Learning Journal

Organized by Sessions with Topic Summaries

Instructions

- Use this document to track your learning sessions.
- Each session entry should include:
  1. Topics covered (briefly).
  2. Key insights or definitions.
  3. Problems attempted (with references or solutions if necessary).
  4. Questions or areas for follow-up.
- After completing a topic, write a summary essay, incorporating insights from session notes and solved examples.

Session 1: Session Date: 14th December, 2024

Main Topic: Set Theory

Resource: Geometrical Anatomy of Physics

Topics Covered

- Maps and bijections
- Maps
- Domain
- Codomain
- Image
- Preimage
- Equivalence Relation
- Equivalence Class

Key Insights

Definitions

Theorems

Takeaways

A map $\phi : A \to B$ applies, by definition, to **all** elements in the domain A. This crucially does not necessarily mean that all elements in B has a corresponding element in A under this map; this property is exactly *surjectivity*.

The set obtained from applying the map to the entire domain is the *image* of the map. If the codomain and the image are equal, then the map is surjective. Thus we can always redefine the codomain to be the image and then the map becomes surjective. But this is often not very interesting.

But the fact that the map is understood to apply to all the elements of the domain is needed to understand why for the map $\phi : A \to B$ we find that

$$\mathrm{preim}_\phi(B) = A$$

where for some $V \subseteq B$ we define the preimage as

$$\mathrm{preim}_\phi(V) = \{a \in A \mid \phi(a) \in V\}$$

Problems Attempted

1. *Problem statement or reference.*
2. *Solution (include partial work if needed).*

Follow-Up Questions

- *Write down any gaps in understanding or questions to revisit.*