

# A Framework for Triple-Entry Bookkeeping

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**ABSTRACT:** Building upon the author's earlier work [1982], which demonstrated that double-entry bookkeeping is not an absolute system defying extensions but is logically extendible to triple-entry bookkeeping, this paper develops a framework for a triple-entry bookkeeping system and illustrates it by means of a simple example which includes a worksheet, journal entries, and three basic financial statements—wealth statement, momentum statement, and force statement. While the earlier work extended the existing two dimensions of bookkeeping (wealth and income) into a third dimension under the same measurement unit, namely dollars, this present paper introduces "momentum accounting" under a related but different measurement unit, namely dollars per time period, such as a month, in such a way that, when mathematically integrated over time, momentum accounting articulates with wealth accounting in every dimension. Dealing with earnings rates per time period associated with assets and liabilities, momentum accounting accounts for earnings rates and their changes. Finally, "force accounting" is introduced as the third layer of the accounting system to explain factors that are judged to be responsible for changes in the earnings rate. Its measurement (dollars per month per month, for example) is induced by the measurement in momentum accounting, which in turn is induced by the measurement in wealth accounting. In this way, the extension of double-entry bookkeeping is carried out under a disciplined framework of measurements, which hopefully will direct management's attention and sensitivity to factors at a level deeper than the level of wealth and income that has been traditionally dealt with by double-entry bookkeeping.

**A**N earlier monograph showed that double-entry bookkeeping is *not* an absolute system, defying any extensions, but is logically extendible to triple-entry bookkeeping by including a set of accounts called "force" in its third axis [Ijiri, 1982]. This paper discusses a framework under which details of triple-entry bookkeeping are now being developed and presents a simple example that illustrates the interrelationship among dimensions in triple-entry bookkeeping. Such an effort is of interest from a theoretical standpoint, enabling us to understand the double-entry bookkeeping system from a higher level of perspective. In addition, the new system appears to have practical feasibility, offering a basis

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under which some of the fundamental concepts in management accounting, especially those in variance analysis, can be extended and integrated with those of financial accounting.

### WEALTH AND INCOME

The key to an extension of any system lies in understanding the logic behind the existing system. In particular, it is important to understand what single-entry bookkeeping was and what was added to it when the system was extended to double-entry. The conclusion on this issue, as elaborated in Ijiri [1982], is the following: Single-entry bookkeeping dealt with only stock accounts, such as assets and liabilities, while double-entry bookkeeping extended it to also include flow accounts, such as revenues and expenses, under an interlocking, articulated framework.<sup>1,2</sup>

The role of flow accounts in double-entry bookkeeping is to explain or to "account for" changes in the net balance in stock accounts based on the underlying causes that are considered to be responsible for the change. A double-entry equation,

$$\Delta Stock_n = Flow_n \quad (1)$$

expresses the requirement that changes (denoted by  $\Delta$ ) in the net balance of stock accounts that occurred in period  $n$  be fully accounted for by a set of flow accounts in the same period. A change in the stock is an observed phenomenon; the flow explains why the change has happened.<sup>3</sup>

Once we understand that the salient feature of double-entry bookkeeping lies in the integration of flow accounts with stock accounts, its logical extension to triple-entry bookkeeping becomes clear, namely, it means an integration of a new set of accounts designed to explain changes in *flow* accounts.

earlier monograph [1982] on triple-entry bookkeeping were most valuable, and to Jeffrey Rickman of Carnegie-Mellon University, whose comments on an earlier draft of this paper from the standpoint of mechanics were very helpful. The author is also indebted to the editor and two anonymous reviewers whose editorial comments and suggestions resulted in a marked improvement of the manuscript.

<sup>1</sup> The integration of income accounts in the bookkeeping framework made double-entry bookkeeping distinctly different from single-entry bookkeeping. See Littleton [1933, pp. 22-40], who emphasizes profit and loss and other nominal accounts as the indispensable component in double-entry bookkeeping. See also Pacioli's [1494] use of revenue and expense accounts and the Profit and Loss (*pro e danno*) account as a summary in the overall bookkeeping framework. The practice of double-entry bookkeeping, however, seems to have existed in Italy for more than a century before Pacioli's *Summa* was published: See, for example, Peragallo [1938] and deRoover [1956]. There are different views on the distinguishing feature of double-entry bookkeeping. See, for example, Yamey [1956, pp. 7-9] and Goldberg [1965, pp. 215-219]. For a more detailed discussion on the nature of double-entry bookkeeping in contrast to single-entry bookkeeping, see Ijiri [1982] and references therein.

