CAGD - Homework 2

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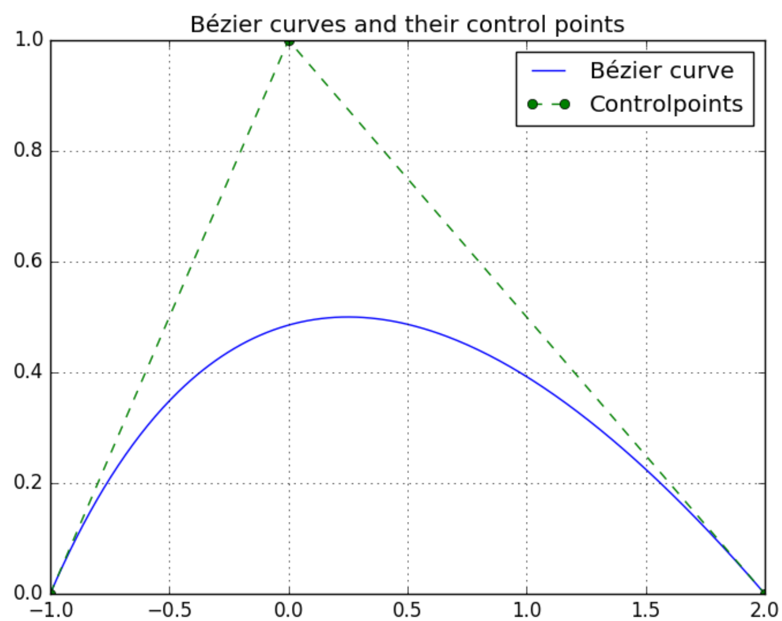
Task 1

In this task we implement subdivision to split a Bézier curve into two different Bézier curves. This is implemented as a method of a Python class, see Appendix 1. Details can be seen in the code. We then test our code by defining a Bézier curve with control points $(-1, 0), (0, 1), (2, 0)$, subdividing at $t = 0.4$. In our test, the control points of the two new Bézier curves were $(-1, 0), (-0.6, 0.4), (-0.04, 0.48)$ and $(-0.04, 0.48), (0.8, 0.6), (2, 0)$. The original curve can be seen in the figure below.



Task 2

In this task we implement degree elevation for the Bézier curve with the same control points as in task 1. This is implemented as a method of a Python class, see Appendix 1. Details can be seen in the code. In our test, the control points of the Bézier curve from task 1 were used, and the degree was increased to four. The control points of the new curve were $(-1, 0), (-0.5, 0.5), (0.17, 0.7), (1, 0.5), (2, 0)$.

Task 3

In this task, we used the trivial reject approach in order to determine the intersection of a Bézier curve and a line. This is implemented as a method of a Python class along with a rectangle class and a line class, see Appendix 1. Details can be seen in the code. In our test, we used the Bézier curve with control points $(0, 0), (9, -4), (7, 5), (2, -4)$ and the line passing through $(4, 5)$ and $(6, -4)$. The intersections found were $(5.32, -0.94)$ and $(5.13, -0.10)$.

