Code

R Notebook

This is an R Markdown (http://rmarkdown.rstudio.com) Notebook. When you execute code within the notebook, the results appear beneath the code.

Try executing this chunk by clicking the *Run* button within the chunk or by placing your cursor inside it and pressing *Cmd+Shift+Enter*.

Add a new chunk by clicking the *Insert Chunk* button on the toolbar or by pressing *Cmd+Option+I*.

When you save the notebook, an HTML file containing the code and output will be saved alongside it (click the *Preview* button or press *Cmd+Shift+K* to preview the HTML file).

The preview shows you a rendered HTML copy of the contents of the editor. Consequently, unlike *Knit*, *Preview* does not run any R code chunks. Instead, the output of the chunk when it was last run in the editor is displayed.

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```
#open exponential.csv file
exp <- read.csv("exponential.csv")
#density function of an exponential distribution with parameter 1/m
exp_density <- function(x, m) {
  1/m * exp(-x/m)
}
#compute the mean of exponential.csv
m_ML <- mean(exp)

#Generate 500 datapoints from an exponential distribution with parameter m_ML
exp_gen <- rexp(500, m_ML)
```

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```
# Generate 3000 samples of U and V i.i.d. following a uniform law in [0,1]
U <- runif(3000)
V <- runif(3000)


#X and Y according to the Box - Muller method
X <- sqrt(-2*log(U))*cos(2*pi*V)
Y <- sqrt(-2*log(U))*sin(2*pi*V)

#plot the histogram of X and Y
hist(X, freq = F, xlim = c(-5, 5), ylim = c(0, 0.5), main = "Histogram of X and Y", x
lab = "x", ylab = "density")
hist(Y, freq = F, xlim = c(-5, 5), ylim = c(0, 0.5), main = "Histogram of X and Y", x
lab = "x", ylab = "density")

#compute the covariance between X and Y
cov(X, Y)
```

Question 12.

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R is given as

$$R = \sqrt{-2\log(U)}$$

where U follow a uniform distribution on [0, 1]

$$\begin{aligned}\mathbb{P}[R \le x] &= \mathbb{P}[-2\log(U) \le x^2] = \\ &= \mathbb{P}[\log(U) \ge -\frac{x^2}{2}] = \\ &= 1 - \mathbb{P}[U < \exp(-\frac{x^2}{2})].\end{aligned}$$

U is a random variable from uniform distribution on [0, 1], so

$$\mathbb{P}[R \le x] = 1 - \int_0^{\exp(-x^2/2)} dx = 1 - \exp(\frac{-x^2}{2})$$

Hence,

$$F_R(x) = 1 - \exp(\frac{-x^2}{2})$$

We can obtain the density by taking the derivative of the CDF as it is differentiable on the interval we the problem is defined on. Thus we have

$$f_R(x) = \exp(\frac{-x^2}{2}) * x$$

for $x > 0$.

Question 13.

Let $V = 2\pi\Theta$. As V is uniformly distributed on [0, 1] it immidiately follows that the density function is

$$f_V(x) = \frac{1}{2\pi}$$

.

Since U and V are independent the joint density is

$$f_{R,\Theta}(x_1, x_2) = f_R(x_1)f_\Theta(x_2) = \frac{1}{2\pi}\exp(-\frac{x_1^2}{2}) \cdot x_1 .$$

Question 14.

Let $(a, b) = g(R, \theta) = (R\cos(\theta), R\sin(\theta))$. Meaning, we have

$$\begin{cases} a = R\cos(\theta) \\ b = R\sin(\theta) \end{cases}$$

Using $\cos^2(\theta) + \sin^2(\theta) = 1$, we have

$$R = \sqrt{a^2 + b^2}$$

.

Question 15.

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FIRST QUESTION IS TBD

$$f_{X,Y}(a,b) = \frac{f_{R,\theta}(g^{-1}(a,b))}{r} = \frac{1}{2\pi}\exp\left(-\frac{a^2+b^2}{2}\right)$$
$$= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{a^2}{2}\right)\cdot\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2}\right)$$

We used

$$R = \sqrt{a^2+b^2}$$

Question 16.

