size(); i++) for (auto e :
        g.g[i])
        if (vis[i] and !vis[e.to] and !e.res)
```

```
            cut.emplace_back(i, e.to);
    return cut;
}
```

3.11 Dominator Tree - Kawakami

```
// Se vira pra usar ai
//
// build - O(n)
// dominates - O(1)
// c80920

int n;

namespace d_tree {
    vector<int> g[MAX];

    // The dominator tree
    vector<int> tree[MAX];
    int dfs_l[MAX], dfs_r[MAX];

    // Auxiliary data
    vector<int> rg[MAX], bucket[MAX];
    int idom[MAX], sdom[MAX], prv[MAX], pre[MAX];
    int ancestor[MAX], label[MAX];
    vector<int> preorder;

    void dfs(int v) {
        static int t = 0;
        pre[v] = ++t;
        sdom[v] = label[v] = v;
        preorder.push_back(v);
        for (int nxt: g[v]) {
            if (sdom[nxt] == -1) {
                prv[nxt] = v;
                dfs(nxt);
            }
            rg[nxt].push_back(v);
        }
    }
    int eval(int v) {
        if (ancestor[v] == -1) return v;
        if (ancestor[ancestor[v]] == -1) return label[v];
        int u = eval(ancestor[v]);
        if (pre[sdom[u]] < pre[sdom[label[v]]]) label[v] = u;
        ancestor[v] = ancestor[u];
        return label[v];
    }
    void dfs2(int v) {
        static int t = 0;
        dfs_l[v] = t++;
        for (int nxt: tree[v]) dfs2(nxt);
        dfs_r[v] = t++;
    }
    void build(int s) {
        for (int i = 0; i < n; i++) {
            sdom[i] = pre[i] = ancestor[i] = -1;
            rg[i].clear();
            tree[i].clear();
            bucket[i].clear();
        }
        preorder.clear();
        dfs(s);
        if (preorder.size() == 1) return;
        for (int i = int(preorder.size()) - 1; i >= 1; i--) {
            int w = preorder[i];
            for (int v: rg[w]) {
                int u = eval(v);
                if (pre[sdom[u]] < pre[sdom[w]]) sdom[w] =
                    sdom[u];
            }
            bucket[sdom[w]].push_back(w);
            ancestor[w] = prv[w];
            for (int v: bucket[prv[w]]) {
                int u = eval(v);
                idom[v] = (u == v) ? sdom[v] : u;
            }
            bucket[prv[w]].clear();
        }
        for (int i = 1; i < preorder.size(); i++) {
            int w = preorder[i];
            if (idom[w] != sdom[w]) idom[w] = idom[idom[w]];
```

```
            tree[idom[w]].push_back(w);
        }
        idom[s] = sdom[s] = -1;
        dfs2(s);
    }

    // Whether every path from s to v passes through u
    bool dominates(int u, int v) {
        if (pre[v] == -1) return 1; // vacuously true
        return dfs_l[u] <= dfs_l[v] && dfs_r[v] <= dfs_r[u];
    }
};
```

3.12 Euler Path / Euler Cycle

```
// Para declarar: 'euler<true> E(n);' se quiser
// direcionado e com 'n' vertices
// As funcoes retornam um par com um booleano
// indicando se possui o cycle/path que voce pediu,
// e um vector de {vertice, id da aresta para chegar no
   vertice}
// Se for get_path, na primeira posicao o id vai ser -1
// get_path(src) tenta achar um caminho ou ciclo euleriano
// comecando no vertice 'src'.
// Se achar um ciclo, o primeiro e ultimo vertice serao
   'src'.
// Se for um P3, um possiveo retorno seria [0, 1, 2, 0]
// get_cycle() acha um ciclo euleriano se o grafo for
   euleriano.
// Se for um P3, um possivel retorno seria [0, 1, 2]
// (vertie inicial nao repete)
//
// O(n+m)
// 7113df

template<bool directed=false> struct euler {
    int n;
    vector<vector<pair<int, int>>> g;
    vector<int> used;

    euler(int n_) : n(n_), g(n) {}
    void add(int a, int b) {
        int at = used.size();
        used.push_back(0);
        g[a].emplace_back(b, at);
        if (!directed) g[b].emplace_back(a, at);
    }
#warning chamar para o src certo!
    pair<bool, vector<pair<int, int>>> get_path(int src) {
        if (!used.size()) return {true, {}};
        vector<int> beg(n, 0);
        for (int& i : used) i = 0;
        // {{vertice, anterior}, label}
        vector<pair<pair<int, int>, int>> ret, st = {{{src,
           -1}, -1}};
        while (st.size()) {
            int at = st.back().first.first;
            int& it = beg[at];
            while (it < g[at].size() and
               used[g[at][it].second]) it++;
            if (it == g[at].size()) {
                if (ret.size() and ret.back().first.second
                   != at)
                    return {false, {}};
                ret.push_back(st.back()), st.pop_back();
            } else {
                st.push_back({{g[at][it].first, at},
                   g[at][it].second});
                used[g[at][it].second] = 1;
            }
        }
        if (ret.size() != used.size()+1) return {false, {}};
        vector<pair<int, int>> ans;
        for (auto i : ret) ans.emplace_back(i.first.first,
           i.second);
        reverse(ans.begin(), ans.end());
        return {true, ans};
    }
    pair<bool, vector<pair<int, int>>> get_cycle() {
        if (!used.size()) return {true, {}};
        int src = 0;
        while (!g[src].size()) src++;
```

```
        auto ans = get_path(src);
        if (!ans.first or ans.second[0].first !=
            ans.second.back().first)
             return {false, {}};
        ans.second[0].second = ans.second.back().second;
        ans.second.pop_back();
        return ans;
    }
};
```

3.13 Euler Tour Tree

```
// Mantem uma floresta enraizada dinamicamente
// e permite queries/updates em sub-arvore
//
// Chamar ETT E(n, v), passando n = numero de vertices
// e v = vector com os valores de cada vertice (se for vazio,
// constroi tudo com 0
//
// link(v, u) cria uma aresta de v pra u, de forma que u se
   torna
// o pai de v (eh preciso que v seja raiz anteriormente)
// cut(v) corta a resta de v para o pai
// query(v) retorna a soma dos valores da sub-arvore de v
// update(v, val) soma val em todos os vertices da
   sub-arvore de v
// update_v(v, val) muda o valor do vertice v para val
// is_in_subtree(v, u) responde se o vertice u esta na
   sub-arvore de v
//
// Tudo O(log(n)) com alta probabilidade
// c97d63

mt19937 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

template<typename T> struct ETT {
    // treap
    struct node {
        node *l, *r, *p;
        int pr, sz;
        T val, sub, lazy;
        int id;
        bool f; // se eh o 'first'
        int qt_f; // numero de firsts na subarvore
        node(int id_, T v, bool f_ = 0) : l(NULL), r(NULL),
            p(NULL), pr(rng()),
             sz(1), val(v), sub(v), lazy(), id(id_), f(f_),
                qt_f(f_) {}
        void prop() {
            if (lazy != T()) {
                if (f) val += lazy;
                sub += lazy*sz;
                if (l) l->lazy += lazy;
                if (r) r->lazy += lazy;
            }
            lazy = T();
        }
        void update() {
            sz = 1, sub = val, qt_f = f;
            if (l) l->prop(), sz += l->sz, sub += l->sub,
                qt_f += l->qt_f;
            if (r) r->prop(), sz += r->sz, sub += r->sub,
                qt_f += r->qt_f;
        }
    };

    node* root;

    int size(node* x) { return x ? x->sz : 0; }
    void join(node* l, node* r, node*& i) { // assume que l
       < r
        if (!l or !r) return void(i = l ? l : r);
        l->prop(), r->prop();
        if (l->pr > r->pr) join(l->r, r, l->r), l->r->p = i
            = l;
        else join(l, r->l, r->l), r->l->p = i = r;
        i->update();
    }
    void split(node* i, node*& l, node*& r, int v, int key =
       0) {
        if (!i) return void(r = l = NULL);
```

```cpp
        i->prop();
        if (key + size(i->l) < v) {
            split(i->r, i->r, r, v, key+size(i->l)+1), l = i;
            if (r) r->p = NULL;
            if (i->r) i->r->p = i;
        } else {
            split(i->l, l, i->l, v, key), r = i;
            if (l) l->p = NULL;
            if (i->l) i->l->p = i;
        }
        i->update();
    }
    int get_idx(node* i) {
        int ret = size(i->l);
        for (; i->p; i = i->p) {
            node* pai = i->p;
            if (i != pai->l) ret += size(pai->l) + 1;
        }
        return ret;
    }
    node* get_min(node* i) {
        if (!i) return NULL;
        return i->l ? get_min(i->l) : i;
    }
    node* get_max(node* i) {
        if (!i) return NULL;
        return i->r ? get_max(i->r) : i;
    }
    // fim da treap

    vector<node*> first, last;

    ETT(int n, vector<T> v = {}) : root(NULL), first(n),
       last(n) {
        if (!v.size()) v = vector<T>(n);
        for (int i = 0; i < n; i++) {
            first[i] = last[i] = new node(i, v[i], 1);
            join(root, first[i], root);
        }
    }
    ETT(const ETT& t) { throw logic_error("Nao copiar a
       ETT!"); }
    ~ETT() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }

    pair<int, int> get_range(int i) {
        return {get_idx(first[i]), get_idx(last[i])};
    }
    void link(int v, int u) { // 'v' tem que ser raiz
        auto [lv, rv] = get_range(v);
        int ru = get_idx(last[u]);

        node* V;
        node *L, *M, *R;
        split(root, M, R, rv+1), split(M, L, M, lv);
        V = M;
        join(L, R, root);

        split(root, L, R, ru+1);
        join(L, V, L);
        join(L, last[u] = new node(u, T() /* elemento neutro
           */), L);
        join(L, R, root);
    }
    void cut(int v) {
        auto [l, r] = get_range(v);

        node *L, *M, *R;
        split(root, M, R, r+1), split(M, L, M, l);
        node *LL = get_max(L), *RR = get_min(R);
        if (LL and RR and LL->id == RR->id) { // remove
           duplicata
            if (last[RR->id] == RR) last[RR->id] = LL;
            node *A, *B;
            split(R, A, B, 1);
            delete A;
            R = B;
```

```
    }
    join(L, R, root);
    join(root, M, root);
}
T query(int v) {
    auto [l, r] = get_range(v);
    node *L, *M, *R;
    split(root, M, R, r+1), split(M, L, M, l);
    T ans = M->sub;
    join(L, M, M), join(M, R, root);
    return ans;
}
void update(int v, T val) { // soma val em todo mundo da
    subarvore
    auto [l, r] = get_range(v);
    node *L, *M, *R;
    split(root, M, R, r+1), split(M, L, M, l);
    M->lazy += val;
    join(L, M, M), join(M, R, root);
}
void update_v(int v, T val) { // muda o valor de v pra
    val
    int l = get_idx(first[v]);
    node *L, *M, *R;
    split(root, M, R, l+1), split(M, L, M, l);
    M->val = M->sub = val;
    join(L, M, M), join(M, R, root);
}
bool is_in_subtree(int v, int u) { // se u ta na subtree
    de v
    auto [lv, rv] = get_range(v);
    auto [lu, ru] = get_range(u);
    return lv <= lu and ru <= rv;
}

void print(node* i) {
    if (!i) return;
    print(i->l);
    cout << i->id+1 << " ";
    print(i->r);
}
void print() { print(root); cout << endl; }
};
```

3.14 Floyd-Warshall

```
// encontra o menor caminho entre todo
// par de vertices e detecta ciclo negativo
// returna 1 sse ha ciclo negativo
// d[i][i] deve ser 0
// para i != j, d[i][j] deve ser w se ha uma aresta
// (i, j) de peso w, INF caso contrario
//
// O(n^3)
// ea05be

int n;
int d[MAX][MAX];

bool floyd_warshall() {
    for (int k = 0; k < n; k++)
    for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        d[i][j] = min(d[i][j], d[i][k] + d[k][j]);

    for (int i = 0; i < n; i++)
        if (d[i][i] < 0) return 1;

    return 0;
}
```

3.15 Functional Graph

```
// rt[i] fala o ID da raiz associada ao vertice i
// d[i] fala a profundidade (0 sse ta no ciclo)
// pos[i] fala a posicao de i no array que eh a concat. dos
    ciclos
// build(f, val) recebe a funcao f e o custo de ir de
// i para f[i] (por default, val = f)
// f_k(i, k) fala onde i vai parar se seguir k arestas
```

```
// path(i, k) fala o custo (soma) seguir k arestas a partir
   de i
// Se quiser outra operacao, da pra alterar facil o codigo
// Codigo um pouco louco, tenho que admitir
//
// build - O(n)
// f_k   - O(log(min(n, k)))
// path  - O(log(min(n, k)))
// 51fabe

namespace func_graph {
    int n;
    int f[MAX], vis[MAX], d[MAX];
    int p[MAX], pp[MAX], rt[MAX], pos[MAX];
    int sz[MAX], comp;
    vector<vector<int>> ciclo;
    ll val[MAX], jmp[MAX], seg[2*MAX];

    ll op(ll a, ll b) { return a+b; }; // mudar a operacao
       aqui
    void dfs(int i, int t = 2) {
        vis[i] = t;
        if (vis[f[i]] >= 2) { // comeca ciclo - f[i] eh o
           rep.
            d[i] = 0, rt[i] = comp;
            sz[comp] = t - vis[f[i]] + 1;
            p[i] = pp[i] = i, jmp[i] = val[i];
            ciclo.emplace_back();
            ciclo.back().push_back(i);
        } else {
            if (!vis[f[i]]) dfs(f[i], t+1);
            rt[i] = rt[f[i]];
            if (sz[comp]+1) { // to no ciclo
                d[i] = 0;
                p[i] = pp[i] = i, jmp[i] = val[i];
                ciclo.back().push_back(i);
            } else { // nao to no ciclo
                d[i] = d[f[i]]+1, p[i] = f[i];
                pp[i] = 2*d[pp[f[i]]] ==
                   d[pp[pp[f[i]]]]+d[f[i]] ? pp[pp[f[i]]] :
                   f[i];
                jmp[i] = pp[i] == f[i] ? val[i] : op(val[i],
                   op(jmp[f[i]], jmp[pp[f[i]]]));
            }
        }
        if (f[ciclo[rt[i]][0]] == i) comp++; // fim do ciclo
        vis[i] = 1;
    }
    void build(vector<int> f_, vector<int> val_ = {}) {
        n = f_.size(), comp = 0;
        if (!val_.size()) val_ = f_;
        for (int i = 0; i < n; i++)
            f[i] = f_[i], val[i] = val_[i], vis[i] = 0,
               sz[i] = -1;

        ciclo.clear();
        for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);
        int t = 0;
        for (auto& c : ciclo) {
            reverse(c.begin(), c.end());
            for (int j : c) {
                pos[j] = t;
                seg[n+t] = val[j];
                t++;
            }
        }
        for (int i = n-1; i; i--) seg[i] = op(seg[2*i],
           seg[2*i+1]);
    }

    int f_k(int i, ll k) {
        while (d[i] and k) {
            int big = d[i] - d[pp[i]];
            if (big <= k) k -= big, i = pp[i];
            else k--, i = p[i];
        }
        if (!k) return i;
        return ciclo[rt[i]][(pos[i] - pos[ciclo[rt[i]][0]] +
           k) % sz[rt[i]]];
    }
    ll path(int i, ll k) {
        auto query = [&](int l, int r) {
            ll q = 0;
            for (l += n, r += n; l <= r; ++l/=2, --r/=2) {
```

```
                if (l%2 == 1) q = op(q, seg[l]);
                if (r%2 == 0) q = op(q, seg[r]);
            }
            return q;
        };
        ll ret = 0;
        while (d[i] and k) {
            int big = d[i] - d[pp[i]];
            if (big <= k) k -= big, ret = op(ret, jmp[i]), i
                = pp[i];
            else k--, ret = op(ret, val[i]), i = p[i];
        }
        if (!k) return ret;
        int first = pos[ciclo[rt[i]][0]], last =
            pos[ciclo[rt[i]].back()];

        // k/sz[rt[i]] voltas completas
        if (k/sz[rt[i]]) ret = op(ret, k/sz[rt[i]] *
            query(first, last));

        k %= sz[rt[i]];
        if (!k) return ret;
        int l = pos[i], r = first + (pos[i] - first + k - 1)
            % sz[rt[i]];
        if (l <= r) return op(ret, query(l, r));
        return op(ret, op(query(l, last), query(first, r)));
    }
}
```

3.16 Heavy-Light Decomposition - aresta

```
// SegTree de soma
// query / update de soma das arestas
//
// Complexidades:
// build - O(n)
// query_path - O(log^2 (n))
// update_path - O(log^2 (n))
// query_subtree - O(log(n))
// update_subtree - O(log(n))

namespace seg { ... }

// 599946
namespace hld {
    vector<pair<int, int> > g[MAX];
    int pos[MAX], sz[MAX];
    int sobe[MAX], pai[MAX];
    int h[MAX], v[MAX], t;

    void build_hld(int k, int p = -1, int f = 1) {
        v[pos[k] = t++] = sobe[k]; sz[k] = 1;
        for (auto& i : g[k]) if (i.first != p) {
            auto [u, w] = i;
            sobe[u] = w; pai[u] = k;
            h[u] = (i == g[k][0] ? h[k] : u);
            build_hld(u, k, f); sz[k] += sz[u];

            if (sz[u] > sz[g[k][0].first] or g[k][0].first
                == p)
                 swap(i, g[k][0]);
        }
        if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
    }
    void build(int root = 0) {
        t = 0;
        build_hld(root);
        seg::build(t, v);
    }
    ll query_path(int a, int b) {
        if (a == b) return 0;
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] == h[b]) return seg::query(pos[b]+1,
            pos[a]);
        return seg::query(pos[h[a]], pos[a]) +
            query_path(pai[h[a]], b);
    }
    void update_path(int a, int b, int x) {
        if (a == b) return;
        if (pos[a] < pos[b]) swap(a, b);
```

```
        if (h[a] == h[b]) return (void)seg::update(pos[b]+1,
            pos[a], x);
        seg::update(pos[h[a]], pos[a], x);
            update_path(pai[h[a]], b, x);
    }
    ll query_subtree(int a) {
        if (sz[a] == 1) return 0;
        return seg::query(pos[a]+1, pos[a]+sz[a]-1);
    }
    void update_subtree(int a, int x) {
        if (sz[a] == 1) return;
        seg::update(pos[a]+1, pos[a]+sz[a]-1, x);
    }
    int lca(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        return h[a] == h[b] ? b : lca(pai[h[a]], b);
    }
}
```

3.17 Heavy-Light Decomposition - vertice

```
// SegTree de soma
// query / update de soma dos vertices
//
// Complexidades:
// build - O(n)
// query_path - O(log^2 (n))
// update_path - O(log^2 (n))
// query_subtree - O(log(n))
// update_subtree - O(log(n))

namespace seg { ... }

// de3d84
namespace hld {
    vector<int> g[MAX];
    int pos[MAX], sz[MAX];
    int peso[MAX], pai[MAX];
    int h[MAX], v[MAX], t;

    void build_hld(int k, int p = -1, int f = 1) {
        v[pos[k] = t++] = peso[k]; sz[k] = 1;
        for (auto& i : g[k]) if (i != p) {
            pai[i] = k;
            h[i] = (i == g[k][0] ? h[k] : i);
            build_hld(i, k, f); sz[k] += sz[i];

            if (sz[i] > sz[g[k][0]] or g[k][0] == p) swap(i,
                g[k][0]);
        }
        if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
    }
    void build(int root = 0) {
        t = 0;
        build_hld(root);
        seg::build(t, v);
    }
    ll query_path(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] == h[b]) return seg::query(pos[b], pos[a]);
        return seg::query(pos[h[a]], pos[a]) +
            query_path(pai[h[a]], b);
    }
    void update_path(int a, int b, int x) {
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] == h[b]) return (void)seg::update(pos[b],
            pos[a], x);
        seg::update(pos[h[a]], pos[a], x);
            update_path(pai[h[a]], b, x);
    }
    ll query_subtree(int a) {
        return seg::query(pos[a], pos[a]+sz[a]-1);
    }
    void update_subtree(int a, int x) {
        seg::update(pos[a], pos[a]+sz[a]-1, x);
    }
    int lca(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        return h[a] == h[b] ? b : lca(pai[h[a]], b);
    }
```

```
}
```

3.18 Heavy-Light Decomposition sem Update

```
// query de min do caminho
//
// Complexidades:
// build - O(n)
// query_path - O(log(n))
// ee6991

namespace hld {
    vector<pair<int, int> > g[MAX];
    int pos[MAX], sz[MAX];
    int sobe[MAX], pai[MAX];
    int h[MAX], v[MAX], t;
    int men[MAX], seg[2*MAX];

    void build_hld(int k, int p = -1, int f = 1) {
        v[pos[k] = t++] = sobe[k]; sz[k] = 1;
        for (auto& i : g[k]) if (i.first != p) {
            sobe[i.first] = i.second; pai[i.first] = k;
            h[i.first] = (i == g[k][0] ? h[k] : i.first);
            men[i.first] = (i == g[k][0] ? min(men[k],
                i.second) : i.second);
            build_hld(i.first, k, f); sz[k] += sz[i.first];

            if (sz[i.first] > sz[g[k][0].first] or
                g[k][0].first == p)
                 swap(i, g[k][0]);
        }
        if (p*f == -1) build_hld(h[k] = k, -1, t = 0);
    }
    void build(int root = 0) {
        t = 0;
        build_hld(root);
        for (int i = 0; i < t; i++) seg[i+t] = v[i];
        for (int i = t-1; i; i--) seg[i] = min(seg[2*i],
            seg[2*i+1]);
    }
    int query_path(int a, int b) {
        if (a == b) return INF;
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] != h[b]) return min(men[a],
            query_path(pai[h[a]], b));
        int ans = INF, x = pos[b]+1+t, y = pos[a]+t;
        for (; x <= y; ++x/=2, --y/=2) ans = min({ans,
            seg[x], seg[y]});
        return ans;
    }
};
```

3.19 Isomorfismo de arvores

```
// thash() retorna o hash da arvore (usando centroids como
   vertices especiais).
// Duas arvores sao isomorfas sse seu hash eh o mesmo
//
// O(|V|.log(|V|))
// 8fb6bb

map<vector<int>, int> mphash;

struct tree {
    int n;
    vector<vector<int>> g;
    vector<int> sz, cs;

    tree(int n_) : n(n_), g(n_), sz(n_) {}

    void dfs_centroid(int v, int p) {
        sz[v] = 1;
        bool cent = true;
        for (int u : g[v]) if (u != p) {
            dfs_centroid(u, v), sz[v] += sz[u];
            if(sz[u] > n/2) cent = false;
        }
        if (cent and n - sz[v] <= n/2) cs.push_back(v);
    }
```

```
    int fhash(int v, int p) {
        vector<int> h;
        for (int u : g[v]) if (u != p) h.push_back(fhash(u,
            v));
        sort(h.begin(), h.end());
        if (!mphash.count(h)) mphash[h] = mphash.size();
        return mphash[h];
    }
    ll thash() {
        cs.clear();
        dfs_centroid(0, -1);
        if (cs.size() == 1) return fhash(cs[0], -1);
        ll h1 = fhash(cs[0], cs[1]), h2 = fhash(cs[1],
            cs[0]);
        return (min(h1, h2) << 30) + max(h1, h2);
    }
};
```

3.20 Kosaraju

```
// O(n + m)
// a4f310

int n;
vector<int> g[MAX];
vector<int> gi[MAX]; // grafo invertido
int vis[MAX];
stack<int> S;
int comp[MAX]; // componente conexo de cada vertice

void dfs(int k) {
    vis[k] = 1;
    for (int i = 0; i < (int) g[k].size(); i++)
        if (!vis[g[k][i]]) dfs(g[k][i]);

    S.push(k);
}

void scc(int k, int c) {
    vis[k] = 1;
    comp[k] = c;
    for (int i = 0; i < (int) gi[k].size(); i++)
        if (!vis[gi[k][i]]) scc(gi[k][i], c);
}

void kosaraju() {
    for (int i = 0; i < n; i++) vis[i] = 0;
    for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);

    for (int i = 0; i < n; i++) vis[i] = 0;
    while (S.size()) {
        int u = S.top();
        S.pop();
        if (!vis[u]) scc(u, u);
    }
}
```

3.21 Kruskal

```
// Gera e retorna uma AGM e seu custo total a partir do
   vetor de arestas (edg)
// do grafo
//
// O(m log(m) + m a(m))
// 864875

vector<tuple<int, int, int>> edg; // {peso,[x,y]}

// DSU em O(a(n))
void dsu_build();
int find(int a);
void unite(int a, int b);

pair<ll, vector<tuple<int, int, int>>> kruskal(int n) {
    dsu_build(n);
    sort(edg.begin(), edg.end());

    ll cost = 0;
    vector<tuple<int, int, int>> mst;
    for (auto [w,x,y] : edg) if (find(x) != find(y)) {
```

```
        mst.emplace_back(w, x, y);
        cost += w;
        unite(x,y);
    }
    return {cost, mst};
}
```

3.22 Kuhn

```
// Computa matching maximo em grafo bipartido
// 'n' e 'm' sao quantos vertices tem em cada particao
// chamar add(i, j) para add aresta entre o cara i
// da particao A, e o cara j da particao B
// (entao i < n, j < m)
// Para recuperar o matching, basta olhar 'ma' e 'mb'
// 'recover' recupera o min vertex cover como um par de
// {caras da particao A, caras da particao B}
//
// O(|V| * |E|)
// Na pratica, parece rodar tao rapido quanto o Dinic

mt19937 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

// b0dda3
struct kuhn {
    int n, m;
    vector<vector<int>> g;
    vector<int> vis, ma, mb;

    kuhn(int n_, int m_) : n(n_), m(m_), g(n),
        vis(n+m), ma(n, -1), mb(m, -1) {}

    void add(int a, int b) { g[a].push_back(b); }

    bool dfs(int i) {
        vis[i] = 1;
        for (int j : g[i]) if (!vis[n+j]) {
            vis[n+j] = 1;
            if (mb[j] == -1 or dfs(mb[j])) {
                ma[i] = j, mb[j] = i;
                return true;
            }
        }
        return false;
    }
    int matching() {
        int ret = 0, aum = 1;
        for (auto& i : g) shuffle(i.begin(), i.end(), rng);
        while (aum) {
            for (int j = 0; j < m; j++) vis[n+j] = 0;
            aum = 0;
            for (int i = 0; i < n; i++)
                if (ma[i] == -1 and dfs(i)) ret++, aum = 1;
        }
        return ret;
    }
};

