An Algorithm for Estimating Field-Aligned Currents from Single Spacecraft Magnetic Field Measurements: A Diagnostic Tool Applied to Freja Satellite Data

Hermann Lühr, Jörg F. Warnecke, and Martin K. A. Rother

***Abstract*— We introduce an algorithm which allows under certain assumptions to estimate the current density distribution along the track from single spacecraft magnetic field measurements. The assumptions are chosen that they meet at least partly the conditions encountered by the Freja satellite on its northernmost orbital segment, namely that all currents are field-aligned, ionospheric currents do not contribute significantly to the magnetic field measurements, and the velocity component perpendicular to the field direction is large (> 5 km/s). Problems arise in the case of moving or temporally varying current systems. In those cases additional data like ground-based observations are needed to resolve the spatio-temporal ambiguity. With the help of simulated data we can show that for most of the encountered stationary current geometries the estimates fall into an uncertainty band of ±20%. Current density estimates are a local quantity with a spatial resolution of order 1 km. They are thus very suitable for use in studying the fine structure of auroral plasma processes.**

I. Introduction

A PRIMARY goal of magnetic field experiments on satellites orbiting the Earth at low altitude is the study of current distribution in the plasma. It is well known that the current distribution cannot be determined uniquely from magnetic field measurements obtained by a single spacecraft. In spite of this limitation space-borne magnetic field observations have contributed remarkably to the understanding of ionospheric/magnetospheric current circuits during the past 30 years. However, these contributions depend on assumptions made regarding the geometry and temporal behavior of the currents.

A good step forward in understanding the role of field-aligned currents (FAC) in the global current circuit was achieved by Iijima and Potemra [10]. They made use of the polar orbiting satellite Triad and assumed that the perpendicular magnetic field variations were caused predominantly by FAC's. Binning a large number of orbits they could determine the average distribution of FAC's in the auroral zone for all sectors of local time.

Zanetti *et al.* [21] went a step further. Besides FAC's, they also included ionospheric Hall and Pedersen currents when interpreting Magsat data. New understanding of the large-scale current systems associated with auroral electrojets emerged as a result of these studies.

The polar orbiting Dynamics Explorers DE-1 and 2 were also used to study the current distribution at high latitudes. Sugiura *et al.* [20] were the first who stated that FAC sheets are made up of many small filaments. However, the temporal resolutions of the magnetic field instruments on DE [9] and Viking [17] were insufficient to probe the lower limit of current sheet thickness. Employing high-resolution Freja data, Lühr *et al.* [14] presented first indications that the minimum thickness of current filaments seem to range around 2 km.

By combining magnetic and electric field measurements of the low altitude spacecraft DE-2, Sugiura [19] and Ishii *et al.* [9] deduced a current/voltage relation in the vicinity of FAC's which was found to be controlled by the ionospheric Pedersen conductivity in the case of large-scale (> 100 km) systems. For these studies it was assumed that the associated current systems were aligned in the east/west direction, thus only the northward component of the electric field and the eastward component of the magnetic field measurements were considered. For small-scale sizes the above relation broke down. In these cases the ratio E/B approached the local Alfvén velocity [7].

Common in all studies mentioned above is that data from polar orbiting spacecraft have been employed and that the main emphasis was on the larger-scale current systems which have much larger correlation lengths perpendicular to the orbit than along it. This condition is favorable for deducing current densities from single spacecraft magnetic field measurements.

In a detailed study, Fung and Hoffmann [5] investigated the effect of finite geometry FAC filaments on current density estimates derived from spacecraft magnetic field measurements. In their paper they describe in mathematical terms how to treat oblique penetrations of current sheets and discuss the influence of off-center crossings.

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In this paper we take advantage of the results cited above and apply it to the conditions of the Freja mission. With its 63°-inclination orbit, Freja is quite different from previous missions. Here the spacecraft crosses the auroral oval at a shallow angle or even flies parallel to it over large portions. Furthermore, the details of the FAC filaments, rather than the large-scale current systems, are of interest in this mission [15].

We regard the current density as an important parameter for studying the electrodynamic coupling between the ionosphere and magnetosphere. The purpose of this paper is therefore to describe the procedure which is used to obtain current estimates from Freja data, show the limits of applicability, and outline the degree of uncertainty.

The data used for the present analysis are from the DC magnetometer on Freja. The fluxgate-type instrument senses all three components of the magnetic field. With its ± 64 000 nT measurements range it provides a least-significant-bit resolution of 2 nT. In a band limited channel for frequencies above 1.5 Hz the resolution is enhanced by a factor of 100. The sampling rate is 128/s in the normal telemetry mode and 256/s in burst mode. Further details of the instrument can be found elsewhere [22].