<sup>2</sup> It is true that in double-entry bookkeeping, assets are recorded as debits, while liabilities (negative assets) are recorded as credits. Also, an asset increase is recorded as a debit, while its negation, an asset decrease, is recorded as a credit. The converse is true with a liability change. Such an expression of negativity by transposition, however, is not essential in double-entry bookkeeping, since nothing whatsoever is lost if all stock accounts (assets and liabilities as well as their changes) are recorded as debits and all flow accounts (revenues and expenses as well as their adjustments) are recorded as credits. It is interesting to note that at the time of Pacioli's [1494] publication, mathematicians did not recognize negative numbers as a full-fledged mathematical entity, which did not come about until nearly 1600 (see Kline [1972, p. 252]). As a result, Pacioli seems to have taken pains to avoid the use of negative numbers, using transposition to mean negativity (see Peters and Emery [1980]). Actually, in this age of personal computers, there is no reason to avoid the use of negative numbers, especially if by doing so a more homogeneous classification of accounts is achieved. However, without transposition it is possible to have journal entries, such as a purchase of inventories, in which all accounts are on the debit side and no accounts show up in the remaining columns. This is not a violation of the double-entry or triple-entry equation since all sides of the entry still add to the same number, zero.

<sup>3</sup> Using the terms in Hempel and Oppenheim [1948],  $\Delta Stock_n$  is the "explanandum" which is explained by  $Flow_n$ , or the "explanans." For example, an enterprise owning only cash had a decrease in cash from \$30,000 to \$20,000. Why did it happen? Flow accounts, collectively

To comprehend what this extension means in the accounting environment, it is useful to translate stock and flow accounts into accounting terms, "wealth" and "income." Wealth is assets less liabilities. Income is revenues less expenses. There are other flow accounts such as contributions by owners or distributions to owners, as well as such flow accounts as purchases, collections, or payments, which do not normally give rise to a change in wealth. But since income-producing activities are the primary source of wealth change of interest, flow accounts other than income will be set aside for simplicity of illustration.

#### MOMENTUM, FORCE, AND IMPULSE

To extend the wealth-income dichotomy to a trichotomy, it is necessary to introduce the concept of "the rate at which wealth is changing," or equivalently "the rate at which income is being earned." The concept will be denoted by *momentum*. While both wealth and income are measured in monetary units such as dollars, momentum is measured in monetary units per period, such as dollars per year or per month. Unlike income, for which measurement requires specifying two points in time, momentum shows a state (of income being earned) at any single point in time. Income is then a realization of momentum as time passes.<sup>4</sup> Under a stable momentum, income is defined as momentum times "duration" of momentum.<sup>5</sup>

The concept of momentum is actually used in business, although normally the difference between momentum and income is not explicitly made. For example, when an executive says "this investment (or that division or that asset) is earning \$60 million a year," what the executive refers to is momentum, not income.<sup>6</sup>

Because momentum may be defined and measured at any single point in time, it is conceivable to prepare a "balance sheet" in which all revenues and expenses are treated as stock accounts and measured in momentum. Such measurements may be based on a detailed analysis of asset and liability items in terms of their ability to contribute to the earning process. In this analysis, momentum for monetary assets and liabilities is easy to determine using the pertinent interest rate. For nonmonetary items, an analogy with the historical cost principle may be taken, and we may adopt what may be called the "historical momentum principle." Under this principle, the momentum of an item acquired is set equal to the momentum of an item given up in exchange, until the change in momentum is confirmed by realization, such as actual sale or lease or else a contract to sell or lease at a fixed price or rate.<sup>7</sup>

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adding to  $-\$10,000$ , explain the reason. The next step will be to treat the loss of  $\$10,000$  as an explanandum and try to find its explanans.

<sup>4</sup> If wealth at time  $T$  is denoted by  $W_T$ , income between time  $T'$  and  $T$  is  $W_T - W_{T'}$ , while momentum is  $dW/dt$  evaluated at  $t = T$ .

<sup>5</sup> An investment of  $\$10,000$  yielding interest at the rate of one percent per month has a momentum of  $\$100/\text{mo}$ . If the momentum is stable at this rate, its realization over a year (a one-year duration) will result in an income of  $\$1,200$ . In mechanics, momentum is defined as mass ( $m$ ) times velocity ( $v$ ). Analogously, in the accounting interpretation, momentum may be defined as investment times yield, e.g.  $\$100/\text{mo} = \$10,000 \times 1\%/\text{mo}$ .

<sup>6</sup> A car driven at 60 miles an hour at a given point in time may or may not go 60 miles in the next hour since the speed need not remain constant.

