Datorseende Assignment 2

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1 Introduction

Solution to the exercises in assignment two of spring term course in Computer Vision 2024.

2 Calibrated vs. Uncalibrated Reconstruction.

2.1 Exercise 1

With projective reconstruction, equal accuracy is achieved regardless of any transformations applied to the camera equation.

$$\hat{X} = H^{-1}X \rightarrow \lambda\hat{X} = PHH^{-1}X = \hat{P}\hat{X}$$

Because $\hat{P} = PH$ is also a valid camera, transformations do not affect the accuracy of our projections in uncalibrated camera projections.

2.2 Computer Exercise 1

2.3 Exercise 2

In the case of euclidan projections, T needs to be a similarity matrix to achieve a valid solution. In euclidian projetcions we try to solve:

$$\lambda_{ij} x_i^{'} j = [R_i \ t_i] X_j$$

When applying a transformation to the rotational part of $[R_i \ t_i]$, a valid solution is only preserved if T is a similarity matrix as:

$$T = \begin{bmatrix} sQ & v \\ 0 & 1 \end{bmatrix}$$

where Q is a rotation. Multiplication with T yields:

$$T[R_i \ t_i]X_j = \begin{bmatrix} R_iQ & \frac{1}{s}(R_iv + t_J) \end{bmatrix} X_j$$

The solution is valid only since R_iQ is a rotational matrix.

In essence, since euclidian projections are euclidian transformations from 3D to 2D points, a transformation T can only have valid solutions as long as T itself is a euclidian transformation. Euclidian transformations are scaling, rotation, or mirrorring which none of distort projections. These euclidian transformations preserve angles and lines.

3 Camera Calibration

3.1 Exercise 3

For an upper triangular 3x3 matrix, the inverse has:

1. Diagonal elements as the reciprocals
2. Other elements such that $AA^{-1} = I$

Which yields:

$$K^{-1} * K = \begin{bmatrix} 1/f * f & 0 & \frac{-x_0}{f} * f + x_0 \\ 0 & 1/f * f & \frac{-y_0}{f} * f + y_0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

We verify A and B by multiplication:

$$AB = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} = K$$

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4 RQ Factorization and Computation of K

4.1 Exercise 5

With

$$K = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} \; and \; R = \begin{bmatrix} R_1^T \\ R_2^T \\ R_3^T \end{bmatrix}$$

simple matrix multiplication yields:

$$KR = \begin{bmatrix} a * R_1^T + b * R_2^T + c * R_3^T \\ d * R_2^T + e * R_3^T \\ f * R_3^T \end{bmatrix}$$

To find R and f, we extract A from the matrix $P = [Aa]$ and establish the matrix equation:

$$A = KR \rightarrow \begin{bmatrix} A_1^T \\ A_2^T \\ A_3^T \end{bmatrix} = \begin{bmatrix} a * R_1^T + b * R_2^T + c * R_3^T \\ d * R_2^T + e * R_3^T \\ f * R_3^T \end{bmatrix}$$

4.1.1 R_3 and f

Since R_3 is orthogonal and thus have length 1, R_3 and f can be calculated with the third row of A:

$$f = ||A_3|| \rightarrow R_3 = \frac{1}{||A_3||} A_3$$

which gives

$$f = 1 \; and \; R_3 = A_3 = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

4.1.2 R_2 and e

Since A_2 is a linear combination of the two vectors $d * R_2$ and $e * R_3$, e can be camputed as $e = A_2^T R_3$ which yields:

$$e = A_2^T R_3 = \begin{bmatrix} -\frac{700}{\sqrt{2}} & 1400 & \frac{700}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 350 + 0 + 350 = 700$$

With e, d is found by solving for:

$$dR_2 = A_2 - eR_3 = \begin{bmatrix} -\frac{700}{\sqrt{2}} \\ 1400 \\ \frac{700}{\sqrt{2}} \end{bmatrix} - 700 \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 0 \end{bmatrix}$$

and because R_2 is orthogonal with length 1:

$$d = 1400 \;\; \rightarrow \;\; R_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

4.1.3 R_1, a, b and c

To finally calculate R_1, a, b and c we solve the equation:

$$A_1 - bR_2 - cR_3 = aR_1$$

from which we get the equations:

$$c = A_1^T R_3 = \begin{bmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = -400 + 0 + 1200 = 800$$

$$b = A_1^T R_2 = \begin{bmatrix} -\frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0 + 0 + 0 = 0$$

and we can finally solve for a and R_1:

$$A_1 - bR_2 - cR_3 = aR_1 = \begin{bmatrix} \frac{800}{\sqrt{2}} \\ 0 \\ \frac{2400}{\sqrt{2}} \end{bmatrix} - \begin{bmatrix} -\frac{800}{\sqrt{2}} \\ 0 \\ \frac{800}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1600}{\sqrt{2}} \\ 0 \\ \frac{1600}{\sqrt{2}} \end{bmatrix} = 1600 \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

With this:

$$a = 1600, b = 0, c = 800, \ and\ R_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

4.1.4 The K matrix

With a-f known we construct the K matrix:

$$K = \begin{bmatrix} \lambda f & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1600 & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{bmatrix}$$

1. focal length = 1400
2. skew = 0
3. aspect ratio = 1600/1400 = 8/7
4. principal point = (800, 700)

4.2 Computer Exercise 2

$$K_{T1} = \begin{bmatrix} 2391.5 & 225.7 & 942.7 \\ 0 & 623.9 & 813.6 \\ 0 & 0 & 1.0 \end{bmatrix} \quad \text{and} \quad K_{T2} = \begin{bmatrix} 2394.0 & 0.0 & 932.4 \\ 0 & 2398.1 & 628.3 \\ 0 & 0 & 1.0 \end{bmatrix}$$

5 Direct Linear Transformation DLT

5.1 Exercise 7

To obtain the original solution P, we simply perform $P = N^{-1}\hat{P}$.

5.2 Computer Exercise 3