In the sections to follow we first describe the algorithm for estimating the current density for ideal conditions, then show how it changes for relaxed constraints. Subsequently the theoretical results are compared to current estimates from simulated flight data. For some selected events we present results from Freja data and discuss the influence of temporal field variations.

II. Mathematical Framework

For the introduction of our algorithm we start with a very simple configuration, then relax the restrictions until we approach the typical geometry of the currents experienced by Freja. Let us first assume that we have an infinitely large plane current sheet of finite thickness and that the current density is stationary and homogeneously distributed. Let the spacecraft cross the sheet at a right angle.

The basic relation for evaluating the current density is Ampère's law

$$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{j} \tag{1}$$

where $\boldsymbol{j}$ is the current density, μ_o the vacuum permeability, and $\boldsymbol{B}$ the magnetic field. We define a coordinate system such that z points in the direction of the current flow, y is normal to the sheet pointing in the direction of the spacecraft velocity, and x lies in the current sheet to complete the triad (see Fig. 1). Solving (1) for j_z, the only nonvanishing component yields

$$j_z = \frac{1}{\mu_o}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right). \tag{2}$$

Since we do not have multi-point measurements, we convert observed temporal variations into spatial gradients by considering the velocity. In the case of our simple setup, for which $B = B_x$ and $v = v_y$, we get

$$j_z = -\frac{1}{\mu_o v_y}\frac{dB_x}{dt}. \tag{3}$$

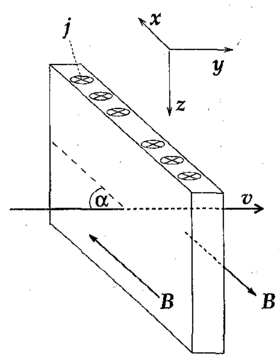


Fig. 1. Basic configuration of an infinitely large current sheet crossed by a spacecraft.

Here it has to be stressed that the spatial gradient can only be determined along the track of the spacecraft. In general therefore the inequality holds

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \neq \frac{1}{v_x}\frac{dB_y}{dt} - \frac{1}{v_y}\frac{dB_x}{dt}. \tag{4}$$

For the treatment of the Freja flight data we use a coordinate system which is guided by the ambient magnetic field and the spacecraft velocity; it is $\boldsymbol{z}'$ along $\boldsymbol{B}, \boldsymbol{x}' = \boldsymbol{v} \times \boldsymbol{B}, \boldsymbol{y}'$ completes the triad and is co-aligned with the velocity component perpendicular to the magnetic field. Inserting the new components into (3) and introducing discrete magnetic field samples yields

$$j_z = \frac{1}{\mu_o \Delta t}\frac{\Delta B'_x}{v_\perp} \tag{5}$$

where $v_\perp$ is the perpendicular velocity component and Δ stands for the difference of two adjacent values. From (5) it is evident that a single spacecraft can only detect a fraction of the actual FAC density, unless the current sheet is perpendicular to the flight pass.

A question we are going to address now is, to what extent the above conditions are fulfilled for the Freja mission and what is the effect of deviations from them? Over the northern hemispheric segment where Freja is tracked by ground stations, practically the total magnetic field variations perpendicular to the background field are caused by FAC's. At the typical height of 1700 km the influence of ionospheric currents is negligible. The influence of the Pedersen currents which close the FAC's in the ionosphere even improves the magnetic field configuration toward that of infinitely long currents strips, and the effect of the Hall currents is attenuated by a factor of order 100 at Freja altitudes compared to ground observations.

While the assumption of a large-scale current sheet may be reasonably well fulfilled along the field direction, its size perpendicular to the field can take almost any value. Also the requirement of stationarity may be violated in some cases, but one should keep in mind that Freja crosses the sheets at a velocity of 7 km/s, so only changes on time scales of seconds or less must be taken into account here. Finally we assumed a crossing of the current sheet at right angle. This configuration will only rarely be encountered. We therefore assess the implications that result from a violation of this requirement.

Along the segment of the Freja orbit in the northern hemisphere where it is tracked, the spacecraft always has the dominant velocity component (> 5 km/s) perpendicular to the magnetic field direction. This situation favors reliable estimates of the FAC density. The angle α between the current sheet and the velocity component perpendicular to the field can, however, take any value. In Fig. 1 α lies in the x-y plane. It is evident from simple considerations that this angular dependence can be taken into account by adding the factor $1/\sin\alpha$.

$$j_z = j_0/\sin\alpha \tag{6}$$

where j_0 is the current density computed according to (5).

