Datorseende Assignment 1

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1 Points in Homogeneous Coordinates

1.1 Exercise 1

1. $x_1 = (2, -1)^T$
2. $x_2 = (-3, 2)^T$
3. $x_3 = (2, -1)^T$

It is the point (4, -2) on the infinity line. It is called a vanished point.

1.2 Computer Exercise 1

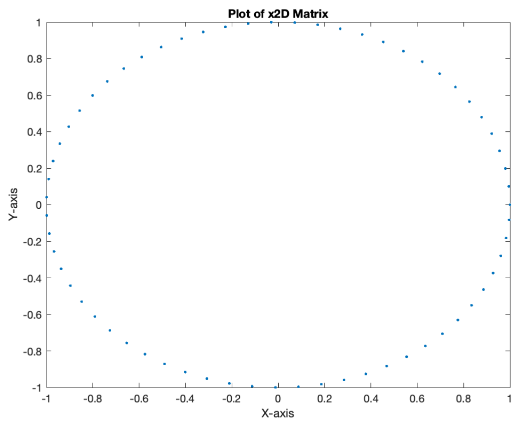

Figure 1: Plot of x2D

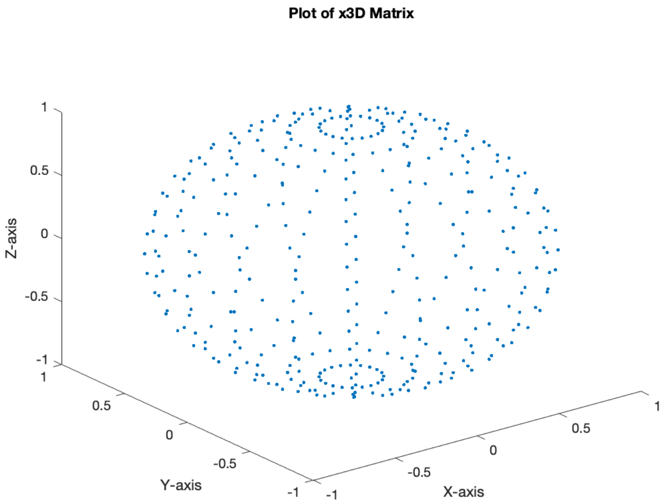

Figure 2: Plot of x3D

2 Lines

2.1 Exercise 2

1. $p_1 = (1, -2)$
2. $p_2 = (-2, 1)$
3. $l_1 = (1, -2)$

p_2 is interpreted as a vanishing point in the direction of (-2, 1)

2.2 Exercise 3

The null space constitutes the vectors which satisfy the relationship $Mx = 0$. Since our intersection satisfy the two equations in Exercise 2 and these equations constitute the rows in M, the null space contains the intersection. Even though $Mx = 0$ has an infinite number of solutions they all correspond to the same point in the projective space P^2, and hence there is no other non-zero point which satisify the equation.

2.3 Computer Exercise 2

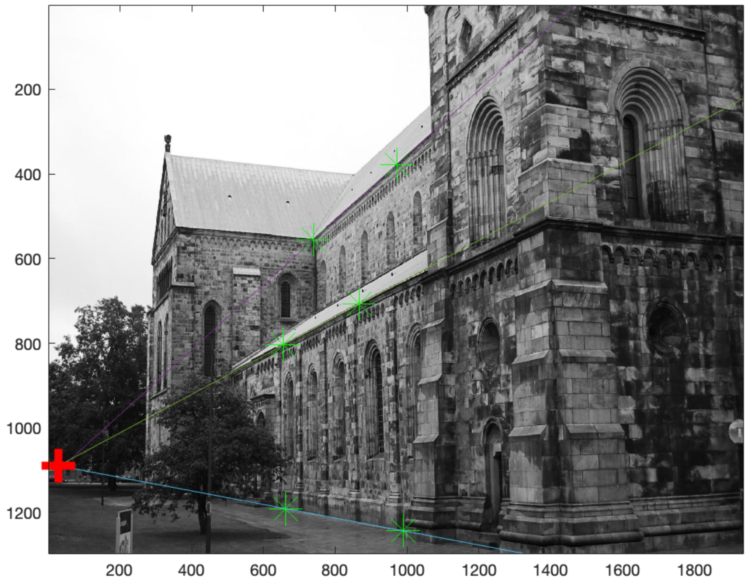

Figure 3: Lines, Points and a Intersection

The computed distance was 8.1950. It is close to zero because the lines are close to parallel in 3D.

2.4 Projective Transformations

2.5 Exercise 4

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Matrix multiplication yields:

$$y_1 \sim Hx_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad y_2 \sim Hx_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

I calculate the lines l_1 and l_2 by performing the cross product between respective x and y.

$$l_1 = y_1 \times y_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad l_2 = x_1 \times x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

W take a line $\mathbf{l}_1^\top \mathbf{x} = 0$. Applying the Matrix $\mathbf{H}$, we get $\mathbf{y} = \mathbf{H}\mathbf{x}$.

