Datorseende Assignment 3

er8251da-s

Febuary 2024

1 Introduction

Solution to the exercises in assignment three of spring term course in Computer Vision 2024.

2 The Fundamental Matrix

2.1 Exercise 1

With $P_1 = [I\ 0]$ and $P_2 = [A\ t]$ we derive F as:

$$F = [t]_x A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix}$$

$$l = Fx = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

To calculate points on the epipolar line we want to satisfy the equation:

$$\hat{x}Fx = 0$$

which translates to

$$\begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = 0 \ \rightarrow\ 2\hat{x}_1 = 4\hat{x}_3$$

In exercise 1, points (2,0) and (2,1) satisfy the equation and are thus possible projections.

2.2 Exercise 2

To compute the camera centers we solve the nullspace of P_1 and P_2. For $P_1 = [I\ 0]$ the camera center is simply [0 0 0 1]. For P_2 we have:

$$\begin{cases} X + Y + Z + 2W = 0 \\ 2Y + 2W = 0 \\ Z = 0 \end{cases} \iff \begin{cases} X = t \\ Y = t \\ Z = 0 \\ W = -t \end{cases} \tag{1}$$

which gives that the camera center is $C_2 = [-1 \ \ -1 \ 0 \ 1]^T$ in homogenous coordinates.

Now we can compute the epipolar lines by $e_1 = P_1C_1$ and $e_2 = P_2C_2$.

$$e_1 = P_1C_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \quad and, \quad e_2 = P_2C_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

The fundamental matrix F can again be computed by $[t]_x A$:

$$F = [t]_x A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix}$$

We verify $e_2^T F = 0$ and $Fe_1 = 0$ with,

$$e_2^T F = \begin{bmatrix} 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & -2 & 2 \end{bmatrix} = \mathbf{0}$$

$$Fe_1 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & -2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \mathbf{0}$$

Lastly, let's verify the determinant of F.

$$det(F) = \begin{vmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & -2 & 2 \end{vmatrix} = 0$$

Computation performed in matlab

2.3 Exercise 3

We can compute the fundamental matrix F by:

$$F = N_2^T \hat{F} N_1$$

2.4 Computer Exercise 1

The unnormalized fundamental matrix is:

$$F = \begin{bmatrix} -3.39 * 10^{-8} & -3.72 * 10^{-6} & 0.0058 \\ 4.67 * 10^{-6} & 2.89 * 10^{-7} & -0.027 \\ -0.0072 & 0.026 & 1 \end{bmatrix}$$