Notice $\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{a^2}{2}\right) = f_X(a)$ and $\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2}\right) = f_Y(b)$, hence we can conclude that X and Y are independent. Furthermore, from their density functions we can see that both N(0,1) distributed.

Question 17.

Question 18.

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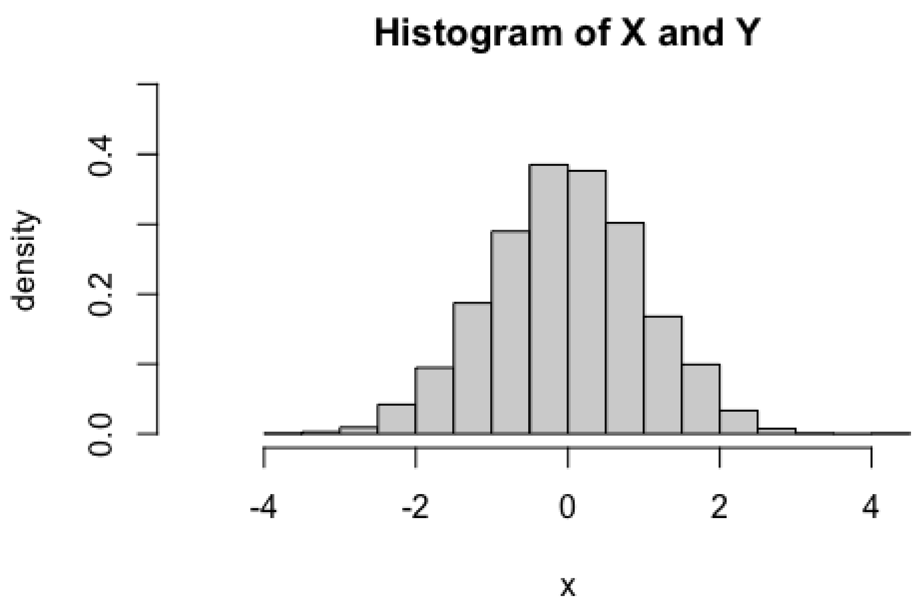
```
# Generate 3000 samples of U and V i.i.d. following a uniform law in [0,1]
U <- runif(3000)
V <- runif(3000)


#X and Y according to the Box - Muller method
X <- sqrt(-2*log(U))*cos(2*pi*V)
Y <- sqrt(-2*log(U))*sin(2*pi*V)

#plot the histogram of X and Y
hist(X, freq = F, xlim = c(-5, 5), ylim = c(0, 0.5), main = "Histogram of X and Y", x
lab = "x", ylab = "density")
```



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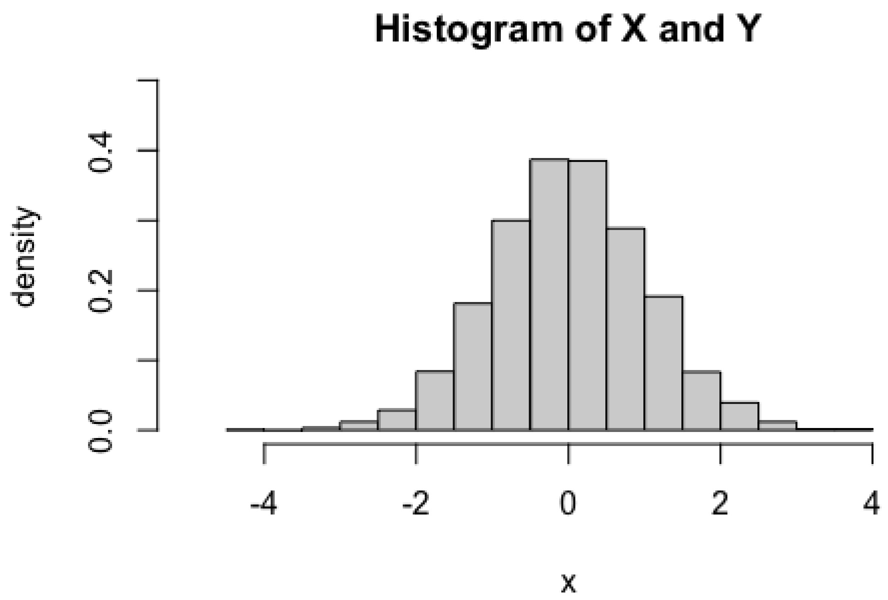
```
hist(Y, freq = F, xlim = c(-5, 5), ylim = c(0, 0.5), main = "Histogram of X and Y", x
lab = "x", ylab = "density")
```



