

Robótica grupo2

Clase 5

Facultad de Ingeniería UNAM

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Conceptos básicos/Elemento base

- Resumen de conceptos
- Elemento base (eslabón)
 - Planteamiento de su modelado cinemático.
 - Modelo cinemático de la posición.
 - Modelo cinemático de las velocidades.
 - Modelos cinemático de sus aceleraciones.

Introducción



Modelo cinemático de la postura

$${}^0\xi_P = {}^0\xi_P(\mathbf{q})$$

$$\mathbf{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$$

$$C = \{q_1, \dots, q_n\}$$



$$\mathbf{F} = {}^0\xi_P - {}^0\xi_P(\mathbf{q}) = 0$$

Introducción



Modelo cinemático directo de las velocidades

$${}^0\dot{\xi}_P = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$$

$$\dot{\mathbf{q}} = \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix}$$



Modelo cinemático directo de las velocidades

$$\dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^+ {}^0\dot{\xi}_P$$

Introducción



Modelo cinemático directo de las aceleraciones

$${}^0\ddot{\xi}_P = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}}$$



Modelo cinemático inverso de las aceleraciones

$$\ddot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^+ {}^0\ddot{\xi}_P + \dot{\mathbf{J}}(\mathbf{q})^+ {}^0\dot{\xi}_P$$

Introducción

Cálculo del par

$$\Gamma = k - u = \sum_{i=1}^n k_i - \sum_{i=1}^n u_i$$

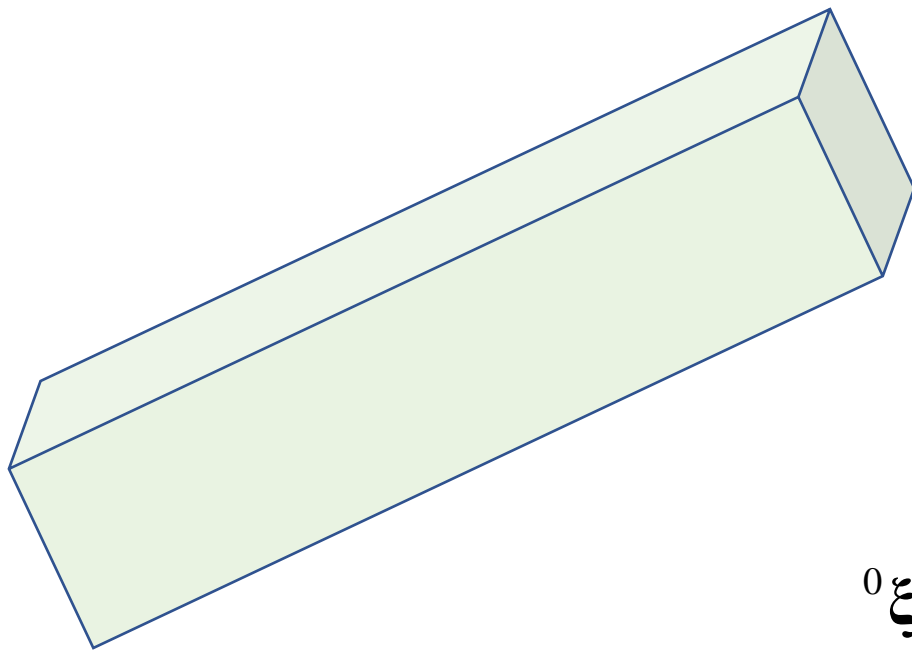
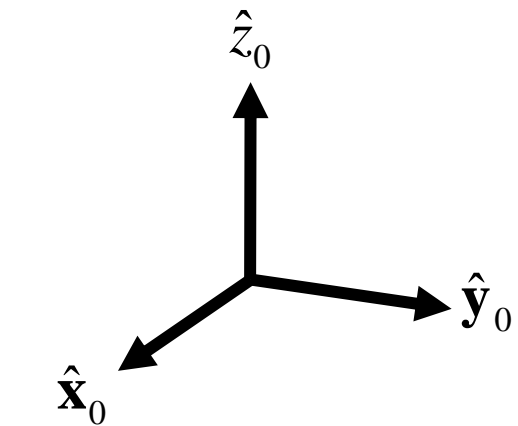
$$\tau_i = \frac{d}{dt} \left(\frac{\partial}{\partial \dot{q}_i} \Gamma \right) - \frac{\partial}{\partial q_i} \Gamma$$

Modelo dinámico

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_{ex} + \boldsymbol{\tau}_{int}$$



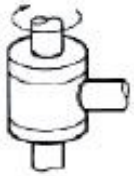
Elemento base



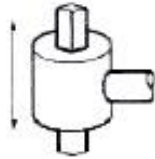
$${}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix}$$

