Learning Journal

Organized by Sessions with Topic Summaries

Instructions

- Use this document to track your learning sessions.
- Each session entry should include:
  1. Topics covered (briefly).
  2. Key insights or definitions.
  3. Problems attempted (with references or solutions if necessary).
  4. Questions or areas for follow-up.

  Make Anki flashcards of whatever makes sense. Even topics if you'd like, just to leverage their spaced repitition algorithm.
- After completing a topic, write a summary essay, incorporating insights from session notes and solved examples.

Resources

Physics

- Eigenchris
- MIT OpenCourseware
- Richard Behiel
- Steve Brunton

Math

- VisualMath on YT
  - Lectures on quantum topology without topology seem really cool! His course notes are also free.
  - Many overview videos
  - Algebraic Topology
  - Algebraic Geometry
  - etc.
- The Bright Side of Mathematics

Learning Plan

This is a first pass on some topics I find interesting. I don't need to understand everything the first time. I don't need to go through all the subtopics the first time. I need to understand the main idea, understand why and when it is useful, and try my hand at some basic problems to use it myself. I can always pick up these topics again since I keep a record here. Just remember to jot down your questions, the things you don't understand (and why) and what you'd like to investigate further in the future. You can ***always*** *come back! The below learning plan is not set in stone. Be open for modifications and new ideas. Follow your curiosity.*

Calculus of Variations in Physics - 21th December

- ~~Understand the derivation of the Euler-Lagrange Equations~~
- Solve the QM problem in Hand and Finch (17th December)
- Answer questions below and synthesize and finalize notes on the topic (20th December)

Questions

- How is it used in modern physics today?
- Does one use different "actions" when working on separate problems. If so, could one find a problem "midway" between those problems and see what the action looks like there? Maybe smoothly interpolate an action between these two problems to gain a deeper understanding of how and why they need different descriptions. Maybe find a general description which they are both special cases of.

Special Relativity, Classical Field Theory and Tensors - 28th December

- Susskind Book (26rd December)
- Learn some tensor notation from Tong Notes
- Solving some basic problems with 4-vectors (27th December)
- Synthesize and finalise (28th December)

Floquet Theory - 31st December

Schuller's Course and self-defined problems - 31st Janurary

Linear Algebra Refresher - 7th February

Complex Analysis Basics - 14th February

MIT Quantum Mechanics Course - 7th February

Misc. things

- Green's Function solution of Poisson's equation to get the electric potential from Griffiths

Session 1: Session Date: 14th December, 2024

Main Topic: Set Theory

Resource: Geometrical Anatomy of Physics

Topics Covered

- Space
- Maps
- Domain
- Codomain
- Image
- Preimage
- Bijection
- Inverse
- Equivalence Relation
- Equivalence Class
- Quotient Space
- What amplitudes are
- What a Positive Grassmannian is

Key Insights

Definitions from Schuller's lecture series

Definition. A **space** is a set with some underlying structure. We often study structure-preserving maps between such spaces.

Definition. A **map** is a relation between to sets. More formally, we can write that given two sets A and B, a map $\phi : A \to B$ is a relation such that for each element $a \in A$ there exists exactly one element $b \in B$ such that $\phi(a, b)$. We write that $a \mapsto \phi(a)$.

Definition. Here, A is the **domain** and B is the **codomain**. The codomain is also called the target.

Definition. The **image** of a set $C \subseteq A$ under a map ϕ is the set one gets, which will be a subset of (or equal to) the codomain, by collecting everything that ϕ maps to when applied to C. We write $\phi(C) \equiv \mathrm{im}_\phi(\mathrm{C}) \coloneqq \{\phi(\mathrm{c}) \mid \mathrm{c} \in \mathrm{C}\}$.