<sup>7</sup> The historical momentum principle implies appreciation or depreciation in book value at a constant rate, hence it conflicts with the historical cost principle which results in a constant book value. A translation of such methods as FIFO, LIFO, the lower-of-cost-or-market method, and various depreciation methods also generates a different result when applied to momentum compared with their application to wealth. Comparisons and reconciliations of the differences between accounting principles and standards as they are applied to wealth and as they are applied to momentum offer an interest-

Consider the fact that momentum itself changes over time. Just as the rate of change in wealth is measured in momentum, the rate of change in momentum must now be measured in another concept.<sup>8</sup> Because there is no such concept currently established in accounting, we may seek a concept in mechanics that fits in the definition. That is the concept of *force*. Force is measured as a rate of change in momentum. Hence, in the accounting interpretation, it is expressed in monetary units per period squared, such as dollars per month per month.<sup>9</sup>

Although both momentum and force are undoubtedly terms that are not familiar to accountants and managers, such an introduction of new terms is unavoidable in an extension of a system like double-entry bookkeeping. Fortunately, both terms have common-sense interpretations that fit very well with the technical meaning we are seeking.<sup>10</sup>

Since momentum shows a state at a given point in time, equation (1) may be applied to momentum, treating it as stock. Namely, we want to account for changes in the net balance of momentum by means of a set of accounts designed to express the reasons for the change in momentum. Here again, since there is no convenient term that currently exists in accounting to describe a set of such accounts, a comparable term in mechanics may be considered.

There are two requirements that the new concept must satisfy. Just as wealth and income are both in the same measurement unit, for example, dollars, momentum and this new concept must be in the same measurement unit, for example, dollars per month. Second, just as income is a realization of momentum and is measured by momentum times duration when the momentum is stable, the new concept is a realization of

force and is measured by force times duration when the force is stable. A concept in mechanics that satisfies both requirements is called *impulse*.<sup>11</sup>

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Table 1 shows the relationship among wealth, momentum, income, force, im-

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ing subject of study, enhancing our understanding of what these accounting principles and standards mean in a broader context.

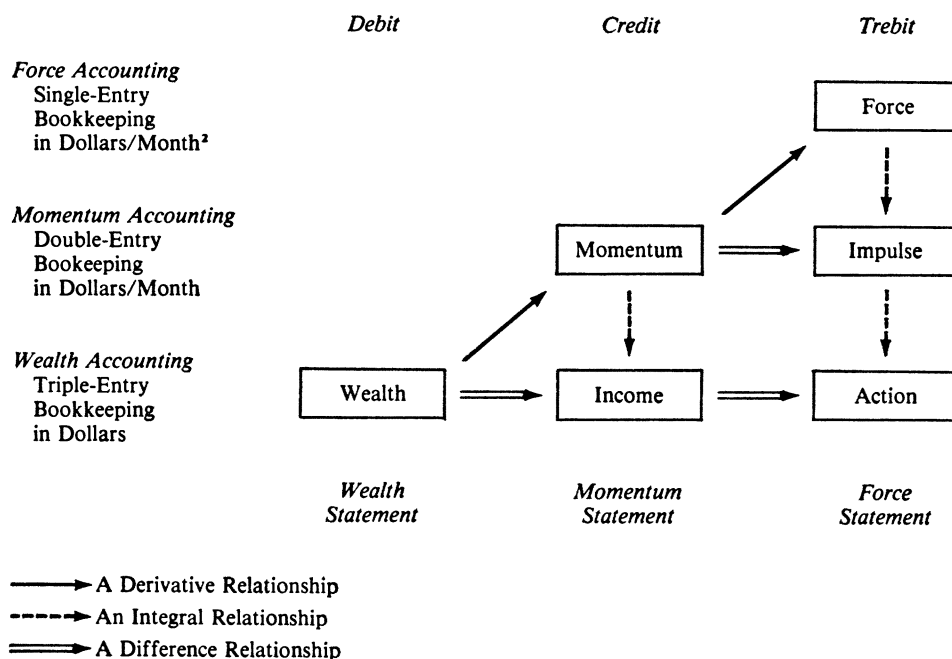
<sup>8</sup> "Why did our momentum go down from \$5 million a month to \$4 million a month?" an executive may ask; "What are the contributing factors?" Often casual explanations are given in monthly management reports to the board of directors. We need to formalize the reasoning behind the attribution to various factors in the form of accounts.

<sup>9</sup> In mechanics, force is defined as mass ( $m$ ) times acceleration ( $a$ ). In the accounting interpretation, force may be defined as investment times acceleration (rate of change in yield), e.g.,  $\$10/\text{mo}^2 = \$10,000 \times 0.1\%/\text{mo}^2$ , namely the yield, stated as a fraction per month, increases by 0.1%/mo for each one-month duration of, say, an inflationary force. If wealth is  $W$ , momentum is  $M$ , and force is  $F$ , then  $M = dW/dt$ , and  $F = dM/dt = d^2W/dt^2$ . Here, the measurement of wealth is the primary measurement and the measurement of momentum and that of force are derived measurements. It has been observed, however, that the "best" measurement for one of the three need not be the "best" measurement for the remaining two. See Ijiri and Noel [1984] for a detailed discussion on the choice of measurement for each of the three dimensions, comparing the historical cost and the current cost measurements.