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```
#compute the covariance between X and Y
cov(X, Y)
```

```
[1] -0.01145399
```

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```
#Generate 3000 samples using the rnorm function of R
Z <- rnorm(3000)

#plot the histogram of Z
hist(Z, freq = F, xlim = c(-5, 5), ylim = c(0, 0.5), main = "Histogram of Z", xlab =
"x", ylab = "density")
```

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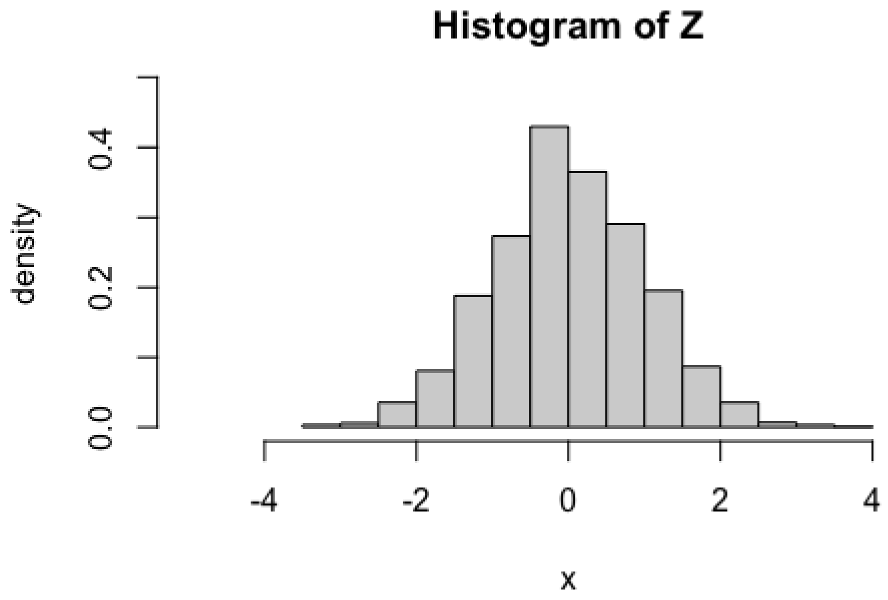