Elemento base

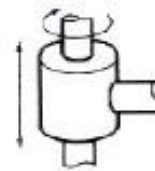
Tipos de Articulaciones



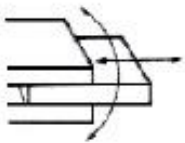
Rotacional
1 GL



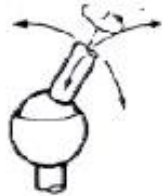
Prismática
1 GL



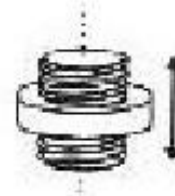
Cilíndrica
2 GL



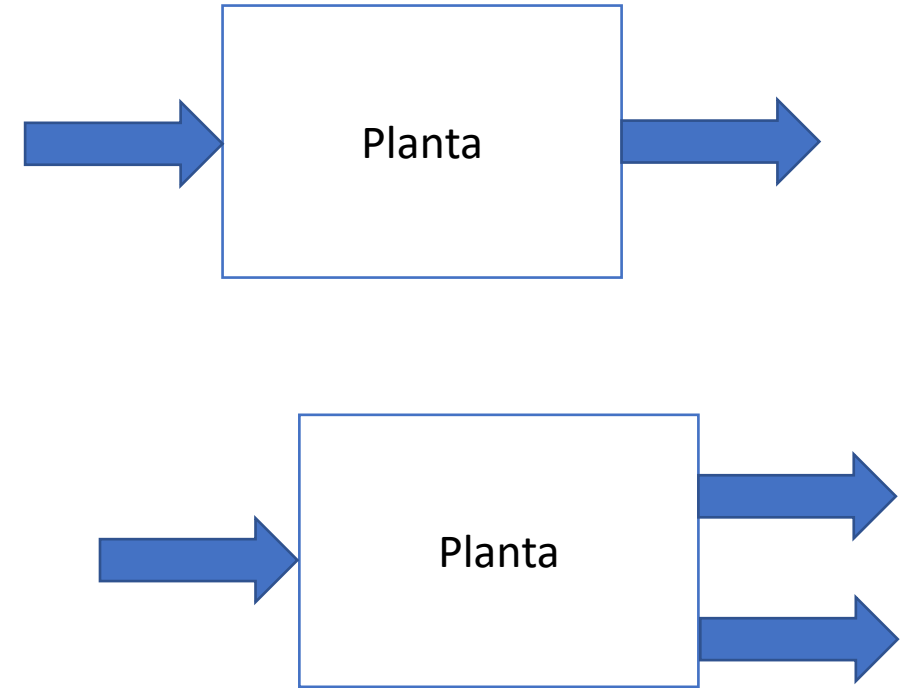
Planar
2 GL



Esférica (rótula)
3 GL



Tornillo
1 GL

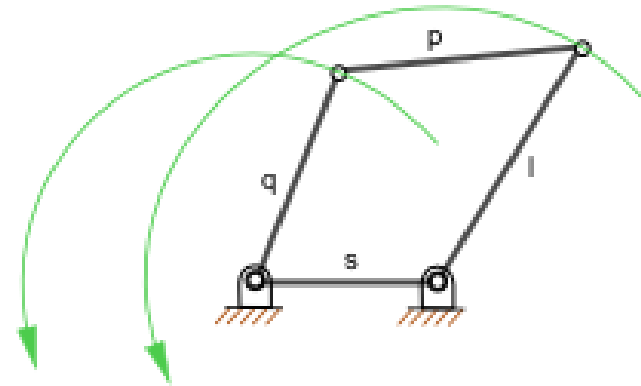
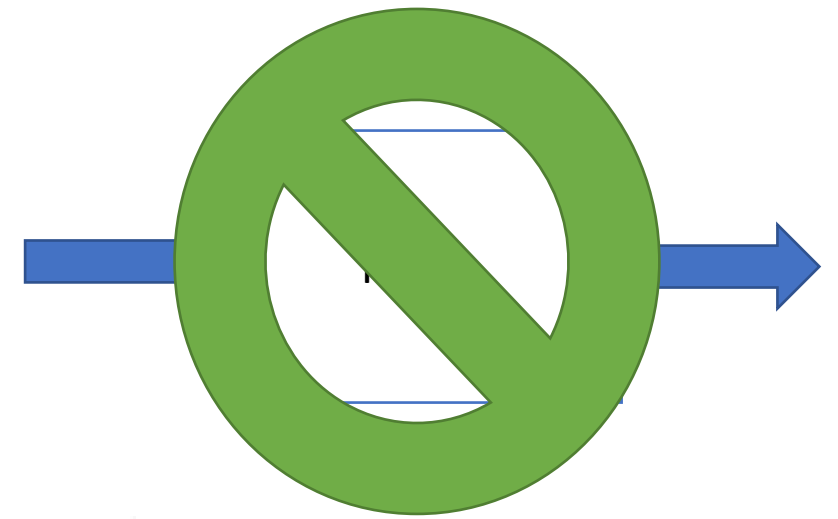


Elemento base

Eslabón

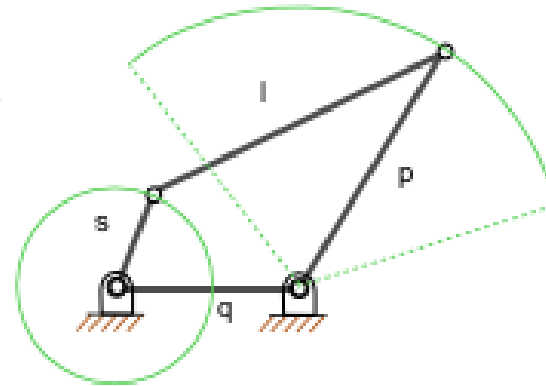
Un eslabón es un cuerpo rígido ideal el cual se relaciona con otros elementos (eslabones) por medio de un arreglo mecánico denominado junta.

Elemento base

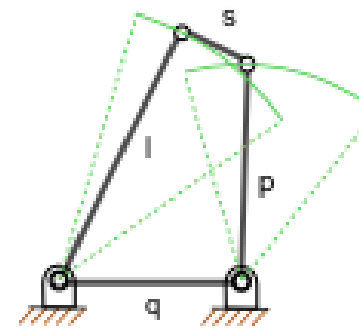


(full revolution,
both links)

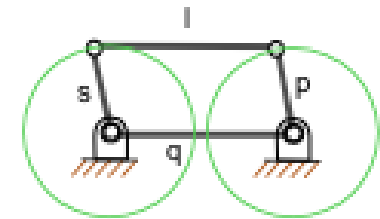
Drag-link
 $s+l \neq p+q$
(continuous motion)



Crank-rocker
 $s+l \neq p+q$
(continuous motion)

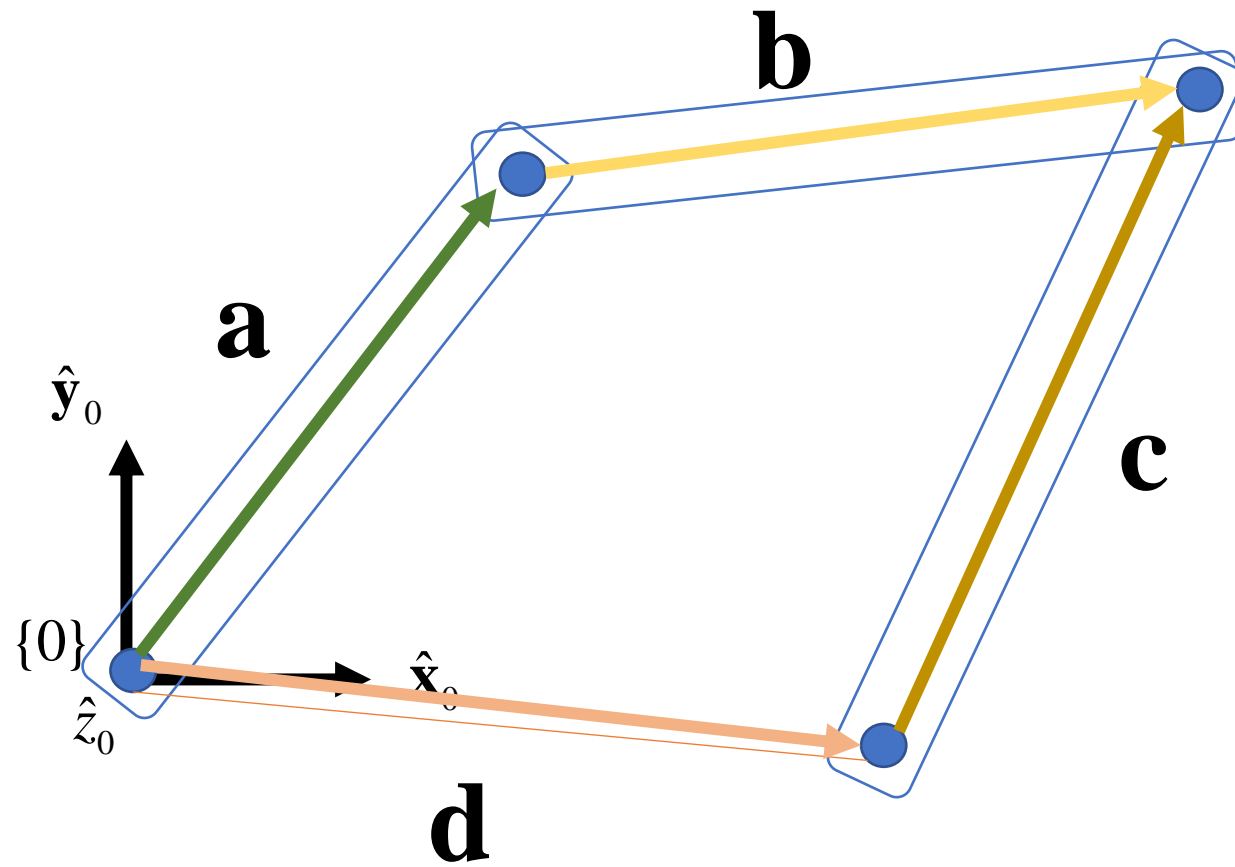


Double-rocker
 $s+l > p+q$
(no continuous motion)

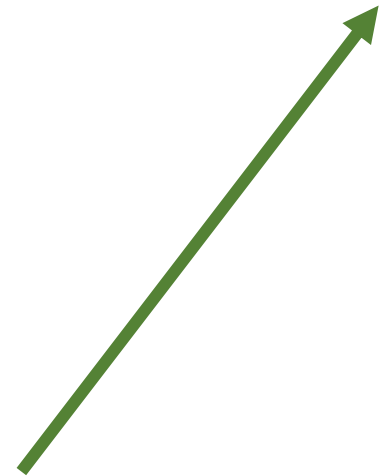


Parallelogram linkage
 $s+l \neq p+q$
(continuous motion)