<sup>10</sup> Concepts and relationships among them in an original field must of course fit in nicely in the applied field; but it is also nice if the concepts and relationships are not inconsistent with their common-sense interpretations.

<sup>11</sup> In mechanics, impulse is measured as force times duration. It can also be shown that it is equivalent to mass times velocity, just as momentum is. Although in mechanics impulse is used in particular for situations where duration is short, in our accounting application we may apply it more generally, irrespective of the length of duration. For a detailed treatment of the concepts and measurements of momentum, force, and impulse in Newtonian mechanics, see, for example, French [1971]. The analogy with Newtonian mechanics should, however, be pursued only to obtain some valuable insights into how the triple-entry bookkeeping framework should be developed. A wholesale translation without regard to its applicability to accounting is dangerous, since after all, physics and accounting are two different fields of endeavor.

TABLE 1  
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pulse, and one more concept yet to be explained, namely action. On the bottom row of Table 1 we have the familiar two dimensions in wealth accounting, that is, wealth and income, setting aside the third dimension of action for now. The two dimensions are tied together by the difference relationship,  $\Delta\text{Wealth} = \text{Income}$ , both of which are stated in monetary units such as dollars.

We now move to the second row of Table 1, which expresses the double-entry equation (1) applied to momentum, namely  $\Delta\text{Momentum} = \text{Impulse}$ . That is, changes in momentum at any two points in time are fully accounted for by impulse, whose accounts express the underlying forces that are judged to be responsible for the momentum change. Both momentum and impulse are stated in monetary units per period.

This particular layer of accounting, called momentum accounting, is analogous to the conventional double-entry framework for wealth and income, except that the primary object of accounting is momentum instead of wealth.

The top row of Table 1 shows the dimension for force accounting. Its only dimension is force, which is stated in monetary units per period squared. While conceptually it is possible to define a concept that deals with a rate of change in force, a higher dimension is not considered at this time. (Such a concept will become necessary if the bookkeeping system needs to be further extended to quadruple-entry bookkeeping.) Hence, force accounting has to be based on a single-entry bookkeeping system.

Two types of relationships exist across



different layers of accounting. One is the derivative relationship that exists between wealth and momentum as well as between momentum and force. One deals with the rate of change in the other, and mathematically one is the time derivative of the other. This relationship is shown in Table 1 by diagonal arrows. The other is an integral relationship connecting concepts vertically in Table 1, such as momentum-income and force-impulse (as well as impulse-action, which will be explained later). For a constant momentum or force, income is momentum times duration and impulse is force times duration, as mentioned earlier. More generally, income is measured as an integration of momentum, and impulse is measured as an integration of force, both over a given period of time.

The diagonal, derivative relationship and the vertical, integral relationship that tie wealth, momentum, and income are the converse of each other if the integration constant is set equal to the beginning balance of wealth. Hence, the equality holds between the wealth dimension and the income dimension when the beginning balance of wealth is added to the latter. Here, the horizontal relationship between wealth and income is a difference relationship, as against the derivative and integral relationships discussed above, since journal entries are made at discrete points in time to change wealth and income accounts.

The same triangular relationship may be observed among momentum, force, and impulse. The derivative relationship between momentum and force, the integral relationship between force and impulse, and the difference relationship between momentum and impulse are all analogous to their counterpart observed in the triangular relationship among wealth, momentum, and income.

One thing that should be emphasized in the vertical linkage of the three types

of accounting (wealth, momentum, and force accounting) is the importance of the timing of momentum change in assessing its impact on wealth. In conventional wealth accounting, one dollar of income increases wealth by exactly one dollar regardless of when it is earned, if we set aside a compounding effect (one dollar's worth of wealth earned earlier in a period may contribute to further earnings during the rest of the period). In momentum accounting, a \$1/month increase in momentum occurring at the beginning of a year yields a \$12 increase in both wealth and income, again setting aside the compounding effect, while the same momentum increase occurring at the end of the year has a zero impact on this year's income.

Normally, an intraperiod timing difference in income recognition has no impact on the income for the period and on the ending wealth. Similarly, an intraperiod timing difference in impulse recognition (recognition of changes in momentum) has no impact on the impulse for the period and on the ending momentum, yet such a timing difference significantly affects the lower level concepts in Table 1, namely income for the period and ending wealth.