Definition. The **preimage** is the set, which will be a subset of (or equal to) the domain, one gets by considering which elements in the domain one has to apply ϕ to to get certain elements in the codomain. Let for example $V \subseteq B$, then $\mathrm{preim}_\phi(\mathrm{V}) := \{\mathrm{a} \in \mathrm{A} \mid \phi(\mathrm{a}) \in \mathrm{V}\}$

Remark. The inverse is only defined for bijections, but the preimage is defined for all maps, and we will often meet it in topology! I was confused at first as to why we know that $preim_\phi(B) = A$ without requiring surjectiveness, but this is because when we write $\phi : A \to B$ we are already stating that ϕ is applied to, or at least makes sense to apply to, all of A. Now the image might not be all of B (it just "lives" in B), but the preimage of B is the set of all of the values in the domain which under the map ϕ ends up in B - but that is of course all of A, since from the definition, applying ϕ to any element in A it will end up B.

Definition. A map is **surjective** if $\phi(A) \equiv \mathrm{im}_\phi(A) = B$ - that is, if all of B is "hit" by applying ϕ to all of A. A map is **injective** if for $a_1, a_2 \in A$ we have that $\phi(a_1) = \phi(a_2) \Rightarrow a_1 = a_2$.

The most important notion: A map is called **bijective** if it is both surjective and injective.

Definition. When a map is bijective, then a unique **inverse** exists. This is the map such that $\phi^{-1} \circ \phi$=$\mathrm{id_A}$ while $\phi \circ \phi^{-1} = \mathrm{id_B}$. In other words, it "undoes" a mapping. Reading $\circ$ as "after" helps to learn the order of application.

Remark. Generically, if there exists one bijection between sets, then there exists many. A bijection is just a "pairing up" of elements - if you can come up with one way, then you can certainly come up with many (unless you try to design a counterexample I guess).

Definition. If there exists any bijection between two sets A and B then we say that they are (set-theoretically) isomorphic ("of the same shape"). We write $A \cong_{set} B$.

Definition. An **equivalence relation** is any relation between elements in a set which is both *reflexive*, *symmetric* and *transative*. Letting $\sim$ denote the relation, we write these as

$$\begin{aligned} &a \sim a \quad \text{(reflexive)} \\ &a \sim b \Leftrightarrow b \sim a \quad \text{(symmetric)} \\ &a \sim b \wedge b \sim c \Rightarrow a \sim c \quad \text{(transative)} \end{aligned}$$

We denote all of the elements of A which are equivalent to some $m \in A$ under the given equivalence relation as $[m] := \{n \in A \mid n \sim m\}$.

Remark. I am very proud since I was able to prove the following:

i) $a \in [m] \Rightarrow [a] = [m]$

ii) Either $[a] = [m]$ *or* $[a] \cap [m] = \emptyset$

The first of these results imply that any member of an equivalence class equally well represents the whole class. The second one implies that an equivalence relation completely splits the set A into disjoint equivalence classes - we say that it "partitions" the set.

Definition. The set of all equivalence classes formed by applying the equivalence relation $\sim$ to the set A is called the **quotient space** and is written as $A\backslash\sim$. Intuitively, the quotient space is what you get when you sort your large set A into smaller sets by using some rules defined by the given equivalence relation. Examples of an equivalence relation is modulo divison by some prime number. One can then take say the quotient space $\mathbb{Z} \setminus \text{mod p}$.

Takeaways

A map $\phi : A \to B$ applies, by definition, to **all** elements in the domain A. This crucially does not necessarily mean that all elements in B has a corresponding element in A under this map; this property is exactly *surjectivity*.

The set obtained from applying the map to the entire domain is the *image* of the map. If the codomain and the image are equal, then the map is surjective. Thus we can always redefine the codomain to be the image and then the map becomes surjective. But this is often not very interesting.

But the fact that the map is understood to apply to all the elements of the domain is needed to understand why for the map $\phi : A \to B$ we find that

$$\text{preim}_\phi(B) = A$$

where for some $V \subseteq B$ we define the preimage as

$$\text{preim}_\phi(V) = \{a \in A \mid \phi(a) \in V\}$$

Amplitudes and the positive Grassmannian

Studying amplitudes is about studying what we expect to happen when fundamental particles interact at very high energies, like when being smashed into each other at the LHC. Physicists then calculate the probabilities related with the particle scattering in different directions with different energies and momenta. These probabilites are precisely the "amplitudes" in "scattering" and "scattering amplitudes". The reason why this is interesting is because if we want to know if our theory is right - or if we want to know precisely when and where it is not - then we need to have very precise expectations from experiments such that we know when they deviate.