Appendix I

Code for task 1-3.

```
"""
This program consists of three different class definitions, a line class,
a rectangle class and a Bzier curve class. The line and rectangle classes
are used to implement the trivial reject method to determine the
    intersection
between a Bzier curve and a line.
"""
import numpy
import scipy
import pylab


class rectangle(object):
    """
    This is a rectangle class.
    """
    def __init__(self, xlow, xhigh, ylow, yhigh):
        """
        An object of the class is initialized with two x-values, one for
            the
        lower bound of the rectangle and one for the upper bound, as
            well as
        two y-values, one for the lower bound of the rectangle and one
            for the
        upper bound.
        """
        # corners of the rectangle
        self.corners = scipy.array([[xlow, ylow],
                                    [xlow, yhigh],
                                    [xhigh, yhigh],
                                    [xhigh, ylow]])
        # lower/higher bounds for x and y
        self.xlow = xlow
        self.xhigh = xhigh
        self.ylow = ylow
        self.yhigh = yhigh

    def plot(self):
        """
        This method plots the rectangle.
        """
        # adding the first corner of the list to the end of the corner
            array,
        # for easier plotting
        rectangle_update = scipy.vstack((self.corners, self.corners[0]))
        pylab.plot(rectangle_update[:, 0], rectangle_update[:, 1])
```

```python
    def get_diagonal_length(self):
        """
        This method calculates and returns the length of the diagonal of
            the
        rectangle, using the two-norm.
        """
        return scipy.linalg.norm(self.corners[0] - self.corners[2], 2)

    def get_center(self):
        """
        This method calculates and returns the center of the rectangle.
        """
        xval = 0.5 * (self.xlow + self.xhigh)
        yval = 0.5 * (self.ylow + self.yhigh)
        return scipy.array([xval, yval])


class line(object):
    """
    This is a line class.
    """
    def __init__(self, p, q):
        """
        An object of the class is initialized with two points through
            which
        the line passes.
        """
        self.p = p
        self.q = q
        self.Lx, self.Ly = self.get_functions_from_points(p, q)

    def get_functions_from_points(self, p, q):
        """
        This method returns two polynomials, describing the line as a
            function
        of x and a function of y, respectively.
        """
        # if the line is parallel with the y axis
        if p[0] == q[0]:
            # coefficients for polynomial of y
            Lycoeff = scipy.polyfit([p[1], q[1]], [p[0], q[0]], 1)
            # no function of x
            Lx = None

            def Ly(y):
                """
                Line as a function of y
                """
                return Lycoeff[0] * y + Lycoeff[1]
```

```
    # if the line is paralell with the x axis
    elif p[1] == q[1]:
        Lxcoeff = scipy.polyfit([p[0], q[0]], [p[1], q[1]], 1)

        def Lx(x):
            """
            Line as a function of x
            """
            return Lxcoeff[0] * x + Lxcoeff[1]

        Ly = None
    # if the line is non-parallel with neither the x or the y axis
    else:
        Lxcoeff = scipy.polyfit([p[0], q[0]], [p[1], q[1]], 1)
        Lycoeff = scipy.polyfit([p[1], q[1]], [p[0], q[0]], 1)

        def Lx(x):
            """
            Line as a function of x
            """
            return Lxcoeff[0] * x + Lxcoeff[1]

        def Ly(y):
            """
            Line as a function of y
            """
            return Lycoeff[0] * y + Lycoeff[1]

    return Lx, Ly

def crosses_line_segment(self, segmentpoints):
    """
    This method checks whether the line intersects a line segment,
        parallel
    to the x or y axis.
    """
    # if the line segment is parallel to the x axis
    if segmentpoints[0, 1] == segmentpoints[1, 1]:
        # if the line is not parallel to the y axis
        if self.Lx:
            # check if line crosses the line segment
            if (
                    (self.Lx(segmentpoints[0, 0]) - segmentpoints[0, 1])
                        *
                    (self.Lx(segmentpoints[1, 0]) - segmentpoints[0,
                        1])) <= 0:
                return True
            else:
                return False
        else:
```

```
            if segmentpoints[0, 0] <= self.p[0] <= segmentpoints[1,
                 0]:
                return True
            else:
                return False
    # if the line segment is parallel to the y axis
    else:
        # if the line is not parallel to the x axis
        if self.Ly:
            if (
                 (self.Ly(segmentpoints[0, 1]) - segmentpoints[0, 0]) *
                 (self.Ly(segmentpoints[1, 1]) - segmentpoints[0, 0]))
                     <= 0:
                return True
            else:
                return False
        else:
            if segmentpoints[0, 1] <= self.p[1] <= segmentpoints[1,
                 1]:
                return True
            else:
                return False

def intersects_rectangle(self, rectangle):
    """
    This method checks whether the line intersects a rectangle.
    """
    result = False
    # check if the line intersects any of the sides of the rectangle,
    # the first three sides
    for i in range(3):
        segmentpoints = rectangle.corners[i:i+2]
        if self.crosses_line_segment(segmentpoints=segmentpoints):
            result = True
    # checking the last side
    segmentpoints = scipy.array([rectangle.corners[0],
                                rectangle.corners[3]])
    if self.crosses_line_segment(segmentpoints=segmentpoints):
        result = True
    return result

def plot(self, parammin, parammax):
    """
    This method plots the line.
    """
    paramlist = scipy.linspace(parammin, parammax, 200)
    if self.Lx:
        vallist = [self.Lx(x) for x in paramlist]
        pylab.plot(paramlist, vallist)
    else:
```

```
            vallist = [self.Ly(y) for y in paramlist]
            pylab.plot(vallist, paramlist)


class beziercurve(object):
    """
    This is a class for Bzier curves.
    """
    def __init__(self, controlpoints):
        """
        An object of the class is initialized with a set of control
            points in
        the plane.
        """
        self.controlpoints = controlpoints
        self.xlow = min(self.controlpoints[:, 0])
        self.xhigh = max(self.controlpoints[:, 0])
        self.ylow = min(self.controlpoints[:, 1])
        self.yhigh = max(self.controlpoints[:, 1])