For this reason, in momentum and force accounting the timing of journal entries recognizing a change in momentum or force is as important as the amount of the entry. Such a sensitivity to timing of the change (another way of stating this is the duration of a given level of momentum or force) is a new element in accounting, observed in momentum and force accounting but not in conventional wealth accounting.<sup>12</sup>

<sup>12</sup> The fact that the intraperiod timing difference in income recognition is not important in conventional wealth accounting does not mean that the underlying mathematical properties of wealth accounting are different from those of momentum or force accounting. They are all identical in terms of their mathematical

Now consider the third dimension in wealth accounting under a triple-entry bookkeeping system. Since  $\Delta \text{Momentum} = \text{Impulse}$ , and momentum integrates to income, impulse should also integrate to something that ties in with income under a suitable selection of integration constants. We shall use the term "action" to designate such a concept that refers to the realization of impulse.<sup>13</sup> Action, therefore, shows an ultimate effect of force and is measured in monetary units just as wealth or income is. In addition, just as income requires beginning wealth as the integration constant to maintain the equality between the wealth dimension and the income dimension, action requires the same beginning wealth as the integration constant. But action requires something more. As stated before, the equality between the momentum dimension and impulse dimension can be maintained only after the beginning momentum is added as the integration constant. Hence, when the impulse is integrated along with the beginning momentum, the result shows a division of income in the current period into two components: (1) the portion of income that is a realization of the momentum that existed at the beginning, and (2) the portion of income that is generated as a result of various changes in the momentum during the period.

Thus, in the action dimension we have two integration constants: (1) the beginning wealth and (2) a realization of the beginning momentum. The difference between the ending wealth and the sum of these two constants is the contribution of forces exerted during the period, measured in terms of their impact on wealth that was realized during the period. Such current period contributions of forces are classified into various force accounts based on the types of forces that are judged to be responsible for the momentum change.

Reading Table 1 column-wise, the six dimensions of triple-entry bookkeeping may be divided into three categories: wealth in the first column (debit); momentum and income in the second column (credit); and force, impulse, and action in the third column (which we shall call "trebit").<sup>14</sup> Each category is summarized in the respective statement (wealth, momentum, and force statements), an example of which is shown later in Table 4. The wealth statement is similar to the assets and liabilities portion of the existing balance sheet. Its bottom line shows the net wealth at the end of the period, classified into various types of assets and liabilities.

The momentum statement has two columns, one for momentum and the other for income. In the momentum column, various revenues and expenses are stated in terms of their respective momentum evaluated at the end of the period. The income column is like the conventional income statement. It shows the realization of revenue and expense

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properties. It just so happens that the concept of an integral of wealth is not widely used in management, except in such areas as interest computations where the timing of deposits and withdrawals is important. In fact, an integral of wealth from the beginning to the end of a period gives rise to the notion of "wealth utilization" during the period, which may have important applications in the determination of the rate of return. (Currently most such needs are met by just taking the average of the beginning and the ending balances of assets or investment.) In such a measurement, an intraperiod timing difference in income recognition can become an important issue.

<sup>13</sup> In mechanics there does not seem to be any concept that specifically refers to the realization of impulse. The term "action" has been chosen here since it describes the managerial aspect of the relationship with wealth and income in the most concise manner, although unfortunately the term has already been used in mechanics to designate another, totally different, concept.

<sup>14</sup> Out of six letters in "trebit," four letters are in the same position as "debit" and also four letters are in the same position as "credit." Hence, the new term has not only a satisfactory connotation for the third column ("tre" for three and "bit" for piece) but also has phonetic consistency with the existing two terms.

momenta during the period along with the beginning balance of owners' equity accounts so that the sum of all items in this column equals the net wealth at the end of the period.

The force statement has three columns, the first one for force, the second one for impulse, and the third one for action. The force column shows the end-of-period balance of various force accounts that are in the process of affecting momentum. They are recorded under a single-entry system. The impulse column shows the impact on momentum of forces exerted during the period. When the beginning momentum is added to the sum of items in the impulse column, the total should agree with the ending momentum balance in the momentum statement. The action column shows the impact on wealth of various forces exerted during the period. The sum of this impact, beginning wealth, and the realization of the beginning momentum during the period equals the ending wealth.

Note that wealth accounting under a triple-entry bookkeeping system can be implemented without the higher-level structure such as momentum accounting and force accounting. After all, judgments on income and action can be made on a purely subjective basis if necessary. The super-structure placed on wealth accounting, however, puts discipline over such judgments and offers a "framework" to add logical strength to the wealth accounting system.

#### AN EXAMPLE

A simple example illustrating the basic relationship among the pertinent dimensions is now presented to provide some idea of the structure of the triple-entry bookkeeping system. After a brief discussion of transactions, a worksheet, journal entries, and financial statements

are presented in Tables 2-4. In Table 5 a list of a variety of force accounts that may be used in triple-entry bookkeeping is provided. Many such items have been used in monthly management reports to the executives and the board of directors as well as in variance analysis of factory costs, describing why income or cost has gone up or down relative to the previous period or relative to the budget. Triple-entry bookkeeping makes such judgments by accountants an integral part of the bookkeeping system so that the judgments are carried out systematically, uniformly, and consistently.