// 55fb67
pair<vector<int>, vector<int>> recover(kuhn& K) {
    K.matching();
    int n = K.n, m = K.m;
    for (int i = 0; i < n+m; i++) K.vis[i] = 0;
    for (int i = 0; i < n; i++) if (K.ma[i] == -1) K.dfs(i);
    vector<int> ca, cb;
    for (int i = 0; i < n; i++) if (!K.vis[i])
       ca.push_back(i);
    for (int i = 0; i < m; i++) if (K.vis[n+i])
       cb.push_back(i);
    return {ca, cb};
}
```

3.23 LCA com binary lifting

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// MAX2 = ceil(log(MAX))
//
// Complexidades:
```

```cpp
// build - O(n log(n))
// lca - O(log(n))

vector<vector<int> > g(MAX);
int n, p;
int pai[MAX2][MAX];
int in[MAX], out[MAX];

void dfs(int k) {
    in[k] = p++;
    for (int i = 0; i < (int) g[k].size(); i++)
        if (in[g[k][i]] == -1) {
            pai[0][g[k][i]] = k;
            dfs(g[k][i]);
        }
    out[k] = p++;
}

void build(int raiz) {
    for (int i = 0; i < n; i++) pai[0][i] = i;
    p = 0, memset(in, -1, sizeof in);
    dfs(raiz);

    // pd dos pais
    for (int k = 1; k < MAX2; k++) for (int i = 0; i < n;
       i++)
        pai[k][i] = pai[k - 1][pai[k - 1][i]];
}

bool anc(int a, int b) { // se a eh ancestral de b
    return in[a] <= in[b] and out[a] >= out[b];
}

int lca(int a, int b) {
    if (anc(a, b)) return a;
    if (anc(b, a)) return b;

    // sobe a
    for (int k = MAX2 - 1; k >= 0; k--)
        if (!anc(pai[k][a], b)) a = pai[k][a];

    return pai[0][a];
}

// Alternativamente:
// 'binary lifting' gastando O(n) de memoria
// Da pra add folhas e fazer queries online
// 3 vezes o tempo do binary lifting normal
//
// build - O(n)
// kth, lca, dist - O(log(n))

int d[MAX], p[MAX], pp[MAX];

void set_root(int i) { p[i] = pp[i] = i, d[i] = 0; }

void add_leaf(int i, int u) {
    p[i] = u, d[i] = d[u]+1;
    pp[i] = 2*d[pp[u]] == d[pp[pp[u]]]+d[u] ? pp[pp[u]] : u;
}

int kth(int i, int k) {
    int dd = max(0, d[i]-k);
    while (d[i] > dd) i = d[pp[i]] >= dd ? pp[i] : p[i];
    return i;
}

int lca(int a, int b) {
    if (d[a] < d[b]) swap(a, b);
    while (d[a] > d[b]) a = d[pp[a]] >= d[b] ? pp[a] : p[a];
    while (a != b) {
        if (pp[a] != pp[b]) a = pp[a], b = pp[b];
        else a = p[a], b = p[b];
    }
    return a;
}

int dist(int a, int b) { return d[a]+d[b]-2*d[lca(a,b)]; }

vector<int> g[MAX];

void build(int i, int pai=-1) {
    if (pai == -1) set_root(i);
    for (int j : g[i]) if (j != pai) {
```

```
        add_leaf(j, i);
        build(j, i);
    }
}
```

3.24 LCA com HLD

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// Para buildar pasta chamar build(root)
// anc(a, b) responde se 'a' eh ancestral de 'b'
//
// Complexidades:
// build - O(n)
// lca - O(log(n))
// anc - O(1)
// fb22c1

vector<int> g[MAX];
int pos[MAX], h[MAX], sz[MAX];
int pai[MAX], t;

void build(int k, int p = -1, int f = 1) {
    pos[k] = t++; sz[k] = 1;
    for (int& i : g[k]) if (i != p) {
        pai[i] = k;
        h[i] = (i == g[k][0] ? h[k] : i);
        build(i, k, f); sz[k] += sz[i];

        if (sz[i] > sz[g[k][0]] or g[k][0] == p) swap(i,
           g[k][0]);
    }
    if (p*f == -1) t = 0, h[k] = k, build(k, -1, 0);
}

int lca(int a, int b) {
    if (pos[a] < pos[b]) swap(a, b);
    return h[a] == h[b] ? b : lca(pai[h[a]], b);
}
bool anc(int a, int b) {
    return pos[a] <= pos[b] and pos[b] <= pos[a]+sz[a]-1;
}
```

3.25 LCA com RMQ

```
// Assume que um vertice eh ancestral dele mesmo, ou seja,
// se a eh ancestral de b, lca(a, b) = a
// dist(a, b) retorna a distancia entre a e b
//
// Complexidades:
// build - O(n)
// lca - O(1)
// dist - O(1)
// 22cde8 - rmq + lca

// 0214e8
template<typename T> struct rmq {
    vector<T> v;
    int n; static const int b = 30;
    vector<int> mask, t;

    int op(int x, int y) { return v[x] < v[y] ? x : y; }
    int msb(int x) { return
       __builtin_clz(1)-__builtin_clz(x); }
    rmq() {}
    rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n),
       t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at<<1)&((1<<b)-1);
            while (at and op(i, i-msb(at&-at)) == i) at ^=
               at&-at;
        }
        for (int i = 0; i < n/b; i++) t[i] =
           b*i+b-1-msb(mask[b*i+b-1]);
        for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0;
           i+(1<<j) <= n/b; i++)
            t[n/b*j+i] = op(t[n/b*(j-1)+i],
               t[n/b*(j-1)+i+(1<<(j-1))]);
    }
```

```
    int small(int r, int sz = b) { return
        r-msb(mask[r]&((1<<sz)-1)); }
    T query(int l, int r) {
        if (r-l+1 <= b) return small(r, r-l+1);
        int ans = op(small(l+b-1), small(r));
        int x = l/b+1, y = r/b-1;
        if (x <= y) {
            int j = msb(y-x+1);
            ans = op(ans, op(t[n/b*j+x],
                t[n/b*j+y-(1<<j)+1]));
        }
        return ans;
    }
};

// 645120
namespace lca {
    vector<int> g[MAX];
    int v[2*MAX], pos[MAX], dep[2*MAX];
    int t;
    rmq<int> RMQ;

    void dfs(int i, int d = 0, int p = -1) {
        v[t] = i, pos[i] = t, dep[t++] = d;
        for (int j : g[i]) if (j != p) {
            dfs(j, d+1, i);
            v[t] = i, dep[t++] = d;
        }
    }
    void build(int n, int root) {
        t = 0;
        dfs(root);
        RMQ = rmq<int>(vector<int>(dep, dep+2*n-1));
    }
    int lca(int a, int b) {
        a = pos[a], b = pos[b];
        return v[RMQ.query(min(a, b), max(a, b))];
    }
    int dist(int a, int b) {
        return dep[pos[a]] + dep[pos[b]] - 2*dep[pos[lca(a,
            b)]];
    }
}
```

3.26 Line Tree

```
// Reduz min-query em arvore para RMQ
// Se o grafo nao for uma arvore, as queries
// sao sobre a arvore geradora maxima
// Queries de minimo
//
// build - O(n log(n))
// query - O(log(n))
// b1f418

int n;

namespace linetree {
    int id[MAX], seg[2*MAX], pos[MAX];
    vector<int> v[MAX], val[MAX];
    vector<pair<int, pair<int, int> > > ar;

    void add(int a, int b, int p) { ar.push_back({p, {a,
        b}}); }
    void build() {
        sort(ar.rbegin(), ar.rend());
        for (int i = 0; i < n; i++) id[i] = i, v[i] = {i},
            val[i].clear();
        for (auto i : ar) {
            int a = id[i.second.first], b =
                id[i.second.second];
            if (a == b) continue;
            if (v[a].size() < v[b].size()) swap(a, b);
            for (auto j : v[b]) id[j] = a, v[a].push_back(j);
            val[a].push_back(i.first);
            for (auto j : val[b]) val[a].push_back(j);
            v[b].clear(), val[b].clear();
        }
        vector<int> vv;
        for (int i = 0; i < n; i++) for (int j = 0; j <
            v[i].size(); j++) {
            pos[v[i][j]] = vv.size();
```

```
            if (j + 1 < v[i].size()) vv.push_back(val[i][j]);
            else vv.push_back(0);
        }
        for (int i = n; i < 2*n; i++) seg[i] = vv[i-n];
        for (int i = n-1; i; i--) seg[i] = min(seg[2*i],
            seg[2*i+1]);
    }
    int query(int a, int b) {
        if (id[a] != id[b]) return 0; // nao estao conectados
        a = pos[a], b = pos[b];
        if (a > b) swap(a, b);
        b--;
        int ans = INF;
        for (a += n, b += n; a <= b; ++a/=2, --b/=2) ans =
            min({ans, seg[a], seg[b]});
        return ans;
    }
};
```

3.27 Link-cut Tree

```
// Link-cut tree padrao
//
// Todas as operacoes sao O(log(n)) amortizado
// e4e663

namespace lct {
    struct node {
        int p, ch[2];
        node() { p = ch[0] = ch[1] = -1; }
    };

    node t[MAX];

    bool is_root(int x) {
        return t[x].p == -1 or (t[t[x].p].ch[0] != x and
            t[t[x].p].ch[1] != x);
    }
    void rotate(int x) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
        bool d = t[p].ch[0] == x;
        t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
        if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
        t[x].p = pp, t[p].p = x;
    }
    void splay(int x) {
        while (!is_root(x)) {
            int p = t[x].p, pp = t[p].p;
            if (!is_root(p)) rotate((t[pp].ch[0] ==
                p)^(t[p].ch[0] == x) ? x : p);
            rotate(x);
        }
    }
    int access(int v) {
        int last = -1;
        for (int w = v; w+1; last = w, splay(v), w = t[v].p)
            splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
        return last;
    }
    int find_root(int v) {
        access(v);
        while (t[v].ch[0]+1) v = t[v].ch[0];
        return splay(v), v;
    }
    void link(int v, int w) { // v deve ser raiz
        access(v);
        t[v].p = w;
    }
    void cut(int v) { // remove aresta de v pro pai
        access(v);
        t[v].ch[0] = t[t[v].ch[0]].p = -1;
    }
    int lca(int v, int w) {
        return access(v), access(w);
    }
}
```

3.28 Link-cut Tree - aresta

```
// Valores nas arestas
// rootify(v) torna v a raiz de sua arvore
// query(v, w) retorna a soma do caminho v--w
// update(v, w, x) soma x nas arestas do caminho v--w
//
// Todas as operacoes sao O(log(n)) amortizado
// 9ce48f

namespace lct {
    struct node {
        int p, ch[2];
        ll val, sub;
        bool rev;
        int sz, ar;
        ll lazy;
        node() {}
        node(int v, int ar_) :
        p(-1), val(v), sub(v), rev(0), sz(ar_), ar(ar_),
            lazy(0) {
             ch[0] = ch[1] = -1;
        }
    };

    node t[2*MAX]; // MAXN + MAXQ
    map<pair<int, int>, int> aresta;
    int sz;

    void prop(int x) {
        if (t[x].lazy) {
            if (t[x].ar) t[x].val += t[x].lazy;
            t[x].sub += t[x].lazy*t[x].sz;
            if (t[x].ch[0]+1) t[t[x].ch[0]].lazy +=
                t[x].lazy;
            if (t[x].ch[1]+1) t[t[x].ch[1]].lazy +=
                t[x].lazy;
        }
        if (t[x].rev) {
            swap(t[x].ch[0], t[x].ch[1]);
            if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
            if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
        }
        t[x].lazy = 0, t[x].rev = 0;
    }
    void update(int x) {
        t[x].sz = t[x].ar, t[x].sub = t[x].val;
        for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
            prop(t[x].ch[i]);
            t[x].sz += t[t[x].ch[i]].sz;
            t[x].sub += t[t[x].ch[i]].sub;
        }
    }
    bool is_root(int x) {
        return t[x].p == -1 or (t[t[x].p].ch[0] != x and
            t[t[x].p].ch[1] != x);
    }
    void rotate(int x) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
        bool d = t[p].ch[0] == x;
        t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
        if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
        t[x].p = pp, t[p].p = x;
        update(p), update(x);
    }
    int splay(int x) {
        while (!is_root(x)) {
            int p = t[x].p, pp = t[p].p;
            if (!is_root(p)) prop(pp);
            prop(p), prop(x);
            if (!is_root(p)) rotate((t[pp].ch[0] ==
                p)^(t[p].ch[0] == x) ? x : p);
            rotate(x);
        }
        return prop(x), x;
    }
    int access(int v) {
        int last = -1;
        for (int w = v; w+1; update(last = w), splay(v), w =
            t[v].p)
             splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
        return last;
    }
    void make_tree(int v, int w=0, int ar=0) { t[v] =
        node(w, ar); }
```

```
int find_root(int v) {
    access(v), prop(v);
    while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
    return splay(v);
}
bool conn(int v, int w) {
    access(v), access(w);
    return v == w ? true : t[v].p != -1;
}
void rootify(int v) {
    access(v);
    t[v].rev ^= 1;
}
ll query(int v, int w) {
    rootify(w), access(v);
    return t[v].sub;
}
void update(int v, int w, int x) {
    rootify(w), access(v);
    t[v].lazy += x;
}
void link_(int v, int w) {
    rootify(w);
    t[w].p = v;
}
void link(int v, int w, int x) { // v--w com peso x
    int id = MAX + sz++;
    aresta[make_pair(v, w)] = id;
    make_tree(id, x, 1);
    link_(v, id), link_(id, w);
}
void cut_(int v, int w) {
    rootify(w), access(v);
    t[v].ch[0] = t[t[v].ch[0]].p = -1;
}
void cut(int v, int w) {
    int id = aresta[make_pair(v, w)];
    cut_(v, id), cut_(id, w);
}
int lca(int v, int w) {
    access(v);
    return access(w);
```

```
    }
}
```

3.29 Link-cut Tree - vertice

```
// Valores nos vertices
// make_tree(v, w) cria uma nova arvore com um
// vertice soh com valor 'w'
// rootify(v) torna v a raiz de sua arvore
// query(v, w) retorna a soma do caminho v--w
// update(v, w, x) soma x nos vertices do caminho v--w
//
// Todas as operacoes sao O(log(n)) amortizado
// f9f489

namespace lct {
    struct node {
        int p, ch[2];
        ll val, sub;
        bool rev;
        int sz;
        ll lazy;
        node() {}
        node(int v) : p(-1), val(v), sub(v), rev(0), sz(1),
            lazy(0) {
             ch[0] = ch[1] = -1;
        }
    };

    node t[MAX];

    void prop(int x) {
        if (t[x].lazy) {
            t[x].val += t[x].lazy, t[x].sub +=
                t[x].lazy*t[x].sz;
            if (t[x].ch[0]+1) t[t[x].ch[0]].lazy +=
                t[x].lazy;
            if (t[x].ch[1]+1) t[t[x].ch[1]].lazy +=
                t[x].lazy;
        }
```

```
    if (t[x].rev) {
        swap(t[x].ch[0], t[x].ch[1]);
        if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
        if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
    }
    t[x].lazy = 0, t[x].rev = 0;
}
void update(int x) {
    t[x].sz = 1, t[x].sub = t[x].val;
    for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
        prop(t[x].ch[i]);
        t[x].sz += t[t[x].ch[i]].sz;
        t[x].sub += t[t[x].ch[i]].sub;
    }
}
bool is_root(int x) {
    return t[x].p == -1 or (t[t[x].p].ch[0] != x and
       t[t[x].p].ch[1] != x);
}
void rotate(int x) {
    int p = t[x].p, pp = t[p].p;
    if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
    bool d = t[p].ch[0] == x;
    t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
    if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
    t[x].p = pp, t[p].p = x;
    update(p), update(x);
}
int splay(int x) {
    while (!is_root(x)) {
        int p = t[x].p, pp = t[p].p;
        if (!is_root(p)) prop(pp);
        prop(p), prop(x);
        if (!is_root(p)) rotate((t[pp].ch[0] ==
           p)^(t[p].ch[0] == x) ? x : p);
        rotate(x);
    }
    return prop(x), x;
}
int access(int v) {
    int last = -1;
    for (int w = v; w+1; update(last = w), splay(v), w =
            t[v].p)
             splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
    return last;
}
void make_tree(int v, int w) { t[v] = node(w); }
int find_root(int v) {
    access(v), prop(v);
    while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
    return splay(v);
}
bool connected(int v, int w) {
    access(v), access(w);
    return v == w ? true : t[v].p != -1;
}
void rootify(int v) {
    access(v);
    t[v].rev ^= 1;
}
ll query(int v, int w) {
    rootify(w), access(v);
    return t[v].sub;
}
void update(int v, int w, int x) {
    rootify(w), access(v);
    t[v].lazy += x;
}
void link(int v, int w) {
    rootify(w);
    t[w].p = v;
}
void cut(int v, int w) {
    rootify(w), access(v);
    t[v].ch[0] = t[t[v].ch[0]].p = -1;
}
int lca(int v, int w) {
    access(v);
    return access(w);
}
}
```

3.30 Max flow com lower bound nas arestas

```
// add(a, b, l, r):
//  adiciona aresta de a pra b, onde precisa passar f de
    fluxo, l <= f <= r
// add(a, b, c):
//  adiciona aresta de a pra b com capacidade c
//
// Mesma complexidade do Dinic
// 5f2379

struct lb_max_flow : dinic {
    vector<int> d;
    lb_max_flow(int n) : dinic(n + 2), d(n, 0) {}
    void add(int a, int b, int l, int r) {
        d[a] -= l;
        d[b] += l;
        dinic::add(a, b, r - l);
    }
    void add(int a, int b, int c) {
        dinic::add(a, b, c);
    }
    bool has_circulation() {
        int n = d.size();

        ll cost = 0;
        for (int i = 0; i < n; i++) {
            if (d[i] > 0) {
                cost += d[i];
                dinic::add(n, i, d[i]);
            } else if (d[i] < 0) {
                dinic::add(i, n+1, -d[i]);
            }
        }

        return (dinic::max_flow(n, n+1) == cost);
    }
    bool has_flow(int src, int snk) {
        dinic::add(snk, src, INF);
        return has_circulation();
    }
    ll max_flow(int src, int snk) {
        if (!has_flow(src, snk)) return -1;
        dinic::F = 0;
        return dinic::max_flow(src, snk);
    }
};
```

3.31 MinCostMaxFlow

```
// min_cost_flow(s, t, f) computa o par (fluxo, custo)
// com max(fluxo) <= f que tenha min(custo)
// min_cost_flow(s, t) -> Fluxo maximo de custo minimo de s
    pra t
// Se for um dag, da pra substituir o SPFA por uma DP pra nao
// pagar O(nm) no comeco
// Se nao tiver aresta com custo negativo, nao precisa do
    SPFA
//
// O(nm + f * m log n)
// 697b4c

template<typename T> struct mcmf {
    struct edge {
        int to, rev, flow, cap; // para, id da reversa,
            fluxo, capacidade
        bool res; // se eh reversa
        T cost; // custo da unidade de fluxo
        edge() : to(0), rev(0), flow(0), cap(0), cost(0),
            res(false) {}
        edge(int to_, int rev_, int flow_, int cap_, T
            cost_, bool res_)
            : to(to_), rev(rev_), flow(flow_), cap(cap_),
                res(res_), cost(cost_) {}
    };

    vector<vector<edge>> g;
    vector<int> par_idx, par;
    T inf;
    vector<T> dist;

    mcmf(int n) : g(n), par_idx(n), par(n),
```

```
    inf(numeric_limits<T>::max()/3) {}

void add(int u, int v, int w, T cost) { // de u pra v
    com cap w e custo cost
    edge a = edge(v, g[v].size(), 0, w, cost, false);
    edge b = edge(u, g[u].size(), 0, 0, -cost, true);

    g[u].push_back(a);
    g[v].push_back(b);
}

vector<T> spfa(int s) { // nao precisa se nao tiver
    custo negativo
    deque<int> q;
    vector<bool> is_inside(g.size(), 0);
    dist = vector<T>(g.size(), inf);

    dist[s] = 0;
    q.push_back(s);
    is_inside[s] = true;

    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        is_inside[v] = false;

        for (int i = 0; i < g[v].size(); i++) {
            auto [to, rev, flow, cap, res, cost] =
                g[v][i];
            if (flow < cap and dist[v] + cost <
                dist[to]) {
                dist[to] = dist[v] + cost;

                if (is_inside[to]) continue;
                if (!q.empty() and dist[to] >
                    dist[q.front()]) q.push_back(to);
                else q.push_front(to);
                is_inside[to] = true;
            }
        }
    }
    return dist;
}
bool dijkstra(int s, int t, vector<T>& pot) {
    priority_queue<pair<T, int>, vector<pair<T, int>>,
        greater<>> q;
    dist = vector<T>(g.size(), inf);
    dist[s] = 0;
    q.emplace(0, s);
    while (q.size()) {
        auto [d, v] = q.top();
        q.pop();
        if (dist[v] < d) continue;
        for (int i = 0; i < g[v].size(); i++) {
            auto [to, rev, flow, cap, res, cost] =
                g[v][i];
            cost += pot[v] - pot[to];
            if (flow < cap and dist[v] + cost <
                dist[to]) {
                dist[to] = dist[v] + cost;
                q.emplace(dist[to], to);
                par_idx[to] = i, par[to] = v;
            }
        }
    }
    return dist[t] < inf;
}

pair<int, T> min_cost_flow(int s, int t, int flow = INF)
    {
    vector<T> pot(g.size(), 0);
    pot = spfa(s); // mudar algoritmo de caminho minimo
        aqui

    int f = 0;
    T ret = 0;
    while (f < flow and dijkstra(s, t, pot)) {
        for (int i = 0; i < g.size(); i++)
            if (dist[i] < inf) pot[i] += dist[i];

        int mn_flow = flow - f, u = t;
        while (u != s){
            mn_flow = min(mn_flow,
                g[par[u]][par_idx[u]].cap -
```

```
                    g[par[u]][par_idx[u]].flow);
                u = par[u];
            }

            ret += pot[t] * mn_flow;

            u = t;
            while (u != s) {
                g[par[u]][par_idx[u]].flow += mn_flow;
                g[u][g[par[u]][par_idx[u]].rev].flow -=
                    mn_flow;
                u = par[u];
            }

            f += mn_flow;
        }

        return make_pair(f, ret);
    }

    // Opcional: retorna as arestas originais por onde passa
        flow = cap
    vector<pair<int,int>> recover() {
        vector<pair<int,int>> used;
        for (int i = 0; i < g.size(); i++) for (edge e :
            g[i])
            if(e.flow == e.cap && !e.res) used.push_back({i,
                e.to});
        return used;
    }
};
```

3.32 Prufer code

```
// Traduz de lista de arestas para prufer code
// e vice-versa
// Os vertices tem label de 0 a n-1
// Todo array com n-2 posicoes e valores de
// 0 a n-1 sao prufer codes validos
//
// O(n)

// d3b324
vector<int> to_prufer(vector<pair<int, int>> tree) {
    int n = tree.size()+1;
    vector<int> d(n, 0);
    vector<vector<int>> g(n);
    for (auto [a, b] : tree) d[a]++, d[b]++,
        g[a].push_back(b), g[b].push_back(a);
    vector<int> pai(n, -1);
    queue<int> q; q.push(n-1);
    while (q.size()) {
        int u = q.front(); q.pop();
        for (int v : g[u]) if (v != pai[u])
            pai[v] = u, q.push(v);
    }
    int idx, x;
    idx = x = find(d.begin(), d.end(), 1) - d.begin();
    vector<int> ret;
    for (int i = 0; i < n-2; i++) {
        int y = pai[x];
        ret.push_back(y);
        if (--d[y] == 1 and y < idx) x = y;
        else idx = x = find(d.begin()+idx+1, d.end(), 1) -
            d.begin();
    }
    return ret;
}

// 765413
vector<pair<int, int>> from_prufer(vector<int> p) {
    int n = p.size()+2;
    vector<int> d(n, 1);
    for (int i : p) d[i]++;
    p.push_back(n-1);
    int idx, x;
    idx = x = find(d.begin(), d.end(), 1) - d.begin();
    vector<pair<int, int>> ret;
    for (int y : p) {
        ret.push_back({x, y});
        if (--d[y] == 1 and y < idx) x = y;
        else idx = x = find(d.begin()+idx+1, d.end(), 1) -
```

```
            d.begin();
    }
    return ret;
}
```

3.33 Sack (DSU em arvores)

```
// Responde queries de todas as sub-arvores
// offline
//
// O(n log(n))
// bb361f

int sz[MAX], cor[MAX], cnt[MAX];
vector<int> g[MAX];

void build(int k, int d=0) {
    sz[k] = 1;
    for (auto& i : g[k]) {
        build(i, d+1); sz[k] += sz[i];
        if (sz[i] > sz[g[k][0]]) swap(i, g[k][0]);
    }
}

void compute(int k, int x, bool dont=1) {
    cnt[cor[k]] += x;
    for (int i = dont; i < g[k].size(); i++)
        compute(g[k][i], x, 0);
}

void solve(int k, bool keep=0) {
    for (int i = int(g[k].size())-1; i >= 0; i--)
        solve(g[k][i], !i);
    compute(k, 1);

    // agora cnt[i] tem quantas vezes a cor
    // i aparece na sub-arvore do k

    if (!keep) compute(k, -1, 0);
}
```

3.34 Tarjan para SCC

```
// O(n + m)
// 573bfa

vector<int> g[MAX];
stack<int> s;
int vis[MAX], comp[MAX];
int id[MAX];

// se quiser comprimir ciclo ou achar ponte em grafo nao
   direcionado,
// colocar um if na dfs para nao voltar pro pai da DFS tree
int dfs(int i, int& t) {
    int lo = id[i] = t++;
    s.push(i);
    vis[i] = 2;

    for (int j : g[i]) {
        if (!vis[j]) lo = min(lo, dfs(j, t));
        else if (vis[j] == 2) lo = min(lo, id[j]);
    }

    // aresta de i pro pai eh uma ponte (no caso nao
       direcionado)
    if (lo == id[i]) while (1) {
        int u = s.top(); s.pop();
        vis[u] = 1, comp[u] = i;
        if (u == i) break;
    }

    return lo;
}

void tarjan(int n) {
    int t = 0;
    for (int i = 0; i < n; i++) vis[i] = 0;

    for (int i = 0; i < n; i++) if (!vis[i]) dfs(i, t);
}
```

3.35 Topological Sort

```
// Retorna uma ordenacaoo topologica de g
// Se g nao for DAG retorna um vetor vazio
//
// O(n + m)
// bdc95e

vector<int> g[MAX];

vector<int> topo_sort(int n) {
    vector<int> ret(n,-1), vis(n,0);

    int pos = n-1, dag = 1;
    function<void(int)> dfs = [&](int v) {
        vis[v] = 1;
        for (auto u : g[v]) {
            if (vis[u] == 1) dag = 0;
            else if (!vis[u]) dfs(u);
        }
        ret[pos--] = v, vis[v] = 2;
    };

    for (int i = 0; i < n; i++) if (!vis[i]) dfs(i);

    if (!dag) ret.clear();
    return ret;
}
```

3.36 Vertex cover

```
// Encontra o tamanho do vertex cover minimo
// Da pra alterar facil pra achar os vertices
// Parece rodar com < 2 s pra N = 90
//
// O(n * 1.38^n)
// 9c5024

namespace cover {
    const int MAX = 96;
    vector<int> g[MAX];
    bitset<MAX> bs[MAX];
    int n;

    void add(int i, int j) {
        if (i == j) return;
        n = max({n, i+1, j+1});
        bs[i][j] = bs[j][i] = 1;
    }

    int rec(bitset<MAX> m) {
        int ans = 0;
        for (int x = 0; x < n; x++) if (m[x]) {
            bitset<MAX> comp;
            function<void(int)> dfs = [&](int i) {
                comp[i] = 1, m[i] = 0;
                for (int j : g[i]) if (m[j]) dfs(j);
            };
            dfs(x);

            int ma, deg = -1, cyc = 1;
            for (int i = 0; i < n; i++) if (comp[i]) {
                int d = (bs[i]&comp).count();
                if (d <= 1) cyc = 0;
                if (d > deg) deg = d, ma = i;
            }
            if (deg <= 2) { // caminho ou ciclo
                ans += (comp.count() + cyc) / 2;
                continue;
            }
            comp[ma] = 0;