Another assumption which cannot be fulfilled, is the requirement of an infinite current sheet. Let us therefore relax the assumption by assuming a sheet of infinite length but finite width. For this generalization an analytical expression is found, if a perpendicular crossings is assumed

$$j_z = j_0\left(\frac{2}{\pi}a\tan\frac{w}{d}\right)^{-1}. \tag{7}$$

Here w is the width perpendicular to the field direction and d the thickness of the current sheet. In the special case of $w = d$, i.e., when the cross section is square, we get $j_z = 2j_0$. This result is also found for a circular cross section.

In the above paragraph it was assumed that the spacecraft crosses the current sheet at its center. If we also relax this condition (7) must be modified.

$$j_z = j_0\left(\frac{2}{\pi}a\tan\frac{w-2s}{d}\right)^{-1}. \tag{8}$$

Here s is the distance from the center of the current sheet to the point where the s/c crosses the sheet. For obvious reasons the condition $2s \leq w$ must be fulfilled.

Outside the current sheet j_z is identically zero. The magnetic components vary in the current free region such that their contributions cancel each other. A simple way to convince oneself is to examine the external magnetic field of a line current (which could be considered as an elementary current filament of an arbitrarily shaped FAC)

$$\begin{aligned} B_x &= -C\frac{y}{x^2+y^2} \\ B_y &= C\frac{x}{x^2+y^2} \end{aligned} \tag{9}$$

and insert it into (2). The value of the constant is $C = \mu_0 I/2\pi$ and I is the current strength. In this respect the current density is a local quantity not affected by other currents in the vicinity. It therefore shows variations quite different from those in the magnetic field data.

III. Model Simulations

To check how well the above theoretical results are reproduced when applying the algorithm given in (5) to spacecraft data we calculated simulated measurements for some simple current models. The model configuration sketched in Fig. 2 is made up of infinitely long cylindrical current tubes of 1 km diameter as elementary currents. By placing these tubes side by side all desired shapes of currents can be formed. The calculated magnetic field was sampled along the spacecraft trajectory at 1 km spacings which corresponds to a sample rate of $8/s$. These discrete values were used as inputs for (5).

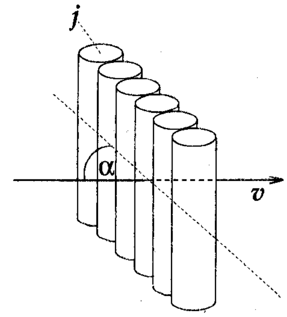


Fig. 2. Current sheet model consisting of a series of infinitely long circular current tubes. The spacecraft crosses the sheet at an angle of incidence, α.

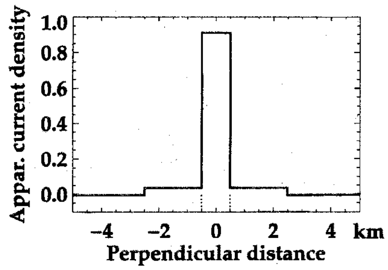


Fig. 3. Current density estimates along the spacecraft obtained from simulated data. The edges of the current sheet are marked by dashed lines.

In a first run we checked the usefulness of our model calculations. Fig. 3 shows the obtained current densities along the satellite track for a basic configuration where the spacecraft crosses an almost infinitely large current sheet of 1 km thickness. Outside the current sheet (marked by vertical dashed lines in Fig. 3) the obtained current density is very small. Only in the immediate vicinity of the edges the curve starts to rise. The peak current density falls short by 9% with respect to the infact current density. This discrepancy is due to our approximation of the current sheet by a row of cylinders. We should keep the 9% deficit found here in mind, because it will show up again in other calculations.

The next step in generalization is to drop the requirement of normal incidence. Fig. 4 shows the dependence of the current density estimates, obtained by applying (5) to the simulated data, on the angle of incidence α (dashed line). The expected dependence according to (6) is also shown (full line). The simulation results at 90° are somewhat lower than the theoretical value (same case as in Fig. 3). The rest of the curve follows the sin α-characteristics reasonable well but at a slightly reduced slope. At 30° it crosses the theoretical curve. From the comparison of the two curves we may conclude that the model results in the angular range 30°–90° track the expectations rather well.

In another set of model runs we investigated the effect of a finite width of the current sheet where the thickness was kept constant. As assumed for (7) the "spacecraft" crossed the

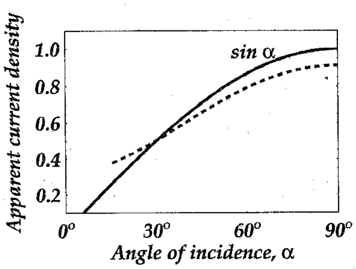


Fig. 4. Dependence of current estimate on angle of incidence (full line: theoretical values, dashed line: simulation results).

