CAGD - Homework 5

Josefine Stål & Erik Ackzell

October 10, 2016

Task 1

In this task we convert between barycentric and homogeneous coordinates. Consider the points

$$p_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad p_1 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

and let

$$q_1 = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.5 \end{pmatrix}$$

in barycentric coordinates with respect to p_0, p_1, p_2. We want to express q_1 in homogeneous coordinates.
First, we express q_1 in Cartesian coordinates

$$q_1 = 0.25p_0 + 0.25p_1 + 0.5p_2 = \begin{pmatrix} 0.25 + 1.5 + 0.25 \\ 0.25 + 1.5 + 0.5 \\ 0.25 + 1.5 + 0.5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2.25 \\ 2.25 \end{pmatrix}.$$

For any $\omega \in \mathbb{R}$, the homogeneous coordinates of q_1 are

$$q_1 = \begin{pmatrix} 2\omega \\ 2.25\omega \\ 2.25\omega \\ \omega \end{pmatrix},$$

which is what we wanted to determine.
Now let

$$q_2 = \begin{pmatrix} 5 \\ 4 \\ 4 \\ 3 \end{pmatrix}$$

in homogeneous coordinates. We wish to express q_2 in barycentric coordinates with respect to p_0, p_1, p_2.
First, we express q_2 in Cartesian coordinates

$$q_2 = \frac{1}{3}\begin{pmatrix} 5 \\ 4 \\ 4 \end{pmatrix}.$$

We now want to determine the coefficients a_0, a_1, a_2 such that

$$\sum_{i=0}^{2} a_i p_i$$

and

$$\sum_{i=0}^{2} a_i = 1.$$

This can be done by solving the linear equation system

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 5 \\ 4 \\ 4 \\ 3 \end{pmatrix},$$

which reduces to

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}.$$

The solution is given by

$$a_0 = 1, \quad a_1 = \frac{1}{3}, \quad a_2 = -\frac{1}{3},$$

so in barycentric coordinates with respect to p_0, p_1, p_2,

$$q_2 = \frac{1}{3}\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix},$$

which is what we wanted to determine.

Appendix I