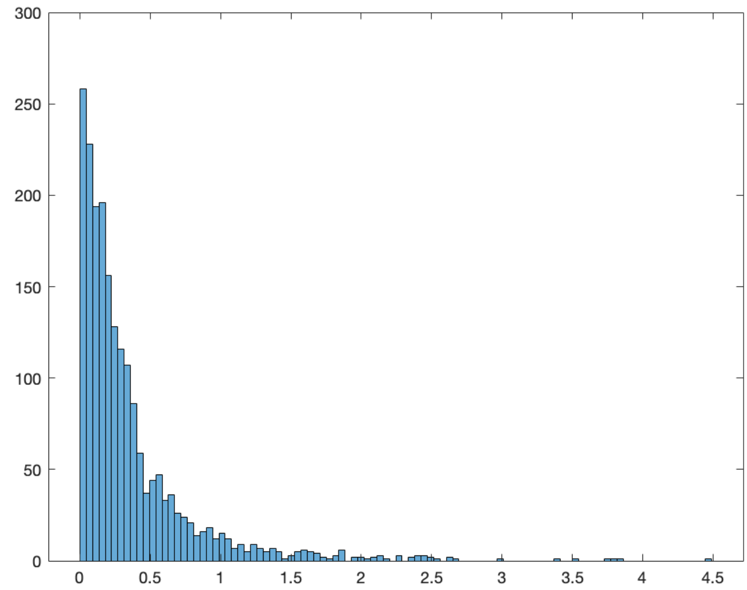

Figure 1: Histogram of mean distance

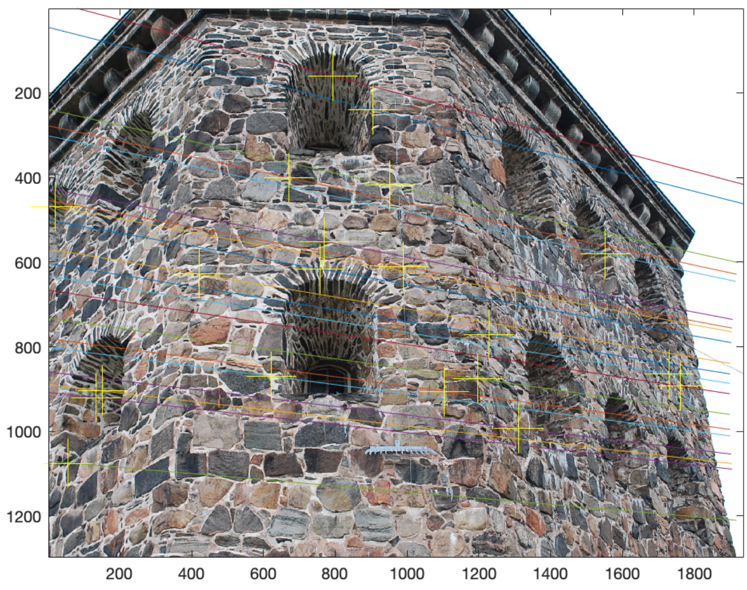

Figure 2: Epipolar lines and plot

The mean difference substantially increased without normalized points from about 0.36 to about 0.48.

2.5 Exercise 4

With a fundamental matrix F, the epipole is found by solving the nullspace of F. We have:

$$F = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

which gives

$$\begin{cases} X + Z = 0 \\ Y = 0 \\ X + Z = 0 \end{cases} \iff \begin{cases} X = t \\ Y = 0 \\ Z = -t \end{cases} \tag{2}$$

In other words our epipole is $[-1\ 0\ 1]$ in homogenous coordinates. and we can extract the second camera as:

$$P2 = [[e_2]_x F\ e_2^T] = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 2 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

Now we check the constraint:

$$\hat{x}_2^T F \hat{x}_1 \quad where, \quad x_i = P_i X$$

For the three 3D scene points we get:

For $X_1 = (1, 2, 3)$: $x_1 = [1\ 2\ 3]$ *and* $x_2 = [-2\ 10\ 0]$

For $X_2 = (3, 2, 1)$: $x_1 = [3\ 2\ 1]$ *and* $x_2 = [-4\ 6\ 2]$

For $X_3 = (1, 0, 1)$: $x_1 = [1\ 0\ 1]$ *and* $x_2 = [-2\ 2\ 0]$

and we can thus verify the constraint $\hat{x}_2^T F \hat{x}_1 = 0$.

For $X_1 = (\frac{1}{3}, \frac{2}{3}, 1)$: $\hat{x}_2^T F \hat{x}_1 = 0$

For $X_2 = (3, 2, 1)$: $\hat{x}_2^T F \hat{x}_1 = 0$

For $X_3 = (1, 0, 1)$: $\hat{x}_2^T F \hat{x}_1 = 0$

Computations performed in matlab

Lastly we calculate the camera center of P_2 by solving $P_2 C_2$'s null-space:

$$P_2 C_2 \rightarrow \begin{cases} -X - W = 0 \\ 2Y + 2Z = 0 \\ -X + W = 0 \end{cases} \iff \begin{cases} X = 0 \\ Y = t \\ Z = -t \\ W = 0 \end{cases}$$