Studying amplitudes is therefore very much at the heart of our most fundamental understanding of nature, and it actually sounds really exciting.

The Grassmannian is a way to group and classify subspaces embedded in larger spaces. For example, $\mathrm{Gr(k, d)}$ is the collection of all k-dimensional subspaces going through the origin in the larger d-dimensional space. I think. The positive Grassmannian is the subspace of the Grassmannian which only has positive minors along (all or certain) axes. I think minors are subdeterminants or something? I'm note sure. But the intuition is that if we are considering the space of all lines in 3D going through the origin $\mathrm{Gr}(1, 3)$, then the positive Grassmannian would be only the lines with positive slope. The Grassmannian kind of "keeps track" of all these distinct geometric objects (lines with different slopes and directions) by only having them as points. The full Grassmannian just discussed would uniquely identify each point on the upper hemisphere with a line (expect for lines going through the "equator" in the (x, y)-plane, if the hemisphere is formed by cutting a sphere in two in the (x, y)-plane).

Apparently, great advances were made in calculating scattering amplitudes by using the positive Grassmannian. And this was just around 10 years ago - so it is still relatively new!. Calculating these amplitudes was (and probably still is) usually is done by adding hundreds of Feynman diagrams for even the simplest calculations - and many thousands for a bit more interesting interactions. And most of these terms sum to zero or something very concise, which is very difficult to understand from the size of the sum. In other words, a lot of redundancies are inherent in the Feynman diagram way of calculating scattering amplitudes, and people are working on more direct ways of doing it since there must be a reason for why many answers come out so beautiful and concise even though the actual calculation is the most messy thing ever.

Problems Attempted

1. Proving the statements regarding equivalence classes (huge victory!)

Follow-Up Questions

- Do a short write up of the proofs above. Remind yourself of the general proof-technique to use if one wants to show a "either - or" statement. Reminder: show $p \Rightarrow \neg q$ because then $\neg(\neg q) \Rightarrow \neg p$ (through contrapositive) which is the same as $q \Rightarrow \neg p$.

Session 2: Session Date: 15th December, 2024

Main Topic: Electromagnetism

Topics Covered

- A small window into amplitudes (Cheung Lecture)
- Fields from electric monopoles, magnetic dipoles and the classical electron radius

Key Insights

Write down definitions, theorems, or takeaways. Use this space for concise notes.

Definitions

Theorems

Takeaways

The Classical Electron Radius: What it is and how to calculate it. What assumption in the calculation makes it so wrong?

When doing the assignment we also found that the closer you integrate both the electric field and the magnetic field to the electric charge and the magnetig dipole respectively, the more energy you find - and this goes towards infinity as the integration approaches all space (by making the excempt sphere around the mono/dipole smaller). Take the electric point charge. This infinite energy makes sense since we can imagine that we take the total charge of the point charge and split it up into smaller, less charged point particles. Now, gathering those point particles in the same point to get the total charge would require us to work an infinite amount against the fields, since the strength of the field becomes infinite as the point charges go to sit on top of each other. The same principle applies with the magnetic dipole. Given a perfect magnetic dipole $\mathbf{m} = I \int d\mathbf{a}$, we see that we need an infinite amound of current to get a finite m since $\int d\mathbf{a}$ is zero for a perfect dipole. But running an infinite current also requires an infinite amount of energy. So that divergence makes sense from Maxwell's theories as well.

Problems Attempted

1. Calculating the "classical electron radius"
2. Outline the method here
3. Remind yourself why it makes sense that the energy stored in the fields explode as you integrate closer and closer to either the elecric charge or the magnetic dipole (something about putting together the total charge of one chareg)

Follow-Up Questions

- Why do we expect Hamilton's principle to work for scalar fields in arbitrary dimensions? We know that it works well in 3 dimensions because we can do experiments - but is it a leap of faith to do it in higher dimensions, or do we have some clue to its validity even in higher dimensions? What if the look of the least action principle is a special case in 3D, and there is a more general mathematical principle - a geometry maybe - which underlies the whole thing in arbritrary D-dimensional space?