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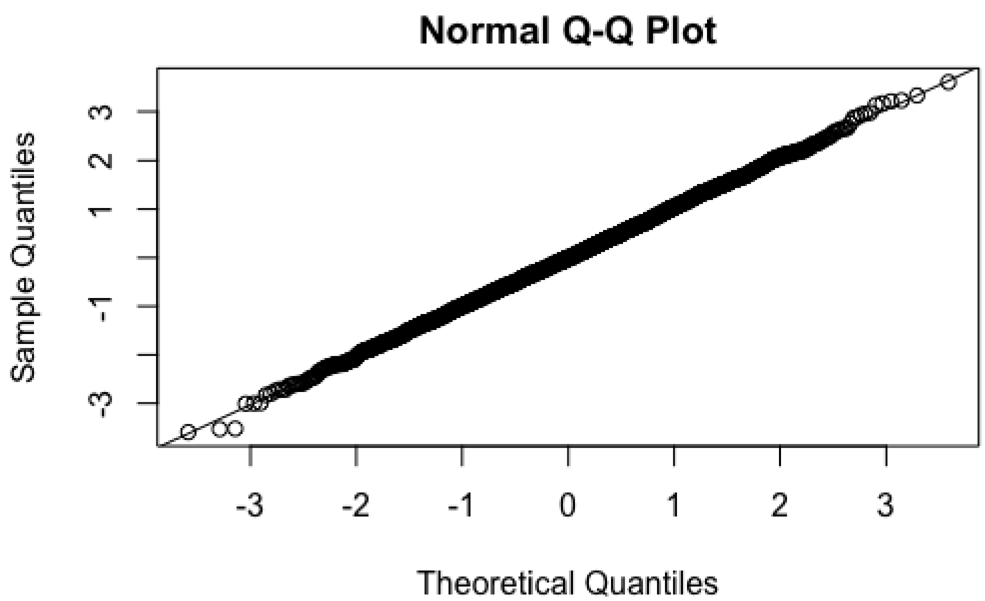
```
#Generate N1,N2 two vectors of 3000 sample of a normal distribution using the funntio
n rnorm
N1 <- rnorm(3000)
N2 <- rnorm(3000)

#Generate E1,E2 a vector of 3000 samples of an exponential distribution using the fun
ction rexp
E1 <- rexp(3000)
E2 <- rexp(3000)

#Plot the quantile-quantile diagrams of N1 and N2
qqnorm(N1)
qqline(N1)
```



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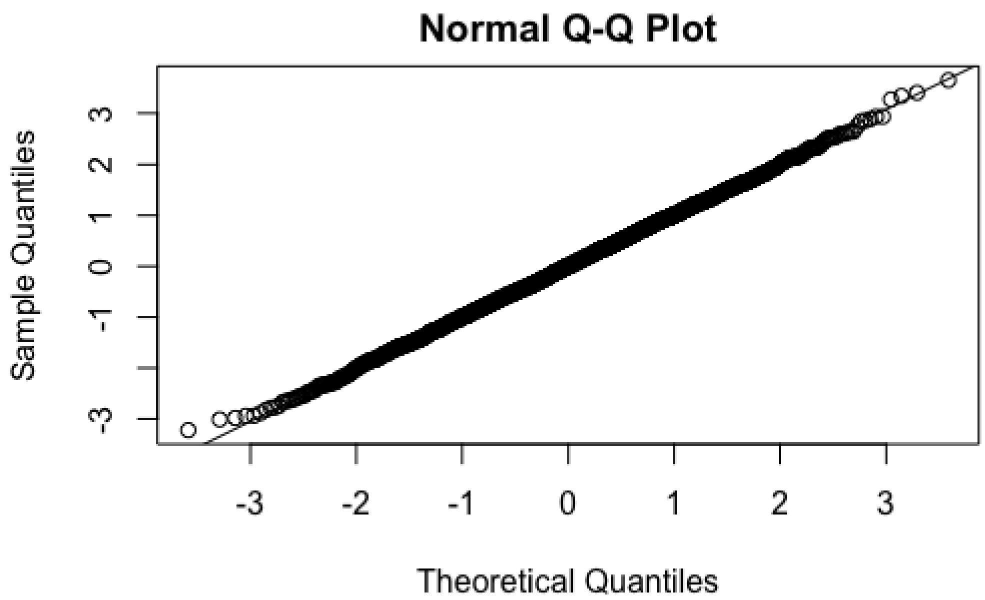
```
qqnorm(N2)
qqline(N2)
```