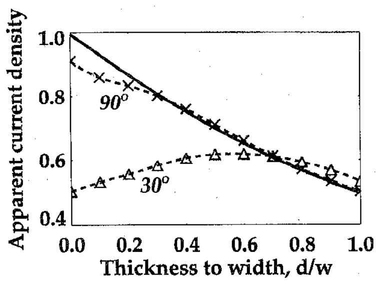


Fig. 5. Dependence of current estimate on the ratio of current sheet thickness to width (full line: theoretical behavior for perpendicular crossing; dashed line, crosses: simulation results for same conditions; dashed line, triangles: simulation results for a 30° incidence).

current sheet at its center. The dashed line marked by crosses in Fig. 5 shows the dependence of the simulation results on the relation between thickness and width, i.e., d/w. The full line again gives the theoretically expected dependence according to (7). In both cases the spacecraft crosses the sheet at right angle. Apart from the known deficit for wide sheets the model curve follows the theoretical trend almost perfectly. In the case of a square cross-section $(d/w = 1)$ the current estimate obtained by (5) is low by a factor of two compared with the actual density.

If we now simulate a traverse of the spacecraft through the current sheet at an angle different from 90°, we get an interesting result. The graph marked by triangles in Fig. 5 shows the dependence of the current estimates on the width of the sheet for a 30° angle of incidence. For an infinite wide sheet we obtain the expected value 0.5. For decreasing widths the values first rise somewhat and then approach 0.5 again for $d/w = 1$. In case of a square cross section the angle of incidence has no influence on the result. The same is obviously true for a circular cross section. All other angles between 30° and 90° give curves lying between the two simulation results shown in Fig. 5. A conclusion we may draw from this exercise is that we cannot simply combine (6) and (7), if a nonperpendicular crossing of a current sheet of finite size is considered.

Our last model run was devoted to the investigation of the effect of a crossing away from the center of the current sheet. The dimensions of the studied current sheet were $w = 10$ km and $d = 1$ km. For a larger w/d ratio the effect will be less pronounced. Again we assume a perpendicular crossing of the current sheet. Fig. 6 shows a comparison between the

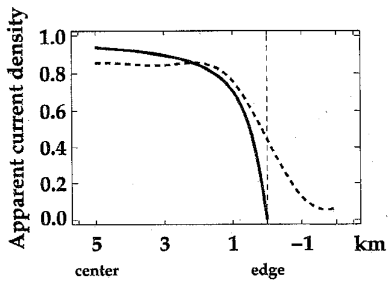


Fig. 6. Current density estimates for crossings offset from the current sheet center (full line: theoretical curve, dashed curve: simulation results) and edge (vertical dashed line).

theoretically expected dependence (full line) according to (8) and the simulation results (dashed line). When crossing at the center (5 km) the model results again fall short by some 9%, as mentioned above. Toward the edge both curves decline. While the theoretical value reaches zero at the edge, the simulated densities exhibit a more gentle transition. This is probably due to the finite sampling distance of 1 km in our model. A remarkable result, in spite of these limitations, is that the current estimate is down to 13% if the trajectory is only 1 km off the current sheet. This convincingly shows that estimates obtained by (5) can actually be treated as local measurements fairly independent of the conditions prevailing more than 1 km away. Fig. 6 furthermore shows that crossing the sheet off the center has only a minor effect on the current estimates.

Summarizing the above presented model calculations we may conclude that current estimates obtained by (5) always fall short whenever the geometry of the current sheet deviates from the ideal configuration assumed in the beginning. If we limit ourselves to the range of 30°–90° for the angle of incidence, the correction factor varies between one and two for any geometry of the current sheet.

IV. Temporal Variations

A rather important assumption that has been made in all above calculations is the stationarity of the current sheet. If this assumption is violated, the interpretation of observed magnetic field variations in terms of spatial gradients is no longer valid. There is, however, no possibility to resolve the spatio-temporal ambiguity from a single spacecraft magnetic field measurement. We therefore will assess the expected uncertainties for the conditions encountered by the Freja spacecraft.

One class of phenomena in this context are moving current sheets. Electrojet current systems sometimes exhibit motions in the north/south direction. The associated velocities are generally below 1 km/s. A reason for this is that the field-aligned flux tubes are anchored in the highly conductive plasma of the polar ionosphere and the velocities there are limited by the ion sound speed. Similar speeds in east/west direction are encountered with auroral structures like omega-bands and evening side patches [1]. In contrast there are also current structures which move much faster. For example westward traveling surges (WTS) [2] or traveling convection vortices (TCV) [6] exhibit typical propagation velocities of 3–10 km/s.