            // ou ta no cover, ou nao ta no cover
            ans += min(1 + rec(comp), deg + rec(comp & ~
               bs[ma]));
        }
        return ans;
    }
    int solve() {
        bitset<MAX> m;
        for (int i = 0; i < n; i++) {
            m[i] = 1;
```

```
            for (int j = 0; j < n; j++)
                if (bs[i][j]) g[i].push_back(j);
        }
        return rec(m);
    }
}
```

3.37 Virtual Tree

```
// Comprime uma arvore dado um conjunto S de vertices, de
   forma que
// o conjunto de vertices da arvore comprimida contenha S e
   seja
// minimal e fechado sobre a operacao de LCA
// Se |S| = k, a arvore comprimida tem menos que 2k vertices
// As arestas de virt possuem a distancia do vertice ate o
   vizinho
// Retorna a raiz da virtual tree
//
// O(k log(k))
// 42d990

vector<pair<int, int>> virt[MAX];

#warning lembrar de buildar o LCA antes
int build_virt(vector<int> v) {
    auto cmp = [&](int i, int j) { return lca::pos[i] <
       lca::pos[j]; };
    sort(v.begin(), v.end(), cmp);
    for (int i = v.size()-1; i; i--)
       v.push_back(lca::lca(v[i], v[i-1]));
    sort(v.begin(), v.end(), cmp);
    v.erase(unique(v.begin(), v.end()), v.end());
    for (int i = 0; i < v.size(); i++) virt[v[i]].clear();
    for (int i = 1; i < v.size(); i++) virt[lca::lca(v[i-1],
       v[i])].clear();
    for (int i = 1; i < v.size(); i++) {
        int parent = lca::lca(v[i-1], v[i]);
        int d = lca::dist(parent, v[i]);
#warning soh to colocando aresta descendo
        virt[parent].emplace_back(v[i], d);
    }
    return v[0];
}
```

4 Estruturas

4.1 BIT

```
// BIT de soma 1-based, v 0-based
// Para mudar o valor da posicao p para x,
// faca: poe(x - query(p, p), p)
// l_bound(x) retorna o menor p tal que
// query(1, p+1) > x    (0 based!)
//
// Complexidades:
// build - O(n)
// poe - O(log(n))
// query - O(log(n))
// l_bound - O(log(n))
// d432a4

int n;
int bit[MAX];
int v[MAX];

void build() {
    bit[0] = 0;
    for (int i = 1; i <= n; i++) bit[i] = v[i - 1];

    for (int i = 1; i <= n; i++) {
        int j = i + (i & -i);
        if (j <= n) bit[j] += bit[i];
    }
}

// soma x na posicao p
void poe(int x, int p) {
    for (; p <= n; p += p & -p) bit[p] += x;
```

```
}

// soma [1, p]
int pref(int p) {
    int ret = 0;
    for (; p; p -= p & -p) ret += bit[p];
    return ret;
}

// soma [a, b]
int query(int a, int b) {
    return pref(b) - pref(a - 1);
}

int l_bound(ll x) {
    int p = 0;
    for (int i = MAX2; i+1; i--) if (p + (1<<i) <= n
        and bit[p + (1<<i)] <= x) x -= bit[p += (1<<i)];
    return p;
}
```

4.2 BIT 2D

```
// BIT de soma, update incrementa posicao
// Tem que construir com um vetor com todos os pontos
// que vc quer um dia atualizar (os pontos q vc vai chamar
    update)
//
// Complexidades:
// construir - O(n log(n))
// update e query - O(log^2(n))
// 6a760a

template<class T = int> struct bit2d {
    vector<T> X;
    vector<vector<T>> Y, t;

    int ub(vector<T>& v, T x) {
        return upper_bound(v.begin(), v.end(), x) -
            v.begin();
    }
    bit2d(vector<pair<T, T>> v) {
        for (auto [x, y] : v) X.push_back(x);
        sort(X.begin(), X.end());
        X.erase(unique(X.begin(), X.end()), X.end());

        t.resize(X.size() + 1);
        Y.resize(t.size());
        sort(v.begin(), v.end(), [](auto a, auto b) {
            return a.second < b.second; });
        for (auto [x, y] : v) for (int i = ub(X, x); i <
            t.size(); i += i&-i)
            if (!Y[i].size() or Y[i].back() != y)
                Y[i].push_back(y);

        for (int i = 0; i < t.size(); i++)
            t[i].resize(Y[i].size() + 1);
    }

    void update(T x, T y, T v) {
        for (int i = ub(X, x); i < t.size(); i += i&-i)
            for (int j = ub(Y[i], y); j < t[i].size(); j +=
                j&-j) t[i][j] += v;
    }

    T query(T x, T y) {
        T ans = 0;
        for (int i = ub(X, x); i; i -= i&-i)
            for (int j = ub(Y[i], y); j; j -= j&-j) ans +=
                t[i][j];
        return ans;
    }
    T query(T x1, T y1, T x2, T y2) {
        return query(x2, y2)-query(x2, y1-1)-query(x1-1,
            y2)+query(x1-1, y1-1);
    }
};
```

4.3 BIT com update em range

```
// Operacoes 0-based
// query(l, r) retorna a soma de v[l..r]
// update(l, r, x) soma x em v[l..r]
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))
// f91737

namespace bit {
	ll bit[2][MAX+2];
	int n;

	void build(int n2, int* v) {
		n = n2;
		for (int i = 1; i <= n; i++)
			bit[1][min(n+1, i+(i&-i))] += bit[1][i] +=
				v[i-1];
	}
	ll get(int x, int i) {
		ll ret = 0;
		for (; i; i -= i&-i) ret += bit[x][i];
		return ret;
	}
	void add(int x, int i, ll val) {
		for (; i <= n; i += i&-i) bit[x][i] += val;
	}
	ll get2(int p) {
		return get(0, p) * p + get(1, p);
	}
	ll query(int l, int r) {
		return get2(r+1) - get2(l);
	}
	void update(int l, int r, ll x) {
		add(0, l+1, x), add(0, r+2, -x);
		add(1, l+1, -x*l), add(1, r+2, x*(r+1));
	}
};
```

4.4 DSU

```
// Une dois conjuntos e acha a qual conjunto um elemento
	pertence por seu id
//
// find e unite: O(a(n)) ~= O(1) amortizado
// 8e197e

struct dsu {
	vector<int> id, sz;

	dsu(int n) : id(n), sz(n, 1) { iota(id.begin(),
		id.end(), 0); }

	int find(int a) { return a == id[a] ? a : id[a] =
		find(id[a]); }

	void unite(int a, int b) {
		a = find(a), b = find(b);
		if (a == b) return;
		if (sz[a] < sz[b]) swap(a, b);
		sz[a] += sz[b], id[b] = a;
	}
};

// DSU de bipartido
//
// Une dois vertices e acha a qual componente um vertice
	pertence
// Informa se a componente de um vertice e bipartida
//
// find e unite: O(log(n))
// 118050

struct dsu {
	vector<int> id, sz, bip, c;

	dsu(int n) : id(n), sz(n, 1), bip(n, 1), c(n) {
		iota(id.begin(), id.end(), 0);
	}

	int find(int a) { return a == id[a] ? a : find(id[a]); }
```

```
    int color(int a) { return a == id[a] ? c[a] : c[a] ^
        color(id[a]); }

    void unite(int a, int b) {
        bool change = color(a) == color(b);
        a = find(a), b = find(b);
        if (a == b) {
            if (change) bip[a] = 0;
            return;
        }

        if (sz[a] < sz[b]) swap(a, b);
        if (change) c[b] = 1;
        sz[a] += sz[b], id[b] = a, bip[a] &= bip[b];
    }
};


// DSU Persistente
//
// Persistencia parcial, ou seja, tem que ir
// incrementando o 't' no une
//
// find e unite: O(log(n))
// 6c63a4

struct dsu {
    vector<int> id, sz, ti;

    dsu(int n) : id(n), sz(n, 1), ti(n, -INF) {
        iota(id.begin(), id.end(), 0);
    }

    int find(int a, int t) {
        if (id[a] == a or ti[a] > t) return a;
        return find(id[a], t);
    }

    void unite(int a, int b, int t) {
        a = find(a, t), b = find(b, t);
        if (a == b) return;
        if (sz[a] < sz[b]) swap(a, b);
        sz[a] += sz[b], id[b] = a, ti[b] = t;
    }
};

// DSU com rollback
//
// checkpoint(): salva o estado atual de todas as variaveis
// rollback(): retorna para o valor das variaveis para
// o ultimo checkpoint
//
// Sempre que uma variavel muda de valor, adiciona na stack
//
// find e unite: O(log(n))
// checkpoint: O(1)
// rollback: O(m) em que m e o numero de vezes que alguma
// variavel mudou de valor desde o ultimo checkpoint
// c6e923

struct dsu {
    vector<int> id, sz;
    stack<stack<pair<int&, int>>> st;

    dsu(int n) : id(n), sz(n, 1) {
        iota(id.begin(), id.end(), 0), st.emplace();
    }

    void save(int &x) { st.top().emplace(x, x); }

    void checkpoint() { st.emplace(); }

    void rollback() {
        while(st.top().size()) {
            auto [end, val] = st.top().top(); st.top().pop();
            end = val;
        }
        st.pop();
    }

    int find(int a) { return a == id[a] ? a : find(id[a]); }

    void unite(int a, int b) {
        a = find(a), b = find(b);
```

```
        if (a == b) return;
        if (sz[a] < sz[b]) swap(a, b);
        save(sz[a]), save(id[b]);
        sz[a] += sz[b], id[b] = a;
    }
};
```

4.5 Li-Chao Tree

```
// Adiciona retas (ax+b), e computa o minimo entre as retas
// em um dado 'x'
// Cuidado com overflow!
// Se tiver overflow, tenta comprimir o 'x' ou usar
// convex hull trick
//
// O(log(MA-MI)), O(n) de memoria
// 59ba68

template<ll MI = ll(-1e9), ll MA = ll(1e9)> struct lichao {
    struct line {
        ll a, b;
        array<int, 2> ch;
        line(ll a_ = 0, ll b_ = LINF) :
            a(a_), b(b_), ch({-1, -1}) {}
        ll operator ()(ll x) { return a*x + b; }
    };
    vector<line> ln;

    int ch(int p, int d) {
        if (ln[p].ch[d] == -1) {
            ln[p].ch[d] = ln.size();
            ln.emplace_back();
        }
        return ln[p].ch[d];
    }
    lichao() { ln.emplace_back(); }

    void add(line s, ll l=MI, ll r=MA, int p=0) {
        ll m = (l+r)/2;
        bool L = s(l) < ln[p](l);
        bool M = s(m) < ln[p](m);
        bool R = s(r) < ln[p](r);
        if (M) swap(ln[p], s), swap(ln[p].ch, s.ch);
        if (s.b == LINF) return;
        if (L != M) add(s, l, m-1, ch(p, 0));
        else if (R != M) add(s, m+1, r, ch(p, 1));
    }
    ll query(int x, ll l=MI, ll r=MA, int p=0) {
        ll m = (l+r)/2, ret = ln[p](x);
        if (ret == LINF) return ret;
        if (x < m) return min(ret, query(x, l, m-1, ch(p,
           0)));
        return min(ret, query(x, m+1, r, ch(p, 1)));
    }
};
```

4.6 MergeSort Tree

```
// Se for construida sobre um array:
//      count(i, j, a, b) retorna quantos
//      elementos de v[i..j] pertencem a [a, b]
//      report(i, j, a, b) retorna os indices dos
//      elementos de v[i..j] que pertencem a [a, b]
//      retorna o vetor ordenado
// Se for construida sobre pontos (x, y):
//      count(x1, x2, y1, x2) retorna quantos pontos
//      pertencem ao retangulo (x1, y1), (x2, y2)
//      report(x1, x2, y1, y2) retorna os indices dos pontos
   que
//      pertencem ao retangulo (x1, y1), (x2, y2)
//      retorna os pontos ordenados lexicograficamente
//      (assume x1 <= x2, y1 <= y2)
//
// kth(y1, y2, k) retorna o indice do ponto com k-esimo menor
// x dentre os pontos que possuem y em [y1, y2] (0 based)
// Se quiser usar para achar k-esimo valor em range,
   construir
// com ms_tree t(v, true), e chamar kth(l, r, k)
//
// Usa O(n log(n)) de memoria
```

```
//
// Complexidades:
// construir - O(n log(n))
// count - O(log(n))
// report - O(log(n) + k) para k indices retornados
// kth - O(log(n))
// 1cef03

template <typename T = int> struct ms_tree {
    vector<tuple<T, T, int>> v;
    int n;
    vector<vector<tuple<T, T, int>>> t; // {y, idx, left}
    vector<T> vy;

    ms_tree(vector<pair<T, T>>& vv) : n(vv.size()), t(4*n),
        vy(n) {
        for (int i = 0; i < n; i++)
            v.push_back({vv[i].first, vv[i].second, i});
        sort(v.begin(), v.end());
        build(1, 0, n-1);
        for (int i = 0; i < n; i++) vy[i] =
            get<0>(t[1][i+1]);
    }
    ms_tree(vector<T>& vv, bool inv = false) { // inv:
        inverte indice e valor
        vector<pair<T, T>> v2;
        for (int i = 0; i < vv.size(); i++)
            inv ? v2.push_back({vv[i], i}) :
                v2.push_back({i, vv[i]});
        *this = ms_tree(v2);
    }
    void build(int p, int l, int r) {
        t[p].push_back({get<0>(v[l]), get<0>(v[r]), 0}); //
            {min_x, max_x, 0}
        if (l == r) return t[p].push_back({get<1>(v[l]),
            get<2>(v[l]), 0});
        int m = (l+r)/2;
        build(2*p, l, m), build(2*p+1, m+1, r);

        int L = 0, R = 0;
        while (t[p].size() <= r-l+1) {
            int left = get<2>(t[p].back());
            if (L > m-l or (R+m+1 <= r and t[2*p+1][1+R] <
                t[2*p][1+L])) {
                t[p].push_back(t[2*p+1][1 + R++]);
                get<2>(t[p].back()) = left;
                continue;
            }
            t[p].push_back(t[2*p][1 + L++]);
            get<2>(t[p].back()) = left+1;
        }
    }

    int get_l(T y) { return lower_bound(vy.begin(),
        vy.end(), y) - vy.begin(); }
    int get_r(T y) { return upper_bound(vy.begin(),
        vy.end(), y) - vy.begin(); }

    int count(T x1, T x2, T y1, T y2) {
        function<int(int, int, int)> dfs = [&](int p, int l,
            int r) {
            if (l == r or x2 < get<0>(t[p][0]) or
                get<1>(t[p][0]) < x1) return 0;
            if (x1 <= get<0>(t[p][0]) and get<1>(t[p][0]) <=
                x2) return r-l;
            int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
            return dfs(2*p, nl, nr) + dfs(2*p+1, l-nl, r-nr);
        };
        return dfs(1, get_l(y1), get_r(y2));
    }
    vector<int> report(T x1, T x2, T y1, T y2) {
        vector<int> ret;
        function<void(int, int, int)> dfs = [&](int p, int
            l, int r) {
            if (l == r or x2 < get<0>(t[p][0]) or
                get<1>(t[p][0]) < x1) return;
            if (x1 <= get<0>(t[p][0]) and get<1>(t[p][0]) <=
                x2) {
                for (int i = l; i < r; i++)
                    ret.push_back(get<1>(t[p][i+1]));
                return;
            }
            int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
            dfs(2*p, nl, nr), dfs(2*p+1, l-nl, r-nr);
```

```
    };
    dfs(1, get_l(y1), get_r(y2));
    return ret;
  }
  int kth(T y1, T y2, int k) {
    function<int(int, int, int)> dfs = [&](int p, int l,
        int r) {
      if (k >= r-l) {
        k -= r-l;
        return -1;
      }
      if (r-l == 1) return get<1>(t[p][l+1]);
      int nl = get<2>(t[p][l]), nr = get<2>(t[p][r]);
      int left = dfs(2*p, nl, nr);
      if (left != -1) return left;
      return dfs(2*p+1, l-nl, r-nr);
    };
    return dfs(1, get_l(y1), get_r(y2));
  }
};
```

4.7 Min queue - deque

```
// Tudo O(1) amortizado
// c13c57

template<class T> struct minqueue {
  deque<pair<T, int>> q;

  void push(T x) {
    int ct = 1;
    while (q.size() and x < q.front().first)
      ct += q.front().second, q.pop_front();
    q.emplace_front(x, ct);
  }
  void pop() {
    if (q.back().second > 1) q.back().second--;
    else q.pop_back();
  }
  T min() { return q.back().first; }
};
```

4.8 Min queue - stack

```
// Tudo O(1) amortizado
// fe0cad

template<class T> struct minstack {
  stack<pair<T, T>> s;

  void push(T x) {
    if (!s.size()) s.push({x, x});
    else s.emplace(x, std::min(s.top().second, x));
  }
  T top() { return s.top().first; }
  T pop() {
    T ans = s.top().first;
    s.pop();
    return ans;
  }
  int size() { return s.size(); }
  T min() { return s.top().second; }
};

template<class T> struct minqueue {
  minstack<T> s1, s2;

  void push(T x) { s1.push(x); }
  void move() {
    if (s2.size()) return;
    while (s1.size()) {
      T x = s1.pop();
      s2.push(x);
    }
  }
  T front() { return move(), s2.top(); }
  T pop() { return move(), s2.pop(); }
  int size() { return s1.size()+s2.size(); }
  T min() {
    if (!s1.size()) return s2.min();
```

```
        else if (!s2.size()) return s1.min();
        return std::min(s1.min(), s2.min());
    }
};
```

4.9 Order Statistic Set

```
// Funciona do C++11 pra cima

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <class T>
    using ord_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

// para declarar:
ord_set<int> s;
// coisas do set normal funcionam:
for (auto i : s) cout << i << endl;
cout << s.size() << endl;
// k-esimo maior elemento O(log|s|):
// k=0: menor elemento
cout << *s.find_by_order(k) << endl;
// quantos sao menores do que k O(log|s|):
cout << s.order_of_key(k) << endl;

// Para fazer um multiset, tem que
// usar ord_set<pair<int, int>> com o
// segundo parametro sendo algo para diferenciar
// os ementos iguais.
// s.order_of_key({k, -INF}) vai retornar o
// numero de elementos < k
```

4.10 Range color

```
// update(l, r, c) colore o range [l, r] com a cor c,
// e retorna os ranges que foram coloridos {l, r, cor}
// query(i) returna a cor da posicao i
//
// Complexidades (para q operacoes):
// update - O(log(q)) amortizado
// query - O(log(q))
// 9e9cab

template<typename T> struct color {
    set<tuple<int, int, T>> se;

    vector<tuple<int, int, T>> update(int l, int r, T val) {
        auto it = se.upper_bound({r, INF, val});
        if (it != se.begin() and get<1>(*prev(it)) > r) {
            auto [L, R, V] = *--it;
            se.erase(it);
            se.emplace(L, r, V), se.emplace(r+1, R, V);
        }
        it = se.lower_bound({l, -INF, val});
        if (it != se.begin() and get<1>(*prev(it)) >= l) {
            auto [L, R, V] = *--it;
            se.erase(it);
            se.emplace(L, l-1, V), it = se.emplace(l, R,
               V).first;
        }
        vector<tuple<int, int, T>> ret;
        for (; it != se.end() and get<0>(*it) <= r; it =
            se.erase(it))
            ret.push_back(*it);
        se.emplace(l, r, val);
        return ret;
    }
    T query(int i) {
        auto it = se.upper_bound({i, INF, T()});
        if (it == se.begin() or get<1>(*--it) < i) return
            -1; // nao tem
        return get<2>(*it);
    }
};
```

4.11 RMQ <O(n), O(1)> - min queue

```
// O(n) pra buildar, query O(1)
// Se tiver varios minimos, retorna
// o de menor indice
// bab412

template<typename T> struct rmq {
	vector<T> v;
	int n; static const int b = 30;
	vector<int> mask, t;

	int op(int x, int y) { return v[x] <= v[y] ? x : y; }
	int msb(int x) { return
		__builtin_clz(1)-__builtin_clz(x); }
	int small(int r, int sz = b) { return
		r-msb(mask[r]&((1<<sz)-1)); }
	rmq() {}
	rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n),
		t(n) {
		for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
			at = (at<<1)&((1<<b)-1);
			while (at and op(i-msb(at&-at), i) == i) at ^=
				at&-at;
		}
		for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
		for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0;
			i+(1<<j) <= n/b; i++)
			t[n/b*j+i] = op(t[n/b*(j-1)+i],
				t[n/b*(j-1)+i+(1<<(j-1))]);
	}
	int index_query(int l, int r) {
		if (r-l+1 <= b) return small(r, r-l+1);
		int x = l/b+1, y = r/b-1;
		if (x > y) return op(small(l+b-1), small(r));
		int j = msb(y-x+1);
		int ans = op(small(l+b-1), op(t[n/b*j+x],
			t[n/b*j+y-(1<<j)+1]));
		return op(ans, small(r));
	}
	T query(int l, int r) { return v[index_query(l, r)]; }
};
```

4.12 SegTreap

```
// Muda uma posicao do plano, e faz query de operacao
// associativa e comutativa em retangulo
// Mudar ZERO e op
// Esparso nas duas coordenadas, inicialmente eh tudo ZERO
//
// Para query com distancia de manhattan <= d, faca
// nx = x+y, ny = x-y
// Update em (nx, ny), query em ((nx-d, ny-d), (nx+d, ny+d))
//
// Valores no X tem que ser de 0 ateh NX
// Para q operacoes, usa O(q log(NX)) de memoria, e as
// operacoes custa O(log(q) log(NX))
// 75f2d0

const int ZERO = INF;
const int op(int l, int r) { return min(l, r); }

mt19937 rng((int)
	chrono::steady_clock::now().time_since_epoch().count());

template<typename T> struct treap {
	struct node {
		node *l, *r;
		int p;
		pair<ll, ll> idx; // {y, x}
		T val, mi;
		node(ll x, ll y, T val_) : l(NULL), r(NULL),
			p(rng()),
			idx(pair(y, x)), val(val_), mi(val) {}
		void update() {
			mi = val;
			if (l) mi = op(mi, l->mi);
			if (r) mi = op(mi, r->mi);
		}
	};

	node* root;

	treap() { root = NULL; }
	~treap() {
```

```
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }
    treap(treap&& t) : treap() { swap(root, t.root); }

    void join(node* l, node* r, node*& i) { // assume que l
       < r
        if (!l or !r) return void(i = l ? l : r);
        if (l->p > r->p) join(l->r, r, l->r), i = l;
        else join(l, r->l, r->l), i = r;
        i->update();
    }
    void split(node* i, node*& l, node*& r, pair<ll, ll>
       idx) {
        if (!i) return void(r = l = NULL);
        if (i->idx < idx) split(i->r, i->r, r, idx), l = i;
        else split(i->l, l, i->l, idx), r = i;
        i->update();
    }
    void update(ll x, ll y, T v) {
        node *L, *M, *R;
        split(root, M, R, pair(y, x+1)), split(M, L, M,
           pair(y, x));
        if (M) M->val = M->mi = v;
        else M = new node(x, y, v);
        join(L, M, M), join(M, R, root);
    }
    T query(ll ly, ll ry) {
        node *L, *M, *R;
        split(root, M, R, pair(ry, LINF)), split(M, L, M,
           pair(ly, 0));
        T ret = M ? M->mi : ZERO;
        join(L, M, M), join(M, R, root);
        return ret;
    }
};
```

```
template<typename T> struct segtreap {
    vector<treap<T>> seg;
    vector<int> ch[2];
    ll NX;

    segtreap(ll NX_) : seg(1), NX(NX_) {
       ch[0].push_back(-1), ch[1].push_back(-1); }

    int get_ch(int i, int d){
        if (ch[d][i] == -1) {
            ch[d][i] = seg.size();
            seg.emplace_back();
            ch[0].push_back(-1), ch[1].push_back(-1);
        }
        return ch[d][i];
    }

    T query(ll lx, ll rx, ll ly, ll ry, int p, ll l, ll r) {
        if (rx < l or r < lx) return ZERO;
        if (lx <= l and r <= rx) return seg[p].query(ly, ry);

        ll m = l + (r-l)/2;
        return op(query(lx, rx, ly, ry, get_ch(p, 0), l, m),
                query(lx, rx, ly, ry, get_ch(p, 1), m+1, r));
    }
    T query(ll lx, ll rx, ll ly, ll ry) { return query(lx,
       rx, ly, ry, 0, 0, NX); }

    void update(ll x, ll y, T val, int p, ll l, ll r) {
        if (l == r) return seg[p].update(x, y, val);
        ll m = l + (r-l)/2;
        if (x <= m) update(x, y, val, get_ch(p, 0), l, m);
        else update(x, y, val, get_ch(p, 1), m+1, r);
        seg[p].update(x, y, val);
    }
    void update(ll x, ll y, T val) { update(x, y, val, 0, 0,
       NX); }
};
```

4.13 SegTree

```
// Recursiva com Lazy Propagation
// Query: soma do range [a, b]
// Update: soma x em cada elemento do range [a, b]
// Pode usar a seguinte funcao para indexar os nohs:
// f(l, r) = (l+r)|(l!=r), usando 2N de memoria
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))

// 0afec1
namespace seg {
    ll seg[4*MAX], lazy[4*MAX];
    int n, *v;

    ll build(int p=1, int l=0, int r=n-1) {
        lazy[p] = 0;
        if (l == r) return seg[p] = v[l];
        int m = (l+r)/2;
        return seg[p] = build(2*p, l, m) + build(2*p+1, m+1,
            r);
    }
    void build(int n2, int* v2) {
        n = n2, v = v2;
        build();
    }
    void prop(int p, int l, int r) {
        seg[p] += lazy[p]*(r-l+1);
        if (l != r) lazy[2*p] += lazy[p], lazy[2*p+1] +=
            lazy[p];
        lazy[p] = 0;
    }
    ll query(int a, int b, int p=1, int l=0, int r=n-1) {
        prop(p, l, r);
        if (a <= l and r <= b) return seg[p];
        if (b < l or r < a) return 0;
        int m = (l+r)/2;
        return query(a, b, 2*p, l, m) + query(a, b, 2*p+1,
            m+1, r);
    }
    ll update(int a, int b, int x, int p=1, int l=0, int
        r=n-1) {
        prop(p, l, r);
        if (a <= l and r <= b) {
            lazy[p] += x;
            prop(p, l, r);
            return seg[p];
        }
        if (b < l or r < a) return seg[p];
        int m = (l+r)/2;
        return seg[p] = update(a, b, x, 2*p, l, m) +
            update(a, b, x, 2*p+1, m+1, r);
    }
};


// Se tiver uma seg de max, da pra descobrir em O(log(n))
// o primeiro e ultimo elemento >= val numa range:

// primeira posicao >= val em [a, b] (ou -1 se nao tem)
// 68c3e5
int get_left(int a, int b, int val, int p=1, int l=0, int
    r=n-1) {
    prop(p, l, r);
    if (b < l or r < a or seg[p] < val) return -1;
    if (r == l) return l;
    int m = (l+r)/2;
    int x = get_left(a, b, val, 2*p, l, m);
    if (x != -1) return x;
    return get_left(a, b, val, 2*p+1, m+1, r);
}

// ultima posicao >= val em [a, b] (ou -1 se nao tem)
// 1b71df
int get_right(int a, int b, int val, int p=1, int l=0, int
    r=n-1) {
    prop(p, l, r);
    if (b < l or r < a or seg[p] < val) return -1;
    if (r == l) return l;
    int m = (l+r)/2;
    int x = get_right(a, b, val, 2*p+1, m+1, r);
    if (x != -1) return x;
    return get_right(a, b, val, 2*p, l, m);
```

```
}

// Se tiver uma seg de soma sobre um array nao negativo v,
    da pra
// descobrir em O(log(n)) o maior j tal que
    v[i]+v[i+1]+...+v[j-1] < val
// 2b8ea7
int lower_bound(int i, ll& val, int p, int l, int r) {
    prop(p, l, r);
    if (r < i) return n;
    if (i <= l and seg[p] < val) {
        val -= seg[p];
        return n;
    }
    if (l == r) return l;
    int m = (l+r)/2;
    int x = lower_bound(i, val, 2*p, l, m);
    if (x != n) return x;
    return lower_bound(i, val, 2*p+1, m+1, r);
}
```

4.14 SegTree 2D Iterativa

```
// Consultas 0-based
// Um valor inicial em (x, y) deve ser colocado em
    seg[x+n][y+n]
// Query: soma do retangulo ((x1, y1), (x2, y2))
// Update: muda o valor da posicao (x, y) para val
// Nao pergunte como que essa coisa funciona
//
// Para query com distancia de manhattan <= d, faca
// nx = x+y, ny = x-y
// Update em (nx, ny), query em ((nx-d, ny-d), (nx+d, ny+d))
//
// Se for de min/max, pode tirar os if's da 'query', e fazer
// sempre as 4 operacoes. Fica mais rapido
//
// Complexidades:
// build - O(n^2)
// query - O(log^2(n))
// update - O(log^2(n))
// 67b9e5

int seg[2*MAX][2*MAX], n;

void build() {
    for (int x = 2*n; x; x--) for (int y = 2*n; y; y--) {
        if (x < n) seg[x][y] = seg[2*x][y] + seg[2*x+1][y];
        if (y < n) seg[x][y] = seg[x][2*y] + seg[x][2*y+1];
    }
}

int query(int x1, int y1, int x2, int y2) {
    int ret = 0, y3 = y1 + n, y4 = y2 + n;
    for (x1 += n, x2 += n; x1 <= x2; ++x1 /= 2, --x2 /= 2)
        for (y1 = y3, y2 = y4; y1 <= y2; ++y1 /= 2, --y2 /=
            2) {
            if (x1%2 == 1 and y1%2 == 1) ret += seg[x1][y1];
            if (x1%2 == 1 and y2%2 == 0) ret += seg[x1][y2];
            if (x2%2 == 0 and y1%2 == 1) ret += seg[x2][y1];
            if (x2%2 == 0 and y2%2 == 0) ret += seg[x2][y2];
        }

    return ret;
}

void update(int x, int y, int val) {
    int y2 = y += n;
    for (x += n; x; x /= 2, y = y2) {
        if (x >= n) seg[x][y] = val;
        else seg[x][y] = seg[2*x][y] + seg[2*x+1][y];

        while (y /= 2) seg[x][y] = seg[x][2*y] +
            seg[x][2*y+1];
    }
}
```

4.15 SegTree Beats

```
// query(a, b) - {{min(v[a..b]), max(v[a..b])}, sum(v[a..b])}
```

```
// updatemin(a, b, x) faz com que v[i] <- min(v[i], x),
// para i em [a, b]
// updatemax faz o mesmo com max, e updatesum soma x
// em todo mundo do intervalo [a, b]
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log^2 (n)) amortizado
// (se nao usar updatesum, fica log(n) amortizado)
// 41672b

#define f first
#define s second

namespace beats {
    struct node {
        int tam;
        ll sum, lazy; // lazy pra soma
        ll mi1, mi2, mi; // mi = #mi1
        ll ma1, ma2, ma; // ma = #ma1

        node(ll x = 0) {
            sum = mi1 = ma1 = x;
            mi2 = LINF, ma2 = -LINF;
            mi = ma = tam = 1;
            lazy = 0;
        }
        node(const node& l, const node& r) {
            sum = l.sum + r.sum, tam = l.tam + r.tam;
            lazy = 0;
            if (l.mi1 > r.mi1) {
                mi1 = r.mi1, mi = r.mi;
                mi2 = min(l.mi1, r.mi2);
            } else if (l.mi1 < r.mi1) {
                mi1 = l.mi1, mi = l.mi;
                mi2 = min(r.mi1, l.mi2);
            } else {
                mi1 = l.mi1, mi = l.mi+r.mi;
                mi2 = min(l.mi2, r.mi2);
            }
            if (l.ma1 < r.ma1) {
                ma1 = r.ma1, ma = r.ma;
                ma2 = max(l.ma1, r.ma2);
            } else if (l.ma1 > r.ma1) {
                ma1 = l.ma1, ma = l.ma;
                ma2 = max(r.ma1, l.ma2);
            } else {
                ma1 = l.ma1, ma = l.ma+r.ma;
                ma2 = max(l.ma2, r.ma2);
            }
        }
        void setmin(ll x) {
            if (x >= ma1) return;
            sum += (x - ma1)*ma;
            if (mi1 == ma1) mi1 = x;
            if (mi2 == ma1) mi2 = x;
            ma1 = x;
        }
        void setmax(ll x) {
            if (x <= mi1) return;
            sum += (x - mi1)*mi;
            if (ma1 == mi1) ma1 = x;
            if (ma2 == mi1) ma2 = x;
            mi1 = x;
        }
        void setsum(ll x) {
            mi1 += x, mi2 += x, ma1 += x, ma2 += x;
            sum += x*tam;
            lazy += x;
        }
    };

    node seg[4*MAX];
    int n, *v;

    node build(int p=1, int l=0, int r=n-1) {
        if (l == r) return seg[p] = {v[l]};
        int m = (l+r)/2;
        return seg[p] = {build(2*p, l, m), build(2*p+1, m+1,
            r)};
    }
    void build(int n2, int* v2) {
        n = n2, v = v2;
```

```
    build();
}
void prop(int p, int l, int r) {
    if (l == r) return;
    for (int k = 0; k < 2; k++) {
        if (seg[p].lazy) seg[2*p+k].setsum(seg[p].lazy);
        seg[2*p+k].setmin(seg[p].ma1);
        seg[2*p+k].setmax(seg[p].mi1);
    }
    seg[p].lazy = 0;
}
pair<pair<ll, ll>, ll> query(int a, int b, int p=1, int
    l=0, int r=n-1) {
    if (b < l or r < a) return {{LINF, -LINF}, 0};
    if (a <= l and r <= b) return {{seg[p].mi1,
        seg[p].ma1}, seg[p].sum};
    prop(p, l, r);
    int m = (l+r)/2;
    auto L = query(a, b, 2*p, l, m), R = query(a, b,
        2*p+1, m+1, r);
    return {{min(L.f.f, R.f.f), max(L.f.s, R.f.s)},
        L.s+R.s};
}
node updatemin(int a, int b, ll x, int p=1, int l=0, int
    r=n-1) {
    if (b < l or r < a or seg[p].ma1 <= x) return seg[p];
    if (a <= l and r <= b and seg[p].ma2 < x) {
        seg[p].setmin(x);
        return seg[p];
    }
    prop(p, l, r);
    int m = (l+r)/2;
    return seg[p] = {updatemin(a, b, x, 2*p, l, m),
                    updatemin(a, b, x, 2*p+1, m+1, r)};
}
node updatemax(int a, int b, ll x, int p=1, int l=0, int
    r=n-1) {
    if (b < l or r < a or seg[p].mi1 >= x) return seg[p];
    if (a <= l and r <= b and seg[p].mi2 > x) {
        seg[p].setmax(x);
        return seg[p];
    }
        prop(p, l, r);
        int m = (l+r)/2;
        return seg[p] = {updatemax(a, b, x, 2*p, l, m),
                        updatemax(a, b, x, 2*p+1, m+1, r)};
    }
    node updatesum(int a, int b, ll x, int p=1, int l=0, int
        r=n-1) {
        if (b < l or r < a) return seg[p];
        if (a <= l and r <= b) {
            seg[p].setsum(x);
            return seg[p];
        }
        prop(p, l, r);
        int m = (l+r)/2;
        return seg[p] = {updatesum(a, b, x, 2*p, l, m),
                        updatesum(a, b, x, 2*p+1, m+1, r)};
    }
};
```

4.16 SegTree Colorida

```
// Cada posicao tem um valor e uma cor
// O construtor receve um vector de {valor, cor}
// e o numero de cores (as cores devem estar em [0, c-1])
// query(c, a, b) retorna a soma dos valores
// de todo mundo em [a, b] que tem cor c
// update(c, a, b, x) soma x em todo mundo em
// [a, b] que tem cor c
// paint(c1, c2, a, b) faz com que todo mundo
// em [a, b] que tem cor c1 passe a ter cor c2
//
// Complexidades:
// construir - O(n log(n)) espaco e tempo
// query - O(log(n))
// update - O(log(n))
// paint - O(log(n)) amortizado
// 2938e8

struct seg_color {
    struct node {
```

```
	node *l, *r;
	int cnt;
	ll val, lazy;
	node() : l(NULL), r(NULL), cnt(0), val(0), lazy(0) {}
	void update() {
		cnt = 0, val = 0;
		for (auto i : {l, r}) if (i) {
			i->prop();
			cnt += i->cnt, val += i->val;
		}
	}
	void prop() {
		if (!lazy) return;
		val += lazy*(ll)cnt;
		for (auto i : {l, r}) if (i) i->lazy += lazy;
		lazy = 0;
	}
};

int n;
vector<node*> seg;

seg_color(vector<pair<int, int>>& v, int c) :
	n(v.size()), seg(c, NULL) {
	for (int i = 0; i < n; i++)
		seg[v[i].second] = insert(seg[v[i].second], i,
			v[i].first, 0, n-1);
}
~seg_color() {
	queue<node*> q;
	for (auto i : seg) q.push(i);
	while (q.size()) {
		auto i = q.front(); q.pop();
		if (!i) continue;
		q.push(i->l), q.push(i->r);
		delete i;
	}
}

node* insert(node* at, int idx, int val, int l, int r) {
	if (!at) at = new node();
	if (l == r) return at->cnt = 1, at->val = val, at;
	int m = (l+r)/2;
	if (idx <= m) at->l = insert(at->l, idx, val, l, m);
	else at->r = insert(at->r, idx, val, m+1, r);
	return at->update(), at;
}
ll query(node* at, int a, int b, int l, int r) {
	if (!at or b < l or r < a) return 0;
	at->prop();
	if (a <= l and r <= b) return at->val;
	int m = (l+r)/2;
	return query(at->l, a, b, l, m) + query(at->r, a, b,
		m+1, r);
}
ll query(int c, int a, int b) { return query(seg[c], a,
	b, 0, n-1); }
void update(node* at, int a, int b, int x, int l, int r)
	{
	if (!at or b < l or r < a) return;
	at->prop();
	if (a <= l and r <= b) {
		at->lazy += x;
		return void(at->prop());
	}
	int m = (l+r)/2;
	update(at->l, a, b, x, l, m), update(at->r, a, b, x,
		m+1, r);
	at->update();
}
void update(int c, int a, int b, int x) { update(seg[c],
	a, b, x, 0, n-1); }
void paint(node*& from, node*& to, int a, int b, int l,
	int r) {
	if (to == from or !from or b < l or r < a) return;
	from->prop();
	if (to) to->prop();
	if (a <= l and r <= b) {
		if (!to) {
			to = from;
			from = NULL;
			return;
		}
		int m = (l+r)/2;
```

```
            paint(from->l, to->l, a, b, l, m),
                paint(from->r, to->r, a, b, m+1, r);
            to->update();
            delete from;
            from = NULL;
            return;
        }
        if (!to) to = new node();
        int m = (l+r)/2;
        paint(from->l, to->l, a, b, l, m), paint(from->r,
            to->r, a, b, m+1, r);
        from->update(), to->update();
    }
    void paint(int c1, int c2, int a, int b) {
        paint(seg[c1], seg[c2], a, b, 0, n-1); }
};
```

4.17 SegTree Esparsa - Lazy

```
// Query: soma do range [a, b]
// Update: flipa os valores de [a, b]
// O MAX tem q ser Q log N para Q updates
//
// Complexidades:
// build - O(1)
// query - O(log(n))
// update - O(log(n))
// dc37e6

namespace seg {
    int seg[MAX], lazy[MAX], R[MAX], L[MAX], ptr;
    int get_l(int i){
        if (L[i] == 0) L[i] = ptr++;
        return L[i];
    }
    int get_r(int i){
        if (R[i] == 0) R[i] = ptr++;
        return R[i];
    }
    void build() { ptr = 2; }

    void prop(int p, int l, int r) {
        if (!lazy[p]) return;
        seg[p] = r-l+1 - seg[p];
        if (l != r) lazy[get_l(p)]^=lazy[p],
            lazy[get_r(p)]^=lazy[p];
        lazy[p] = 0;
    }

    int query(int a, int b, int p=1, int l=0, int r=N-1) {
        prop(p, l, r);
        if (b < l or r < a) return 0;
        if (a <= l and r <= b) return seg[p];

        int m = (l+r)/2;
        return query(a, b, get_l(p), l, m)+query(a, b,
            get_r(p), m+1, r);
    }

    int update(int a, int b, int p=1, int l=0, int r=N-1) {
        prop(p, l, r);
        if (b < l or r < a) return seg[p];
        if (a <= l and r <= b) {
            lazy[p] ^= 1;
            prop(p, l, r);
            return seg[p];
        }
        int m = (l+r)/2;
        return seg[p] = update(a, b, get_l(p), l,
            m)+update(a, b, get_r(p), m+1, r);
    }
};
```

4.18 SegTree Esparsa - O(q) memoria

```
// Query: min do range [a, b]
// Update: troca o valor de uma posicao
// Usa O(q) de memoria para q updates
//
```

```
// Complexidades:
// query - O(log(n))
// update - O(log(n))
// 072a21

template<typename T> struct seg {
    struct node {
        node* ch[2];
        char d;
        T v;

        T mi;

        node(int d_, T v_, T val) : d(d_), v(v_) {
            ch[0] = ch[1] = NULL;
            mi = val;
        }
        node(node* x) : d(x->d), v(x->v), mi(x->mi) {
            ch[0] = x->ch[0], ch[1] = x->ch[1];
        }
        void update() {
            mi = numeric_limits<T>::max();
            for (int i = 0; i < 2; i++) if (ch[i])
                mi = min(mi, ch[i]->mi);
        }
    };

    node* root;
    char n;

    seg() : root(NULL), n(0) {}
    ~seg() {
        std::vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->ch[0]), q.push_back(x->ch[1]);
            delete x;
        }
    }

    char msb(T v, char l, char r) { // msb in range (l, r]
        for (char i = r; i > l; i--) if (v>>i&1) return i;
        return -1;
    }
    void cut(node* at, T v, char i) {
        char d = msb(v ^ at->v, at->d, i);
        if (d == -1) return; // no need to split
        node* nxt = new node(at);
        at->ch[v>>d&1] = NULL;
        at->ch[!(v>>d&1)] = nxt;
        at->d = d;
    }

    node* update(node* at, T idx, T val, char i) {
        if (!at) return new node(-1, idx, val);
        cut(at, idx, i);
        if (at->d == -1) { // leaf
            at->mi = val;
            return at;
        }
        bool dir = idx>>at->d&1;
        at->ch[dir] = update(at->ch[dir], idx, val, at->d-1);
        at->update();
        return at;
    }
    void update(T idx, T val) {
        while (idx>>n) n++;
        root = update(root, idx, val, n-1);
    }

    T query(node* at, T a, T b, T l, T r, char i) {
        if (!at or b < l or r < a) return
            numeric_limits<T>::max();
        if (a <= l and r <= b) return at->mi;
        T m = l + (r-l)/2;
        if (at->d < i) {
            if ((at->v>>i&1) == 0) return query(at, a, b, l,
                m, i-1);
            else return query(at, a, b, m+1, r, i-1);
        }
        return min(query(at->ch[0], a, b, l, m, i-1),
            query(at->ch[1], a, b, m+1, r, i-1));
    }
```

```
    T query(T l, T r) { return query(root, l, r, 0,
        (1<<n)-1, n-1); }
};
```

4.19 SegTree Iterativa

```
// Consultas 0-based
// Valores iniciais devem estar em (seg[n], ... , seg[2*n-1])
// Query: soma do range [a, b]
// Update: muda o valor da posicao p para x
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))
// 779519

int seg[2 * MAX];
int n;

void build() {
    for (int i = n - 1; i; i--) seg[i] = seg[2*i] +
        seg[2*i+1];
}

int query(int a, int b) {
    int ret = 0;
    for(a += n, b += n; a <= b; ++a /= 2, --b /= 2) {
        if (a % 2 == 1) ret += seg[a];
        if (b % 2 == 0) ret += seg[b];
    }
    return ret;
}

void update(int p, int x) {
    seg[p += n] = x;
    while (p /= 2) seg[p] = seg[2*p] + seg[2*p+1];
}
```

4.20 SegTree Iterativa com Lazy Propagation

```
// Query: soma do range [a, b]
// Update: soma x em cada elemento do range [a, b]
// Para mudar, mudar as funcoes junta, poe e query
// LOG = ceil(log2(MAX))
//
// Complexidades:
// build - O(n)
// query - O(log(n))
// update - O(log(n))
// 6dc475

namespace seg {
    ll seg[2*MAX], lazy[2*MAX];
    int n;

    ll junta(ll a, ll b) {
        return a+b;
    }

    // soma x na posicao p de tamanho tam
    void poe(int p, ll x, int tam, bool prop=1) {
        seg[p] += x*tam;
        if (prop and p < n) lazy[p] += x;
    }

    // atualiza todos os pais da folha p
    void sobe(int p) {
        for (int tam = 2; p /= 2; tam *= 2) {
            seg[p] = junta(seg[2*p], seg[2*p+1]);
            poe(p, lazy[p], tam, 0);
        }
    }

    // propaga o caminho da raiz ate a folha p
    void prop(int p) {
        int tam = 1 << (LOG-1);
        for (int s = LOG; s; s--, tam /= 2) {
            int i = p >> s;
            if (lazy[i]) {
                poe(2*i, lazy[i], tam);
```

```
                poe(2*i+1, lazy[i], tam);
                lazy[i] = 0;
            }
        }
    }

    void build(int n2, int* v) {
        n = n2;
        for (int i = 0; i < n; i++) seg[n+i] = v[i];
        for (int i = n-1; i; i--) seg[i] = junta(seg[2*i],
            seg[2*i+1]);
        for (int i = 0; i < 2*n; i++) lazy[i] = 0;
    }

    ll query(int a, int b) {
        ll ret = 0;
        for (prop(a+=n), prop(b+=n); a <= b; ++a/=2, --b/=2)
            {
            if (a%2 == 1) ret = junta(ret, seg[a]);
            if (b%2 == 0) ret = junta(ret, seg[b]);
        }
        return ret;
    }

    void update(int a, int b, int x) {
        int a2 = a += n, b2 = b += n, tam = 1;
        for (; a <= b; ++a/=2, --b/=2, tam *= 2) {
            if (a%2 == 1) poe(a, x, tam);
            if (b%2 == 0) poe(b, x, tam);
        }
        sobe(a2), sobe(b2);
    }
};
```

4.21 SegTree PA

```
// Segtree de PA
// update_set(l, r, A, R) seta [l, r] para PA(A, R),
// update_add soma PA(A, R) em [l, r]
// query(l, r) retorna a soma de [l, r]
//
// PA(A, R) eh a PA: [A+R, A+2R, A+3R, ... ]
//
// Complexidades:
// construir - O(n)
// update_set, update_add, query - O(log(n))
// bc4746

struct seg_pa {
    struct Data {
        ll sum;
        ll set_a, set_r, add_a, add_r;
        Data() : sum(0), set_a(LINF), set_r(0), add_a(0),
            add_r(0) {}
    };
    vector<Data> seg;
    int n;

    seg_pa(int n_) {
        n = n_;
        seg = vector<Data>(4*n);
    }

    void prop(int p, int l, int r) {
        int tam = r-l+1;
        ll &sum = seg[p].sum, &set_a = seg[p].set_a, &set_r
            = seg[p].set_r,
            &add_a = seg[p].add_a, &add_r = seg[p].add_r;

        if (set_a != LINF) {
            set_a += add_a, set_r += add_r;
            sum = set_a*tam + set_r*tam*(tam+1)/2;
            if (l != r) {
                int m = (l+r)/2;

                seg[2*p].set_a = set_a;
                seg[2*p].set_r = set_r;
                seg[2*p].add_a = seg[2*p].add_r = 0;

                seg[2*p+1].set_a = set_a + set_r * (m-l+1);
                seg[2*p+1].set_r = set_r;
                seg[2*p+1].add_a = seg[2*p+1].add_r = 0;
```

```
			}
			set_a = LINF, set_r = 0;
			add_a = add_r = 0;
		} else if (add_a or add_r) {
			sum += add_a*tam + add_r*tam*(tam+1)/2;
			if (l != r) {
				int m = (l+r)/2;

				seg[2*p].add_a += add_a;
				seg[2*p].add_r += add_r;

				seg[2*p+1].add_a += add_a + add_r * (m-l+1);
				seg[2*p+1].add_r += add_r;
			}
			add_a = add_r = 0;
		}
	}

	int inter(pair<int, int> a, pair<int, int> b) {
		if (a.first > b.first) swap(a, b);
		return max(0, min(a.second, b.second) - b.first + 1);
	}
	ll set(int a, int b, ll aa, ll rr, int p, int l, int r) {
		prop(p, l, r);
		if (b < l or r < a) return seg[p].sum;
		if (a <= l and r <= b) {
			seg[p].set_a = aa;
			seg[p].set_r = rr;
			prop(p, l, r);
			return seg[p].sum;
		}
		int m = (l+r)/2;
		int tam_l = inter({l, m}, {a, b});
		return seg[p].sum = set(a, b, aa, rr, 2*p, l, m) +
			set(a, b, aa + rr * tam_l, rr, 2*p+1, m+1, r);
	}
	void update_set(int l, int r, ll aa, ll rr) {
		set(l, r, aa, rr, 1, 0, n-1);
	}
	ll add(int a, int b, ll aa, ll rr, int p, int l, int r) {
		prop(p, l, r);
		if (b < l or r < a) return seg[p].sum;
		if (a <= l and r <= b) {
			seg[p].add_a += aa;
			seg[p].add_r += rr;
			prop(p, l, r);
			return seg[p].sum;
		}
		int m = (l+r)/2;
		int tam_l = inter({l, m}, {a, b});
		return seg[p].sum = add(a, b, aa, rr, 2*p, l, m) +
			add(a, b, aa + rr * tam_l, rr, 2*p+1, m+1, r);
	}
	void update_add(int l, int r, ll aa, ll rr) {
		add(l, r, aa, rr, 1, 0, n-1);
	}
	ll query(int a, int b, int p, int l, int r) {
		prop(p, l, r);
		if (b < l or r < a) return 0;
		if (a <= l and r <= b) return seg[p].sum;
		int m = (l+r)/2;
		return query(a, b, 2*p, l, m) + query(a, b, 2*p+1,
			m+1, r);
	}
	ll query(int l, int r) { return query(l, r, 1, 0, n-1); }
};
```

4.22 SegTree Persistente

```
// SegTree de soma, update de somar numa posicao
//
// query(a, b, t) retorna a query de [a, b] na versao t
// update(a, x, t) faz um update v[a]+=x a partir da
// versao de t, criando uma nova versao e retornando seu id
// Por default, faz o update a partir da ultima versao
//
// build - O(n)
// query - O(log(n))
// update - O(log(n))
// 50ab73

const int MAX = 1e5+10, UPD = 1e5+10, LOG = 18;
```

```
const int MAXS = 2*MAX+UPD*LOG;

namespace perseg {
    ll seg[MAXS];
    int rt[UPD], L[MAXS], R[MAXS], cnt, t;
    int n, *v;

    ll build(int p, int l, int r) {
        if (l == r) return seg[p] = v[l];
        L[p] = cnt++, R[p] = cnt++;
        int m = (l+r)/2;
        return seg[p] = build(L[p], l, m) + build(R[p], m+1,
           r);
    }
    void build(int n2, int* v2) {
        n = n2, v = v2;
        rt[0] = cnt++;
        build(0, 0, n-1);
    }
    ll query(int a, int b, int p, int l, int r) {
        if (b < l or r < a) return 0;
        if (a <= l and r <= b) return seg[p];
        int m = (l+r)/2;
        return query(a, b, L[p], l, m) + query(a, b, R[p],
           m+1, r);
    }
    ll query(int a, int b, int tt) {
        return query(a, b, rt[tt], 0, n-1);
    }
    ll update(int a, int x, int lp, int p, int l, int r) {
        if (l == r) return seg[p] = seg[lp]+x;
        int m = (l+r)/2;
        if (a <= m)
            return seg[p] = update(a, x, L[lp], L[p]=cnt++,
               l, m) + seg[R[p]=R[lp]];
        return seg[p] = seg[L[p]=L[lp]] + update(a, x,
           R[lp], R[p]=cnt++, m+1, r);
    }
    int update(int a, int x, int tt=t) {
        update(a, x, rt[tt], rt[++t]=cnt++, 0, n-1);
        return t;
    }
};
```

4.23 Sparse Table

```
// Resolve RMQ
// MAX2 = log(MAX)
//
// Complexidades:
// build - O(n log(n))
// query - O(1)
// 7aa4c9

namespace sparse {
    int m[MAX2][MAX], n;
    void build(int n2, int* v) {
        n = n2;
        for (int i = 0; i < n; i++) m[0][i] = v[i];
        for (int j = 1; (1<<j) <= n; j++) for (int i = 0;
           i+(1<<j) <= n; i++)
            m[j][i] = min(m[j-1][i], m[j-1][i+(1<<(j-1))]);
    }
    int query(int a, int b) {
        int j = __builtin_clz(1) - __builtin_clz(b-a+1);
        return min(m[j][a], m[j][b-(1<<j)+1]);
    }
}
```

4.24 Sparse Table Disjunta

```
// Resolve qualquer operacao associativa
// MAX2 = log(MAX)
//
// Complexidades:
// build - O(n log(n))
// query - O(1)
// fd81ae

namespace sparse {
```

```
    int m[MAX2][2*MAX], n, v[2*MAX];
    int op(int a, int b) { return min(a, b); }
    void build(int n2, int* v2) {
        n = n2;
        for (int i = 0; i < n; i++) v[i] = v2[i];
        while (n&(n-1)) n++;
        for (int j = 0; (1<<j) < n; j++) {
            int len = 1<<j;
            for (int c = len; c < n; c += 2*len) {
                m[j][c] = v[c], m[j][c-1] = v[c-1];
                for (int i = c+1; i <  c+len; i++) m[j][i] =
                   op(m[j][i-1], v[i]);
                for (int i = c-2; i >= c-len; i--) m[j][i] =
                   op(v[i], m[j][i+1]);
            }
        }
    }
    int query(int l, int r) {
        if (l == r) return v[l];
        int j = __builtin_clz(1) - __builtin_clz(l^r);
        return op(m[j][l], m[j][r]);
    }
}
```

4.25 Splay Tree