and $C_2 = [0\ 1\ \ -1\ 0]$

2.6 Computer Exercise 2

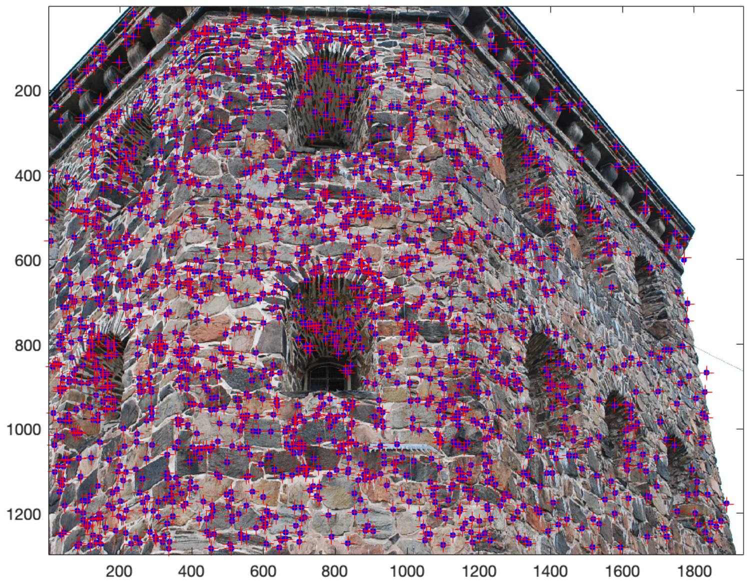

Figure 3: Projected (red cross) and actual points (blue dots) for image 1

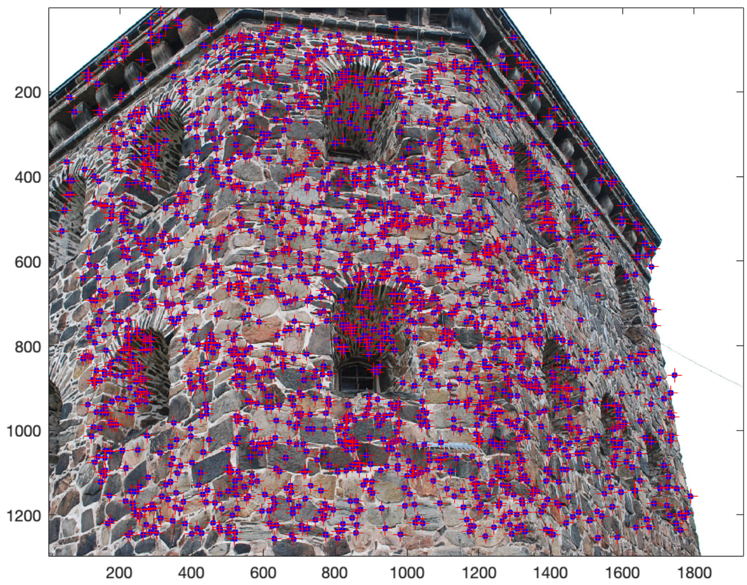

Figure 4: Projected (red cross) and actual points (blue dots) for image 2

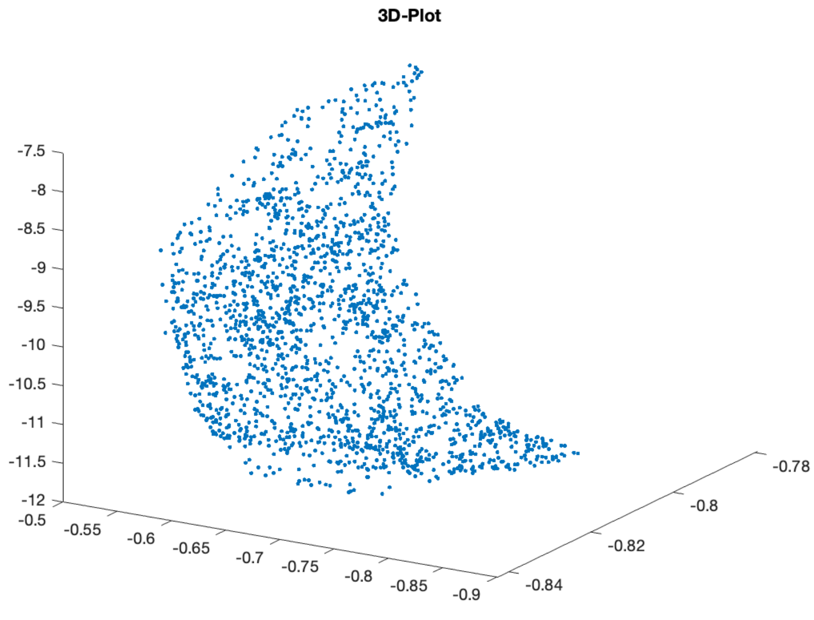

Figure 5: Wrong 3D model

3 The Essential Matrix

3.1 Computer Exercise 3

The essential matrix was:

$$E = \begin{bmatrix} -8.9 & -1005.81 & 377.08 \\ 1252.52 & 78.37 & -2448.17 \\ -472.79 & 2550.19 & 1 \end{bmatrix}$$