For a proper current estimate the velocity of the current structure must be added to the Freja orbital velocity component perpendicular to the field lines. The velocity of current structures can reliably be determined from ground-based magnetometer networks [13]. Studies of FAC densities associated with TCV or WTS can thus only be conducted if reliable velocity estimates from ground-based observations or multiple spacecraft measurements, are available. In the case of electrojet studies corrections are not so important because the orbital velocity is generally much faster than the electrojet motion, so errors are generally less than 20%.

Another class of transient phenomena are magnetic pulsations. At high magnetic latitudes (> 60°) which are of interest in this study, Pc5 pulsations (150 s $> T >$ 600 s) are the dominating type. Since the amplitude spectrum falls off faster than $1/f$, the maximum slew rate is determined by this type of pulsation [18]. Assuming an amplitude A = 100 nT and a period T = 300 s the maximum slew rate $(dB/dt)_{\max} = 2\pi A/T$ is $(dB/dt)_{\max}$ = 2 nT/s. Inserting this into (3) and applying the orbital velocity of 7 km/s an apparent current density of 2.3 $\mu\mathrm{A/m}^2$ results. This number can, however, only be taken as an indication for the order of magnitude, since it is overestimated in many respects: 1) Pulsation events exhibiting that large amplitudes are very rare, 2) the magnetic variations observed on the ground are caused primarily by Hall currents while Freja senses the current densities in the FAC/Pedersen current circuit which are typically only half as strong, 3) the dominant mode of these pulsation is toroidal, hence the current density false readings will be furthermore reduced by the fact that the magnetic variations above the ionosphere and the spacecraft velocity are almost parallel. As a conclusion we may state that the error in our current density estimate caused by geomagnetic pulsations is well below 1 $\mu\mathrm{A/m}^2$.

There is another kind of magnetic field variation which more severly affects our current estimates. Above the polar ionosphere Alfvén waves with small transverse wave length (1–10 km) are often encountered [3]. Candidates for these small-scale waves are kinetic Alfvén waves and/or standing Alfvén waves which are reflected between the ionosphere and the auroral acceleration region [16]. Typical frequencies of the latter waves are of order 1 Hz. Further complications arise because the spatial variations of the small-scale FAC filaments are Doppler-shifted into the same frequency range as the temporal variations of the wave field.

Due to the fairly high slew rate of these waves, apparent current densities of tens or even hundreds of micro amps per square meter may arise. To unravel the spatio-temporal ambiguity caused by this kind of wave, extensive study including electric field and particle measurements is required [11], [12]. In summary, the small-scale waves represent a much more severe source of errors than the pulsations.