```
// SEMPRE QUE DESCER NA ARVORE, DAR SPLAY NO
// NODE MAIS PROFUNDO VISITADO
// Todas as operacoes sao O(log(n)) amortizado
// Se quiser colocar mais informacao no node,
// mudar em 'update'
// 4ff2b3

template<typename T> struct splaytree {
    struct node {
        node *ch[2], *p;
        int sz;
        T val;
        node(T v) {
            ch[0] = ch[1] = p = NULL;
            sz = 1;
            val = v;
        }
        void update() {
            sz = 1;
            for (int i = 0; i < 2; i++) if (ch[i]) {
                sz += ch[i]->sz;
            }
        }
    };

    node* root;

    splaytree() { root = NULL; }
    splaytree(const splaytree& t) {
        throw logic_error("Nao copiar a splaytree!");
    }
    ~splaytree() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->ch[0]), q.push_back(x->ch[1]);
            delete x;
        }
    }

    void rotate(node* x) { // x vai ficar em cima
        node *p = x->p, *pp = p->p;
        if (pp) pp->ch[pp->ch[1] == p] = x;
        bool d = p->ch[0] == x;
        p->ch[!d] = x->ch[d], x->ch[d] = p;
        if (p->ch[!d]) p->ch[!d]->p = p;
        x->p = pp, p->p = x;
        p->update(), x->update();
    }
    node* splay(node* x) {
        if (!x) return x;
        root = x;
        while (x->p) {
            node *p = x->p, *pp = p->p;
            if (!pp) return rotate(x), x; // zig
```

```
        if ((pp->ch[0] == p)^(p->ch[0] == x))
            rotate(x), rotate(x); // zigzag
        else rotate(p), rotate(x); // zigzig
    }
    return x;
}
node* insert(T v, bool lb=0) {
    if (!root) return lb ? NULL : root = new node(v);
    node *x = root, *last = NULL;;
    while (1) {
        bool d = x->val < v;
        if (!d) last = x;
        if (x->val == v) break;
        if (x->ch[d]) x = x->ch[d];
        else {
            if (lb) break;
            x->ch[d] = new node(v);
            x->ch[d]->p = x;
            x = x->ch[d];
            break;
        }
    }
    splay(x);
    return lb ? splay(last) : x;
}
int size() { return root ? root->sz : 0; }
int count(T v) { return insert(v, 1) and root->val == v;
  }
node* lower_bound(T v) { return insert(v, 1); }
void erase(T v) {
    if (!count(v)) return;
    node *x = root, *l = x->ch[0];
    if (!l) {
        root = x->ch[1];
        if (root) root->p = NULL;
        return delete x;
    }
    root = l, l->p = NULL;
    while (l->ch[1]) l = l->ch[1];
    splay(l);
    l->ch[1] = x->ch[1];
    if (l->ch[1]) l->ch[1]->p = l;
        delete x;
        l->update();
    }
    int order_of_key(T v) {
        if (!lower_bound(v)) return root ? root->sz : 0;
        return root->ch[0] ? root->ch[0]->sz : 0;
    }
    node* find_by_order(int k) {
        if (k >= size()) return NULL;
        node* x = root;
        while (1) {
            if (x->ch[0] and x->ch[0]->sz >= k+1) x =
               x->ch[0];
            else {
                if (x->ch[0]) k -= x->ch[0]->sz;
                if (!k) return splay(x);
                k--, x = x->ch[1];
            }
        }
    }
    T min() {
        node* x = root;
        while (x->ch[0]) x = x->ch[0]; // max -> ch[1]
        return splay(x)->val;
    }
};
```

4.26 Splay Tree Implicita

```
// vector da NASA
// Um pouco mais rapido q a treap
// O construtor a partir do vector
// eh linear, todas as outras operacoes
// custam O(log(n)) amortizado
// a3575a

template<typename T> struct splay {
    struct node {
        node *ch[2], *p;
        int sz;
```

```
    T val, sub, lazy;
    bool rev;
    node(T v) {
        ch[0] = ch[1] = p = NULL;
        sz = 1;
        sub = val = v;
        lazy = 0;
        rev = false;
    }
    void prop() {
        if (lazy) {
            val += lazy, sub += lazy*sz;
            if (ch[0]) ch[0]->lazy += lazy;
            if (ch[1]) ch[1]->lazy += lazy;
        }
        if (rev) {
            swap(ch[0], ch[1]);
            if (ch[0]) ch[0]->rev ^= 1;
            if (ch[1]) ch[1]->rev ^= 1;
        }
        lazy = 0, rev = 0;
    }
    void update() {
        sz = 1, sub = val;
        for (int i = 0; i < 2; i++) if (ch[i]) {
            ch[i]->prop();
            sz += ch[i]->sz;
            sub += ch[i]->sub;
        }
    }
};

node* root;

splay() { root = NULL; }
splay(node* x) {
    root = x;
    if (root) root->p = NULL;
}
splay(vector<T> v) { // O(n)
    root = NULL;
    for (T i : v) {
            node* x = new node(i);
            x->ch[0] = root;
            if (root) root->p = x;
            root = x;
            root->update();
        }
    }
    splay(const splay& t) {
        throw logic_error("Nao copiar a splay!");
    }
    ~splay() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->ch[0]), q.push_back(x->ch[1]);
            delete x;
        }
    }

    int size(node* x) { return x ? x->sz : 0; }
    void rotate(node* x) { // x vai ficar em cima
        node *p = x->p, *pp = p->p;
        if (pp) pp->ch[pp->ch[1] == p] = x;
        bool d = p->ch[0] == x;
        p->ch[!d] = x->ch[d], x->ch[d] = p;
        if (p->ch[!d]) p->ch[!d]->p = p;
        x->p = pp, p->p = x;
        p->update(), x->update();
    }
    node* splaya(node* x) {
        if (!x) return x;
        root = x, x->update();
        while (x->p) {
            node *p = x->p, *pp = p->p;
            if (!pp) return rotate(x), x; // zig
            if ((pp->ch[0] == p)^(p->ch[0] == x))
                rotate(x), rotate(x); // zigzag
            else rotate(p), rotate(x); // zigzig
        }
        return x;
    }
```

```cpp
node* find(int v) {
    if (!root) return NULL;
    node *x = root;
    int key = 0;
    while (1) {
        x->prop();
        bool d = key + size(x->ch[0]) < v;
        if (key + size(x->ch[0]) != v and x->ch[d]) {
            if (d) key += size(x->ch[0])+1;
            x = x->ch[d];
        } else break;
    }
    return splaya(x);
}
int size() { return root ? root->sz : 0; }
void join(splay<T>& l) { // assume que l < *this
    if (!size()) swap(root, l.root);
    if (!size() or !l.size()) return;
    node* x = l.root;
    while (1) {
        x->prop();
        if (!x->ch[1]) break;
        x = x->ch[1];
    }
    l.splaya(x), root->prop(), root->update();
    x->ch[1] = root, x->ch[1]->p = x;
    root = l.root, l.root = NULL;
    root->update();
}
node* split(int v) { // retorna os elementos < v
    if (v <= 0) return NULL;
    if (v >= size()) {
        node* ret = root;
        root = NULL;
        ret->update();
        return ret;
    }
    find(v);
    node* l = root->ch[0];
    root->ch[0] = NULL;
    if (l) l->p = NULL;
    root->update();
        return l;
    }
    T& operator [](int i) {
        find(i);
        return root->val;
    }
    void push_back(T v) { // O(1)
        node* r = new node(v);
        r->ch[0] = root;
        if (root) root->p = r;
        root = r, root->update();
    }
    T query(int l, int r) {
        splay<T> M(split(r+1));
        splay<T> L(M.split(l));
        T ans = M.root->sub;
        M.join(L), join(M);
        return ans;
    }
    void update(int l, int r, T s) {
        splay<T> M(split(r+1));
        splay<T> L(M.split(l));
        M.root->lazy += s;
        M.join(L), join(M);
    }
    void reverse(int l, int r) {
        splay<T> M(split(r+1));
        splay<T> L(M.split(l));
        M.root->rev ^= 1;
        M.join(L), join(M);
    }
    void erase(int l, int r) {
        splay<T> M(split(r+1));
        splay<T> L(M.split(l));
        join(L);
    }
};
```

4.27 Split-Merge Set

```
// Representa um conjunto de inteiros nao negativos
// Todas as operacoes custam O(log(N)),
// em que N = maior elemento do set,
// exceto o merge, que custa O(log(N)) amortizado
// Usa O(min(N, n log(N))) de memoria, sendo 'n' o
// numero de elementos distintos no set
// 2d2d8a

template<typename T, bool MULTI=false, typename SIZE_T=int>
   struct sms {
    struct node {
        node *l, *r;
        SIZE_T cnt;
        node() : l(NULL), r(NULL), cnt(0) {}
        void update() {
            cnt = 0;
            if (l) cnt += l->cnt;
            if (r) cnt += r->cnt;
        }
    };

    node* root;
    T N;

    sms() : root(NULL), N(0) {}
    sms(T v) : sms() { while (v >= N) N = 2*N+1; }
    sms(const sms& t) : root(NULL), N(t.N) {
        for (SIZE_T i = 0; i < t.size(); i++) {
            T at = t[i];
            SIZE_T qt = t.count(at);
            insert(at, qt);
            i += qt-1;
        }
    }
    sms(initializer_list<T> v) : sms() { for (T i : v)
       insert(i); }
    ~sms() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }

    friend void swap(sms& a, sms& b) {
        swap(a.root, b.root), swap(a.N, b.N);
    }
    sms& operator =(const sms& v) {
        sms tmp = v;
        swap(tmp, *this);
        return *this;
    }
    SIZE_T size() const { return root ? root->cnt : 0; }
    SIZE_T count(node* x) const { return x ? x->cnt : 0; }
    void clear() {
        sms tmp;
        swap(*this, tmp);
    }
    void expand(T v) {
        for (; N < v; N = 2*N+1) if (root) {
            node* nroot = new node();
            nroot->l = root;
            root = nroot;
            root->update();
        }
    }

    node* insert(node* at, T idx, SIZE_T qt, T l, T r) {
        if (!at) at = new node();
        if (l == r) {
            at->cnt += qt;
            if (!MULTI) at->cnt = 1;
            return at;
        }
        T m = l + (r-l)/2;
        if (idx <= m) at->l = insert(at->l, idx, qt, l, m);
        else at->r = insert(at->r, idx, qt, m+1, r);
        return at->update(), at;
    }
    void insert(T v, SIZE_T qt=1) { // insere 'qt'
       ocorrencias de 'v'
        if (qt <= 0) return erase(v, -qt);
```

```
    assert(v >= 0);
    expand(v);
    root = insert(root, v, qt, 0, N);
}

node* erase(node* at, T idx, SIZE_T qt, T l, T r) {
    if (!at) return at;
    if (l == r) at->cnt = at->cnt < qt ? 0 : at->cnt -
        qt;
    else {
        T m = l + (r-l)/2;
        if (idx <= m) at->l = erase(at->l, idx, qt, l,
            m);
        else at->r = erase(at->r, idx, qt, m+1, r);
        at->update();
    }
    if (!at->cnt) delete at, at = NULL;
    return at;
}
void erase(T v, SIZE_T qt=1) { // remove 'qt'
    ocorrencias de 'v'
    if (v < 0 or v > N or !qt) return;
    if (qt < 0) insert(v, -qt);
    root = erase(root, v, qt, 0, N);
}
void erase_all(T v) { // remove todos os 'v'
    if (v < 0 or v > N) return;
    root = erase(root, v, numeric_limits<SIZE_T>::max(),
        0, N);
}

SIZE_T count(node* at, T a, T b, T l, T r) const {
    if (!at or b < l or r < a) return 0;
    if (a <= l and r <= b) return at->cnt;
    T m = l + (r-l)/2;
    return count(at->l, a, b, l, m) + count(at->r, a, b,
        m+1, r);
}
SIZE_T count(T v) const { return count(root, v, v, 0,
    N); }
SIZE_T order_of_key(T v) { return count(root, 0, v-1, 0,
    N); }
SIZE_T lower_bound(T v) { return order_of_key(v); }

const T operator [](SIZE_T i) const { // i-esimo menor
    elemento
    assert(i >= 0 and i < size());
    node* at = root;
    T l = 0, r = N;
    while (l < r) {
        T m = l + (r-l)/2;
        if (count(at->l) > i) at = at->l, r = m;
        else {
            i -= count(at->l);
            at = at->r; l = m+1;
        }
    }
    return l;
}

node* merge(node* l, node* r) {
    if (!l or !r) return l ? l : r;
    if (!l->l and !l->r) { // folha
        if (MULTI) l->cnt += r->cnt;
        delete r;
        return l;
    }
    l->l = merge(l->l, r->l), l->r = merge(l->r, r->r);
    l->update(), delete r;
    return l;
}
void merge(sms& s) { // mergeia dois sets
    if (N > s.N) swap(*this, s);
    expand(s.N);
    root = merge(root, s.root);
    s.root = NULL;
}

node* split(node*& x, SIZE_T k) {
    if (k <= 0 or !x) return NULL;
    node* ret = new node();
    if (!x->l and !x->r) x->cnt -= k, ret->cnt += k;
    else {
        if (k <= count(x->l)) ret->l = split(x->l, k);
```

```
            else {
                ret->r = split(x->r, k - count(x->l));
                swap(x->l, ret->l);
            }
            ret->update(), x->update();
        }
        if (!x->cnt) delete x, x = NULL;
        return ret;
    }
    void split(SIZE_T k, sms& s) { // pega os 'k' menores
        s.clear();
        s.root = split(root, min(k, size()));
        s.N = N;
    }
    // pega os menores que 'k'
    void split_val(T k, sms& s) { split(order_of_key(k), s);
        }
};
```

4.28 Split-Merge Set - Lazy

```
// Representa um conjunto de inteiros nao negativos
// Todas as operacoes custam O(log(N)),
// em que N = maior elemento do set,
// exceto o merge e o insert_range, que custa O(log(N))
   amortizado
// Usa O(min(N, n log(N))) de memoria, sendo 'n' o
// numero de elementos distintos no set
// 3828d0

template<typename T> struct sms {
    struct node {
        node *l, *r;
        int cnt;
        bool flip;
        node() : l(NULL), r(NULL), cnt(0), flip(0) {}
        void update() {
            cnt = 0;
            if (l) cnt += l->cnt;
            if (r) cnt += r->cnt;
        }
    };

    void prop(node* x, int size) {
        if (!x or !x->flip) return;
        x->flip = 0;
        x->cnt = size - x->cnt;
        if (size > 1) {
            if (!x->l) x->l = new node();
            if (!x->r) x->r = new node();
            x->l->flip ^= 1;
            x->r->flip ^= 1;
        }
    }

    node* root;
    T N;

    sms() : root(NULL), N(0) {}
    sms(T v) : sms() { while (v >= N) N = 2*N+1; }
    sms(sms& t) : root(NULL), N(t.N) {
        for (int i = 0; i < t.size(); i++) insert(t[i]);
    }
    sms(initializer_list<T> v) : sms() { for (T i : v)
       insert(i); }
    void destroy(node* r) {
        vector<node*> q = {r};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }
    ~sms() { destroy(root); }

    friend void swap(sms& a, sms& b) {
        swap(a.root, b.root), swap(a.N, b.N);
    }
    sms& operator =(const sms& v) {
        sms tmp = v;
        swap(tmp, *this);
```

```
        return *this;
    }
    int count(node* x, T size) {
        if (!x) return 0;
        prop(x, size);
        return x->cnt;
    }
    int size() { return count(root, N+1); }
    void clear() {
        sms tmp;
        swap(*this, tmp);
    }
    void expand(T v) {
        for (; N < v; N = 2*N+1) if (root) {
            prop(root, N+1);
            node* nroot = new node();
            nroot->l = root;
            root = nroot;
            root->update();
        }
    }

    node* insert(node* at, T idx, T l, T r) {
        if (!at) at = new node();
        else prop(at, r-l+1);
        if (l == r) {
            at->cnt = 1;
            return at;
        }
        T m = l + (r-l)/2;
        if (idx <= m) at->l = insert(at->l, idx, l, m);
        else at->r = insert(at->r, idx, m+1, r);
        return at->update(), at;
    }
    void insert(T v) {
        assert(v >= 0);
        expand(v);
        root = insert(root, v, 0, N);
    }

    node* erase(node* at, T idx, T l, T r) {
        if (!at) return at;
        prop(at, r-l+1);
        if (l == r) at->cnt = 0;
        else {
            T m = l + (r-l)/2;
            if (idx <= m) at->l = erase(at->l, idx, l, m);
            else at->r = erase(at->r, idx, m+1, r);
            at->update();
        }
        return at;
    }
    void erase(T v) {
        if (v < 0 or v > N) return;
        root = erase(root, v, 0, N);
    }

    int count(node* at, T a, T b, T l, T r) {
        if (!at or b < l or r < a) return 0;
        prop(at, r-l+1);
        if (a <= l and r <= b) return at->cnt;
        T m = l + (r-l)/2;
        return count(at->l, a, b, l, m) + count(at->r, a, b,
           m+1, r);
    }
    int count(T v) { return count(root, v, v, 0, N); }
    int order_of_key(T v) { return count(root, 0, v-1, 0,
      N); }
    int lower_bound(T v) { return order_of_key(v); }

    const T operator [](int i) { // i-esimo menor elemento
        assert(i >= 0 and i < size());
        node* at = root;
        T l = 0, r = N;
        while (l < r) {
            prop(at, r-l+1);
            T m = l + (r-l)/2;
            if (count(at->l, m-l+1) > i) at = at->l, r = m;
            else {
                i -= count(at->l, r-m);
                at = at->r; l = m+1;
            }
        }
        return l;
```

```
    }

    node* merge(node* a, node* b, T tam) {
        if (!a or !b) return a ? a : b;
        prop(a, tam), prop(b, tam);
        if (b->cnt == tam) swap(a, b);
        if (tam == 1 or a->cnt == tam) {
            destroy(b);
            return a;
        }
        a->l = merge(a->l, b->l, tam>>1), a->r = merge(a->r,
           b->r, tam>>1);
        a->update(), delete b;
        return a;
    }
    void merge(sms& s) { // mergeia dois sets
        if (N > s.N) swap(*this, s);
        expand(s.N);
        root = merge(root, s.root, N+1);
        s.root = NULL;
    }

    node* split(node*& x, int k, T tam) {
        if (k <= 0 or !x) return NULL;
        prop(x, tam);
        node* ret = new node();
        if (tam == 1) x->cnt = 0, ret->cnt = 1;
        else {
            if (k <= count(x->l, tam>>1)) ret->l =
               split(x->l, k, tam>>1);
            else {
                ret->r = split(x->r, k - count(x->l,
                   tam>>1), tam>>1);
                swap(x->l, ret->l);
            }
            ret->update(), x->update();
        }
        return ret;
    }
    void split(int k, sms& s) { // pega os 'k' menores
        s.clear();
        s.root = split(root, min(k, size()), N+1);
        s.N = N;
    }
    // pega os menores que 'k'
    void split_val(T k, sms& s) { split(order_of_key(k), s);
       }

    void flip(node*& at, T a, T b, T l, T r) {
        if (!at) at = new node();
        else prop(at, r-l+1);
        if (a <= l and r <= b) {
            at->flip ^= 1;
            prop(at, r-l+1);
            return;
        }
        if (r < a or b < l) return;
        T m = l + (r-l)/2;
        flip(at->l, a, b, l, m), flip(at->r, a, b, m+1, r);
        at->update();
    }
    void flip(T l, T r) { // flipa os valores em [l, r]
        assert(l >= 0 and l <= r);
        expand(r);
        flip(root, l, r, 0, N);
    }
    // complemento considerando que o universo eh [0, lim]
    void complement(T lim) {
        assert(lim >= 0);
        if (lim > N) expand(lim);
        flip(root, 0, lim, 0, N);
        sms tmp;
        split_val(lim+1, tmp);
        swap(*this, tmp);
    }
    void insert_range(T l, T r) { // insere todo os valores
       em [l, r]
        sms tmp;
        tmp.flip(l, r);
        merge(tmp);
    }
};
```

4.29 SQRT Tree

```
// RMQ em O(log log n) com O(n log log n) pra buildar
// Funciona com qualquer operacao associativa
// Tao rapido quanto a sparse table, mas usa menos memoria
// (log log (1e9) < 5, entao a query eh praticamente O(1))
//
// build - O(n log log n)
// query - O(log log n)
// 8ff986

namespace sqrtTree {
    int n, *v;
    int pref[4][MAX], sulf[4][MAX], getl[4][MAX],
       entre[4][MAX], sz[4];

    int op(int a, int b) { return min(a, b); }
    inline int getblk(int p, int i) { return
       (i-getl[p][i])/sz[p]; }
    void build(int p, int l, int r) {
        if (l+1 >= r) return;
        for (int i = l; i <= r; i++) getl[p][i] = l;
        for (int L = l; L <= r; L += sz[p]) {
            int R = min(L+sz[p]-1, r);
            pref[p][L] = v[L], sulf[p][R] = v[R];
            for (int i = L+1; i <= R; i++) pref[p][i] =
               op(pref[p][i-1], v[i]);
            for (int i = R-1; i >= L; i--) sulf[p][i] =
               op(v[i], sulf[p][i+1]);
            build(p+1, L, R);
        }
        for (int i = 0; i <= sz[p]; i++) {
            int at = entre[p][l+i*sz[p]+i] =
               sulf[p][l+i*sz[p]];
            for (int j = i+1; j <= sz[p]; j++)
               entre[p][l+i*sz[p]+j] = at =
                     op(at, sulf[p][l+j*sz[p]]);
        }
    }
    void build(int n2, int* v2) {
        n = n2, v = v2;
        for (int p = 0; p < 4; p++) sz[p] = n2 = sqrt(n2);
        build(0, 0, n-1);
    }
    int query(int l, int r) {
        if (l+1 >= r) return l == r ? v[l] : op(v[l], v[r]);
        int p = 0;
        while (getblk(p, l) == getblk(p, r)) p++;
        int ans = sulf[p][l], a = getblk(p, l)+1, b =
           getblk(p, r)-1;
        if (a <= b) ans = op(ans,
           entre[p][getl[p][l]+a*sz[p]+b]);
        return op(ans, pref[p][r]);
    }
}
```

4.30 Treap

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade, exceto meld
// meld custa O(log^2 n) amortizado com alta prob.,
// e permite unir duas treaps sem restricao adicional
// Na pratica, esse meld tem constante muito boa e
// o pior caso eh meio estranho de acontecer
// bd93e2

mt19937 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

template<typename T> struct treap {
    struct node {
        node *l, *r;
        int p, sz;
        T val, mi;
        node(T v) : l(NULL), r(NULL), p(rng()), sz(1),
           val(v), mi(v) {}
        void update() {
            sz = 1;
            mi = val;
            if (l) sz += l->sz, mi = min(mi, l->mi);
            if (r) sz += r->sz, mi = min(mi, r->mi);
        }
```

```
};

node* root;

treap() { root = NULL; }
treap(const treap& t) {
    throw logic_error("Nao copiar a treap!");
}
~treap() {
    vector<node*> q = {root};
    while (q.size()) {
        node* x = q.back(); q.pop_back();
        if (!x) continue;
        q.push_back(x->l), q.push_back(x->r);
        delete x;
    }
}

int size(node* x) { return x ? x->sz : 0; }
int size() { return size(root); }
void join(node* l, node* r, node*& i) { // assume que l
   < r
    if (!l or !r) return void(i = l ? l : r);
    if (l->p > r->p) join(l->r, r, l->r), i = l;
    else join(l, r->l, r->l), i = r;
    i->update();
}
void split(node* i, node*& l, node*& r, T v) {
    if (!i) return void(r = l = NULL);
    if (i->val < v) split(i->r, i->r, r, v), l = i;
    else split(i->l, l, i->l, v), r = i;
    i->update();
}
void split_leq(node* i, node*& l, node*& r, T v) {
    if (!i) return void(r = l = NULL);
    if (i->val <= v) split_leq(i->r, i->r, r, v), l = i;
    else split_leq(i->l, l, i->l, v), r = i;
    i->update();
}
int count(node* i, T v) {
    if (!i) return 0;
    if (i->val == v) return 1;
    if (v < i->val) return count(i->l, v);
    return count(i->r, v);
}
void index_split(node* i, node*& l, node*& r, int v, int
   key = 0) {
    if (!i) return void(r = l = NULL);
    if (key + size(i->l) < v) index_split(i->r, i->r, r,
       v, key+size(i->l)+1), l = i;
    else index_split(i->l, l, i->l, v, key), r = i;
    i->update();
}
int count(T v) {
    return count(root, v);
}
void insert(T v) {
    if (count(v)) return;
    node *L, *R;
    split(root, L, R, v);
    node* at = new node(v);
    join(L, at, L);
    join(L, R, root);
}
void erase(T v) {
    node *L, *M, *R;
    split_leq(root, M, R, v), split(M, L, M, v);
    if (M) delete M;
    M = NULL;
    join(L, R, root);
}
void meld(treap& t) { // segmented merge
    node *L = root, *R = t.root;
    root = NULL;
    while (L or R) {
        if (!L or (L and R and L->mi > R->mi))
            std::swap(L, R);
        if (!R) join(root, L, root), L = NULL;
        else if (L->mi == R->mi) {
            node* LL;
            split(L, LL, L, R->mi+1);
            delete LL;
        } else {
            node* LL;
```

```
            split(L, LL, L, R->mi);
            join(root, LL, root);
        }
    }
    t.root = NULL;
  }
};
```

4.31 Treap Implicita

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade
// 63ba4d

mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());

template<typename T> struct treap {
    struct node {
        node *l, *r;
        int p, sz;
        T val, sub, lazy;
        bool rev;
        node(T v) : l(NULL), r(NULL), p(rng()), sz(1),
            val(v), sub(v), lazy(0), rev(0) {}
        void prop() {
            if (lazy) {
                val += lazy, sub += lazy*sz;
                if (l) l->lazy += lazy;
                if (r) r->lazy += lazy;
            }
            if (rev) {
                swap(l, r);
                if (l) l->rev ^= 1;
                if (r) r->rev ^= 1;
            }
            lazy = 0, rev = 0;
        }
        void update() {
            sz = 1, sub = val;
            if (l) l->prop(), sz += l->sz, sub += l->sub;
            if (r) r->prop(), sz += r->sz, sub += r->sub;
        }
    };

    node* root;

    treap() { root = NULL; }
    treap(const treap& t) {
        throw logic_error("Nao copiar a treap!");
    }
    ~treap() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }

    int size(node* x) { return x ? x->sz : 0; }
    int size() { return size(root); }
    void join(node* l, node* r, node*& i) { // assume que l
        < r
        if (!l or !r) return void(i = l ? l : r);
        l->prop(), r->prop();
        if (l->p > r->p) join(l->r, r, l->r), i = l;
        else join(l, r->l, r->l), i = r;
        i->update();
    }
    void split(node* i, node*& l, node*& r, int v, int key =
        0) {
        if (!i) return void(r = l = NULL);
        i->prop();
        if (key + size(i->l) < v) split(i->r, i->r, r, v,
            key+size(i->l)+1), l = i;
        else split(i->l, l, i->l, v, key), r = i;
        i->update();
    }
    void push_back(T v) {
        node* i = new node(v);
```