- When is the Fourier Transform a smart thing to use? I remember Brian said something about that an instinctive response when seeing a function $f(r_i - r_j)$ should be to Fourier Transform. How come? Are there other "forms" of functions where it immediately simplifies the problem (most of the time). What is k-space, and why does the Fourier Transform take us there?

  Also, look at the picture you took of the blackboard last week when Jens Paaske wrote something about Fourier Transforming a single wave packet. Try it out for yourself with the normal distribution as the wave. Here you of course have to figure out how to write it "as a wave" (problably just replace x with $x - vt$) as well as how to integrate it properly. Exciting!

Session 3: Session Date: 16th December, 2024

Main Topic: Electromagnetism

Topics Covered

- Total momentum in fields from momentum density
- Angular momentum in the fields from momentum density
- Complex current from complex impedances

Key Insights

Takeaways

If one has a field whose magnitude only depends on r but whose direction is given by $\hat{\phi}$, then any integral one full revolution in the ϕ-direction will of course give $\mathbf{0}$. This makes great sense both when you think about it geometrically (you will add up as many components in one direction as in any other, hence summing up to zero) or if you write it out in cartesian components you will integrate $\sin\phi$ and $\cos\phi$ around a full period which gives zero.

Follow-Up Questions

- How does one derive rest energy? I don't remember where it comes from.

Session 4: Session Date: 17th December, 2024

Main Topic: Electromagnetism

Topics Covered

- Maxwell's equations in terms of potentials only
- Gauge Transformations
- 4-vector notation
- d'Alembert operator
- Retarded potentials

Key Insights

Derivation Recap

i) $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$

ii) $\nabla \cdot \mathbf{B} = 0$

iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

ii) allows us to write

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

since the divergence of any curl is zero. This allows us to write

$$\begin{aligned}\nabla \times \mathbf{E} &= -\nabla \times \frac{\partial \mathbf{A}}{\partial t} \\ \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) &= 0\end{aligned}$$

And since the curl of any gradient is zero too, we know that we can write the above as the (negative) gradient of a potential too:

$$-\nabla V = \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}$$

or

$$\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}}$$

such that Gauss' law becomes

$$\begin{aligned}\nabla \cdot \left(-\nabla V - \frac{\partial \mathbf{A}}{\partial t}\right) = -\nabla^2 V - \nabla \cdot \frac{\partial \mathbf{A}}{\partial t} &= \frac{\rho}{\epsilon_0} \\ \Rightarrow \nabla^2 V + \nabla \cdot \frac{\partial \mathbf{A}}{\partial t} &= -\rho/\epsilon_0\end{aligned}$$

In the static case ($\partial_t \mathbf{A} = 0$) this reduces to Laplace's equation

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

We can also rewrite the Ampére-Maxwell law, keeping the source on the right and moving anything else (the fields or potentials) to the left:

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

which in terms of the potentials gives

$$\begin{aligned}
\nabla \times (\nabla \times \mathbf{A}) - \frac{1}{c^2}\frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial \mathbf{A}}{\partial t}\right) &= \mu_0 \mathbf{J} \\
= \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} + \frac{1}{c^2}\nabla\left(\frac{\partial V}{\partial t}\right) + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} \\
= \nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}\right) - \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}
\end{aligned}$$

Defining the d'Alembertian as

$$\square^2 \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

and letting

$$L \equiv \nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}$$

we can succintly write the Ampére-Maxwell law as:

$$\boxed{\square^2 \mathbf{A} - \nabla L = -\mu_0 \mathbf{J}}$$

Notice how we can use this in Gauss' law:

$$\begin{aligned}
\nabla^2 V + \nabla \cdot \frac{\partial \mathbf{A}}{\partial t} &= -\rho/\epsilon_0 \\
\Rightarrow \nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} + \frac{\partial}{\partial t}\left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial V}{\partial t}\right) &= -\rho/\epsilon_0 \\
= \boxed{\square^2 V + \frac{\partial L}{\partial t} = -\rho/\epsilon_0}
\end{aligned}$$

Such that Maxwell's equation in terms of potentials become

Follow-Up Questions

- How come the retarded potential formulation is not equivalent to putting a heaviside inside the integration.
- Why will you get all modes below the cutoff frequency inside wave guides?