V. An Example from Spacecraft Measurements

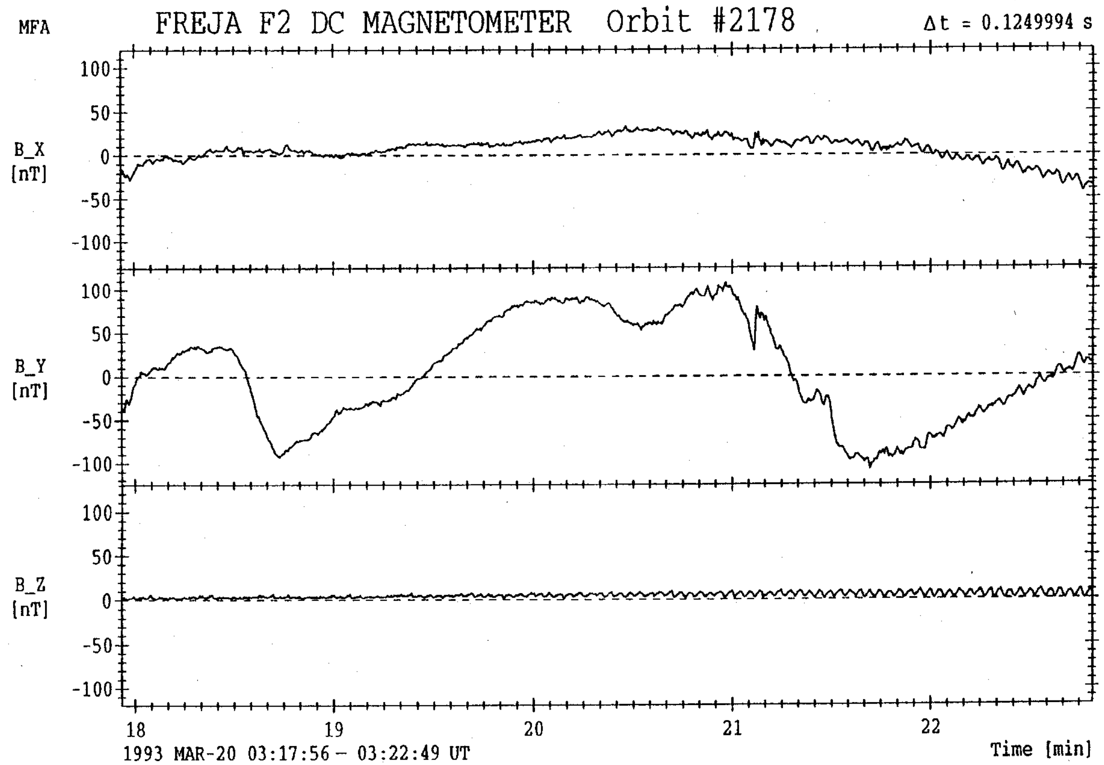
As an example for FAC density estimates deduced from Freja magnetic field measurements we have chosen orbit 2178 on 20 March 1993. The spacecraft traversed the auroral oval in the premidnight sector. A more detailed description of this pass is given by Haerendel *et al.* [8] labled there "event (2)." Fig. 7 shows in the upper half the magnetic field measurements in mean-field-aligned (MFA) coordinates. In this system the B_z component is aligned with the background magnetic field direction, B_y points eastward, perpendicular to the magnetic meridian, and B_x completes the triad pointing predominantly northward. The mean field as determined by the IGRF model has been subtracted from B_z. It is obvious from this figure that all relevant magnetic field variations are confined to the perpendicular components. The sinusoidal variations at 6 s period, most prominent in B_z, are residuals of the spin tone.

The measured magnetic variations are mainly confined to the B_y component, which means that the FAC sheets were aligned predominantly in the east/west direction. Freja flew in the southeast direction, crossing the current sheets at an angle of about 40°. In the lower half of Fig. 7 electron fluxes detected simultaneously by the Freja F7 electron spectrometer [4] are shown. In the top panel we see the fluxes of precipitating electrons coded in grey scale. Below are the perpendicular and the upward streaming electrons. In the bottom panel the FAC densities estimated from the magnetic field measurements are plotted. Positive values represent downward directed currents. The small-scale scatter of this signal is caused by the quantization noise of the magnetic field measurements.

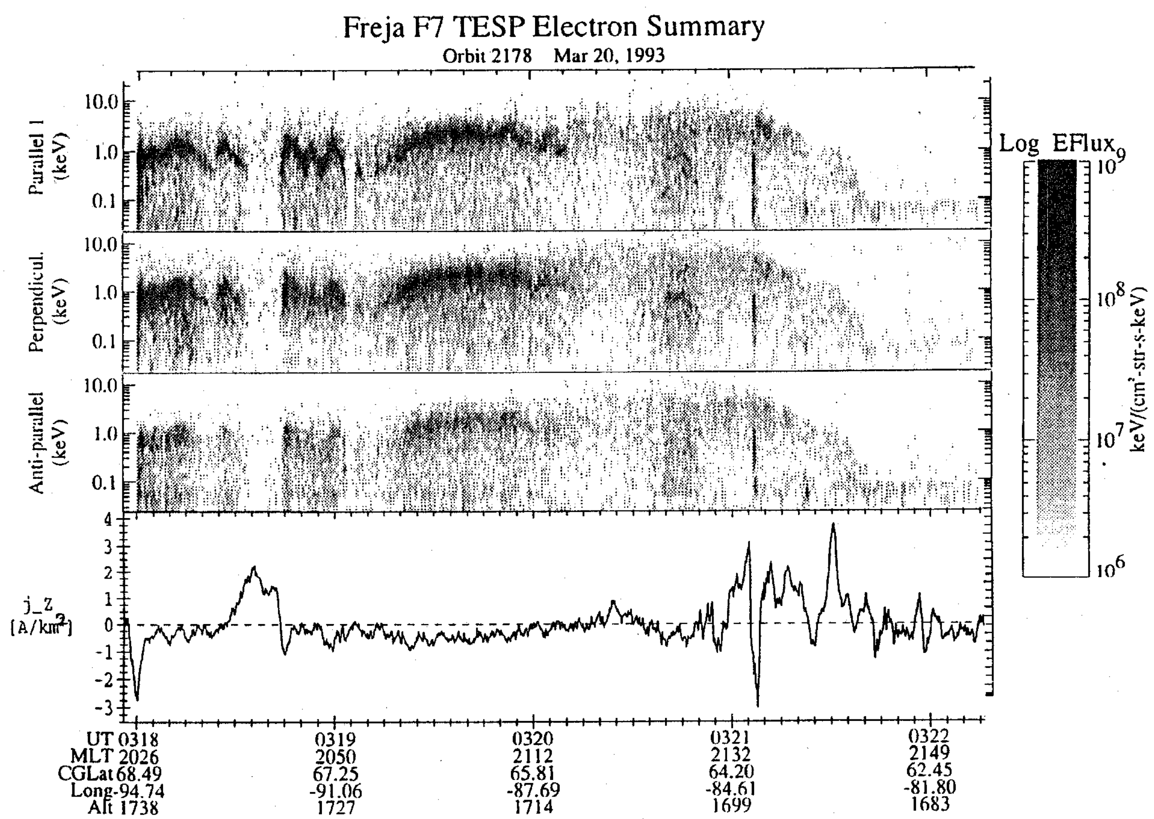
It is apparent from this figure that the presence of precipitating electrons is always accompanied by upward flowing currents. Downward currents are generally associated with a partial or total drop-out of energetic electrons, which is particularly evident around 0318:40 UT. The displayed period is rather interesting, since it comprises regions of various precipitation characteristics. During the first half of the plot the electrons show a clear "inverted-V" characteristic. It is worth noting how little net current is carried by these electrons. In the second half the energies are lower but higher current densities are observed. Rather striking is the burst of electrons at 0321:08 UT lasting for less than 2 s. Well synchronized with it we find a peak in upward current density. Another, albeit weaker, burst is seen 15 s later. This again is accompanied by a negative deflection in current density. The remarkable agreement between the observed electron fluxes and the obtained current density estimates provides convincing evidence for our computer algorithm. The example presented here furthermore shows how well the three displayed quantities complement eachother in characterizing the various types of FAC's in the auroral region.