```
        join(root, i, root);
    }
    T query(int l, int r) {
        node *L, *M, *R;
        split(root, M, R, r+1), split(M, L, M, l);
        T ans = M->sub;
        join(L, M, M), join(M, R, root);
        return ans;
    }
    void update(int l, int r, T s) {
        node *L, *M, *R;
        split(root, M, R, r+1), split(M, L, M, l);
        M->lazy += s;
        join(L, M, M), join(M, R, root);
    }
    void reverse(int l, int r) {
        node *L, *M, *R;
        split(root, M, R, r+1), split(M, L, M, l);
        M->rev ^= 1;
        join(L, M, M), join(M, R, root);
    }
};
```

4.32 Treap Persistent Implicita

```
// Todas as operacoes custam
// O(log(n)) com alta probabilidade
// fb8013

mt19937_64 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

struct node {
    node *l, *r;
    ll sz, val, sub;
    node(ll v) : l(NULL), r(NULL), sz(1), val(v), sub(v) {}
    node(node* x) : l(x->l), r(x->r), sz(x->sz),
       val(x->val), sub(x->sub) {}
    void update() {
        sz = 1, sub = val;
        if (l) sz += l->sz, sub += l->sub;
        if (r) sz += r->sz, sub += r->sub;
        sub %= MOD;
    }
};

ll size(node* x) { return x ? x->sz : 0; }
void update(node* x) { if (x) x->update(); }
node* copy(node* x) { return x ? new node(x) : NULL; }

node* join(node* l, node* r) {
    if (!l or !r) return l ? copy(l) : copy(r);
    node* ret;
    if (rng() % (size(l) + size(r)) < size(l)) {
        ret = copy(l);
        ret->r = join(ret->r, r);
    } else {
        ret = copy(r);
        ret->l = join(l, ret->l);
    }
    return update(ret), ret;
}

void split(node* x, node*& l, node*& r, ll v, ll key = 0) {
    if (!x) return void(l = r = NULL);
    if (key + size(x->l) < v) {
        l = copy(x);
        split(l->r, l->r, r, v, key+size(l->l)+1);
    } else {
        r = copy(x);
        split(r->l, l, r->l, v, key);
    }
    update(l), update(r);
}

vector<node*> treap;

void init(const vector<ll>& v) {
    treap = {NULL};
    for (auto i : v) treap[0] = join(treap[0], new node(i));
}
```

4.33 Wavelet Tree

```
// Usa O(sigma + n log(sigma)) de memoria,
// onde sigma = MAXN - MINN
// Depois do build, o v fica ordenado
// count(i, j, x, y) retorna o numero de elementos de
// v[i, j) que pertencem a [x, y]
// kth(i, j, k) retorna o elemento que estaria
// na poscicao k-1 de v[i, j), se ele fosse ordenado
// sum(i, j, x, y) retorna a soma dos elementos de
// v[i, j) que pertencem a [x, y]
// sumk(i, j, k) retorna a soma dos k-esimos menores
// elementos de v[i, j) (sum(i, j, 1) retorna o menor)
//
// Complexidades:
// build - O(n log(sigma))
// count - O(log(sigma))
// kth   - O(log(sigma))
// sum   - O(log(sigma))
// sumk  - O(log(sigma))
// 782344

int n, v[MAX];
vector<int> esq[4*(MAXN-MINN)], pref[4*(MAXN-MINN)];

void build(int b = 0, int e = n, int p = 1, int l = MINN,
    int r = MAXN) {
     int m = (l+r)/2; esq[p].push_back(0);
         pref[p].push_back(0);
     for (int i = b; i < e; i++) {
          esq[p].push_back(esq[p].back()+(v[i]<=m));
          pref[p].push_back(pref[p].back()+v[i]);
     }
     if (l == r) return;
     int m2 = stable_partition(v+b, v+e, [=](int i){return i
         <= m;}) - v;
     build(b, m2, 2*p, l, m), build(m2, e, 2*p+1, m+1, r);
}

int count(int i, int j, int x, int y, int p = 1, int l =
    MINN, int r = MAXN) {
     if (y < l or r < x) return 0;
     if (x <= l and r <= y) return j-i;
     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
     return count(ei, ej, x, y, 2*p, l, m)+count(i-ei, j-ej,
         x, y, 2*p+1, m+1, r);
}

int kth(int i, int j, int k, int p=1, int l = MINN, int r =
    MAXN) {
     if (l == r) return l;
     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
     if (k <= ej-ei) return kth(ei, ej, k, 2*p, l, m);
     return kth(i-ei, j-ej, k-(ej-ei), 2*p+1, m+1, r);
}

int sum(int i, int j, int x, int y, int p = 1, int l = MINN,
    int r = MAXN) {
     if (y < l or r < x) return 0;
     if (x <= l and r <= y) return pref[p][j]-pref[p][i];
     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
     return sum(ei, ej, x, y, 2*p, l, m) + sum(i-ei, j-ej, x,
         y, 2*p+1, m+1, r);
}

int sumk(int i, int j, int k, int p = 1, int l = MINN, int r
    = MAXN) {
     if (l == r) return l*k;
     int m = (l+r)/2, ei = esq[p][i], ej = esq[p][j];
     if (k <= ej-ei) return sumk(ei, ej, k, 2*p, l, m);
     return pref[2*p][ej]-pref[2*p][ei]+sumk(i-ei, j-ej,
         k-(ej-ei), 2*p+1, m+1, r);
}
```

5 Matematica

5.1 2-SAT

```
// solve() retorna um par, o first fala se eh possivel
// atribuir, o second fala se cada variavel eh verdadeira
//
```

```
// O(|V|+|E|) = O(#variaveis + #restricoes)
// ef6b3b

struct sat {
    int n, tot;
    vector<vector<int>> g;
    vector<int> vis, comp, id, ans;
    stack<int> s;

    sat() {}
    sat(int n_) : n(n_), tot(n), g(2*n) {}

    int dfs(int i, int& t) {
        int lo = id[i] = t++;
        s.push(i), vis[i] = 2;
        for (int j : g[i]) {
            if (!vis[j]) lo = min(lo, dfs(j, t));
            else if (vis[j] == 2) lo = min(lo, id[j]);
        }
        if (lo == id[i]) while (1) {
            int u = s.top(); s.pop();
            vis[u] = 1, comp[u] = i;
            if ((u>>1) < n and ans[u>>1] == -1) ans[u>>1] = ~
                u&1;
            if (u == i) break;
        }
        return lo;
    }

    void add_impl(int x, int y) { // x -> y = !x ou y
        x = x >= 0 ? 2*x : -2*x-1;
        y = y >= 0 ? 2*y : -2*y-1;
        g[x].push_back(y);
        g[y^1].push_back(x^1);
    }
    void add_cl(int x, int y) { // x ou y
        add_impl(~x, y);
    }
    void add_xor(int x, int y) { // x xor y
        add_cl(x, y), add_cl(~x, ~y);
    }
    void add_eq(int x, int y) { // x = y
        add_xor(~x, y);
    }
    void add_true(int x) { // x = T
        add_impl(~x, x);
    }
    void at_most_one(vector<int> v) { // no max um verdadeiro
        g.resize(2*(tot+v.size()));
        for (int i = 0; i < v.size(); i++) {
            add_impl(tot+i, ~v[i]);
            if (i) {
                add_impl(tot+i, tot+i-1);
                add_impl(v[i], tot+i-1);
            }
        }
        tot += v.size();
    }

    pair<bool, vector<int>> solve() {
        ans = vector<int>(n, -1);
        int t = 0;
        vis = comp = id = vector<int>(2*tot, 0);
        for (int i = 0; i < 2*tot; i++) if (!vis[i]) dfs(i,
            t);
        for (int i = 0; i < tot; i++)
            if (comp[2*i] == comp[2*i+1]) return {false, {}};
        return {true, ans};
    }
};
```

5.2 Algoritmo de Euclides estendido

```
// Acha x e y tal que ax + by = mdc(a, b) (nao eh unico)
// Assume a, b >= 0
//
// O(log(min(a, b)))
// 35411d

tuple<ll, ll, ll> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b%a, a);
```

```
    return {g, y - b/a*x, x};
}
```

5.3 Avaliacao de Interpolacao

```
// Dado 'n' pontos (i, y[i]), i \in [0, n),
// avalia o polinomio de grau n-1 que passa
// por esses pontos em 'x'
// Tudo modular, precisa do mint
//
// O(n)
// 4fe929

mint evaluate_interpolation(int x, vector<mint> y) {
    int n = y.size();

    vector<mint> sulf(n+1, 1), fat(n, 1), ifat(n);
    for (int i = n-1; i >= 0; i--) sulf[i] = sulf[i+1] * (x - i);
    for (int i = 1; i < n; i++) fat[i] = fat[i-1] * i;
    ifat[n-1] = 1/fat[n-1];
    for (int i = n-2; i >= 0; i--) ifat[i] = ifat[i+1] * (i + 1);

    mint pref = 1, ans = 0;
    for (int i = 0; i < n; pref *= (x - i++)) {
        mint num = pref * sulf[i+1];

        mint den = ifat[i] * ifat[n-1 - i];
        if ((n-1 - i)%2) den *= -1;

        ans += y[i] * num * den;
    }
    return ans;
}
```

5.4 Berlekamp-Massey

```
// guess_kth(s, k) chuta o k-esimo (0-based) termo
// de uma recorrencia linear que gera s
// Para uma rec. lin. de ordem x, se passar 2x termos
// vai gerar a certa
// Usar aritmetica modular
//
// O(n^2 log k), em que n = |s|
// 8644e3

template<typename T> T evaluate(vector<T> c, vector<T> s, ll k) {
    int n = c.size();
    assert(c.size() <= s.size());

    auto mul = [&](const vector<T> &a, const vector<T> &b) {
        vector<T> ret(a.size() + b.size() - 1);
        for (int i = 0; i < a.size(); i++) for (int j = 0; j < b.size(); j++)
            ret[i+j] += a[i] * b[j];
        for (int i = ret.size()-1; i >= n; i--) for (int j = n-1; j >= 0; j--)
            ret[i-j-1] += ret[i] * c[j];
        ret.resize(min<int>(ret.size(), n));
        return ret;
    };

    vector<T> a = n == 1 ? vector<T>({c[0]}) : vector<T>({0, 1}), x = {1};
    while (k) {
        if (k&1) x = mul(x, a);
        a = mul(a, a), k >>= 1;
    }
    x.resize(n);

    T ret = 0;
    for (int i = 0; i < n; i++) ret += x[i] * s[i];
    return ret;
}

template<typename T> vector<T> berlekamp_massey(vector<T> s) {
    int n = s.size(), l = 0, m = 1;
```

```
    vector<T> b(n), c(n);
    T ld = b[0] = c[0] = 1;
    for (int i = 0; i < n; i++, m++) {
        T d = s[i];
        for (int j = 1; j <= l; j++) d += c[j] * s[i-j];
        if (d == 0) continue;
        vector<T> temp = c;
        T coef = d / ld;
        for (int j = m; j < n; j++) c[j] -= coef * b[j-m];
        if (2 * l <= i) l = i + 1 - l, b = temp, ld = d, m =
            0;
    }
    c.resize(l + 1);
    c.erase(c.begin());
    for (T& x : c) x = -x;
    return c;
}

template<typename T> T guess_kth(const vector<T>& s, ll k) {
    auto c = berlekamp_massey(s);
    return evaluate(c, s, k);
}
```

5.5 Binomial Distribution

```
// binom(n, k, p) retorna a probabilidade de k sucessos
// numa binomial(n, p)
// 00d38f

double logfact[MAX];

void calc() {
    logfact[0] = 0;
    for (int i = 1; i < MAX; i++) logfact[i] = logfact[i-1]
        + log(i);
}

double binom(int n, int k, double p) {
    return exp(logfact[n] - logfact[k] - logfact[n-k] + k *
        log(p) + (n-k) * log(1 - p));
}
```

5.6 Convolucao de GCD / LCM

```
// O(n log(n))

// multiple_transform(a)[i] = \sum_d a[d * i]
// 338be8
template<typename T> void multiple_transform(vector<T>& v,
    bool inv = false) {
    vector<int> I(v.size()-1);
    iota(I.begin(), I.end(), 1);
    if (inv) reverse(I.begin(), I.end());
    for (int i : I) for (int j = 2; i*j < v.size(); j++)
        v[i] += (inv ? -1 : 1) * v[i*j];
}

// gcd_convolution(a, b)[k] = \sum_{gcd(i, j) = k} a_i * b_j
// 984f53
template<typename T> vector<T> gcd_convolution(vector<T> a,
    vector<T> b) {
    multiple_transform(a), multiple_transform(b);
    for (int i = 0; i < a.size(); i++) a[i] *= b[i];
    multiple_transform(a, true);
    return a;
}

// divisor_transform(a)[i] = \sum_{d|i} a[i/d]
// aa74e5
template<typename T> void divisor_transform(vector<T>& v,
    bool inv = false) {
    vector<int> I(v.size()-1);
    iota(I.begin(), I.end(), 1);
    if (!inv) reverse(I.begin(), I.end());
    for (int i : I) for (int j = 2; i*j < v.size(); j++)
        v[i*j] += (inv ? -1 : 1) * v[i];
}

// lcm_convolution(a, b)[k] = \sum_{lcm(i, j) = k} a_i * b_j
// f5acc1
```

```cpp
template<typename T> vector<T> lcm_convolution(vector<T> a,
    vector<T> b) {
     divisor_transform(a), divisor_transform(b);
     for (int i = 0; i < a.size(); i++) a[i] *= b[i];
     divisor_transform(a, true);
     return a;
}
```

5.7 Deteccao de ciclo - Tortoise and Hare

```cpp
// Linear no tanto que tem que andar pra ciclar,
// O(1) de memoria
// Retorna um par com o tanto que tem que andar
// do f0 ate o inicio do ciclo e o tam do ciclo
// 899f20

pair<ll, ll> find_cycle() {
    ll tort = f(f0);
    ll hare = f(f(f0));
    ll t = 0;
    while (tort != hare) {
        tort = f(tort);
        hare = f(f(hare));
        t++;
    }
    ll st = 0;
    tort = f0;
    while (tort != hare) {
        tort = f(tort);
        hare = f(hare);
        st++;
    }

    ll len = 1;
    hare = f(tort);
    while (tort != hare) {
        hare = f(hare);
        len++;
    }
    return {st, len};
}
```

5.8 Division Trick

```cpp
// Gera o conjunto n/i, pra todo i, em O(sqrt(n))
// copiei do github do tfg50

for(int l = 1, r; l <= n; l = r + 1) {
    r = n / (n / l);
    // n / i has the same value for l <= i <= r
}
```

5.9 Eliminacao Gaussiana

```cpp
// Resolve sistema linear
// Retornar um par com o numero de solucoes
// e alguma solucao, caso exista
//
// O(n^2 * m)
// 1d10b5

template<typename T>
pair<int, vector<T>> gauss(vector<vector<T>> a, vector<T> b)
    {
    const double eps = 1e-6;
    int n = a.size(), m = a[0].size();
    for (int i = 0; i < n; i++) a[i].push_back(b[i]);

    vector<int> where(m, -1);
    for (int col = 0, row = 0; col < m and row < n; col++) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs(a[i][col]) > abs(a[sel][col])) sel = i;
        if (abs(a[sel][col]) < eps) continue;
        for (int i = col; i <= m; i++)
            swap(a[sel][i], a[row][i]);
        where[col] = row;
```

```
        for (int i = 0; i < n; i++) if (i != row) {
            T c = a[i][col] / a[row][col];
            for (int j = col; j <= m; j++)
                a[i][j] -= a[row][j] * c;
        }
        row++;
    }

    vector<T> ans(m, 0);
    for (int i = 0; i < m; i++) if (where[i] != -1)
        ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i = 0; i < n; i++) {
        T sum = 0;
        for (int j = 0; j < m; j++)
            sum += ans[j] * a[i][j];
        if (abs(sum - a[i][m]) > eps)
            return pair(0, vector<T>());
    }

    for (int i = 0; i < m; i++) if (where[i] == -1)
        return pair(INF, ans);
    return pair(1, ans);
}
```

5.10 Eliminacao Gaussiana Z2

```
// D eh dimensao do espaco vetorial
// add(v) - adiciona o vetor v na base (retorna se ele jah
   pertencia ao span da base)
// coord(v) - retorna as coordenadas (c) de v na base atual
   (basis^T.c = v)
// recover(v) - retorna as coordenadas de v nos vetores na
   ordem em que foram inseridos
// coord(v).first e recover(v).first - se v pertence ao span
//
// Complexidade:
// add, coord, recover: O(D^2 / 64)
// d0a4b3

template<int D> struct Gauss_z2 {
    bitset<D> basis[D], keep[D];
    int rk, in;
    vector<int> id;

    Gauss_z2 () : rk(0), in(-1), id(D, -1) {};

    bool add(bitset<D> v) {
        in++;
        bitset<D> k;
        for (int i = D - 1; i >= 0; i--) if (v[i]) {
            if (basis[i][i]) v ^= basis[i], k ^= keep[i];
            else {
                k[i] = true, id[i] = in, keep[i] = k;
                basis[i] = v, rk++;
                return true;
            }
        }
        return false;
    }
    pair<bool, bitset<D>> coord(bitset<D> v) {
        bitset<D> c;
        for (int i = D - 1; i >= 0; i--) if (v[i]) {
            if (basis[i][i]) v ^= basis[i], c[i] = true;
            else return {false, bitset<D>()};
        }
        return {true, c};
    }
    pair<bool, vector<int>> recover(bitset<D> v) {
        auto [span, bc] = coord(v);
        if (not span) return {false, {}};
        bitset<D> aux;
        for (int i = D - 1; i >= 0; i--) if (bc[i]) aux ^=
           keep[i];
        vector<int> oc;
        for (int i = D - 1; i >= 0; i--) if (aux[i])
           oc.push_back(id[i]);
        return {true, oc};
    }
};
```

5.11 Equacao Diofantina Linear

```
// Encontra o numero de solucoes de a*x + b*y = c,
// em que x \in [lx, rx] e y \in [ly, ry]
// Usar o comentario para recuperar as solucoes
// (note que o b ao final eh b/gcd(a, b))
// Cuidado com overflow! Tem que caber o quadrado dos valores
//
// O(log(min(a, b)))
// 2e8259

template<typename T> tuple<ll, T, T> ext_gcd(ll a, ll b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd<T>(b%a, a);
    return {g, y - b/a*x, x};
}

// numero de solucoes de a*[lx, rx] + b*[ly, ry] = c
template<typename T = ll> // usar __int128 se for ate 1e18
ll diophantine(ll a, ll b, ll c, ll lx, ll rx, ll ly, ll ry)
    {
    if (lx > rx or ly > ry) return 0;
    if (a == 0 and b == 0) return c ? 0 :
        (rx-lx+1)*(ry-ly+1);
    auto [g, x, y] = ext_gcd<T>(abs(a), abs(b));
    if (c % g != 0) return 0;
    if (a == 0) return (rx-lx+1)*(ly <= c/b and c/b <= ry);
    if (b == 0) return (ry-ly+1)*(lx <= c/a and c/a <= rx);
    x *= a/abs(a) * c/g, y *= b/abs(b) * c/g, a /= g, b /= g;

    auto shift = [&](T qt) { x += qt*b, y -= qt*a; };
    auto test = [&](T& k, ll mi, ll ma, ll coef, int t) {
        shift((mi - k)*t / coef);
        if (k < mi) shift(coef > 0 ? t : -t);
        if (k > ma) return pair<T, T>(rx+2, rx+1);
        T x1 = x;
        shift((ma - k)*t / coef);
        if (k > ma) shift(coef > 0 ? -t : t);
        return pair<T, T>(x1, x);
    };

    auto [l1, r1] = test(x, lx, rx, b, 1);
    auto [l2, r2] = test(y, ly, ry, a, -1);
    if (l2 > r2) swap(l2, r2);
    T l = max(l1, l2), r = min(r1, r2);
    if (l > r) return 0;
    ll k = (r-l) / abs(b) + 1;
    return k; // solucoes: x = l + [0, k)*|b|
}
```

5.12 Exponenciacao rapida

```
// (x^y mod m) em O(log(y))

ll pow(ll x, ll y, ll m) { // iterativo
    ll ret = 1;
    while (y) {
        if (y & 1) ret = (ret * x) % m;
        y >>= 1;
        x = (x * x) % m;
    }
    return ret;
}

ll pow(ll x, ll y, ll m) { // recursivo
    if (!y) return 1;
    ll ans = pow(x*x%m, y/2, m);
    return y%2 ? x*ans%m : ans;
}
```

5.13 Fast Walsh Hadamard Transform

```
// FWHT<'|'>(f) eh SOS DP
// FWHT<'&'>(f) eh soma de superset DP
// Se chamar com ^, usar tamanho potencia de 2!!
//
// O(n log(n))
// 50e84f

template<char op, class T> vector<T> FWHT(vector<T> f, bool
    inv = false) {
```

```
    int n = f.size();
    for (int k = 0; (n-1)>>k; k++) for (int i = 0; i < n;
        i++) if (i>>k&1) {
        int j = i^(1<<k);
        if (op == '^') f[j] += f[i], f[i] = f[j] - 2*f[i];
        if (op == '|') f[i] += (inv ? -1 : 1) * f[j];
        if (op == '&') f[j] += (inv ? -1 : 1) * f[i];
    }
    if (op == '^' and inv) for (auto& i : f) i /= n;
    return f;
}
```

5.14 FFT

```
// Chamar convolution com vector<complex<double>> para FFT
// Precisa do mint para NTT
//
// O(n log(n))

// Para FFT
// de56b9
void get_roots(bool f, int n, vector<complex<double>>&
    roots) {
    const static double PI = acosl(-1);
    for (int i = 0; i < n/2; i++) {
        double alpha = i*((2*PI)/n);
        if (f) alpha = -alpha;
        roots[i] = {cos(alpha), sin(alpha)};
    }
}

// Para NTT
// 91cd08
template<int p>
void get_roots(bool f, int n, vector<mod_int<p>>& roots) {
    mod_int<p> r;
    int ord;
    if (p == 998244353) {
        r = 102292;
        ord = (1 << 23);
    } else if (p == 754974721) {
        r = 739831874;
        ord = (1 << 24);
    } else if (p == 167772161) {
        r = 243;
        ord = (1 << 25);
    } else assert(false);

    if (f) r = r^(p - 1 -ord/n);
    else r = r^(ord/n);
    roots[0] = 1;
    for (int i = 1; i < n/2; i++) roots[i] = roots[i-1]*r;
}

// d5c432
template<typename T> void fft(vector<T> &a, bool f, int N,
    vector<int> &rev) {
    for (int i = 0; i < N; i++) if (i < rev[i]) swap(a[i],
        a[rev[i]]);
    int l, r, m;
    vector<T> roots(N);
    for (int n = 2; n <= N; n *= 2) {
        get_roots(f, n, roots);

        for (int pos = 0; pos < N; pos += n) {
            l = pos+0, r = pos+n/2, m = 0;
            while (m < n/2) {
                auto t = roots[m]*a[r];
                a[r] = a[l] - t;
                a[l] = a[l] + t;
                l++; r++; m++;
            }
        }
    }
    if (f) {
        auto invN = T(1)/T(N);
        for (int i = 0; i < N; i++) a[i] = a[i]*invN;
    }
}
template<typename T> vector<T> convolution(vector<T> &a,
    vector<T> &b) {
    vector<T> l(a.begin(), a.end());
```

```
    vector<T> r(b.begin(), b.end());
    int ln = l.size(), rn = r.size();
    int N = ln+rn-1;
    int n = 1, log_n = 0;
    while (n <= N) { n <<= 1; log_n++; }
    vector<int> rev(n);
    for (int i = 0; i < n; ++i) {
        rev[i] = 0;
        for (int j = 0; j < log_n; ++j)
            if (i & (1<<j)) rev[i] |= 1 << (log_n-1-j);
    }
    assert(N <= n);
    l.resize(n);
    r.resize(n);
    fft(l, false, n, rev);
    fft(r, false, n, rev);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    fft(l, true, n, rev);
    l.resize(N);
    return l;
}

// NTT
// 3bf256
template<int p, typename T> vector<mod_int<p>>
    ntt(vector<T>& a, vector<T>& b) {
    vector<mod_int<p>> A(a.begin(), a.end()), B(b.begin(),
        b.end());
    return convolution(A, B);
}

// Convolucao de inteiro
//
// Precisa do CRT
//
// Tabela de valores:
// [0,1]         - <int, 1>
// [-1e5, 1e5] - <ll, 2>
// [-1e9, 1e9] - <__int128, 3>
//
// 053a7d
template<typename T, int mods>
vector<T> int_convolution(vector<int>& a, vector<int>& b) {
    static const int M1 = 998244353, M2 = 754974721, M3 =
        167772161;

    auto c1 = ntt<M1>(a, b);
    auto c2 = (mods >= 2 ? ntt<M2>(a, b) :
        vector<mod_int<M2>>());
    auto c3 = (mods >= 3 ? ntt<M3>(a, b) :
        vector<mod_int<M3>>());

    vector<T> ans;
    for (int i = 0; i < c1.size(); i++) {
        crt<T> at(c1[i].v, M1);
        if (mods >= 2) at = at * crt<T>(c2[i].v, M2);
        if (mods >= 3) at = at * crt<T>(c3[i].v, M3);
        ans.push_back(at.a);
        if (at.a > at.m/2) ans.back() -= at.m;
    }
    return ans;
}
```

5.15 Integracao Numerica - Metodo de Simpson 3/8

```
// Integra f no intervalo [a, b], erro cresce proporcional a
    (b - a)^5

const int N = 3*100; // multiplo de 3
ld integrate(ld a, ld b, function<ld(ld)> f) {
    ld s = 0, h = (b - a)/N;
    for (int i = 1 ; i < N; i++) s += f(a + i*h)*(i%3 ? 3 :
        2);
    return (f(a) + s + f(b))*3*h/8;
}
```

5.16 Inverso Modular

```
// Computa o inverso de a modulo b
// Se b eh primo, basta fazer
```

```
// a^(b-2)

ll inv(ll a, ll b) {
    return a > 1 ? b - inv(b%a, a)*b/a : 1;
}

// computa o inverso modular de 1..MAX-1 modulo um primo
ll inv[MAX]:
inv[1] = 1;
for (int i = 2; i < MAX; i++) inv[i] = MOD -
    MOD/i*inv[MOD%i]%MOD;
```

5.17 Karatsuba

```
// Os pragmas podem ajudar
// Para n ~ 2e5, roda em < 1 s
//
// O(n^1.58)
// 8065d6