We then take a new line. $\mathbf{l}_2^\top \mathbf{y} = 0$. Let $\mathbf{l}_2^\top = \mathbf{l}_1^\top \mathbf{H}^{-1}$, then:

$$\mathbf{l}_2^\top \mathbf{y} = \mathbf{l}_1^\top \mathbf{H}^{-1} \mathbf{H}\mathbf{x} = \mathbf{l}_1^\top \mathbf{x} = 0. \tag{1}$$

2.6 Computer Exercise 3

Transformation	Properties
H_1	Similarity, Euclidean, Affine, Projective
H_2	Similarity, Affine, Projective
H_3	Affine, Projective
H_4	Projective

Table 1: Properties of the different H-matrices plotted below

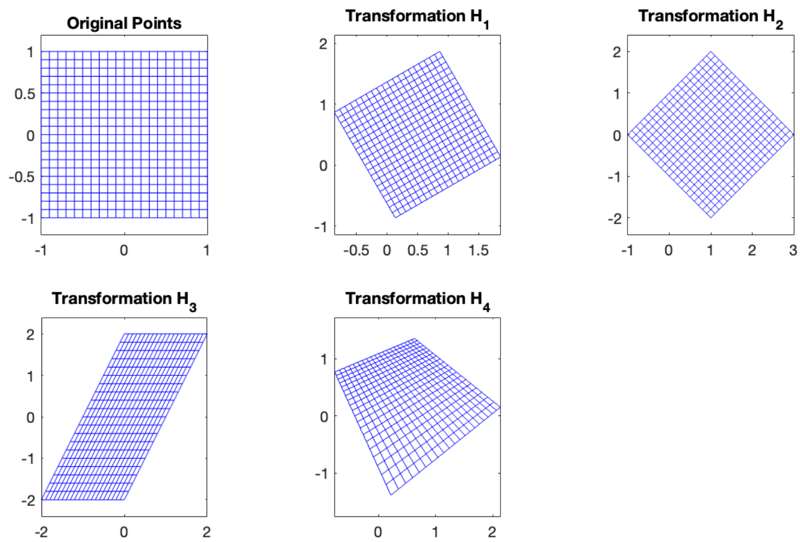

Figure 4: H transformations

3 The Pinhole Camera

3.1 Exercise 5

$$PX_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad PX_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad PX_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

PX_3 is a vanishing point in the direction of (1,1).

With the camera center defined as $C = -R^T t$ we get:

$$-R^\top \mathbf{t} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

The viewing direction is R_3^T, i.e the third row of the rotation matrix.

$$R_3^\top = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3.2 Computer Exercise 4

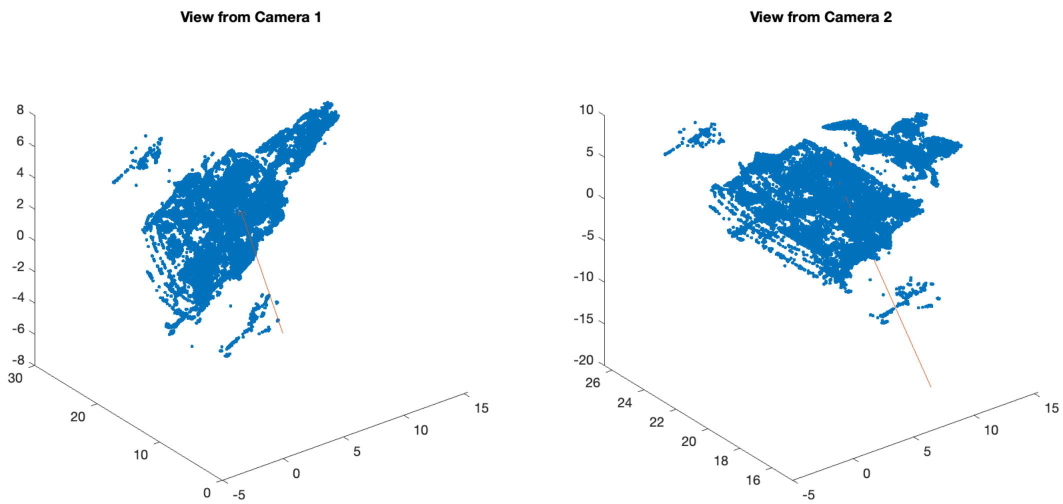

Figure 5: Modeling of Statue

For Camera 1 the camera center and viewing direction is:

$$C_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} V_1 = \begin{bmatrix} 0.312922810572239 \\ 0.946084666794662 \\ 0.0836846334735666 \end{bmatrix}$$

For Camera 2 the camera center and viewing direction is:

$$C_2 = \begin{bmatrix} 6.63520389848774 \\ 14.8459791894476 \\ -15.0691158460852 \\ 1 \end{bmatrix} V_2 = \begin{bmatrix} 0.0318638434603584 \\ 0.340165416823134 \\ 0.939825613971804 \end{bmatrix}$$