VI. Discussion and Conclusion

In the previous sections we have outlined our technique for obtaining current density estimates from single spacecraft magnetic field measurements. It is obvious that reliable current densities can only be obtained if a number of assumptions are fulfilled. The first set of assumptions refers to the geometry of the current sheet. We found out that any deviation from a perpendicular crossing through an infinitely large sheet will result in an underestimation of the actual current density. This underestimation is, however, confined to factors between 1 and



(a)



(b)

Fig. 7. Example of Freja measurements. Upper half: magnetic field data in field-aligned coordinates over 5 min; lower half: electron fluxes in the downward, perpendicular and upward channels; bottom panel: FAC density estimates deduced from the magnetic field measurements.

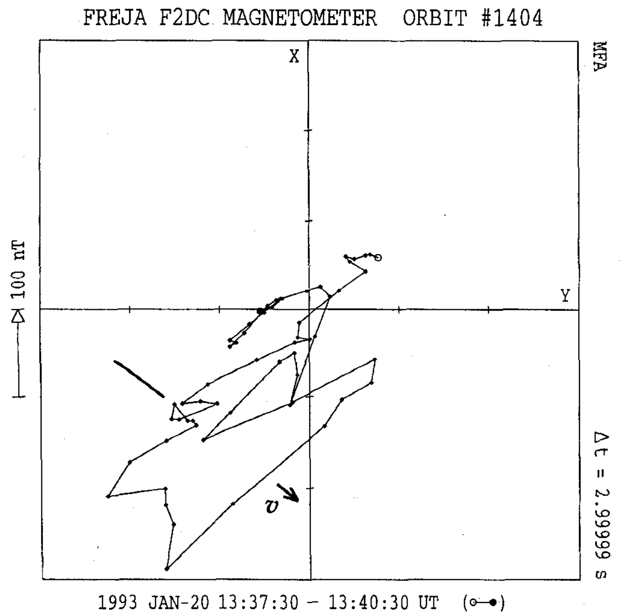


Fig. 8. Hodogram of the perpendicular magnetic field components over 3 min. From the direction of maximum variance the orientation of the current sheet can be determined. The arrow denoted "v" shows the projection of the spacecraft velocity.

2 for any kind of geometry, as long as the angle of incidence is larger than 30°. To check this condition the orientation of the current sheet can be determined by plotting the hodogram of the perpendicular magnetic field variations. As an example, Fig. 8 shows the hodogram of a crossing through a pair of oppositely directed FAC sheets. Also included is the spacecraft velocity vector. The current sheets are aligned with the axis of maximum variance in the hodogram, which makes an angle of 75° with the velocity in this case.