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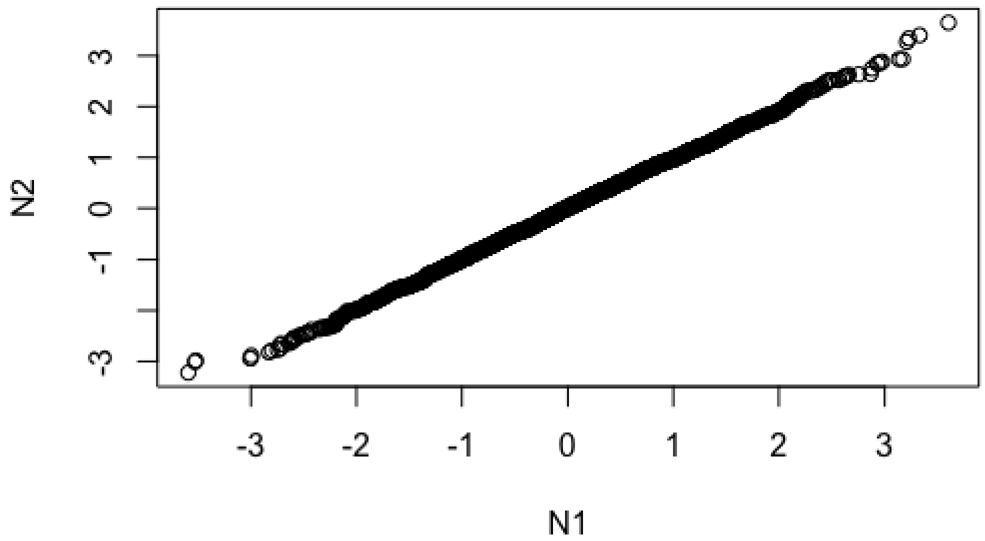
```
qqplot(N1,N2)
```



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```
#Plot the quantile-quantile diagrams of E1 and E2
qqnorm(E1)
qqline(E1)
```

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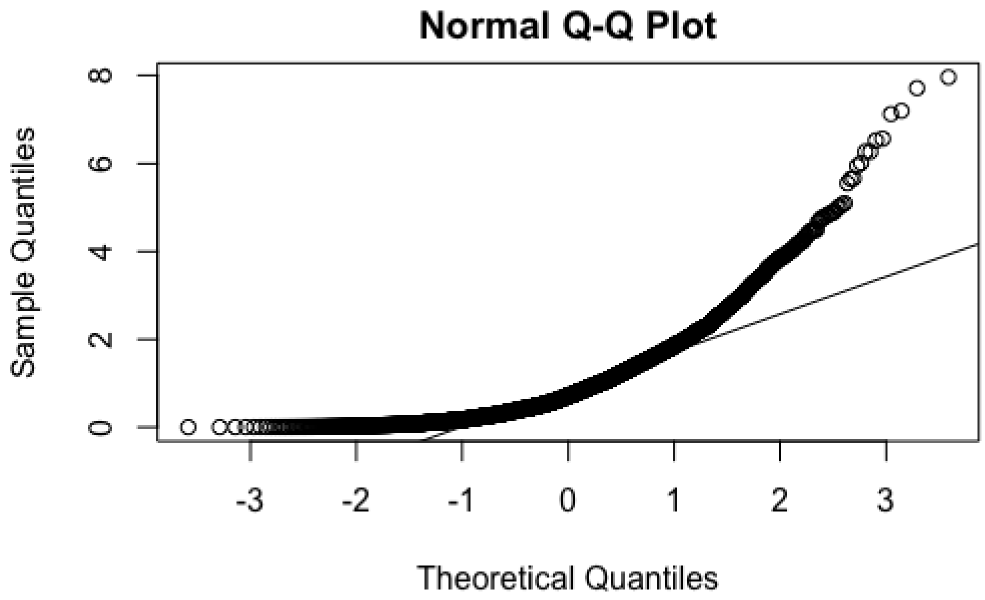

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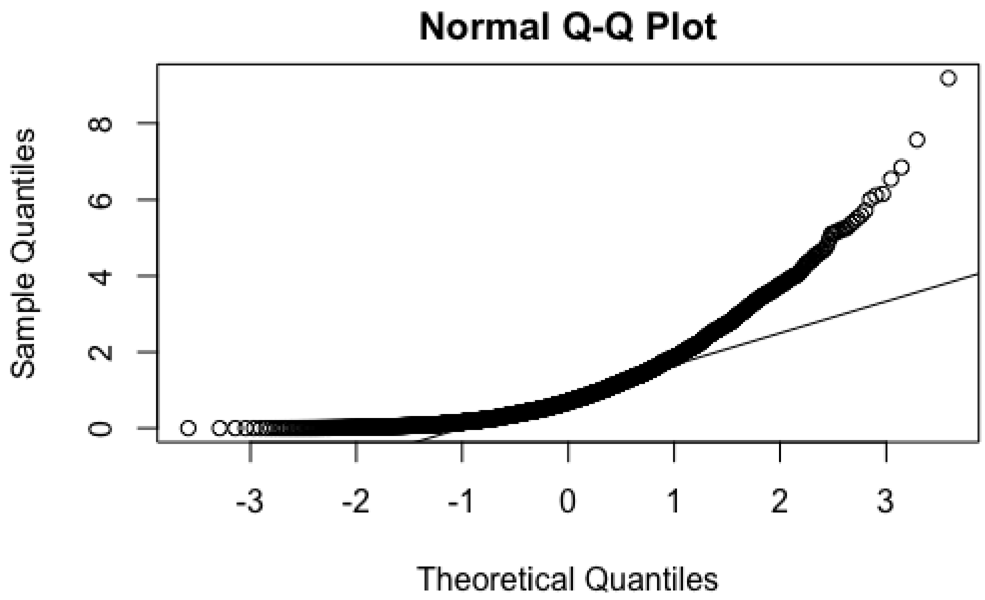
```
qqnorm(E2)
qqline(E2)
```



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```
qqplot(E1,E2)
```