XYZ Company engages in retailing of bottled mineral water, including deliveries. At the end of month zero, the company's assets are \$30 in cash, \$180 in receivables, and \$340 in inventories, while its liabilities are \$50 in payables (000's omitted.) There are no fixed asset accounts since the warehouse, the office building, and delivery trucks are all leased. The company's wealth at the end of month zero is, therefore, \$500. (See the first row in Table 2.) In addition, as of the end of month zero, the company has a net momentum of \$10/month, which is \$100/mo in sales less \$90/mo in expenses. The latter consists of \$50/mo in cost of sales, \$30/mo in operating expenses, and \$10/mo in income taxes. (See the first column in Table 2.) Thus, in month one management inherits the enterprise legacy in the form of a \$500 wealth and a \$10/mo momentum.

At the beginning of month one, management plans to introduce a new product, carbonated mineral water. Management is most interested in knowing the impact of this action on momentum as well as on wealth. Also, management wants to keep an eye on three factors, namely (1) margin, (2) volume, and (3) efficiency of delivery workers, so that corrective actions may be taken immediately when



they are needed. Margin is gross margin per unit of sales and volume is sales volume. In addition to these two key elements of operation, efficiency of delivery workers is an important factor, since it causes a significant fluctuation in the operating expense. The wage rate has been fixed by a union contract and is not expected to change in the near future. Other operating expenses are expected to be stable, and the tax rate is set at 50 percent.

In view of these factors in the management environment, it is decided to open four force accounts: New Product, Margin Variance, Volume Variance, and Efficiency Variance. Normally, there is a need for a miscellaneous force account which takes care of insignificant or unexplainable changes in momentum, but in this example, it is assumed that the above four accounts are sufficient. Since management's concern is after-tax figures, all force accounts are expressed in terms of their impact on the after-tax momentum and wealth.<sup>15</sup>

It is also decided to sequence force accounts in exactly the way in which they are shown in Table 2 so that any joint effects would be grouped together in the force account that appears earlier in the sequence. Hence, for example, all of the volume effect of new product introduction is included in the new product account so that the volume variance account shows only the impact of volume change in the old product. Similarly, the efficiency variance account shows the impact of labor usage after adjusting for the impact of the new product introduction and the volume change in the old product. One month has been selected as the basic time unit by which to measure momentum. The accounting period in this example is considered to be only two months long instead of a more standard 12 months.

Activities in month one are summarized on the top half of Table 2. It shows an increase in momentum from \$10/mo to \$15/mo, to a large extent due to the new product. For simplicity, it is assumed that all momentum changes occur at the beginning of each month. Table 2 also shows that the momentum and its change increased wealth by \$15 in month one. Activities in month two are likewise summarized in the same table. The bottom portion of the table summarizes the impact of force on momentum as well as on wealth. Table 3 shows a trial balance along with aggregate journal entries in each of the two months. Table 4 gives wealth, momentum, and force statements covering the two-month period. Table 5 provides examples of force accounts, classified by their types.

#### SUMMARY

In summary, this paper introduced three layers of accounting (wealth, momentum, and force), each with its own measurement unit (dollars, dollars per month, and dollars per month per month), which interlock with each other through the derivative/integral relationship. The third dimension of triple-entry bookkeeping has thus been developed not only at the dollar measurement level, which articulates with the conventional wealth and income dimensions, but also at the level of its first and second derivatives.

The conventional accounting identity,  $\text{debit} = \text{credit}$ , is therefore extended "horizontally" along the same measurement unit to  $\text{debit} = \text{credit} = \text{trebit}$ ; at the

<sup>15</sup> Although margin, volume, and efficiency variances seem to have no direct relationship with forces or actions in management as such, they are used here because of their familiarity in cost accounting. As we gain more experience in dealing with force accounts, they may be regrouped to tie in more closely with specific managerial forces and actions as shown by examples in Table 5.

TABLE 2  
WORKSHEET FOR WEALTH, MOMENTUM, AND FORCE

	Force Accounts				Wealth Accounts						
	Beginning Momentum	New Prod.	Margin Var.	Volume Var.	Effic. Var.	Ending Momentum	Cash	Receivables	Inventories	Payables	Total Wealth
Month 1											
Beginning Wealth											
Income Accounts											
Sales	\$100/mo	\$20/mo	\$12/mo	\$8/mo		\$140/mo	\$30	\$180	\$340	-\$50	\$500
Cost of Sales	-50	-8	-10	-4		-72	40	100	-72		140
Operating Expenses	-30	-4		-2	-\$2/mo	-38				-38	-38
Income Taxes	-10	-4	-1	-1	1	-15				-15	-15
Net Income	\$10/mo	\$4/mo	\$1/mo	\$1/mo	-\$1/mo	\$15/mo					
Other Flow Accounts											
Purchases									80	-80	
Collections							90	-90		70	
Payments							-70				
Ending Wealth							\$90	\$190	\$348	-\$113	\$515
Month 2											
Beginning Wealth											
Income Accounts											
Sales	\$140/mo	\$6/mo	-\$8/mo	\$12/mo		\$150/mo	\$90	\$190	\$348	-113	\$515
Cost of Sales	-72	-2	2	-6		-78	30	120	-78		150
Operating Expenses	-38			-2	\$4/mo	-36				-36	-36
Income Taxes	-15	-2	3	-2	-2	-18				-18	-18
Net Income	\$15/mo	\$2/mo	-\$3/mo	\$2/mo	\$2/mo	\$18/mo					
Other Flow Accounts											
Purchases									75	-75	
Collections							110	-110		122	
Payments							-122				
Ending Wealth							\$108	\$200	\$345	-\$120	\$533