It is, however, not always advisable to use this angle directly for corrections according to (6), since that equation is only valid for infinitely large sheets. The effects of a finite sheet size and of the angle of attack are coupled, as is evident from Fig. 5, and it is not possible to correct them independently. For the standard Freja data processing we therefore decided to multiply (5) by a factor of $\sqrt{2}$ which is the geometric mean between one and two, and we also limit the analysis to cases with angles of incidence larger than 30°. We believe that the overwhelming majority of current estimates determined in this way are reliable within an uncertainty band of ±20°.

An interesting feature of the current density estimates is that they are, in contrast to magnetic field measurements, local quantities. This makes them very valuable for comparisons with electric field and density measurements. The above simulations have shown that a separation of 1 km from the current sheet is sufficient to let the current estimates drop close to zero. Combining the current density estimates with the electric field and density measurements allows to study effectively the fine structure of FAC current sheets, as has been shown by Lühr *et al.* [14]. The magnetic field measurements provide the larger scale context in which these structures are embedded.

Motions of the FAC sheet may affect the current estimates. With the uncertainty limits outlined above speeds of the current system up to 1 km/s are of no concern. Special corrections, however, are required, if phenomena like WTS or TCV are encountered. In several studies it has been demonstrated that magnetometer networks can effectively be used to determine the speed of ionospheric current structures [13]. In the case of the above mentioned phenomena concurrent recordings from magnetometer networks would also be needed to identify them [6].

Finally we should discuss the influence of the small-scale Alfvén waves on the current estimates. The presence of these waves can severely affect the current density estimates, since the employed algorithm cannot distinguish between magnetic field variations caused by spatial gradients or temporal changes. The apparent current densities in association with such small-scale magnetic field variations have amplitudes of order 10 $\mu A/m^2$ as can be seen in Fig. 3 of Lühr *et al.* [14] (0726:24 UT). Within the Freja data set there are also examples where such small-scale signals caused current density readings exceeding 100 $\mu A/m^2$. Since it is not a simple task to reliably resolve the spatio-temporal ambiguity, we recommend averaging the results over an integer number of wave periods to get rid of the ambiguity. The resulting loss in temporal resolution is not too severe, since the apparent frequency of these structures is generally above 1 Hz.

From the arguments presented above we may conclude that it is possible to derive reliable FAC density estimates from the Freja magnetic field measurements if certain not-too-severe constraints are taken into account. The current density is a very useful diagnostic tool. Together with other *in situ* Freja measurements like electric field, plasma density and particle fluxes it can be effectively used to study the fine structure of plasma dynamics in low plasma beta environment.

VII. SUMMARY

The special features of the Freja orbit make this mission particularly suitable for deriving FAC densities from the magnetic field measurements. Conditions are most favorable around the northern hemisphere apogee where the spacecraft is directly tracked. At the orbital altitude of about 1700 km the influence of the ionospheric currents is negligible and the plasma beta is still low enough that the magnetic variations for all practical reasons are confined to directions perpendicular to the field lines. In the auroral region the satellite velocity component perpendicular to the field direction is the dominant component, which improves the reliability of current density estimates from a single spacecraft.

The algorithm employed is based on Ampère's law. Reliable current density estimates can only be determined from single spacecraft magnetic field measurements if the geometry of the current and its temporal variation are known. In the case of Freja we assume that all currents encountered are FAC's. The uncertainty in the estimates resulting from the shape of FAC layer ranges between the factors one and two if we limit ourselves to cases where the angle between the velocity vector and current sheet is larger than 30° To reduce overall least-square errors, we multiply all estimates by a factor of $\sqrt{2}$, since the shape of the layer is not known.

Model calculations have shown that the algorithm can distinguish very well between measurements inside and outside a current filament. The current estimates can be regarded as local measurements with spatial resolutions of less than 1 km.

Current estimates will also be affected if the structures are not at rest. Due to the high orbital velocity, current system speeds of less than 1 km/s, common for electrojet systems, will not cause errors greater than 20%. For the study of fast moving structures like WTS or TCV the velocity determined by ground-based observations must be taken into account.

Another reason for erroneous current density estimates are temporal field variations. It has been shown that the effect of geomagnetic pulsations is generally negligible. Attention must be paid, however, to Alfvén waves with small transverse wave lengths ($\sim$1 km). To circumvent this problem we recommend averaging the measurements over an integer number of wave periods.

Estimates of the FAC density constitute an important diagnostic tool for space plasma physics. The technique presented here could also be applied to future missions like FAST or ØRSTED to help enhance the scientific return from these projects.

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