- Show what Gauge invariance means
- Do an example from the blackboar
- Rederive boundary conditions for the fields (both free and in matter)
- Learn more about the 4-vector formulation and how $g_{\mu\nu}$ can "raise or lower indecies"
- Lorentz tansformationts as rotations in 4-space (Mogen's notes)
- Coloumb Gauge ($\nabla \cdot \mathbf{A}$)
- Lorenz Gauge ($L = 0$)
- Walk through the full retarded potential derivation.

Session 5: Session Date: 18th December, 2024

Main Topic: Electromagnetism

Topics Covered

Cool way to do propagation of errors Variance is equal to σ^2. If our function is

$$f = xy$$

the law of propagation of errors gives

$$\begin{aligned} V(f) &= \frac{\partial f}{\partial x} V(x) + \frac{\partial f}{\partial y} V(y) \\ &= yV(x) + xV(y) \end{aligned}$$

such that

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$$

which also works for fractions.

Understood how we can conclude that the E and B fields are in phase for monochromatic plane waves. We derive that

$$\begin{aligned} k(\tilde{E}_0)_x &= \omega(\tilde{B}_0)_y \\ -k(\tilde{E}_0)_y &= \omega(\tilde{B}_0)_x \end{aligned}$$

But since k and ω are real, the only way that these scalings can hold all along the wave, then the waves have to hit zero and peak at the same time, which means that they are in phase! Since we from above have the neat writing that

$$\tilde{\mathbf{B}}_0 = \frac{k}{\omega}(\hat{z} \times \tilde{\mathbf{E}}_0) = \frac{1}{c}(\hat{z} \times \tilde{\mathbf{E}}_0)$$

taking the modulus we get that

$$B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0$$

even for the real wave.

Session 6: Session Date: 19th December, 2024

Main Topic: Electromagnetism

Topics Covered

- Potential formulation
- Gauge Transformations

Key Insights

Write down definitions, theorems, or takeaways. Use this space for concise notes.

Definitions

Theorems

Takeaways

Problems Attempted

10.3 in Griffiths was about finding the fields *and* sources given $V(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$. I immediately recognized that the electric field found corresponded to a point charge (at the origin), while the magnetic field was zero. I then began using the potential formulation to find the sources by using the d'Alembertian etc., but this was of course silly, it simply follows that if $\mathbf{B} = \mathbf{0}$ everywhere, then $\mathbf{J} = \mathbf{0}$ everywhere too, while for a point charge $\rho = q\delta^3(\mathbf{r})$. I then saw how a simple gauge transformation changed the funny potentials into what we'd expect to have for a point charge at the origin and no magnetic fields.

10.4 in Griffiths was given a "wave" potential. Derived the fields. If they were to satisfy Maxwell's equation in vacuum, we need to have the condition that $k^2 = \mu_0\epsilon_0\omega^2$ or equivalently, $v = c = \frac{\omega}{k}$. Thus, for the given potential, the electromagnetic fields (which we could see satisfy the wave equation and are thus "waves") simply *have* to propagate at the speed of light to satisfy Maxwell's equations, which we know from experiments that all electric and magnetic fields do. This might not be a suprise since we found that with the way that the electromagnetic fields satisfy the wave equation, they must have a speed of c

$$\nabla^2\mathbf{E} = \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}, \qquad \nabla^2\mathbf{B} = \mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2}$$

9.30 in Griffiths was about figuring out a specific frequency range if one only wanted to excite a single TE-mode. When solving the original wave guide problem, we found that

$$k_x = \frac{m\pi}{a}, \quad m \in \mathbb{Z} \setminus 0$$

(or something similar). This lead to a derivation of the cutoff frequency. As long as the driving frequency is above the cutof frequency, then waves can propagate. But since the wave equation is linear, any sum of possible waves will also satisfy the equations. Thus we expect the solution of the wave propagating in the wave guide to be a sum of all the waves which has a k corresponding to a cutoff-frequency below the current driving frequency. Or something like that. It is the idea anyway. So we found the frequency gap between the two TE-modes with the lowest cutoff-frequency, since this is the range where only 1 is excited. If we pass the threshold, then 2 will be excited and so forth up to an infinitude, I guess? Except for energy considerations.