TABLE 2—(Continued)

<i>Summary: Force (Ending Balance)</i>		$\$2/\text{mo}^1$	$-\$3/\text{mo}^2$	$\$2/\text{mo}^2$	$\$2/\text{mo}^2$
<i>Impulse (Impact on Momentum)</i>					
Month 1		$\$4/\text{mo}$	$\$1/\text{mo}$	$\$1/\text{mo}$	$-\$1/\text{mo}$
Month 2		2	-3	2	2
Total	$\$10/\text{mo}$	$\$6/\text{mo}$	$-\$2/\text{mo}$	$\$3/\text{mo}$	$\$1/\text{mo}$
<i>Action (Impact on Wealth)</i>					$\$18/\text{mo}$
Month 1	$\$10$	$\$4$	$\$1$	$\$1$	$-\$1$
Month 2	$\$10$	6	-2	3	1
Total	$\$20$	$\$10$	$-\$1$	$\$4$	$\$0$
					Ending Wealth
					Beginning Wealth
					Net Increase
					$\$533$
					500
					$\$33$

All momentum changes are assumed to have occurred at the beginning of each month.

TABLE 3  
TRIAL BALANCE AND JOURNAL ENTRIES

<i>Debit (Wealth)</i>		<i>Credit (Income)</i>		<i>Trebit (Action)</i>	
<i>Trial Balance at the End of Month 0</i>					
Cash	\$30	Beginning Wealth	\$500	Beginning Wealth	\$500
Receivables	180				
Inventories	340				
Payables	-50				
Total	<u>\$500</u>	Total	<u>\$500</u>	Total	<u>\$500</u>
<i>Journal Entries in Month 1 (Total)</i>					
Cash	\$60	Sales	\$140	Beginning Momentum	\$10
Receivables	10	Cost of Sales	-72	New Product	4
Inventories	8	Operating Expenses	-38	Margin Variance	1
Payables	-63	Income Taxes	-15	Volume Variance	1
				Efficiency Variance	-1
Total	<u>\$15</u>	Total	<u>\$15</u>	Total	<u>\$15</u>
<i>Journal Entries in Month 2 (Total)</i>					
Cash	\$18	Sales	\$150	Beginning Momentum	\$10
Receivables	10	Cost of Sales	-78	New Product	6
Inventories	-3	Operating Expenses	-36	Margin Variance	-2
Payables	-7	Income Taxes	-18	Volume Variance	3
				Efficiency Variance	1
Total	<u>\$18</u>	Total	<u>\$18</u>	Total	<u>\$18</u>
<i>Trial Balance at the End of Month 2</i>					
		Beginning Wealth	\$500	Beginning Wealth	\$500
Cash	\$108	Sales	290	Beginning Momentum	20
Receivables	200	Cost of Sales	-150	New Product	10
Inventories	345	Operating Expenses	-74	Margin Variance	-1
Payables	-120	Income Taxes	-33	Volume Variance	4
				Efficiency Variance	0
Total	<u>\$533</u>	Total	<u>\$533</u>	Total	<u>\$533</u>

same time a new accounting identity is introduced to link measurements "vertically" along the derivative/integral relationship using the fact that rate of change times duration of the rate equals the amount of change.

In this way, the extension of double-entry bookkeeping to triple-entry book-

keeping is carried out under a disciplined framework of measurements, which hopefully will direct management's attention and sensitivity to factors at a level deeper than the level of wealth and income that has been traditionally dealt with by double-entry accounting.