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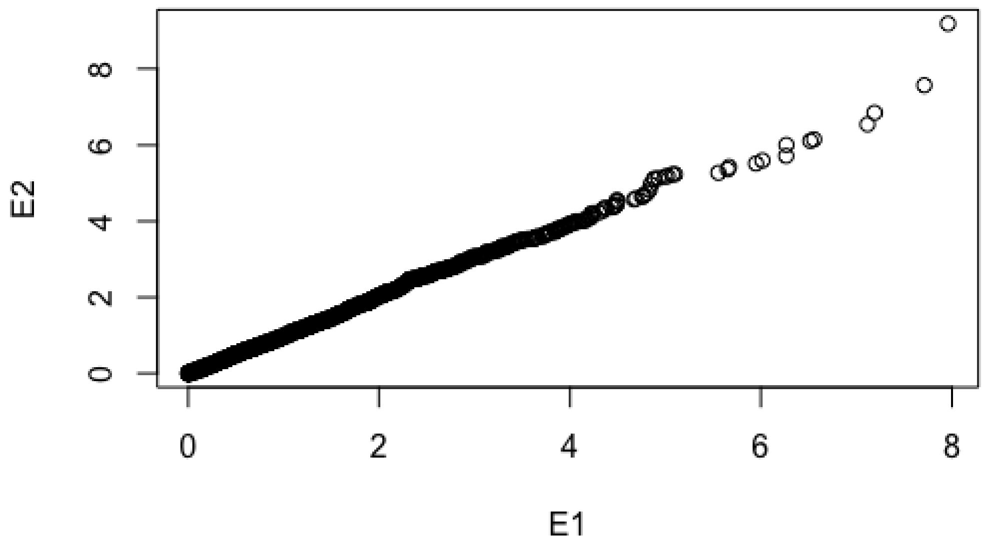