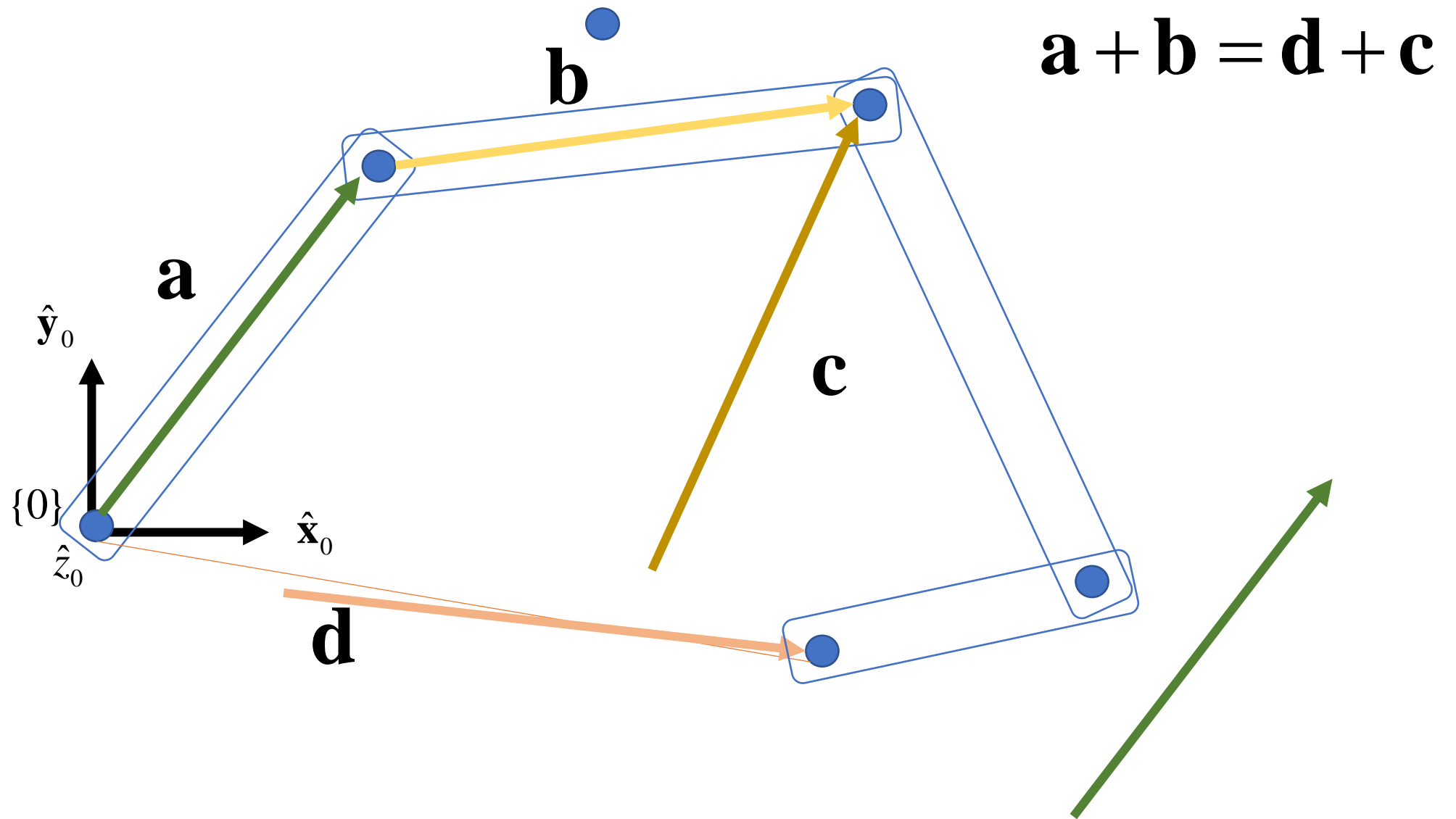
Elemento base



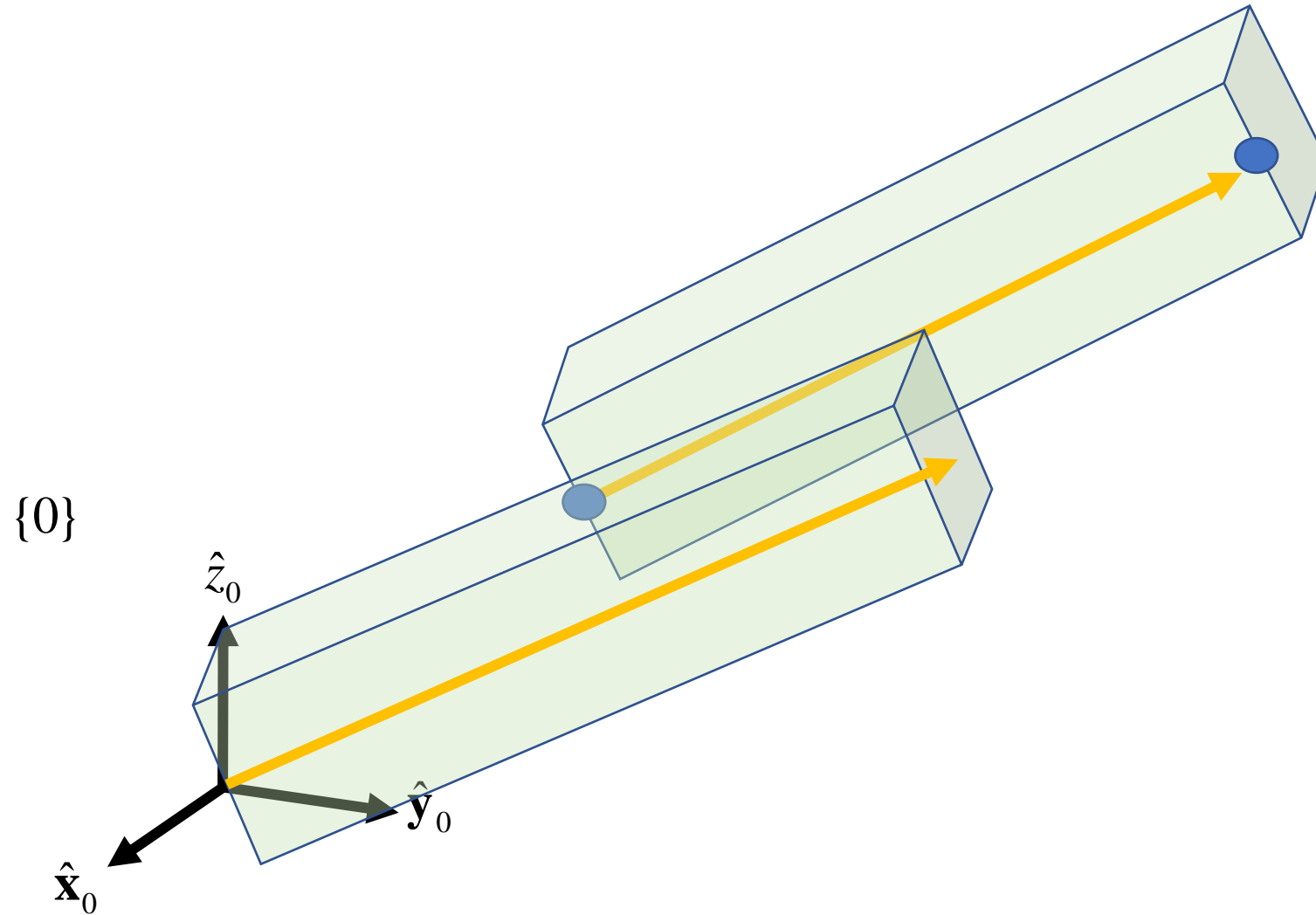
$$\mathbf{a} + \mathbf{b} = \mathbf{d} + \mathbf{c}$$