9.31 in Griffiths was about showing how the velocity of the wave for mode TE_{mn} is in fact the group velocity. This was a very calculation dense task, and I did not succeed. I do need to practice calculations, but this will be practiced just by doing *more* exercises. But a few takeaways from the exercise are listed here: Review these ideas. Prob 9.12 f.eks..why we might want

$$v = \frac{\int d\mathbf{a} \cdot \langle \mathbf{S} \rangle}{\int \langle u \rangle d\tau}$$

and not just

$$v = \frac{\mathbf{S}}{u}$$

9.42 in Griffiths was about finding the ω_{lmn} frequency in a closed "wave box". This was a very difficult exercise for me, but also very instructive in systematically applying boundary conditions and attacking the problem. Go through this problem in depth. Start with Maxwell's equations and get an inhomogenous Laplacian for the electric field (three equations). Then, use an ansatz of a separable solution in each coordinate. Find a solution to these. Apply boundary conditions systematically to remove constants. Use the final solution to determine ω_{lmn}. Review

Follow-Up Questions

- Why do we have to change V and $\mathbf{A}$ simultaneously to have a proper Gauge transformation? Try to write out the potential formulation of Maxwell's equations and walk through the conditions imposed by the look of the Gauge Transformation. What happens if you only shift say V. What about only shifting $\mathbf{A}$?
- Why might we want

  $$v = \frac{\int d\mathbf{a} \cdot \langle \mathbf{S} \rangle}{\int \langle u \rangle d\tau}$$

  and not just

  $$v = \frac{\mathbf{S}}{u}$$

  . Maybe it makes sense that a group velocity is determined from the average of these quantitites. But does that mean that

Session 7: Session Date: 20th December, 2024

Main Topic: Green's Functions

Resource: Mathemaniac and Andrew Dotson YouTube Videos

Topics Covered

Green's Functions Only had time to learn a little bit about Green's functions from a high level perspective. The main idea is that given a (inhomogenous) differential equation

$$\mathcal{L}u(x) = f(x)$$

where $\mathcal{L}$ is a linear operator, we find a function, the Green's function, which satisfies

$$\mathcal{L}G(x, x') = \delta(x - x')$$

such that

$$\begin{aligned} f(x)\mathcal{L}G(x, x') &= f(x)\delta(x - x') \\ \Rightarrow \mathcal{L}\left(f(x)G(x, x')\right) &= f(x)\delta(x - x') \end{aligned}$$

where I guess the linear operator above is w.r.t. x' such that we can move $f(x)$ inside. Integrating both sides, we get

$$\int \mathcal{L}f(x)G(x, x')dx' = \int f(x)\delta(x - x')dx' = f(x)$$

and since $\mathbf{L}$ is a linear operator, we can pull it out of the integral

$$\mathcal{L}\left(\int f(x)G(x, x')dx'\right) = f(x)$$

where comparison with the original DE shows that we have in fact found the solution!

$$u(x) = \int f(x)G(x, x')dx'$$

The equation which the Green's function satisfies

$$\mathcal{L}G(x, x') = \delta(x - x')$$

shows how the Green's function can be intuitively thought of as the system's response to a single pulse-like pertubation. And since the operator is linear, the total response to the driving force $f(x)$ can be found by "adding up" weighted pulse values which in combination give the total pertubation $f(x)$.

I think this is the main idea anyway. I am looking forward to seeing its connection to scatteing.

Follow-Up Questions

- Walk through Mathemaniac derivation again. Read Dotson's resource from Arizona state. Do the Mathemaniac problem.