//#pragma GCC optimize("Ofast")
//#pragma GCC target ("avx,avx2")
template<typename T> void kar(T* a, T* b, int n, T* r, T*
    tmp) {
    if (n <= 64) {
        for (int i = 0; i < n; i++) for (int j = 0; j < n;
            j++)
            r[i+j] += a[i] * b[j];
        return;
    }
    int mid = n/2;
    T *atmp = tmp, *btmp = tmp+mid, *E = tmp+n;
    memset(E, 0, sizeof(E[0])*n);
    for (int i = 0; i < mid; i++) {
        atmp[i] = a[i] + a[i+mid];
        btmp[i] = b[i] + b[i+mid];
    }
    kar(atmp, btmp, mid, E, tmp+2*n);
    kar(a, b, mid, r, tmp+2*n);
    kar(a+mid, b+mid, mid, r+n, tmp+2*n);
    for (int i = 0; i < mid; i++) {
        T temp = r[i+mid];
        r[i+mid] += E[i] - r[i] - r[i+2*mid];
        r[i+2*mid] += E[i+mid] - temp - r[i+3*mid];
    }
}

template<typename T> vector<T> karatsuba(vector<T> a,
    vector<T> b) {
    int n = max(a.size(), b.size());
    while (n&(n-1)) n++;
    a.resize(n), b.resize(n);
    vector<T> ret(2*n), tmp(4*n);
    kar(&a[0], &b[0], n, &ret[0], &tmp[0]);
    return ret;
}
```

5.18 Logaritmo Discreto

```
// Resolve logaritmo discreto com o algoritmo baby step
    giant step
// Encontra o menor x tal que a^x = b (mod m)
// Se nao tem, retorna -1
//
// O(sqrt(m) * log(sqrt(m))
// 739fa8

int dlog(int b, int a, int m) {
    if (a == 0) return b ? -1 : 1; // caso nao definido

    a %= m, b %= m;
    int k = 1, shift = 0;
    while (1) {
        int g = gcd(a, m);
        if (g == 1) break;

        if (b == k) return shift;
        if (b % g) return -1;
        b /= g, m /= g, shift++;
        k = (ll) k * a / g % m;
```

```
    }

    int sq = sqrt(m)+1, giant = 1;
    for (int i = 0; i < sq; i++) giant = (ll) giant * a % m;

    vector<pair<int, int>> baby;
    for (int i = 0, cur = b; i <= sq; i++) {
        baby.emplace_back(cur, i);
        cur = (ll) cur * a % m;
    }
    sort(baby.begin(), baby.end());

    for (int j = 1, cur = k; j <= sq; j++) {
        cur = (ll) cur * giant % m;
        auto it = lower_bound(baby.begin(), baby.end(),
           pair(cur, INF));
        if (it != baby.begin() and (--it)->first == cur)
             return sq * j - it->second + shift;
    }

    return -1;
}
```

5.19 Miller-Rabin

```
// Testa se n eh primo, n <= 3 * 10^18
//
// O(log(n)), considerando multiplicacao
// e exponenciacao constantes
// 4ebecc

ll mul(ll a, ll b, ll m) {
    ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
    return ret < 0 ? ret+m : ret;
}

ll pow(ll x, ll y, ll m) {
    if (!y) return 1;
    ll ans = pow(mul(x, x, m), y/2, m);
    return y%2 ? mul(x, ans, m) : ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;
    ll r = __builtin_ctzll(n - 1), d = n >> r;

    // com esses primos, o teste funciona garantido para n
       <= 2^64
    // funciona para n <= 3*10^24 com os primos ate 41
    for (int a : {2, 325, 9375, 28178, 450775, 9780504,
       795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}
```

5.20 Pollard's Rho Alg

```
// Usa o algoritmo de deteccao de ciclo de Floyd
// com uma otimizacao na qual o gcd eh acumulado
// A fatoracao nao sai necessariamente ordenada
// O algoritmo rho encontra um fator de n,
// e funciona muito bem quando n possui um fator pequeno
//
// Complexidades (considerando mul constante):
// rho - esperado O(n^(1/4)) no pior caso
// fact - esperado menos que O(n^(1/4) log(n)) no pior caso
// b00653

ll mul(ll a, ll b, ll m) {
    ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
```

```
    return ret < 0 ? ret+m : ret;
}

ll pow(ll x, ll y, ll m) {
    if (!y) return 1;
    ll ans = pow(mul(x, x, m), y/2, m);
    return y%2 ? mul(x, ans, m) : ans;
}

bool prime(ll n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;

    ll r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504,
       795265022}) {
        ll x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;

        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

ll rho(ll n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](ll x) {return mul(x, x, n) + 1;};

    ll x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x==y) x = ++x0, y = f(x);
        q = mul(prd, abs(x-y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<ll> fact(ll n) {
    if (n == 1) return {};
    if (prime(n)) return {n};
    ll d = rho(n);
    vector<ll> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}
```

5.21 Produto de dois long long mod m

```
// O(1)
// 260e72

ll mul(ll a, ll b, ll m) { // a*b % m
    ll ret = a*b - ll((long double)1/m*a*b+0.5)*m;
    return ret < 0 ? ret+m : ret;
}
```

5.22 Simplex

```
// Maximiza c^T x s.t. Ax <= b, x >= 0
//
// O(2^n), porem executa em O(n^3) no caso medio
// 3a08e5

const double eps = 1e-7;

namespace Simplex {
    vector<vector<double>> T;
    int n, m;
    vector<int> X, Y;

    void pivot(int x, int y) {
        swap(X[y], Y[x-1]);
        for (int i = 0; i <= m; i++) if (i != y) T[x][i] /=
           T[x][y];
```

```
        T[x][y] = 1/T[x][y];
        for (int i = 0; i <= n; i++) if (i != x and
            abs(T[i][y]) > eps) {
            for (int j = 0; j <= m; j++) if (j != y) T[i][j]
                -= T[i][y] * T[x][j];
            T[i][y] = -T[i][y] * T[x][y];
        }
    }

    // Retorna o par (valor maximo, vetor solucao)
    pair<double, vector<double>> simplex(
            vector<vector<double>> A, vector<double> b,
                vector<double> c) {
        n = b.size(), m = c.size();
        T = vector(n + 1, vector<double>(m + 1));
        X = vector<int>(m);
        Y = vector<int>(n);
        for (int i = 0; i < m; i++) X[i] = i;
        for (int i = 0; i < n; i++) Y[i] = i+m;
        for (int i = 0; i < m; i++) T[0][i] = -c[i];
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) T[i+1][j] = A[i][j];
            T[i+1][m] = b[i];
        }
        while (true) {
            int x = -1, y = -1;
            double mn = -eps;
            for (int i = 1; i <= n; i++) if (T[i][m] < mn)
                mn = T[i][m], x = i;
            if (x < 0) break;
            for (int i = 0; i < m; i++) if (T[x][i] < -eps)
                { y = i; break; }

            if (y < 0) return {-1e18, {}}; // sem solucao
                para Ax <= b
            pivot(x, y);
        }
        while (true) {
            int x = -1, y = -1;
            double mn = -eps;
            for (int i = 0; i < m; i++) if (T[0][i] < mn) mn
                = T[0][i], y = i;
            if (y < 0) break;
            mn = 1e200;
            for (int i = 1; i <= n; i++) if (T[i][y] > eps
                and T[i][m] / T[i][y] < mn)
                 mn = T[i][m] / T[i][y], x = i;

            if (x < 0) return {1e18, {}}; // c^T x eh
                ilimitado
            pivot(x, y);
        }
        vector<double> r(m);
        for(int i = 0; i < n; i++) if (Y[i] < m) r[Y[i]] =
            T[i+1][m];
        return {T[0][m], r};
    }
}
```

5.23 Teorema Chines do Resto

```
// Combina equacoes modulares lineares: x = a (mod m)
// O m final eh o lcm dos m's, e a resposta eh unica mod o
   lcm
// Os m nao precisam ser coprimos
// Se nao tiver solucao, o 'a' vai ser -1
// 7cd7b3

template<typename T> tuple<T, T, T> ext_gcd(T a, T b) {
    if (!a) return {b, 0, 1};
    auto [g, x, y] = ext_gcd(b%a, a);
    return {g, y - b/a*x, x};
}

template<typename T = ll> struct crt {
    T a, m;

    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator * (crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g) a = -1;
```

```
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m/g*C.m;
        T ans = a + (x*(C.a-a)/g % (C.m/g))*m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};
```

5.24 Totiente

```
// O(sqrt(n))
// faeca3

int tot(int n){
    int ret = n;

    for (int i = 2; i*i <= n; i++) if (n % i == 0) {
        while (n % i == 0) n /= i;
        ret -= ret / i;
    }
    if (n > 1) ret -= ret / n;

    return ret;
}
```

5.25 Variacoes do crivo de Eratosthenes

```
// "O" crivo
//
// Encontra maior divisor primo
// Um numero eh primo sse divi[x] == x
// fact fatora um numero <= lim
// A fatoracao sai ordenada
//
// crivo - O(n log(log(n)))
// fact - O(log(n))

int divi[MAX];

void crivo(int lim) {
    for (int i = 1; i <= lim; i++) divi[i] = 1;

    for (int i = 2; i <= lim; i++) if (divi[i] == 1)
        for (int j = i; j <= lim; j += i) divi[j] = i;
}

void fact(vector<int>& v, int n) {
    if (n != divi[n]) fact(v, n/divi[n]);
    v.push_back(divi[n]);
}

// Crivo linear
//
// Mesma coisa que o de cima, mas tambem
// calcula a lista de primos
//
// O(n)

int divi[MAX];
vector<int> primes;

void crivo(int lim) {
    divi[1] = 1;
    for (int i = 2; i <= lim; i++) {
        if (divi[i] == 0) divi[i] = i, primes.push_back(i);
        for (int j : primes) {
            if (j > divi[i] or i*j > lim) break;
            divi[i*j] = j;
        }
    }
}

// Crivo de divisores
//
// Encontra numero de divisores
// ou soma dos divisores
//
// O(n log(n))

int divi[MAX];
```

```
void crivo(int lim) {
	for (int i = 1; i <= lim; i++) divi[i] = 1;

	for (int i = 2; i <= lim; i++)
		for (int j = i; j <= lim; j += i) {
			// para numero de divisores
			divi[j]++;
			// para soma dos divisores
			divi[j] += i;
		}
}

// Crivo de totiente
//
// Encontra o valor da funcao
// totiente de Euler
//
// O(n log(log(n)))

int tot[MAX];

void crivo(int lim) {
	for (int i = 1; i <= lim; i++) {
		tot[i] += i;
		for (int j = 2*i; j <= lim; j += i)
			tot[j] -= tot[i];
	}
}

// Crivo de funcao de mobius
//
// O(n log(log(n)))

char meb[MAX];

void crivo(int lim) {
	for (int i = 2; i <= lim; i++) meb[i] = 2;
	meb[1] = 1;
	for (int i = 2; i <= lim; i++) if (meb[i] == 2)
		for (int j = i; j <= lim; j += i) if (meb[j]) {
			if (meb[j] == 2) meb[j] = 1;
			meb[j] *= j/i%i ? -1 : 0;
		}
}

// Crivo linear de funcao multiplicativa
//
// Computa f(i) para todo 1 <= i <= n, sendo f
// uma funcao multiplicativa (se gcd(a,b) = 1,
// entao f(a*b) = f(a)*f(b))
// f_prime tem que computar f de um primo, e
// add_prime tem que computar f(p^(k+1)) dado f(p^k) e p
// Se quiser computar f(p^k) dado p e k, usar os comentarios
//
// O(n)

vector<int> primes;
int f[MAX], pot[MAX];
//int expo[MAX];

void sieve(int lim) {
	// Funcoes para soma dos divisores:
	auto f_prime = [](int p) { return p+1; };
	auto add_prime = [](int fpak, int p) { return fpak*p+1;
		};
	//auto f_pak = [](int p, int k) {};

	f[1] = 1;
	for (int i = 2; i <= lim; i++) {
		if (!pot[i]) {
			primes.push_back(i);
			f[i] = f_prime(i), pot[i] = i;
			//expo[i] = 1;
		}
		for (int p : primes) {
			if (i*p > lim) break;
			if (i%p == 0) {
				f[i*p] = f[i / pot[i]] *
					add_prime(f[pot[i]], p);
				// se for descomentar, tirar a linha de cima
					tambem
				//f[i*p] = f[i / pot[i]] * f_pak(p,
					expo[i]+1);
				//expo[i*p] = expo[i]+1;
```

```
                pot[i*p] = pot[i] * p;
                break;
            } else {
                f[i*p] = f[i] * f[p];
                pot[i*p] = p;
                //expo[i*p] = 1;
            }
        }
    }
}
```

6 DP

6.1 Convex Hull Trick (Rafael)

```
// adds tem que serem feitos em ordem de slope
// queries tem que ser feitas em ordem de x
//
// linear
// 30323e

struct CHT {
    int it;
    vector<ll> a, b;
    CHT():it(0){}
    ll eval(int i, ll x){
        return a[i]*x + b[i];
    }
    bool useless(){
        int sz = a.size();
        int r = sz-1, m = sz-2, l = sz-3;
        return  (b[l] - b[r])*(a[m] - a[l]) <
            (b[l] - b[m])*(a[r] - a[l]);
    }
    void add(ll A, ll B){
        a.push_back(A); b.push_back(B);
        while (!a.empty()){
            if ((a.size() < 3) || !useless()) break;
            a.erase(a.end() - 2);
            b.erase(b.end() - 2);
        }
    }
    ll get(ll x){
        it = min(it, int(a.size()) - 1);
        while (it+1 < a.size()){
            if (eval(it+1, x) > eval(it, x)) it++;
            else break;
        }
        return eval(it, x);
    }
};
```

6.2 Convex Hull Trick Dinamico

```
// para double, use LINF = 1/.0, div(a, b) = a/b
// update(x) atualiza o ponto de intersecao da reta x
// overlap(x) verifica se a reta x sobrepoe a proxima
// add(a, b) adiciona reta da forma ax + b
// query(x) computa maximo de ax + b para entre as retas
//
// O(log(n)) amortizado por insercao
// O(log(n)) por query
// 978376

struct Line {
    mutable ll a, b, p;
    bool operator<(const Line& o) const { return a < o.a; }
    bool operator<(ll x) const { return p < x; }
};

struct dynamic_hull : multiset<Line, less<>> {
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 and a % b);
    }

    void update(iterator x) {
        if (next(x) == end()) x->p = LINF;
        else if (x->a == next(x)->a) x->p = x->b >=
            next(x)->b ? LINF : -LINF;
```

```
        else x->p = div(next(x)->b - x->b, x->a -
            next(x)->a);
    }

    bool overlap(iterator x) {
        update(x);
        if (next(x) == end()) return 0;
        if (x->a == next(x)->a) return x->b >= next(x)->b;
        return x->p >= next(x)->p;
    }

    void add(ll a, ll b) {
        auto x = insert({a, b, 0});
        while (overlap(x)) erase(next(x)), update(x);
        if (x != begin() and !overlap(prev(x))) x = prev(x),
            update(x);
        while (x != begin() and overlap(prev(x)))
             x = prev(x), erase(next(x)), update(x);
    }

    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

6.3 Divide and Conquer DP

```
// Particiona o array em k subarrays
// minimizando o somatorio das queries
//
// O(k n log n), assumindo quer query(l, r) eh O(1)
// 4efe6b

ll dp[MAX][2];

void solve(int k, int l, int r, int lk, int rk) {
    if (l > r) return;
    int m = (l+r)/2, p = -1;
    auto& ans = dp[m][k&1] = LINF;
    for (int i = max(m, lk); i <= rk; i++) {
        int at = dp[i+1][~k&1] + query(m, i);
        if (at < ans) ans = at, p = i;
    }
    solve(k, l, m-1, lk, p), solve(k, m+1, r, p, rk);
}

ll DC(int n, int k) {
    dp[n][0] = dp[n][1] = 0;
    for (int i = 0; i < n; i++) dp[i][0] = LINF;
    for (int i = 1; i <= k; i++) solve(i, 0, n-i, 0, n-i);
    return dp[0][k&1];
}
```

6.4 Longest Common Subsequence

```
// Computa a LCS entre dois arrays usando
// o algoritmo de Hirschberg para recuperar
//
// O(n*m), O(n+m) de memoria
// 337bb3

int lcs_s[MAX], lcs_t[MAX];
int dp[2][MAX];

// dp[0][j] = max lcs(s[li...ri], t[lj, lj+j])
void dp_top(int li, int ri, int lj, int rj) {
    memset(dp[0], 0, (rj-lj+1)*sizeof(dp[0][0]));
    for (int i = li; i <= ri; i++) {
        for (int j = rj; j >= lj; j--)
            dp[0][j - lj] = max(dp[0][j - lj],
            (lcs_s[i] == lcs_t[j]) + (j > lj ? dp[0][j-1 -
                lj] : 0));
        for (int j = lj+1; j <= rj; j++)
            dp[0][j - lj] = max(dp[0][j - lj], dp[0][j-1
                -lj]);
    }
}
```

```
// dp[1][j] = max lcs(s[li...ri], t[lj+j, rj])
void dp_bottom(int li, int ri, int lj, int rj) {
    memset(dp[1], 0, (rj-lj+1)*sizeof(dp[1][0]));
    for (int i = ri; i >= li; i--) {
        for (int j = lj; j <= rj; j++)
            dp[1][j - lj] = max(dp[1][j - lj],
            (lcs_s[i] == lcs_t[j]) +  (j < rj ? dp[1][j+1 -
               lj] : 0));
        for (int j = rj-1; j >= lj; j--)
            dp[1][j - lj] = max(dp[1][j - lj], dp[1][j+1 -
               lj]);
    }
}

void solve(vector<int>& ans, int li, int ri, int lj, int rj)
   {
    if (li == ri){
        for (int j = lj; j <= rj; j++)
            if (lcs_s[li] == lcs_t[j]){
                ans.push_back(lcs_t[j]);
                break;
            }
        return;
    }
    if (lj == rj){
        for (int i = li; i <= ri; i++){
            if (lcs_s[i] == lcs_t[lj]){
                ans.push_back(lcs_s[i]);
                break;
            }
        }
        return;
    }
    int mi = (li+ri)/2;
    dp_top(li, mi, lj, rj), dp_bottom(mi+1, ri, lj, rj);

    int j_ = 0, mx = -1;

    for (int j = lj-1; j <= rj; j++) {
        int val = 0;
        if (j >= lj) val += dp[0][j - lj];
        if (j < rj) val += dp[1][j+1 - lj];
        if (val >= mx) mx = val, j_ = j;
    }
    if (mx == -1) return;
    solve(ans, li, mi, lj, j_), solve(ans, mi+1, ri, j_+1,
       rj);
}

vector<int> lcs(const vector<int>& s, const vector<int>& t) {
    for (int i = 0; i < s.size(); i++) lcs_s[i] = s[i];
    for (int i = 0; i < t.size(); i++) lcs_t[i] = t[i];
    vector<int> ans;
    solve(ans, 0, s.size()-1, 0, t.size()-1);
    return ans;
}
```

6.5 Mochila

```
// Resolve mochila, recuperando a resposta
//
// O(n * cap), O(n + cap) de memoria
// 400885

int v[MAX], w[MAX]; // valor e peso
int dp[2][MAX_CAP];

// DP usando os itens [l, r], com capacidade = cap
void get_dp(int x, int l, int r, int cap) {
    memset(dp[x], 0, (cap+1)*sizeof(dp[x][0]));
    for (int i = l; i <= r; i++) for (int j = cap; j >= 0;
       j--)
        if (j - w[i] >= 0) dp[x][j] = max(dp[x][j], v[i] +
           dp[x][j - w[i]]);
}

void solve(vector<int>& ans, int l, int r, int cap) {
    if (l == r) {
        if (w[l] <= cap) ans.push_back(l);
        return;
    }
```

```
    int m = (l+r)/2;
    get_dp(0, l, m, cap), get_dp(1, m+1, r, cap);
    int left_cap = -1, opt = -INF;
    for (int j = 0; j <= cap; j++)
        if (int at = dp[0][j] + dp[1][cap - j]; at > opt)
            opt = at, left_cap = j;
    solve(ans, l, m, left_cap), solve(ans, m+1, r, cap -
        left_cap);
}

vector<int> knapsack(int n, int cap) {
    vector<int> ans;
    solve(ans, 0, n-1, cap);
    return ans;
}
```

6.6 SOS DP

```
// O(n 2^n)

// soma de sub-conjunto
vector<ll> sos_dp(vector<ll> f) {
    int N = __builtin_ctz(f.size());
    assert((1<<N) == f.size());

    for (int i = 0; i < N; i++) for (int mask = 0; mask <
        (1<<N); mask++)
        if (mask>>i&1) f[mask] += f[mask^(1<<i)];
    return f;
}

// soma de super-conjunto
vector<ll> sos_dp(vector<ll> f) {
    int N = __builtin_ctz(f.size());
    assert((1<<N) == f.size());

    for (int i = 0; i < N; i++) for (int mask = 0; mask <
        (1<<N); mask++)
        if (~mask>>i&1) f[mask] += f[mask^(1<<i)];
    return f;
}
```

7 Strings

7.1 Aho-corasick

```
// query retorna o somatorio do numero de matches de
// todas as stringuinhas na stringona
//
// insert - O(|s| log(SIGMA))
// build - O(N), onde N = somatorio dos tamanhos das strings
// query - O(|s|)
// a30d6e

namespace aho {
    map<char, int> to[MAX];
    int link[MAX], idx, term[MAX], exit[MAX], sobe[MAX];

    void insert(string& s) {
        int at = 0;
        for (char c : s) {
            auto it = to[at].find(c);
            if (it == to[at].end()) at = to[at][c] = ++idx;
            else at = it->second;
        }
        term[at]++, sobe[at]++;
    }
#warning nao esquece de chamar build() depois de inserir
    void build() {
        queue<int> q;
        q.push(0);
        link[0] = exit[0] = -1;
        while (q.size()) {
            int i = q.front(); q.pop();
            for (auto [c, j] : to[i]) {
                int l = link[i];
                while (l != -1 and !to[l].count(c)) l =
                    link[l];
                link[j] = l == -1 ? 0 : to[l][c];
```

```
            exit[j] = term[link[j]] ? link[j] :
                exit[link[j]];
            if (exit[j]+1) sobe[j] += sobe[exit[j]];
            q.push(j);
        }
    }
}
int query(string& s) {
    int at = 0, ans = 0;
    for (char c : s){
        while (at != -1 and !to[at].count(c)) at =
            link[at];
        at = at == -1 ? 0 : to[at][c];
        ans += sobe[at];
    }
    return ans;
}
}
```

7.2 Algoritmo Z

```
// z[i] = lcp(s, s[i..n))
//
// Complexidades:
// z - O(|s|)
// match - O(|s| + |p|)
// 74a9e1

vector<int> get_z(string s) {
    int n = s.size();
    vector<int> z(n, 0);

    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n and s[z[i]] == s[i + z[i]])
            z[i]++;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```

7.3 Automato de Sufixo

```
// Automato que aceita os sufixos de uma string
// Todas as funcoes sao lineares
// c37a72

namespace sam {
    int cur, sz, len[2*MAX], link[2*MAX], acc[2*MAX];
    int nxt[2*MAX][26];

    void add(int c) {
        int at = cur;
        len[sz] = len[cur]+1, cur = sz++;
        while (at != -1 and !nxt[at][c]) nxt[at][c] = cur,
            at = link[at];
        if (at == -1) { link[cur] = 0; return; }
        int q = nxt[at][c];
        if (len[q] == len[at]+1) { link[cur] = q; return; }
        int qq = sz++;
        len[qq] = len[at]+1, link[qq] = link[q];
        for (int i = 0; i < 26; i++) nxt[qq][i] = nxt[q][i];
        while (at != -1 and nxt[at][c] == q) nxt[at][c] =
            qq, at = link[at];
        link[cur] = link[q] = qq;
    }
    void build(string& s) {
        cur = 0, sz = 0, len[0] = 0, link[0] = -1, sz++;
        for (auto i : s) add(i-'a');
        int at = cur;
        while (at) acc[at] = 1, at = link[at];
    }

    // coisas que da pra fazer:
    ll distinct_substrings() {
        ll ans = 0;
        for (int i = 1; i < sz; i++) ans += len[i] -
            len[link[i]];
```

```
        return ans;
    }
    string longest_common_substring(string& S, string& T) {
        build(S);
        int at = 0, l = 0, ans = 0, pos = -1;
        for (int i = 0; i < T.size(); i++) {
            while (at and !nxt[at][T[i]-'a']) at = link[at],
               l = len[at];
            if (nxt[at][T[i]-'a']) at = nxt[at][T[i]-'a'],
               l++;
            else at = 0, l = 0;
            if (l > ans) ans = l, pos = i;
        }
        return T.substr(pos-ans+1, ans);
    }
    ll dp[2*MAX];
    ll paths(int i) {
        auto& x = dp[i];
        if (x) return x;
        x = 1;
        for (int j = 0; j < 26; j++) if (nxt[i][j]) x +=
           paths(nxt[i][j]);
        return x;
    }
    void kth_substring(int k, int at=0) { // k=1 : menor
       substring lexicog.
        for (int i = 0; i < 26; i++) if (k and nxt[at][i]) {
            if (paths(nxt[at][i]) >= k) {
                cout << char('a'+i);
                kth_substring(k-1, nxt[at][i]);
                return;
            }
            k -= paths(nxt[at][i]);
        }
    }
};
```

7.4 eertree

```
// Constroi a eertree, caractere a caractere
// Inicializar com a quantidade de caracteres maxima
// size() retorna a quantidade de substrings pal. distintas
// depois de chamar propagate(), cada substring palindromica
// ocorre qt[i] vezes. O propagate() retorna o numero de
// substrings pal. com repeticao
//
// O(n) amortizado, considerando alfabeto O(1)
// a2e693

struct eertree {
    vector<vector<int>> t;
    int n, last, sz;
    vector<int> s, len, link, qt;

    eertree(int N) {
        t = vector(N+2, vector(26, int()));
        s = len = link = qt = vector<int>(N+2);
        s[0] = -1;
        link[0] = 1, len[0] = 0, link[1] = 1, len[1] = -1;
        sz = 2, last = 0, n = 1;
    }

    void add(char c) {
        s[n++] = c -= 'a';
        while (s[n-len[last]-2] != c) last = link[last];
        if (!t[last][c]) {
            int prev = link[last];
            while (s[n-len[prev]-2] != c) prev = link[prev];
            link[sz] = t[prev][c];
            len[sz] = len[last]+2;
            t[last][c] = sz++;
        }
        qt[last = t[last][c]]++;
    }
    int size() { return sz-2; }
    ll propagate() {
        ll ret = 0;
        for (int i = n; i > 1; i--) {
            qt[link[i]] += qt[i];
            ret += qt[i];
        }
        return ret;
```

```
    }
};
```

7.5 KMP

```
// mathcing(s, t) retorna os indices das ocorrencias
// de s em t
// autKMP constroi o automato do KMP
//
// Complexidades:
// pi - O(n)
// match - O(n + m)
// construir o automato - O(|sigma|*n)
// n = |padrao| e m = |texto|

// f50359
template<typename T> vector<int> pi(T s) {
    vector<int> p(s.size());
    for (int i = 1, j = 0; i < s.size(); i++) {
        while (j and s[j] != s[i]) j = p[j-1];
        if (s[j] == s[i]) j++;
        p[i] = j;
    }
    return p;
}

// c82524
template<typename T> vector<int> matching(T& s, T& t) {
    vector<int> p = pi(s), match;
    for (int i = 0, j = 0; i < t.size(); i++) {
        while (j and s[j] != t[i]) j = p[j-1];
        if (s[j] == t[i]) j++;
        if (j == s.size()) match.push_back(i-j+1), j =
            p[j-1];
    }
    return match;
}