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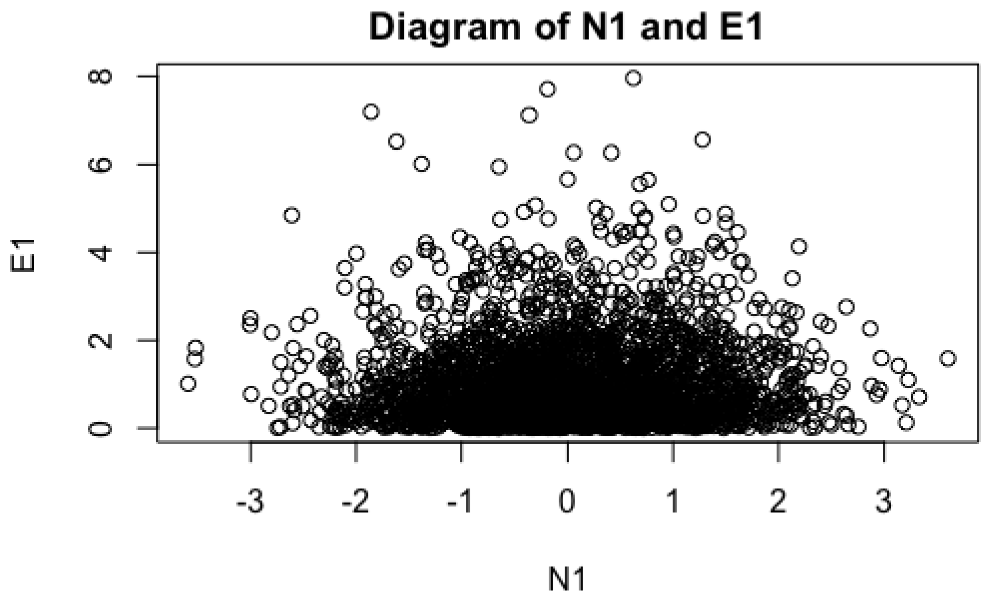
```
#Plot the diagram of N1 and E1
plot(N1, E1, main = "Diagram of N1 and E1", xlab = "N1", ylab = "E1")
```



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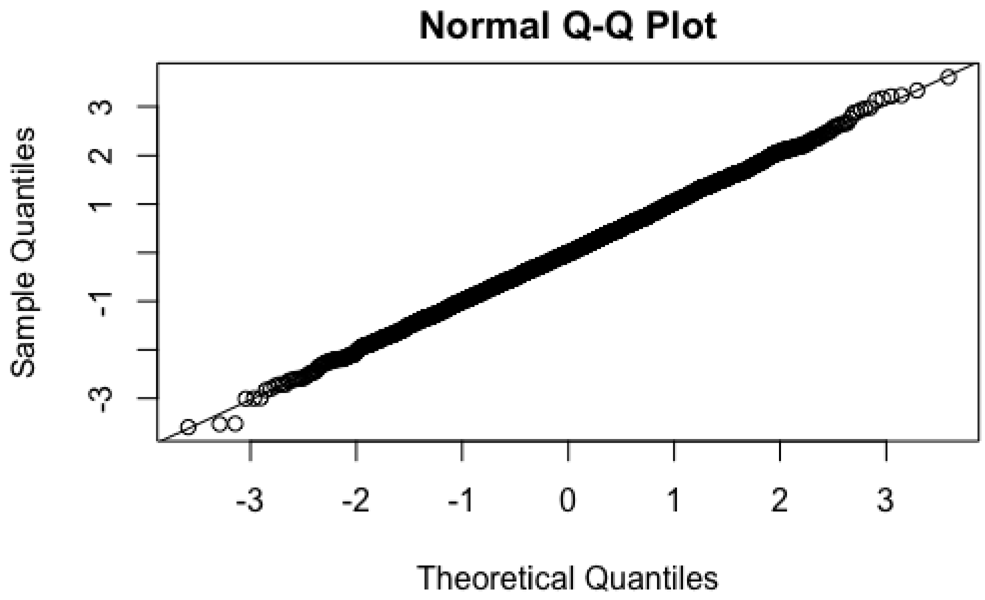
```
#Plot the quantile-quantile diagrams of N1 and E1
qqnorm(N1)
qqline(N1)
```