TABLE 4  
WEALTH, MOMENTUM, AND FORCE STATEMENTS

Wealth Statement			
Wealth Accounts		Wealth Ending Balance	
Cash		\$108	
Receivables		200	
Inventories		345	
Payables		-120	
Total		<u>\$533</u>	
Momentum Statement			
Momentum Accounts	Momentum: Ending Balance	Income: Impact on Wealth	
Sales	\$150/mo	\$290	
Cost of Sales	-78	-150	
Operating Expenses	-36	-74	
Income Taxes	-18	-33	
Total	<u>\$18/mo</u>	<u>\$33</u>	
Beginning Wealth		500	
Ending Wealth		<u>\$533</u>	
Force Statement			
Force Accounts	Force: Ending Balance	Impulse: Impact on Momentum	Action: Impact on Wealth
New Product	\$2/mo <sup>2</sup>	\$6/mo	\$10
Margin Variance	-3	-2	-1
Volume Variance	2	3	4
Efficiency Variance	2	1	0
Total	<u>\$3/mo<sup>2</sup></u>	<u>\$8/mo</u>	<u>\$13</u>
Beginning Momentum		10	20
Ending Momentum		<u>\$18/mo</u>	
Net Income			\$33
Beginning Wealth			500
Ending Wealth			<u>\$533</u>

TABLE 5  
EXAMPLES OF FORCE ACCOUNTS

<i>A. Internal Forces</i>	<i>B. External Forces</i>
<ol style="list-style-type: none"> <li>1. <i>Investment Forces</i> <ol style="list-style-type: none"> <li>a. Acquisitions</li> <li>b. Plant Openings</li> <li>c. Plant Expansions</li> <li>d. Securities Investments</li> </ol> </li> <li>2. <i>Divestment Forces</i> <ol style="list-style-type: none"> <li>a. Segment Disposals</li> <li>b. Plant Closings</li> <li>c. Product Discontinuance</li> <li>d. Inventory Clearance</li> </ol> </li> <li>3. <i>R&amp;D Forces</i> <ol style="list-style-type: none"> <li>a. R&amp;D Discoveries</li> <li>b. Patent Licensing</li> <li>c. Product Development</li> <li>d. Process Development</li> <li>e. Mineral Discoveries</li> </ol> </li> <li>4. <i>Labor Forces</i> <ol style="list-style-type: none"> <li>a. Hiring</li> <li>b. Firing</li> <li>c. Training</li> <li>d. Strikes</li> <li>e. Productivity</li> <li>f. Wage Rates</li> </ol> </li> <li>5. <i>Production Forces</i> <ol style="list-style-type: none"> <li>a. Process Improvements</li> <li>b. Product Quality Changes</li> <li>c. Crash Production</li> <li>d. Industrial Accidents</li> <li>e. Contaminations</li> <li>f. Spoilage</li> <li>g. Stockouts</li> </ol> </li> <li>6. <i>Marketing Forces</i> <ol style="list-style-type: none"> <li>a. New Product Campaigns</li> <li>b. Advertising Campaigns</li> <li>c. Price Changes</li> <li>d. Market Research</li> <li>e. Designs and Packaging</li> </ol> </li> <li>7. <i>Financing Forces</i> <ol style="list-style-type: none"> <li>a. Borrowing</li> <li>b. Refunding</li> <li>c. Stock Issues</li> <li>d. Stock Retirements</li> <li>e. Conversions</li> <li>f. Debt Restructuring</li> <li>g. Hedging</li> </ol> </li> </ol>	<ol style="list-style-type: none"> <li>8. <i>Competitive Forces</i> <ol style="list-style-type: none"> <li>a. Competitors' New Products</li> <li>b. Competitors' New Services</li> <li>c. Competitors' Promotions</li> <li>d. Price Wars</li> </ol> </li> <li>9. <i>Economic Forces</i> <ol style="list-style-type: none"> <li>a. Seasonal</li> <li>b. Cyclical</li> <li>c. Growth Trend</li> <li>d. Inflation Rate</li> <li>e. Unemployment Rate</li> </ol> </li> <li>10. <i>Governmental Forces</i> <ol style="list-style-type: none"> <li>a. Tax Reforms</li> <li>b. Subsidies</li> <li>c. Interest Rates</li> <li>d. Product Safety Control</li> <li>e. Environmental Control</li> <li>f. Zoning and Permits</li> <li>g. Consent Decrees</li> </ol> </li> <li>11. <i>International Forces</i> <ol style="list-style-type: none"> <li>a. Wars</li> <li>b. Nationalization</li> <li>c. Embargoes</li> <li>d. Currency Restrictions</li> <li>e. Quotas</li> <li>f. Import Duties</li> <li>g. Foreign Exchange Rates</li> <li>h. Foreign Interest Rates</li> <li>i. Foreign Inflation Rates</li> </ol> </li> <li>12. <i>Natural Forces</i> <ol style="list-style-type: none"> <li>a. Earthquakes</li> <li>b. Tornadoes</li> <li>c. Hurricane</li> <li>d. Severe Weather</li> <li>e. Lightning</li> </ol> </li> </ol> <p style="text-align: center;"><i>C. Residual Forces</i></p> <ol style="list-style-type: none"> <li>13. <i>Unattributed Forces</i> <ol style="list-style-type: none"> <li>a. Price Variance</li> <li>b. Quantity Variance</li> <li>c. Volume Variance</li> <li>d. Spending Variance</li> <li>e. Efficiency Variance</li> <li>f. Miscellaneous</li> </ol> </li> </ol>

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