Elemento base

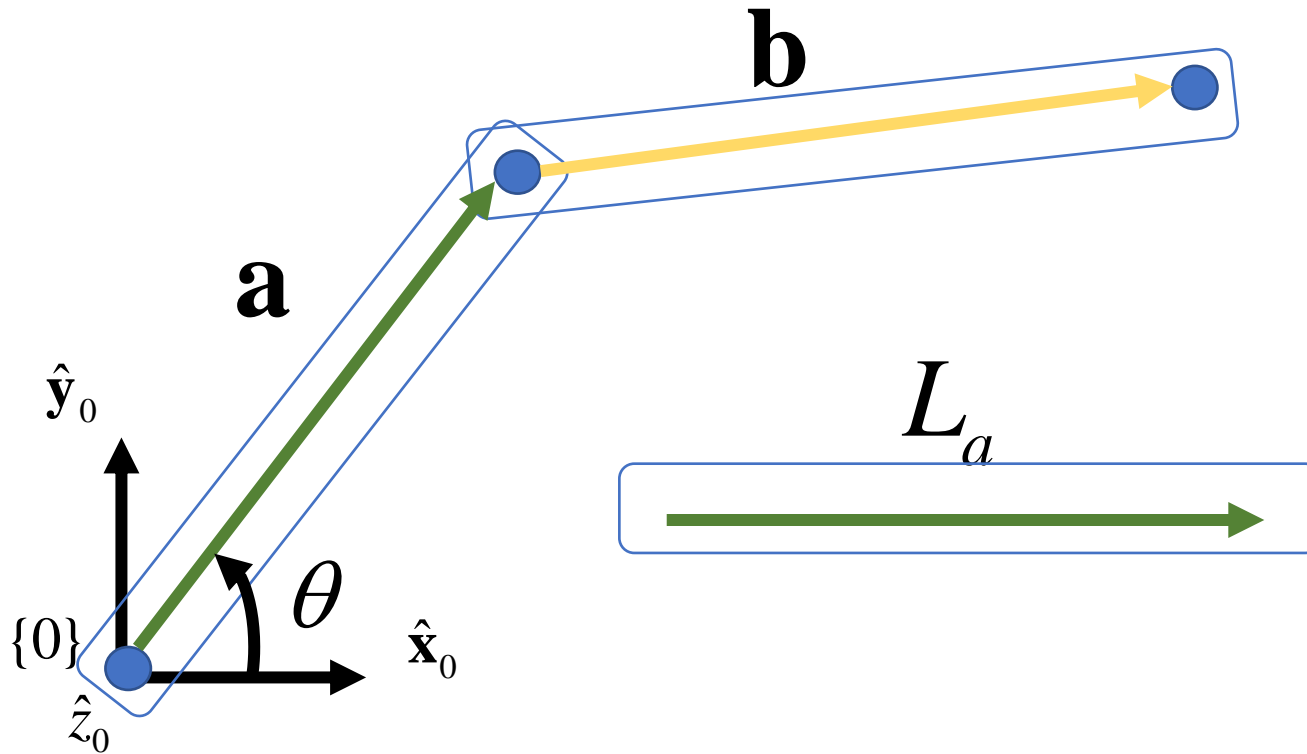


Elemento base



Elemento base

$$\mathbf{a} = \begin{pmatrix} x_a \\ y_a \\ 0 \end{pmatrix}$$



$$\mathbf{a} = \mathbf{R}(\theta) \mathbf{x}_a = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L_a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

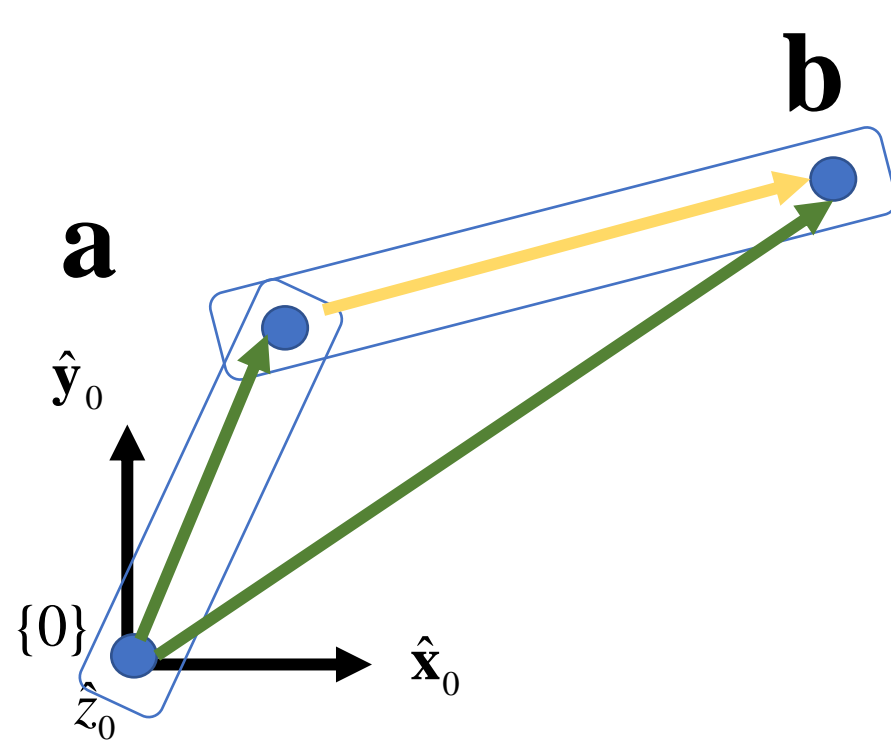
$$\mathbf{a} + \mathbf{b} = \mathbf{d} + \mathbf{c}$$

$$\mathbf{a} = \mathbf{R}(\theta) \mathbf{x}_a$$

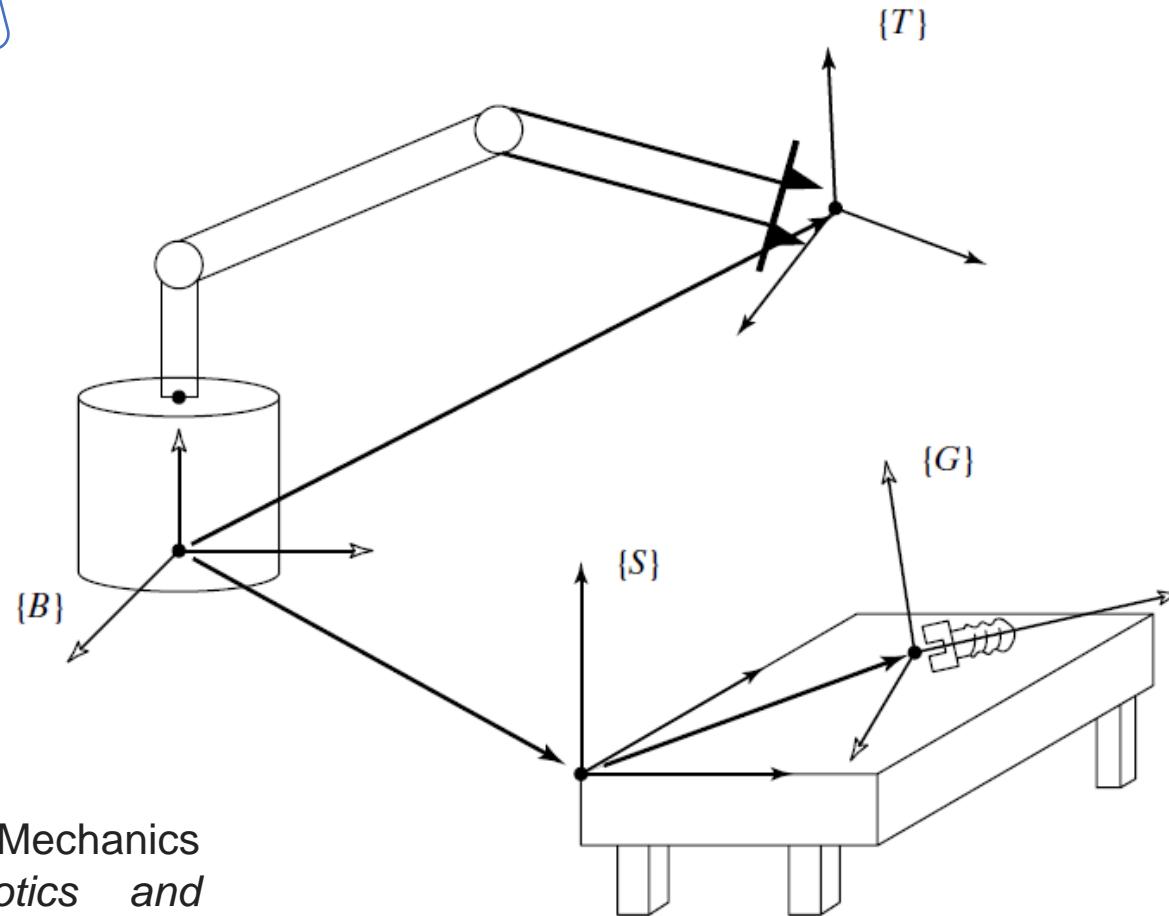
$$\mathbf{x}_a = \begin{pmatrix} L_a \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

