

# CAGD - Homework 5

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## Task 1

In this task we convert between barycentric and homogeneous coordinates. Consider the points

$$p_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad p_1 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}, \quad p_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

and let

$$q_1 = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.5 \end{pmatrix}$$

in barycentric coordinates with respect to  $p_0, p_1, p_2$ . We want to express  $q_1$  in homogeneous coordinates.

First, we express  $q_1$  in Cartesian coordinates

$$q_1 = 0.25p_0 + 0.25p_1 + 0.5p_2 = \begin{pmatrix} 0.25 + 1.5 + 0.25 \\ 0.25 + 1.5 + 0.5 \\ 0.25 + 1.5 + 0.5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2.25 \\ 2.25 \end{pmatrix}.$$

For any  $\omega \in \mathbb{R}$ , the homogeneous coordinates of  $q_1$  are

$$q_1 = \begin{pmatrix} 2\omega \\ 2.25\omega \\ 2.25\omega \\ \omega \end{pmatrix},$$

which is what we wanted to show.

Now let

$$q_2 = \begin{pmatrix} 5 \\ 4 \\ 4 \\ 3 \end{pmatrix}$$

in homogeneous coordinates. We wish to express  $q_2$  in barycentric coordinates with respect to  $p_0, p_1, p_2$ .

First, we express  $q_2$  in Cartesian coordinates

$$q_2 = \begin{pmatrix} \frac{5}{3} \\ \frac{3}{4} \\ \frac{3}{3} \end{pmatrix}.$$

We now want to determine the coefficients  $a_0, a_1, a_2$  such that

$$\sum_{i=0}^2 a_i p_i$$

and

$$\sum_{i=0}^2 a_i = 1.$$

This can be done by solving the linear equation system

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

## Appendix I