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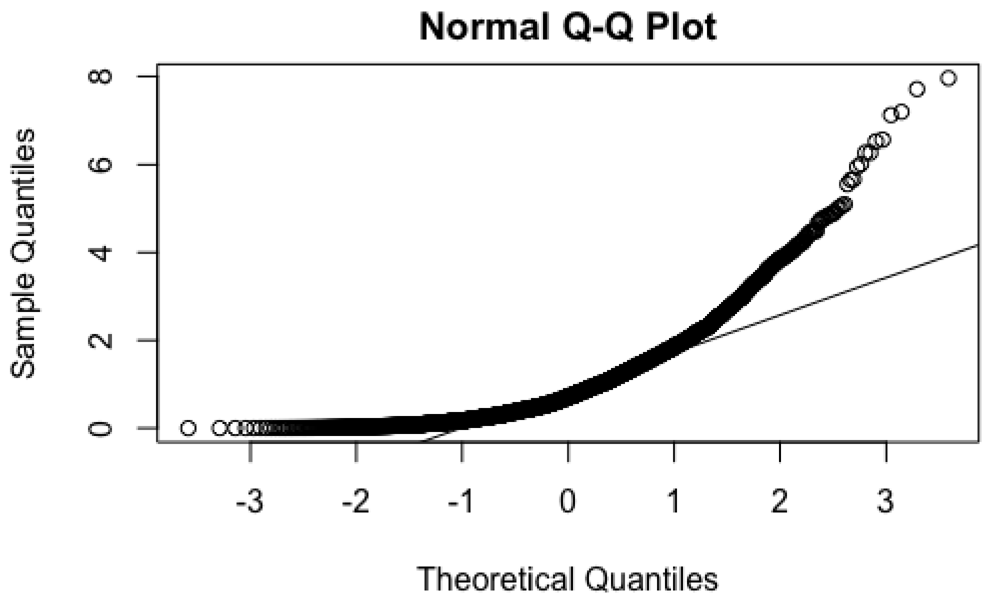
```
qqnorm(E1)
qqline(E1)
```



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```
qqplot(N1,E1)
```

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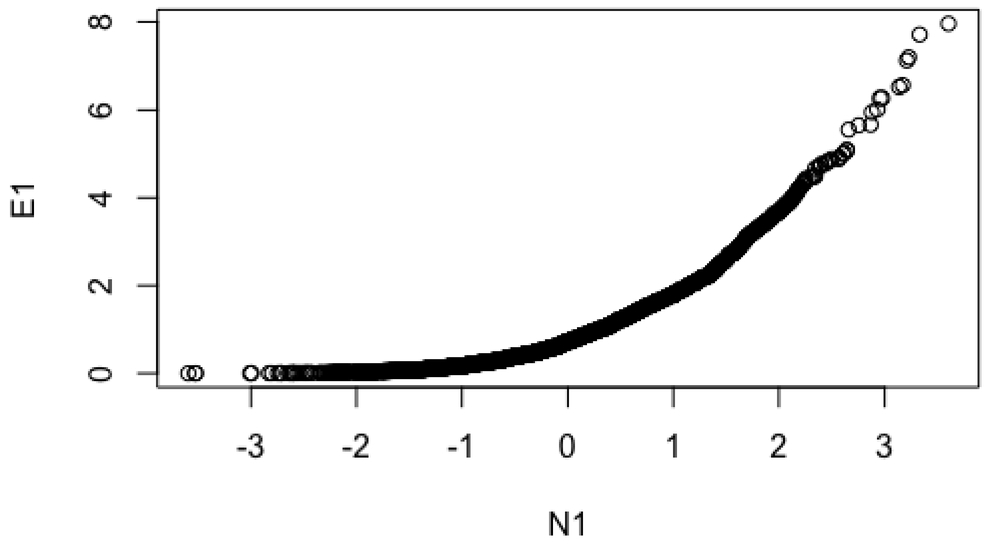

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