Elemento base



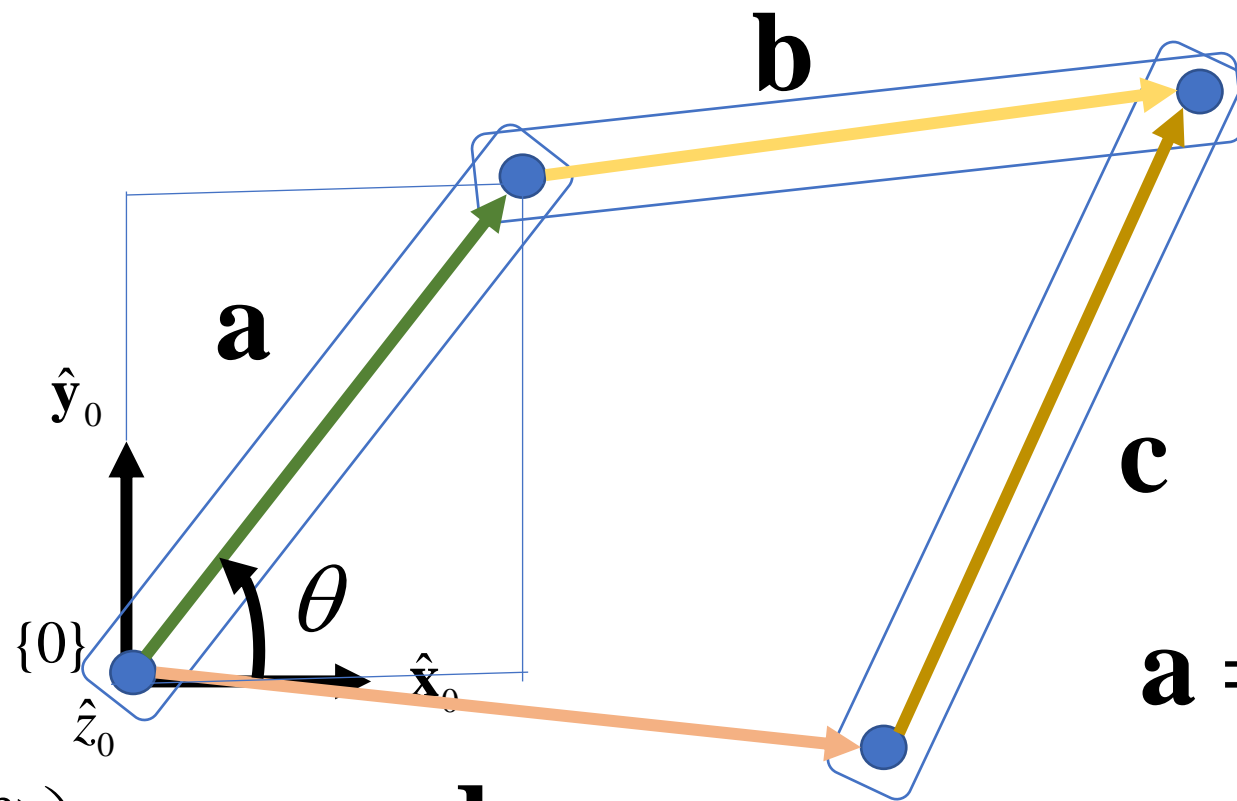
$$\mathbf{a} + \mathbf{b} = \mathbf{x}$$



Merat, F. (1987). Introduction to robotics: Mechanics and control. *IEEE Journal on Robotics and Automation*, 3(2), 166-166.

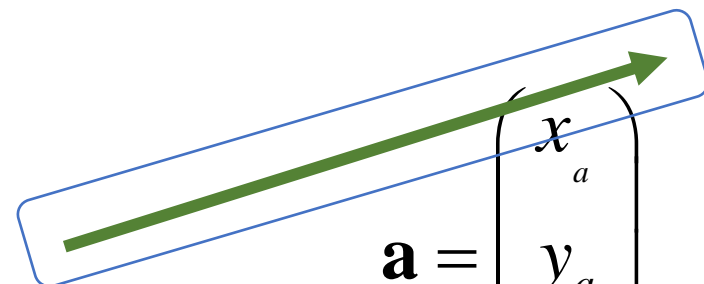
FIGURE 2.16: Manipulator reaching for a bolt.

Elemento base



$$\mathbf{a} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

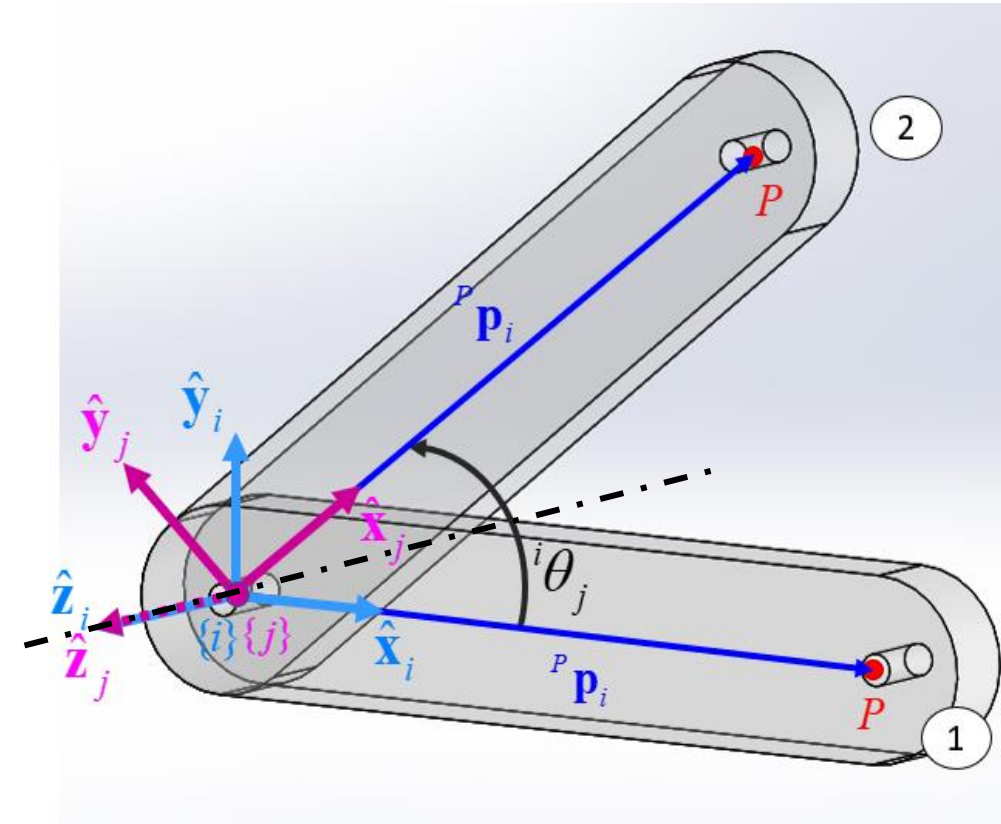
$$\mathbf{a} + \mathbf{b} = \mathbf{d} + \mathbf{c}$$