    def __call__(self, t):
        """
        This method returns the point on the line for some t.
        """
        deCasteljauArray = self.get_deCasteljauArray(t)
        return deCasteljauArray[-1, -2:]

    def subdivision(self, t):
        """
        This method implements subdivision at t.
        """
        # getting the de Casteljau array using t
        deCasteljauArray = self.get_deCasteljauArray(t)
        # extracting the new controlpoints from the array
        controlpoints1 = scipy.array([deCasteljauArray[i, 2 * i:2 * i+2]
                                     for i in
                                         range(len(self.controlpoints))])
        controlpoints2 = scipy.array([deCasteljauArray[-1, 2 * i:2 * i+2]
                                     for i in
                                         range(len(self.controlpoints))])
        controlpoints2 = controlpoints2[::-1]
        curve1 = beziercurve(controlpoints1)
        curve2 = beziercurve(controlpoints2)

        return (curve1, curve2)

    def get_deCasteljauArray(self, t):
        """
        This method calculates and returns a matrix with the lower left
            corner
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```
        containing the de Casteljau array, calculated for the specified
            t.
        """
        # initializing the array
        deCasteljauArray = scipy.column_stack((
                                numpy.copy(self.controlpoints),
                                scipy.zeros((len(self.controlpoints),
                                            2 * len(self.controlpoints) -
                                                2))
                                           ))
        # filling the array
        for i in range(1, len(deCasteljauArray)):
            for j in range(1, i + 1):
                deCasteljauArray[i, j*2:j*2+2] = (
                        (1 - t) * deCasteljauArray[i-1, (j-1)*2:(j-1)*2+2]
                            +
                        t * deCasteljauArray[i, (j-1)*2:(j-1)*2+2])
        return deCasteljauArray

    def intersects_line(self, line1):
        """
        This method checks if the curve intersects a line.
        """
        # rectangle with sides parallel to the x and y axis, containing
            the
        # convex hull of the control points of the curve
        rectangle1 = rectangle(xlow=self.xlow,
                               xhigh=self.xhigh,
                               ylow=self.ylow,
                               yhigh=self.yhigh)
        # initial check
        if not line1.intersects_rectangle(rectangle1):
            return False
        else:
            # list of rectangles
            rectangle_list = [rectangle1]
            # list of intersections
            intersection_list = []
            # list of curves
            curve_list = [self]
            # as long as there are rectangles in the rectangle list
            while rectangle_list:
                # list to update curve list with
                updated_curve_list = []
                # list to update rectangle list with
                updated_rectangle_list = []
                # going through all the curves in the list
                for C in curve_list:
                    # subdividing
                    C1, C2 = C.subdivision(t=0.5)
```

```
                # getting rectangles corresponding to the new curves
                R1 = rectangle(xlow=C1.xlow,
                               xhigh=C1.xhigh,
                               ylow=C1.ylow,
                               yhigh=C1.yhigh)
                R2 = rectangle(xlow=C2.xlow,
                               xhigh=C2.xhigh,
                               ylow=C2.ylow,
                               yhigh=C2.yhigh)
                # checking if the line intersects any of the new
                    rectangles
                for RC in [(R1, C1), (R2, C2)]:
                    # if intersection, use to update rectangle/curve
                        list
                    if line1.intersects_rectangle(RC[0]):
                        updated_rectangle_list.append(RC[0])
                        updated_curve_list.append(RC[1])
            # update curve and rectangle lists
            curve_list = updated_curve_list
            rectangle_list = updated_rectangle_list
            # list of indices of rectangles and curves to remove from
            # the rectangle and curve list
            poplist = []
            for i, R in enumerate(rectangle_list):
                # if the rectangles are very small
                if R.get_diagonal_length() < 1e-7:
                    intersection_list.append(R.get_center())
                    poplist.append(i)
            # remove elements from rectangle and curve lists
            for i in poplist[::-1]:
                rectangle_list.pop(i)
                curve_list.pop(i)
        # list of indices of calculated intersections to remove
        poplist = []
        for i, I in enumerate(intersection_list[:-2]):
            for j, I2 in enumerate(intersection_list[i + 1:]):
                # if the distance of intersections is too small
                if scipy.linalg.norm(I - I2, 2) < 1e-7:
                    poplist.append(j + i)
        # remove duplicates
        for i in poplist[::-1]:
            intersection_list.pop(i)
        return intersection_list

def degree_elevation(self):
    """
    This method implements degree elevation.
    """
    n = len(self.controlpoints)
    # initializing the array to hold control points
```

```
        new_controlpoints = scipy.zeros((n + 1, 2))
        # the first and last control points are the same as before
        new_controlpoints[0] = numpy.copy(self.controlpoints[0])
        new_controlpoints[-1] = numpy.copy(self.controlpoints[-1])
        # calculating the new control points
        for i in range(1, n):
            new_controlpoints[i] = (
                        (1 - i/n) * numpy.copy(self.controlpoints[i]) +
                        (i/n) * numpy.copy(self.controlpoints[i - 1])
                                  )
        return beziercurve(new_controlpoints)

    def plot(self, controlpoints=True):
        """
        This method plots the curve.
        """
        # list of u values for which to plot
        tlist = scipy.linspace(0, 1, 300)
        pylab.plot(*zip(*[self(t) for t in tlist]), label='Bzier curve')
        title = 'Bzier curves'
        if controlpoints: # checking whether to plot control points
            pylab.plot(*zip(*self.controlpoints), 'o--',
                label='Controlpoints')
            title += ' and their control points'
        pylab.legend()
        pylab.title(title)
```