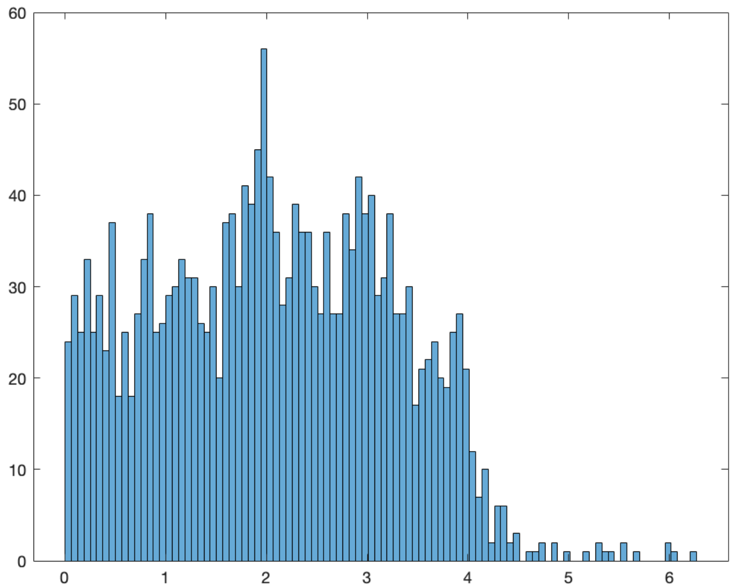

Figure 6: Histogram over distance between points and lines

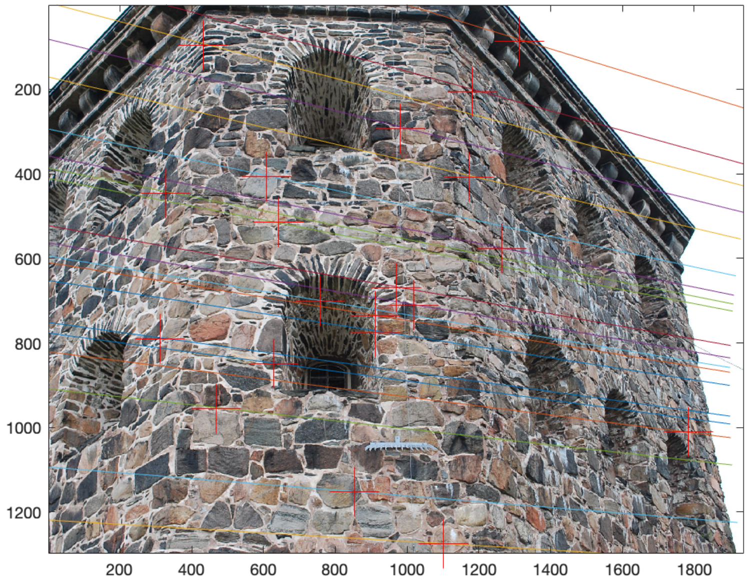

Figure 7: Epilines and projected points

The mean distance was 2.803 with the essential matrix. A substantial improvement compared to exercise 1.

3.2 Exercise 6

Determinant of UV^T verified through matlab.

By $E = Udiag([110])V^T$

$$E = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix}$$

Two corresponding points x_1 and x_2 for two cameras with an essential matrix E, will fulfill the equation:

$$x_2^T E x_1 = 0$$

In this exercise: $x_1 = (0, 0), x_2 = (1, 1)$. We verify by

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

Computation performed in matlab

Any points projected into Camera 1 follows the projection formula:

$$x = PX$$

with x_1 and P_1 this yields

$$x_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X(s)$$

where $X(s)$ is

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix}$$

In a camera projections from R^3 to R^2 the fourth coordinate s does not affect camera projections. Accordingly we can say that x_1 is a projection of X(s) in camera 1 since $x_1 = (0, 0, 1)$ in homogeneous coordinates.