$$\mathbf{a} = \begin{pmatrix} x_a \\ y_a \\ 0 \end{pmatrix}$$

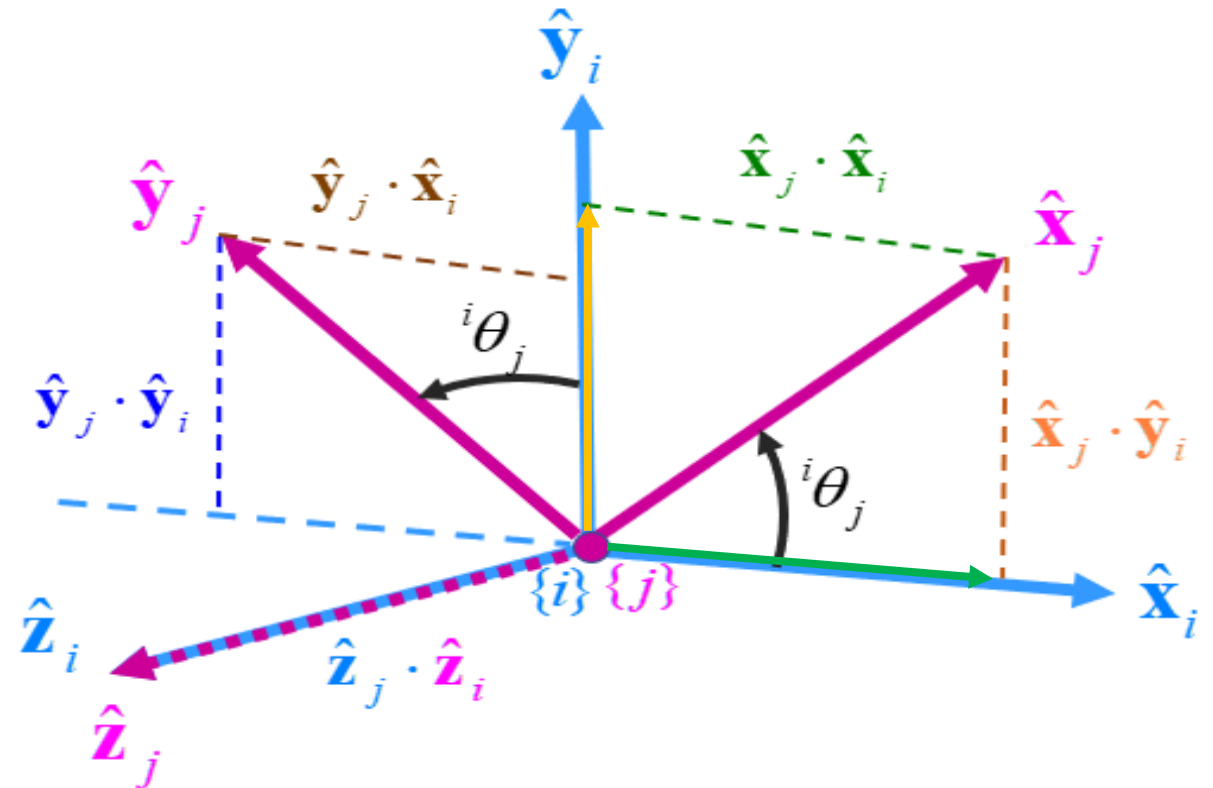
$$\mathbf{a} = \mathbf{R}(\theta) \mathbf{x}_a$$

$$\mathbf{b} = \mathbf{R}(\theta) \mathbf{x}_b$$

Elemento base



$$\hat{z}_j \cdot \hat{z}_i = \|\hat{z}_j\| \|\hat{z}_i\| \cos(0) = 1$$



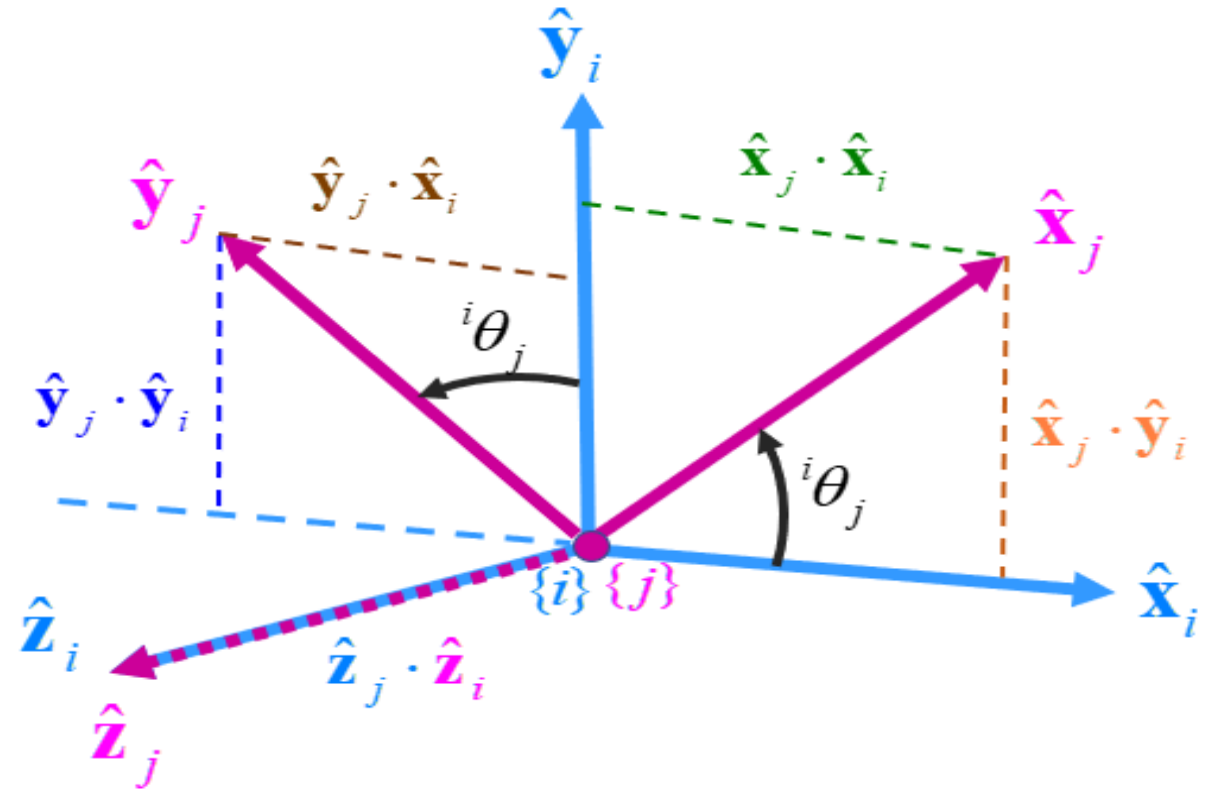
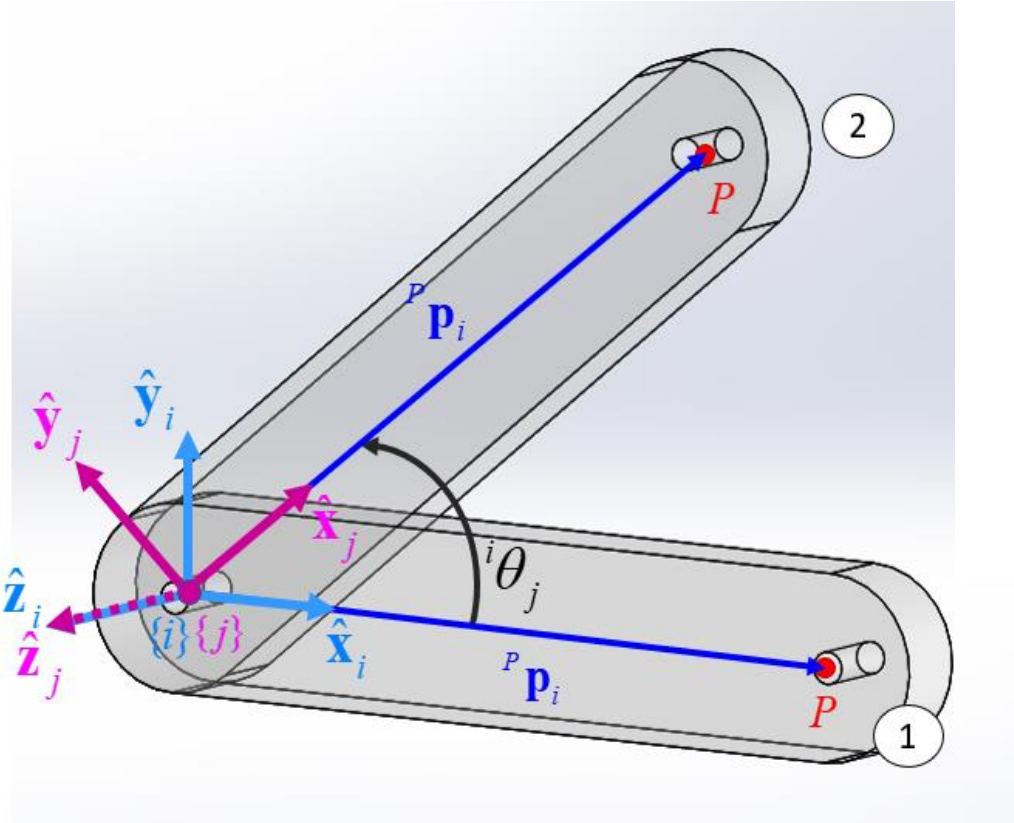
$$\hat{x}_j \cdot \hat{x}_i = \|\hat{x}_j\| \|\hat{x}_i\| \cos({}^i\theta_j) = \cos({}^i\theta_j)$$