// 79bd9e
struct KMPaut : vector<vector<int>> {
    KMPaut(){}
    KMPaut (string& s) : vector<vector<int>>(26,
        vector<int>(s.size()+1)) {
        vector<int> p = pi(s);
        auto& aut = *this;
        aut[s[0]-'a'][0] = 1;
        for (char c = 0; c < 26; c++)
            for (int i = 1; i <= s.size(); i++)
                aut[c][i] = s[i]-'a' == c ? i+1 :
                    aut[c][p[i-1]];
    }
};
```

7.6 Manacher

```
// manacher recebe um vetor de T e retorna o vetor com
   tamanho dos palindromos
// ret[2*i] = tamanho do maior palindromo centrado em i
// ret[2*i+1] = tamanho maior palindromo centrado em i e i+1
//
// Complexidades:
// manacher - O(n)
// palindrome - <O(n), O(1)>
// pal_end - O(n)

// ebb184
template<typename T> vector<int> manacher(const T& s) {
    int l = 0, r = -1, n = s.size();
    vector<int> d1(n), d2(n);
    for (int i = 0; i < n; i++) {
        int k = i > r ? 1 : min(d1[l+r-i], r-i);
        while (i+k < n && i-k >= 0 && s[i+k] == s[i-k]) k++;
        d1[i] = k--;
        if (i+k > r) l = i-k, r = i+k;
    }
    l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        int k = i > r ? 0 : min(d2[l+r-i+1], r-i+1); k++;
        while (i+k <= n && i-k >= 0 && s[i+k-1] == s[i-k])
            k++;
```

```
        d2[i] = --k;
        if (i+k-1 > r) l = i-k, r = i+k-1;
    }
    vector<int> ret(2*n-1);
    for (int i = 0; i < n; i++) ret[2*i] = 2*d1[i]-1;
    for (int i = 0; i < n-1; i++) ret[2*i+1] = 2*d2[i+1];
    return ret;
}

// 60c6f5
// verifica se a string s[i..j] eh palindromo
template<typename T> struct palindrome {
    vector<int> man;

    palindrome(const T& s) : man(manacher(s)) {}
    bool query(int i, int j) {
        return man[i+j] >= j-i+1;
    }
};

// 8bd4d5
// tamanho do maior palindromo que termina em cada posicao
template<typename T> vector<int> pal_end(const T& s) {
    vector<int> ret(s.size());
    palindrome<T> p(s);
    ret[0] = 1;
    for (int i = 1; i < s.size(); i++) {
        ret[i] = min(ret[i-1]+2, i+1);
        while (!p.query(i-ret[i]+1, i)) ret[i]--;
    }
    return ret;
}
```

7.7 Min/max suffix/cyclic shift

```
// Computa o indice do menor/maior sufixo/cyclic shift
// da string, lexicograficamente
//
// O(n)
// af0367

template<typename T> int max_suffix(T s, bool mi = false) {
    s.push_back(*min_element(s.begin(), s.end())-1);
    int ans = 0;
    for (int i = 1; i < s.size(); i++) {
        int j = 0;
        while (ans+j < i and s[i+j] == s[ans+j]) j++;
        if (s[i+j] > s[ans+j]) {
            if (!mi or i != s.size()-2) ans = i;
        } else if (j) i += j-1;
    }
    return ans;
}

template<typename T> int min_suffix(T s) {
    for (auto& i : s) i *= -1;
    s.push_back(*max_element(s.begin(), s.end())+1);
    return max_suffix(s, true);
}

template<typename T> int max_cyclic_shift(T s) {
    int n = s.size();
    for (int i = 0; i < n; i++) s.push_back(s[i]);
    return max_suffix(s);
}

template<typename T> int min_cyclic_shift(T s) {
    for (auto& i : s) i *= -1;
    return max_cyclic_shift(s);
}
```

7.8 String Hashing

```
// Complexidades:
// construtor - O(|s|)
// operator() - O(1)

mt19937 rng((int)
    chrono::steady_clock::now().time_since_epoch().count());
```

```
int uniform(int l, int r) {
    uniform_int_distribution<int> uid(l, r);
    return uid(rng);
}

template<int MOD> struct str_hash { // 116fcb
    static int P;
    vector<ll> h, p;
    str_hash(string s) : h(s.size()), p(s.size()) {
        p[0] = 1, h[0] = s[0];
        for (int i = 1; i < s.size(); i++)
            p[i] = p[i - 1]*P%MOD, h[i] = (h[i - 1]*P +
               s[i])%MOD;
    }
    ll operator()(int l, int r) { // retorna hash s[l...r]
        ll hash = h[r] - (l ? h[l - 1]*p[r - l + 1]%MOD : 0);
        return hash < 0 ? hash + MOD : hash;
    }
};
template<int MOD> int str_hash<MOD>::P = uniform(256, MOD -
   1); // l > |sigma|
```

7.9 String Hashing - modulo 2^61 - 1

```
// Quase duas vezes mais lento
//
// Complexidades:
// build - O(|s|)
// operator() - O(1)
//
// d3c0f0

const ll MOD = (1ll<<61) - 1;
ll mulmod(ll a, ll b) {
    const static ll LOWER = (1ll<<30) - 1, GET31 = (1ll<<31)
       - 1;
    ll l1 = a&LOWER, h1 = a>>30, l2 = b&LOWER, h2 = b>>30;
    ll m = l1*h2 + l2*h1, h = h1*h2;
    ll ans = l1*l2 + (h>>1) + ((h&1)<<60) + (m>>31) +
       ((m&GET31)<<30) + 1;
    ans = (ans&MOD) + (ans>>61), ans = (ans&MOD) + (ans>>61);
    return ans - 1;
}

mt19937_64
   rng(chrono::steady_clock::now().time_since_epoch().count());

ll uniform(ll l, ll r) {
    uniform_int_distribution<ll> uid(l, r);
    return uid(rng);
}

struct str_hash {
    static ll P;
    vector<ll> h, p;
    str_hash(string s) : h(s.size()), p(s.size()) {
        p[0] = 1, h[0] = s[0];
        for (int i = 1; i < s.size(); i++)
            p[i] = mulmod(p[i - 1], P), h[i] = (mulmod(h[i -
               1], P) + s[i])%MOD;
    }
    ll operator()(int l, int r) { // retorna hash s[l...r]
        ll hash = h[r] - (l ? mulmod(h[l - 1], p[r - l + 1])
           : 0);
        return hash < 0 ? hash + MOD : hash;
    }
};
ll str_hash::P = uniform(256, MOD - 1); // l > |sigma|
```

7.10 Suffix Array - O(n log n)

```
// kasai recebe o suffix array e calcula lcp[i],
// o lcp entre s[sa[i],...,n-1] e s[sa[i+1],..,n-1]
//
// Complexidades:
// suffix_array - O(n log(n))
// kasai - O(n)
// d3a6ce

vector<int> suffix_array(string s) {
```

```
    s += "$";
    int n = s.size(), N = max(n, 260);
    vector<int> sa(n), ra(n);
    for(int i = 0; i < n; i++) sa[i] = i, ra[i] = s[i];

    for(int k = 0; k < n; k ? k *= 2 : k++) {
        vector<int> nsa(sa), nra(n), cnt(N);

        for(int i = 0; i < n; i++) nsa[i] = (nsa[i]-k+n)%n,
           cnt[ra[i]]++;
        for(int i = 1; i < N; i++) cnt[i] += cnt[i-1];
        for(int i = n-1; i+1; i--) sa[--cnt[ra[nsa[i]]]] =
           nsa[i];

        for(int i = 1, r = 0; i < n; i++) nra[sa[i]] = r +=
           ra[sa[i]] !=
            ra[sa[i-1]] or ra[(sa[i]+k)%n] !=
               ra[(sa[i-1]+k)%n];
        ra = nra;
        if (ra[sa[n-1]] == n-1) break;
    }
    return vector<int>(sa.begin()+1, sa.end());
}

vector<int> kasai(string s, vector<int> sa) {
    int n = s.size(), k = 0;
    vector<int> ra(n), lcp(n);
    for (int i = 0; i < n; i++) ra[sa[i]] = i;

    for (int i = 0; i < n; i++, k -= !!k) {
        if (ra[i] == n-1) { k = 0; continue; }
        int j = sa[ra[i]+1];
        while (i+k < n and j+k < n and s[i+k] == s[j+k]) k++;
        lcp[ra[i]] = k;
    }
    return lcp;
}
```

7.11 Suffix Array - O(n)

```
// Rapidao
// Computa o suffix array em 'sa', o rank em 'rnk'
// e o lcp em 'lcp'
// query(i, j) retorna o LCP entre s[i..n-1] e s[j..n-1]
//
// Complexidades
// O(n) para construir
// query - O(1)

// bab412
template<typename T> struct rmq {
    vector<T> v;
    int n; static const int b = 30;
    vector<int> mask, t;

    int op(int x, int y) { return v[x] <= v[y] ? x : y; }
    int msb(int x) { return
       __builtin_clz(1)-__builtin_clz(x); }
    int small(int r, int sz = b) { return
       r-msb(mask[r]&((1<<sz)-1)); }
    rmq() {}
    rmq(const vector<T>& v_) : v(v_), n(v.size()), mask(n),
       t(n) {
        for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
            at = (at<<1)&((1<<b)-1);
            while (at and op(i-msb(at&-at), i) == i) at ^=
               at&-at;
        }
        for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
        for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0;
           i+(1<<j) <= n/b; i++)
            t[n/b*j+i] = op(t[n/b*(j-1)+i],
               t[n/b*(j-1)+i+(1<<(j-1))]);
    }
    int index_query(int l, int r) {
        if (r-l+1 <= b) return small(r, r-l+1);
        int x = l/b+1, y = r/b-1;
        if (x > y) return op(small(l+b-1), small(r));
        int j = msb(y-x+1);
        int ans = op(small(l+b-1), op(t[n/b*j+x],
           t[n/b*j+y-(1<<j)+1]));
        return op(ans, small(r));
```

```
    }
    T query(int l, int r) { return v[index_query(l, r)]; }
};

struct suffix_array {
    string s;
    int n;
    vector<int> sa, cnt, rnk, lcp;
    rmq<int> RMQ;

    bool cmp(int a1, int b1, int a2, int b2, int a3=0, int
       b3=0) {
        return a1 != b1 ? a1 < b1 : (a2 != b2 ? a2 < b2 : a3
            < b3);
    }
    template<typename T> void radix(int* fr, int* to, T* r,
       int N, int k) {
        cnt = vector<int>(k+1, 0);
        for (int i = 0; i < N; i++) cnt[r[fr[i]]]++;
        for (int i = 1; i <= k; i++) cnt[i] += cnt[i-1];
        for (int i = N-1; i+1; i--) to[--cnt[r[fr[i]]]] =
            fr[i];
    }
    void rec(vector<int>& v, int k) {
        auto &tmp = rnk, &m0 = lcp;
        int N = v.size()-3, sz = (N+2)/3, sz2 = sz+N/3;
        vector<int> R(sz2+3);
        for (int i = 1, j = 0; j < sz2; i += i%3) R[j++] = i;

        radix(&R[0], &tmp[0], &v[0]+2, sz2, k);
        radix(&tmp[0], &R[0], &v[0]+1, sz2, k);
        radix(&R[0], &tmp[0], &v[0]+0, sz2, k);

        int dif = 0;
        int l0 = -1, l1 = -1, l2 = -1;
        for (int i = 0; i < sz2; i++) {
            if (v[tmp[i]] != l0 or v[tmp[i]+1] != l1 or
               v[tmp[i]+2] != l2)
                l0 = v[tmp[i]], l1 = v[tmp[i]+1], l2 =
                    v[tmp[i]+2], dif++;
            if (tmp[i]%3 == 1) R[tmp[i]/3] = dif;
            else R[tmp[i]/3+sz] = dif;
```

```
        }

        if (dif < sz2) {
            rec(R, dif);
            for (int i = 0; i < sz2; i++) R[sa[i]] = i+1;
        } else for (int i = 0; i < sz2; i++) sa[R[i]-1] = i;

        for (int i = 0, j = 0; j < sz2; i++) if (sa[i] < sz)
           tmp[j++] = 3*sa[i];
        radix(&tmp[0], &m0[0], &v[0], sz, k);
        for (int i = 0; i < sz2; i++)
            sa[i] = sa[i] < sz ? 3*sa[i]+1 : 3*(sa[i]-sz)+2;

        int at = sz2+sz-1, p = sz-1, p2 = sz2-1;
        while (p >= 0 and p2 >= 0) {
            if ((sa[p2]%3==1 and cmp(v[m0[p]], v[sa[p2]],
               R[m0[p]/3],
                R[sa[p2]/3+sz])) or (sa[p2]%3==2 and
                    cmp(v[m0[p]], v[sa[p2]],
                v[m0[p]+1], v[sa[p2]+1], R[m0[p]/3+sz],
                    R[sa[p2]/3+1])))
                sa[at--] = sa[p2--];
            else sa[at--] = m0[p--];
        }
        while (p >= 0) sa[at--] = m0[p--];
        if (N%3==1) for (int i = 0; i < N; i++) sa[i] =
           sa[i+1];
    }

    suffix_array(const string& s_) : s(s_), n(s.size()),
       sa(n+3),
            cnt(n+1), rnk(n), lcp(n-1) {
        vector<int> v(n+3);
        for (int i = 0; i < n; i++) v[i] = i;
        radix(&v[0], &rnk[0], &s[0], n, 256);
        int dif = 1;
        for (int i = 0; i < n; i++)
            v[rnk[i]] = dif += (i and s[rnk[i]] !=
               s[rnk[i-1]]);
        if (n >= 2) rec(v, dif);
        sa.resize(n);
```

```
    for (int i = 0; i < n; i++) rnk[sa[i]] = i;
    for (int i = 0, k = 0; i < n; i++, k -= !!k) {
        if (rnk[i] == n-1) {
            k = 0;
            continue;
        }
        int j = sa[rnk[i]+1];
        while (i+k < n and j+k < n and s[i+k] == s[j+k])
            k++;
        lcp[rnk[i]] = k;
    }
    RMQ = rmq<int>(lcp);
}
// hash ateh aqui (sem o RMQ): 1ff700

int query(int i, int j) {
    if (i == j) return n-i;
    i = rnk[i], j = rnk[j];
    return RMQ.query(min(i, j), max(i, j)-1);
}
pair<int, int> next(int L, int R, int i, char c) {
    int l = L, r = R+1;
    while (l < r) {
        int m = (l+r)/2;
        if (i+sa[m] >= n or s[i+sa[m]] < c) l = m+1;
        else r = m;
    }
    if (l == R+1 or s[i+sa[l]] > c) return {-1, -1};
    L = l;

    l = L, r = R+1;
    while (l < r) {
        int m = (l+r)/2;
        if (i+sa[m] >= n or s[i+sa[m]] <= c) l = m+1;
        else r = m;
    }
    R = l-1;
    return {L, R};
}
// quantas vezes 't' ocorre em 's' - O(|t| log n)
int count_substr(string& t) {
    int L = 0, R = n-1;
    for (int i = 0; i < t.size(); i++) {
        tie(L, R) = next(L, R, i, t[i]);
        if (L == -1) return 0;
    }
    return R-L+1;
}

// exemplo de f que resolve o problema
//
   https://codeforces.com/edu/course/2/lesson/2/5/practice/con
ll f(ll k) { return k*(k+1)/2; }

ll dfs(int L, int R, int p) { // dfs na suffix tree
   chamado em pre ordem
    int ext = L != R ? RMQ.query(L, R-1) : n - sa[L];

    // Tem 'ext - p' substrings diferentes que ocorrem
       'R-L+1' vezes
    // O LCP de todas elas eh 'ext'
    ll ans = (ext-p)*f(R-L+1);

    // L eh terminal, e folha sse L == R
    if (sa[L]+ext == n) L++;

    /* se for um SA de varias strings separadas como
       s#t$u&, usar no lugar do if de cima
        (separadores < 'a', diferentes e inclusive no
           final)
    while (L <= R && (sa[L]+ext == n || s[sa[L]+ext] <
       'a')) {
        L++;
    } */

    while (L <= R) {
        int idx = L != R ? RMQ.index_query(L, R-1) : -1;
        if (idx == -1 or lcp[idx] != ext) idx = R;

        ans += dfs(L, idx, ext);
        L = idx+1;
    }
    return ans;
}
```

```
    // sum over substrings: computa, para toda substring t
        distinta de s,
    // \sum f(# ocorrencias de t em s) - O (n)
    ll sos() { return dfs(0, n-1, 0); }
};
```

7.12 Suffix Array Dinamico

```
// Mantem o suffix array, lcp e rank de uma string,
// premitindo push_front e pop_front
// O operador [i] return um par com sa[i] e lcp[i]
// lcp[i] tem o lcp entre sa[i] e sa[i-1] (lcp[0] = 0)
//
// Complexidades:
// Construir sobre uma string de tamanho n: O(n log n)
// push_front e pop_front: O(log n) amortizado
// 4c2a2e

struct dyn_sa {
    struct node {
        int sa, lcp;
        node *l, *r, *p;
        int sz, mi;
        node(int sa_, int lcp_, node* p_) : sa(sa_),
            lcp(lcp_),
            l(NULL), r(NULL), p(p_), sz(1), mi(lcp) {}
        void update() {
            sz = 1, mi = lcp;
            if (l) sz += l->sz, mi = min(mi, l->mi);
            if (r) sz += r->sz, mi = min(mi, r->mi);
        }
    };

    node* root;
    vector<ll> tag; // tag of a suffix (reversed id)
    string s; // reversed

    dyn_sa() : root(NULL) {}
    dyn_sa(string s_) : dyn_sa() {
        reverse(s_.begin(), s_.end());
        for (char c : s_) push_front(c);
    }
    ~dyn_sa() {
        vector<node*> q = {root};
        while (q.size()) {
            node* x = q.back(); q.pop_back();
            if (!x) continue;
            q.push_back(x->l), q.push_back(x->r);
            delete x;
        }
    }

    int size(node* x) { return x ? x->sz : 0; }
    int mirror(int i) { return s.size()-1 - i; }
    bool cmp(int i, int j) {
        if (s[i] != s[j]) return s[i] < s[j];
        if (i == 0 or j == 0) return i < j;
        return tag[i-1] < tag[j-1];
    }
    void fix_path(node* x) { while (x) x->update(), x =
        x->p; }
    void flatten(vector<node*>& v, node* x) {
        if (!x) return;
        flatten(v, x->l);
        v.push_back(x);
        flatten(v, x->r);
    }
    void build(vector<node*>& v, node*& x, node* p, int L,
        int R, ll l, ll r) {
        if (L > R) return void(x = NULL);
        int M = (L+R)/2;
        ll m = (l+r)/2;
        x = v[M];
        x->p = p;
        tag[x->sa] = m;
        build(v, x->l, x, L, M-1, l, m-1), build(v, x->r, x,
            M+1, R, m+1, r);
        x->update();
    }
    void fix(node*& x, node* p, ll l, ll r) {
        if (3*max(size(x->l), size(x->r)) <= 2*size(x))
```

```cpp
        return x->update();
    vector<node*> v;
    flatten(v, x);
    build(v, x, p, 0, v.size()-1, l, r);
}
node* next(node* x) {
    if (x->r) {
        x = x->r;
        while (x->l) x = x->l;
        return x;
    }
    while (x->p and x->p->r == x) x = x->p;
    return x->p;
}
node* prev(node* x) {
    if (x->l) {
        x = x->l;
        while (x->r) x = x->r;
        return x;
    }
    while (x->p and x->p->l == x) x = x->p;
    return x->p;
}

int get_lcp(node* x, node* y) {
    if (!x or !y) return 0; // change defaut value here
    if (s[x->sa] != s[y->sa]) return 0;
    if (x->sa == 0 or y->sa == 0) return 1;
    return 1 + query(mirror(x->sa-1), mirror(y->sa-1));
}
void add_suf(node*& x, node* p, int id, ll l, ll r) {
    if (!x) {
        x = new node(id, 0, p);
        node *prv = prev(x), *nxt = next(x);
        int lcp_cur = get_lcp(prv, x), lcp_nxt =
            get_lcp(x, nxt);
        if (nxt) nxt->lcp = lcp_nxt, fix_path(nxt);
        x->lcp = lcp_cur;
        tag[id] = (l+r)/2;
        x->update();
        return;
    }
    if (cmp(id, x->sa)) add_suf(x->l, x, id, l,
        tag[x->sa]-1);
    else add_suf(x->r, x, id, tag[x->sa]+1, r);
    fix(x, p, l, r);
}
void push_front(char c) {
    s += c;
    tag.push_back(-1);
    add_suf(root, NULL, s.size() - 1, 0, 1e18);
}

void rem_suf(node*& x, int id) {
    if (x->sa != id) {
        if (tag[id] < tag[x->sa]) return rem_suf(x->l,
            id);
        return rem_suf(x->r, id);
    }
    node* nxt = next(x);
    if (nxt) nxt->lcp = min(nxt->lcp, x->lcp),
        fix_path(nxt);

    node *p = x->p, *tmp = x;
    if (!x->l or !x->r) {
        x = x->l ? x->l : x->r;
        if (x) x->p = p;
    } else {
        for (tmp = x->l, p = x; tmp->r; tmp = tmp->r) p
            = tmp;
        x->sa = tmp->sa, x->lcp = tmp->lcp;
        if (tmp->l) tmp->l->p = p;
        if (p->l == tmp) p->l = tmp->l;
        else p->r = tmp->l;
    }
    fix_path(p);
    delete tmp;
}
void pop_front() {
    if (!s.size()) return;
    s.pop_back();
    rem_suf(root, s.size());
    tag.pop_back();
}
```

```
int query(node* x, ll l, ll r, ll a, ll b) {
	if (!x or tag[x->sa] == -1 or r < a or b < l) return
		s.size();
	if (a <= l and r <= b) return x->mi;
	int ans = s.size();
	if (a <= tag[x->sa] and tag[x->sa] <= b) ans =
		min(ans, x->lcp);
	ans = min(ans, query(x->l, l, tag[x->sa]-1, a, b));
	ans = min(ans, query(x->r, tag[x->sa]+1, r, a, b));
	return ans;
}
int query(int i, int j) { // lcp(s[i..], s[j..])
	if (i == j) return s.size() - i;
	ll a = tag[mirror(i)], b = tag[mirror(j)];
	int ret = query(root, 0, 1e18, min(a, b)+1, max(a,
		b));
	return ret;
}
// optional: get rank[i], sa[i] and lcp[i]
int rank(int i) {
	i = mirror(i);
	node* x = root;
	int ret = 0;
	while (x) {
		if (tag[x->sa] < tag[i]) {
			ret += size(x->l)+1;
			x = x->r;
		} else x = x->l;
	}
	return ret;
}
pair<int, int> operator[](int i) {
	node* x = root;
	while (1) {
		if (i < size(x->l)) x = x->l;
		else {
			i -= size(x->l);
			if (!i) return {mirror(x->sa), x->lcp};
			i--, x = x->r;
		}
	}
}
};
```

7.13 Trie

```
// trie T() constroi uma trie para o alfabeto das letras
	minusculas
// trie T(tamanho do alfabeto, menor caracter) tambem pode
	ser usado
//
// T.insert(s) - O(|s|*sigma)
// T.erase(s) - O(|s|)
// T.find(s) retorna a posicao, 0 se nao achar - O(|s|)
// T.count_pref(s) numero de strings que possuem s como
	prefixo - O(|s|)
//
// Nao funciona para string vazia
// 979609

struct trie {
	vector<vector<int>> to;
	vector<int> end, pref;
	int sigma; char norm;
	trie(int sigma_=26, char norm_='a') : sigma(sigma_),
		norm(norm_) {
		to = {vector<int>(sigma)};
		end = {0}, pref = {0};
	}
	void insert(string s) {
		int x = 0;
		for(auto c : s) {
			int &nxt = to[x][c-norm];
			if(!nxt) {
				nxt = to.size();
				to.push_back(vector<int>(sigma));
				end.push_back(0), pref.push_back(0);
			}
			x = nxt, pref[x]++;
		}
		end[x]++;
```

```
    }
    void erase(string s) {
        int x = 0;
        for(char c : s) {
            int &nxt = to[x][c-norm];
            x = nxt, pref[x]--;
            if(!pref[x]) nxt = 0;
        }
        end[x]--;
    }
    int find(string s) {
        int x = 0;
        for(auto c : s) {
            x = to[x][c-norm];
            if(!x) return 0;
        }
        return x;
    }
    int count_pref(string s) {
        return pref[find(s)];
    }
};
```

8 Extra

8.1 debug.cpp

```
void debug_out(string s, int line) { cerr << endl; }
template<typename H, typename... T>
void debug_out(string s, int line, H h, T... t) {
    if (s[0] != ',') cerr << "Line(" << line << ") ";
    do { cerr << s[0]; s = s.substr(1);
    } while (s.size() and s[0] != ',');
    cerr << " = " << h;
    debug_out(s, line, t...);
}
#ifdef DEBUG
#define debug(...) debug_out(#__VA_ARGS__, __LINE__,
   __VA_ARGS__)
#else
#define debug(...)
#endif
```

8.2 template.cpp

```
#include <bits/stdc++.h>

using namespace std;

#define _ ios_base::sync_with_stdio(0);cin.tie(0);
#define endl '\n'

typedef long long ll;

const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3fll;

int main() { _
    exit(0);
}
```

8.3 vimrc

```
set ts=4 si ai sw=4 nu mouse=a undofile
syntax on
```

8.4 hash.sh

```
# Para usar (hash das linhas [l1, l2]):
# ./hash.sh arquivo.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P
   -fpreprocessed | tr -d '[:space:]' | md5sum | cut -c-6
```

8.5 makefile

```
CXX = g++
CXXFLAGS = -fsanitize=address,undefined
   -fno-omit-frame-pointer -g -Wall -Wshadow -std=c++17
   -Wno-unused-result -Wno-sign-compare -Wno-char-subscripts
   #-fuse-ld=gold
```

8.6 stress.sh

```
P=a
make ${P} ${P}2 gen || exit 1
for ((i = 1; ; i++)) do
    ./gen $i > in
    ./${P} < in > out
    ./${P}2 < in > out2
    if (! cmp -s out out2) then
        echo "--> entrada:"
        cat in
        echo "--> saida1:"
        cat out
        echo "--> saida2:"
        cat out2
        break;
    fi
    echo $i
done
```

8.7 rand.cpp

```
mt19937 rng((int)
   chrono::steady_clock::now().time_since_epoch().count());

int uniform(int l, int r){
    uniform_int_distribution<int> uid(l, r);
    return uid(rng);
}
```

8.8 fastIO.cpp

```
int read_int() {
    bool minus = false;
    int result = 0;
    char ch;
    ch = getchar();
    while (1) {
        if (ch == '-') break;
        if (ch >= '0' && ch <= '9') break;
        ch = getchar();
    }
    if (ch == '-') minus = true;
    else result = ch-'0';
    while (1) {
        ch = getchar();
        if (ch < '0' || ch > '9') break;
        result = result*10 + (ch - '0');
    }
    if (minus) return -result;
    else return result;
}
```

8.9 timer.cpp

```
// timer T; T() -> retorna o tempo em ms desde que declarou
using namespace chrono;
struct timer : high_resolution_clock {
    const time_point start;
    timer(): start(now()) {}
    int operator()() {
        return duration_cast<milliseconds>(now() -
            start).count();
    }
};
```