To continue, let's calculate the matrix multiplications.

$$UWV^T = \begin{bmatrix} \frac{-1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} \\ \frac{1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} \\ 0 & -1 & 0 \end{bmatrix}$$

$$UW^TV^T = \begin{bmatrix} \frac{1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} \\ \frac{-1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} \\ 0 & -1 & 0 \end{bmatrix}$$

For the point $x_2 = (1, 1, 1)$ in homogeneous coordinate the projection X(s) is a valid projection if the resulting point x_i is λx_1, i.e is proportional.

With matrix multiplications performed, I compute P_{2_i} and calculate $x_{2_i} = P_{2_i}X(s)$ and $x_{1_i} = P_{1_i}X(s)$ for each of the cases.

Case 1

$$P_{2_1}X(s) = \begin{bmatrix} UWV^T & u_3 \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} & 0 \\ \frac{1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ s \end{bmatrix}$$

Case 2

$$P_{2_2}X(s) = \begin{bmatrix} UWV^T & -u_3 \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} & 0 \\ \frac{1}{\sqrt{w}} & 0 & \frac{-1}{\sqrt{w}} & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ -s \end{bmatrix}$$

Case 3

$$P_{2_3}X(s) = \begin{bmatrix} UW^TV^T & u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} & 0 \\ \frac{-1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ s \end{bmatrix}$$

Case 4

$$P_{2_4}X(s) = \begin{bmatrix} UW^TV^T & -u_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} & 0 \\ \frac{-1}{\sqrt{w}} & 0 & \frac{1}{\sqrt{w}} & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -s \end{bmatrix}$$

Computation performed in matlab

Since all valid projections must result in $\lambda x_1 = \lambda[1\ 1\ 1]^T$, I can solve for s and thus get:

Case 1: s = $\frac{-1}{\sqrt{2}}$

Case 2: s = $\frac{1}{\sqrt{2}}$

Case 3: s = $\frac{1}{\sqrt{2}}$

Case 4: s = $\frac{-1}{\sqrt{2}}$

To calculate the location of the camera pairs in relation to the two cameras I calculate their depth. The formula for calculating depth is:

$$depth(P, \mathbf{X}) = \frac{sign(det(A))}{||A_3||}\lambda, \quad where \quad \lambda = \begin{bmatrix} A_3^T & a_3 \end{bmatrix} \mathbf{X}, \quad and \quad P = \begin{bmatrix} A & a \end{bmatrix}$$

Which for our 4 cases results in:

Case 1: P2-depth = 1 and P1-depth = -1.4142

Case 2: P2-depth = -1 and P1-depth = 1.4142

Case 3: P2-depth = 1 and P1-depth = 1.4142

Case 4: P2-depth = -1 and P1-depth = -1.4142

Computation performed in matlab

We can see that only in case 3 do we have positive depth and the camera pairs are infront.

3.3 Computer Exercise 4

Figure 8: Projection (red cross) and actual points (blue dots) for image 1

Figure 9: Projection (red cross) and actual points (blue dots) for image 2 with projection 1

Figure 10: Projection (red cross) and actual points (blue dots) for image 2 with projection 2

Figure 11: Projection (red cross) and actual points (blue dots) for image 2 with projection 3

Figure 12: Projection (red cross) and actual points (blue dots) for image 2 with projection 4

I’ve refined the calculations for the above plots, and the error look very small to me.

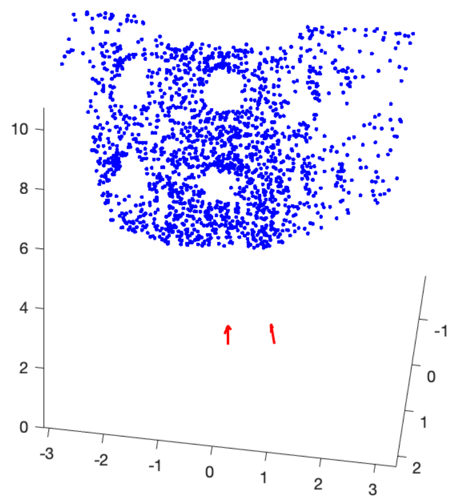

Figure 13: 3D projected model