$$\hat{y}_j \cdot \hat{x}_i = \|\hat{y}_j\| \|\hat{x}_i\| \sin({}^i\theta_j) = -\sin({}^i\theta_j)$$

$$\hat{x}_j \cdot \hat{y}_i = \|\hat{x}_j\| \|\hat{y}_i\| \cos\left(\frac{\pi}{2} - {}^i\theta_j\right) = \|\hat{x}_j\| \|\hat{y}_i\| \sin({}^i\theta_j) = \sin({}^i\theta_j)$$

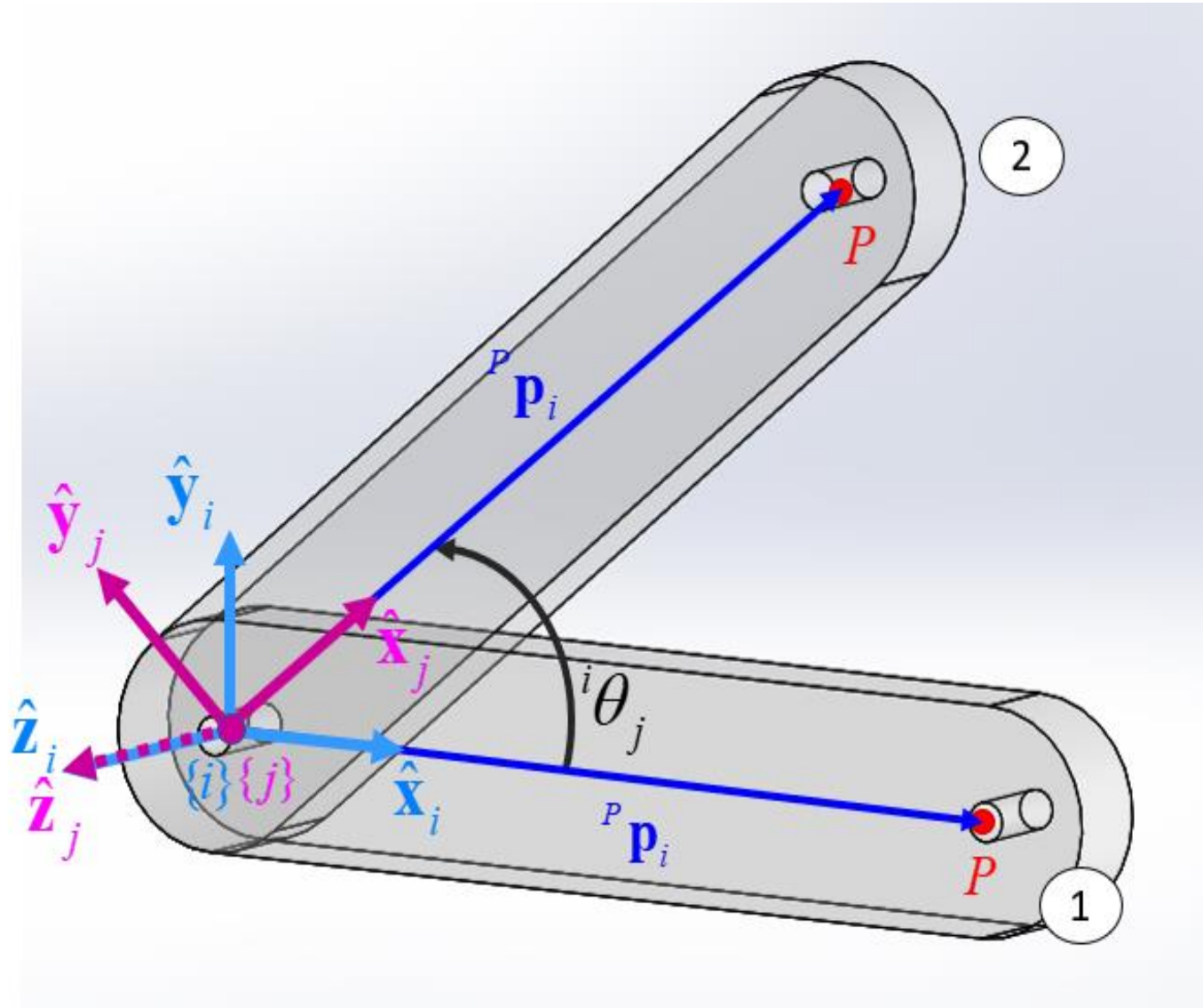
$$\hat{y}_j \cdot \hat{y}_i = \|\hat{y}_j\| \|\hat{y}_i\| \cos({}^i\theta_j) = \cos({}^i\theta_j)$$

Elemento base



$${}^i\mathbf{R}_j = \begin{pmatrix} \hat{x}_j \cdot \hat{x}_i & \hat{y}_j \cdot \hat{x}_i & \hat{z}_j \cdot \hat{x}_i \\ \hat{y}_j \cdot \hat{x}_i & \hat{y}_j \cdot \hat{y}_i & \hat{z}_j \cdot \hat{y}_i \\ \hat{z}_j \cdot \hat{x}_i & \hat{z}_j \cdot \hat{y}_i & \hat{z}_j \cdot \hat{z}_i \end{pmatrix}$$

