Quiz 1A

1 Exercise 1

For $n \geq 1$ define $f_n : \mathbb{R} \to \mathbb{R}$, $x \to (-1)^n \frac{x^2+n}{n^2}$.

1. Show that the series $\sum_1^\infty f_n$ is pointwise convergent. We note $f(x) = \sum_1^\infty f_n(x)$.
2. Show that the series $\sum_1^\infty f_n$ converges uniformly in every bounded interval. Deduce that f is continuous on $\mathbb{R}$.

2 Exercise 2

For $n \geq 1$ define $f_n : [0, \infty) \to \mathbb{R}$, $x \to \frac{1}{x+n^2}$.

1. Show that the series $\sum_1^\infty f_n$ converges uniformly to a function f.
2. Show that f is continuous on $[0, \infty)$.
3. Show that the series $\sum_1^\infty f_n'$ is uniformly convergent on $[0, \infty)$.
4. Conclude that f is C^1 on $[0, \infty)$ and calculate f'.