Elemento base

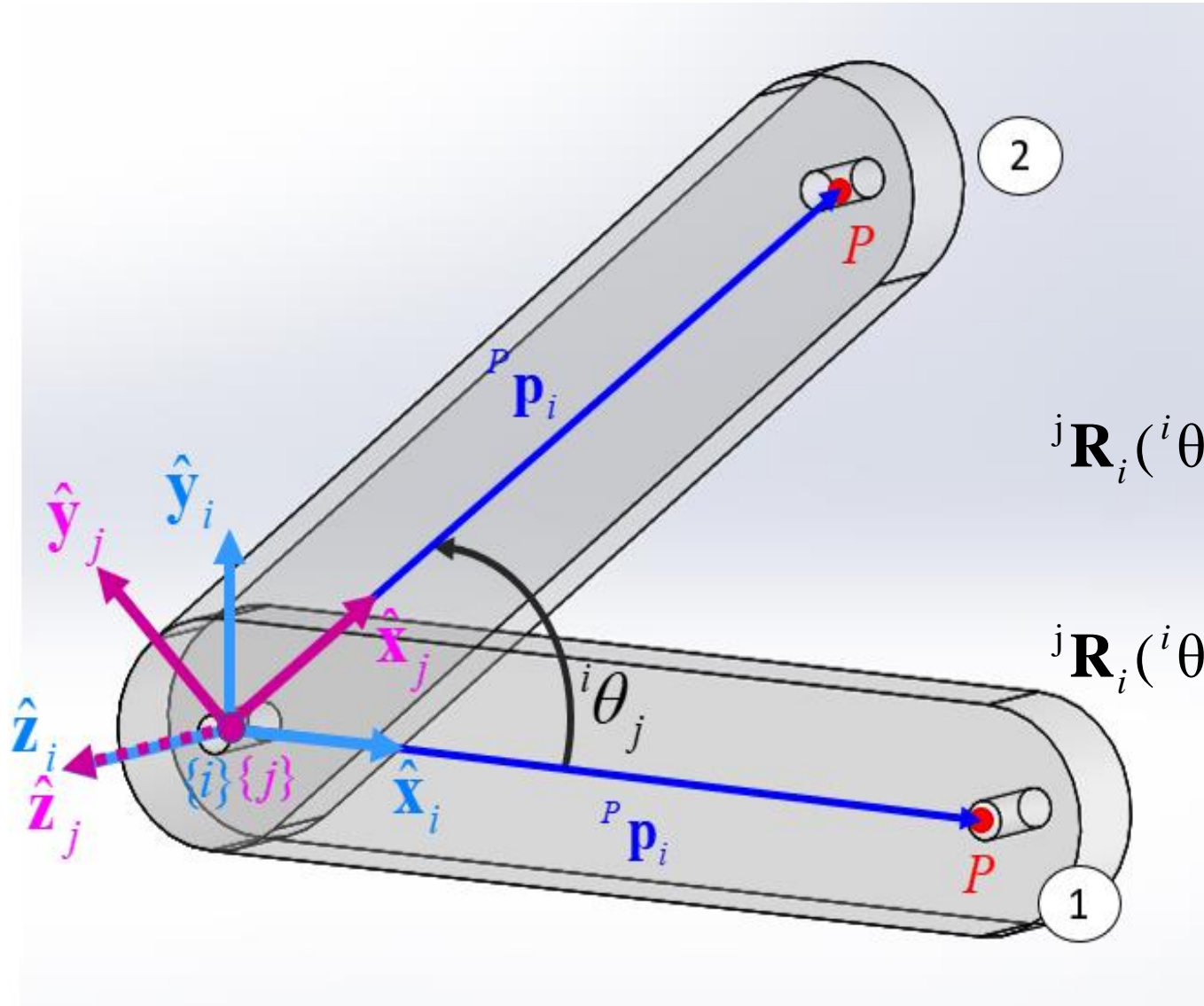


$${}^i\mathbf{R}_j = \begin{pmatrix} \hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i \end{pmatrix}$$

$${}^i\mathbf{R}_j({}^i\theta_j) = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^j\mathbf{p}_P = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$$

Elemento base



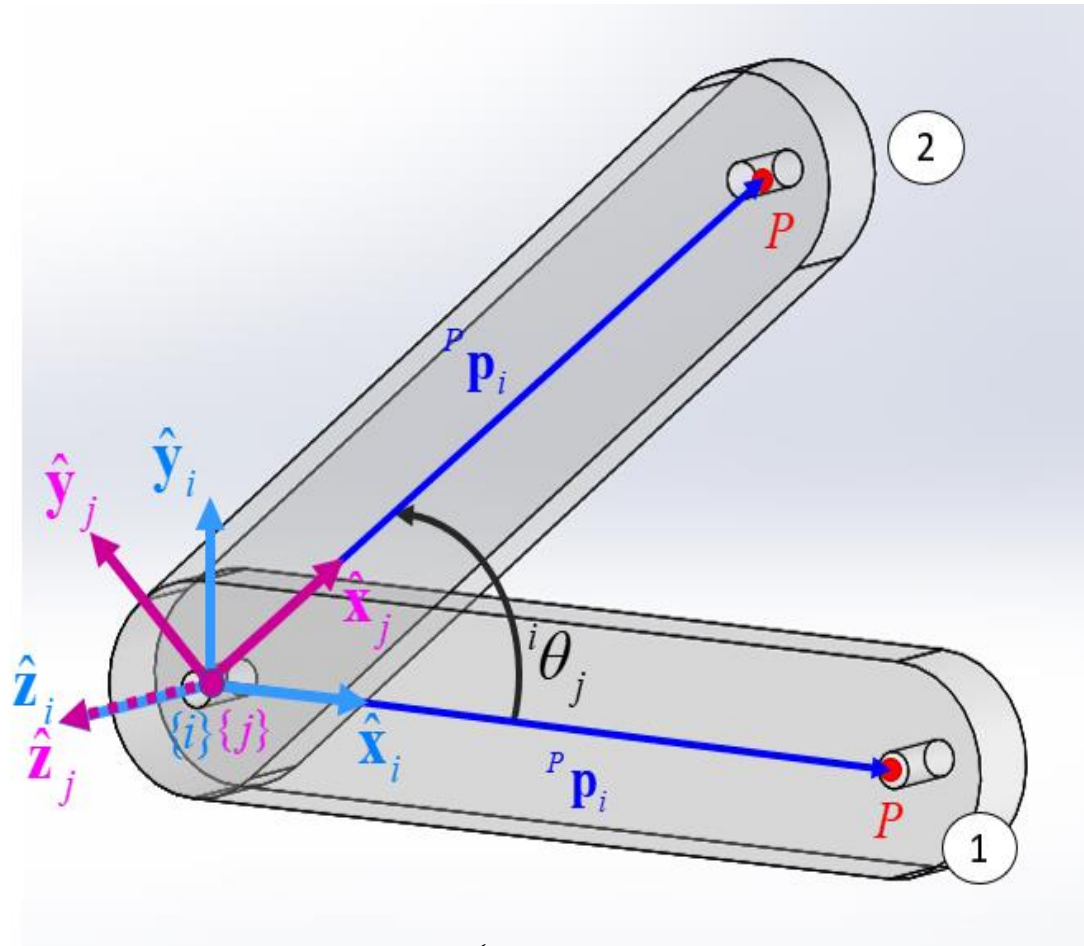
$${}^i\mathbf{R}_j({}^i\theta_j) = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^j\mathbf{R}_i({}^i\theta_j) = {}^i\mathbf{R}_j({}^i\theta_j)^{-1} = \begin{pmatrix} \cos({}^i\theta_j) & \sin({}^i\theta_j) & 0 \\ -\sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^j\mathbf{R}_i({}^i\theta_j) = {}^i\mathbf{R}_j({}^i\theta_j)^{-1} = {}^i\mathbf{R}_j({}^i\theta_j)^T$$

$${}^j\mathbf{p}_P = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$$

Elemento base



$${}^i\mathbf{T}_j({}^i\theta_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^i\mathbf{R}_j({}^i\theta_j) & {}^i\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & {}^iy_j \\ 0 & 0 & 1 \end{pmatrix}$$

Elemento base

$${}^i\mathbf{T}_j({}^i\theta_j, {}^i x_j, {}^i y_j) = \begin{pmatrix} {}^i\mathbf{R}_j({}^i\theta_j) & {}^i\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & {}^i x_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & {}^i y_j \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^i\mathbf{T}_P = {}^i\mathbf{T}_j {}^j\mathbf{T}_P$$

$${}^i\mathbf{T}_j = {}^i\mathbf{T}_j({}^i\theta_j, 0, 0) = \begin{pmatrix} {}^i\mathbf{R}_j({}^i\theta_j) & {}^i\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^j\mathbf{T}_P = {}^j\mathbf{T}_j(0, L, 0) = \begin{pmatrix} 1 & 0 & L_i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^i\mathbf{T}_P = {}^i\mathbf{T}_j {}^j\mathbf{T}_P = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & L_i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & L_i \cos({}^i\theta_j) \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & L_i \sin({}^i\theta_j) \\ 0 & 0 & 1 \end{pmatrix}$$

