Robótica grupo2
Clase 14

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Derechos reservados

Repaso/Elemento base (caso de estudio)

- Repaso de la clases anteriores
  - Planteamiento del modelado del elementos base
    - Planteamiento del modelo de la postura
      - Transformaciones homogéneas
      - Composición de transformaciones
    - Planteamiento del modelo cinemático de las velocidades
    - Planteamiento del modelo cinemático de las aceleraciones
    - Plantemiento dinámico
      - Planteamiento del modelo dinámico directo
      - Planteamiento del modelo dinámico inverso

- Planteamiento del elemento base en la robótica

Modelo cinemático de la postura

$$
{}^{0}\mathbf{T}_{P} = {}^{0}\mathbf{T}_{1}\,{}^{1}\mathbf{T}_{2}\,{}^{2}\mathbf{T}_{3}\,{}^{3}\mathbf{T}_{P} = \begin{pmatrix} {}^{0}\mathbf{R}_{P} & {}^{0}\mathbf{p}_{P} \\ \mathbf{0}^{T} & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix} \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & -\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
q = \{{}^{0}\theta_1, {}^{1}\theta_2, {}^{2}\theta_3\}
$$

$$
\mathbf{q}^{T} = \begin{pmatrix} {}^{0}\theta_1 & {}^{1}\theta_2 & {}^{2}\theta_3 \end{pmatrix}
$$

$$
{}^{0}\boldsymbol{\xi}_{P}(q) = \begin{pmatrix} {}^{0}\mathbf{p}_{P} \\ {}^{0}\boldsymbol{\theta}_{P} \end{pmatrix} = \begin{pmatrix} {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3 \end{pmatrix} \qquad {}^{0}\boldsymbol{\xi}_{P} = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix}
$$

Modelo cinemático de la postura

$$ {}^{0}\boldsymbol{\xi}_P = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P(q) = \begin{pmatrix} {}^{0}\mathbf{p}_P \\ {}^{0}\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3 \end{pmatrix} $$

$$ \mathbf{F} = {}^{0}\boldsymbol{\xi}_P - {}^{0}\boldsymbol{\xi}_P(q) = \mathbf{0} = \begin{pmatrix} {}^{0}x_P - {}^{0}x_1 - L_1\cos({}^{0}\theta_1) - L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_P - {}^{0}y_1 - L_1\sin({}^{0}\theta_1) - L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_P - {}^{0}\theta_1 - {}^{1}\theta_2 - {}^{2}\theta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P = {}^{0}\boldsymbol{\xi}_P(q) $$

Modelo cinemático directo de las velocidades

Modelo de la postura

$$^{0}\boldsymbol{\xi}_P = {}^{0}\boldsymbol{\xi}_P(q)$$

$$\frac{d}{dt}{}^{0}\boldsymbol{\xi}_P = \frac{d}{dt}\begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix} = \frac{\partial}{\partial {}^{0}x_P}\begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix}{}^{0}\dot{x}_P + \frac{\partial}{\partial {}^{0}y_P}\begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix}{}^{0}\dot{y}_P + \frac{\partial}{\partial {}^{0}\theta_P}\begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix}{}^{0}\dot{\theta}_P =$$

$$= \begin{pmatrix} {}^{0}\dot{x}_P \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ {}^{0}\dot{y}_P \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ {}^{0}\dot{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}\dot{y}_P \\ {}^{0}\dot{\theta}_P \end{pmatrix}$$

Derivada del modelo

$$\frac{d}{dt}{}^{0}\boldsymbol{\xi}_P = \frac{d}{dt}{}^{0}\boldsymbol{\xi}_P(q)$$

$$^{0}\dot{\boldsymbol{\xi}}_P = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}\dot{y}_P \\ {}^{0}\dot{\theta}_P \end{pmatrix}$$

Modelo cinemático directo de las velocidades

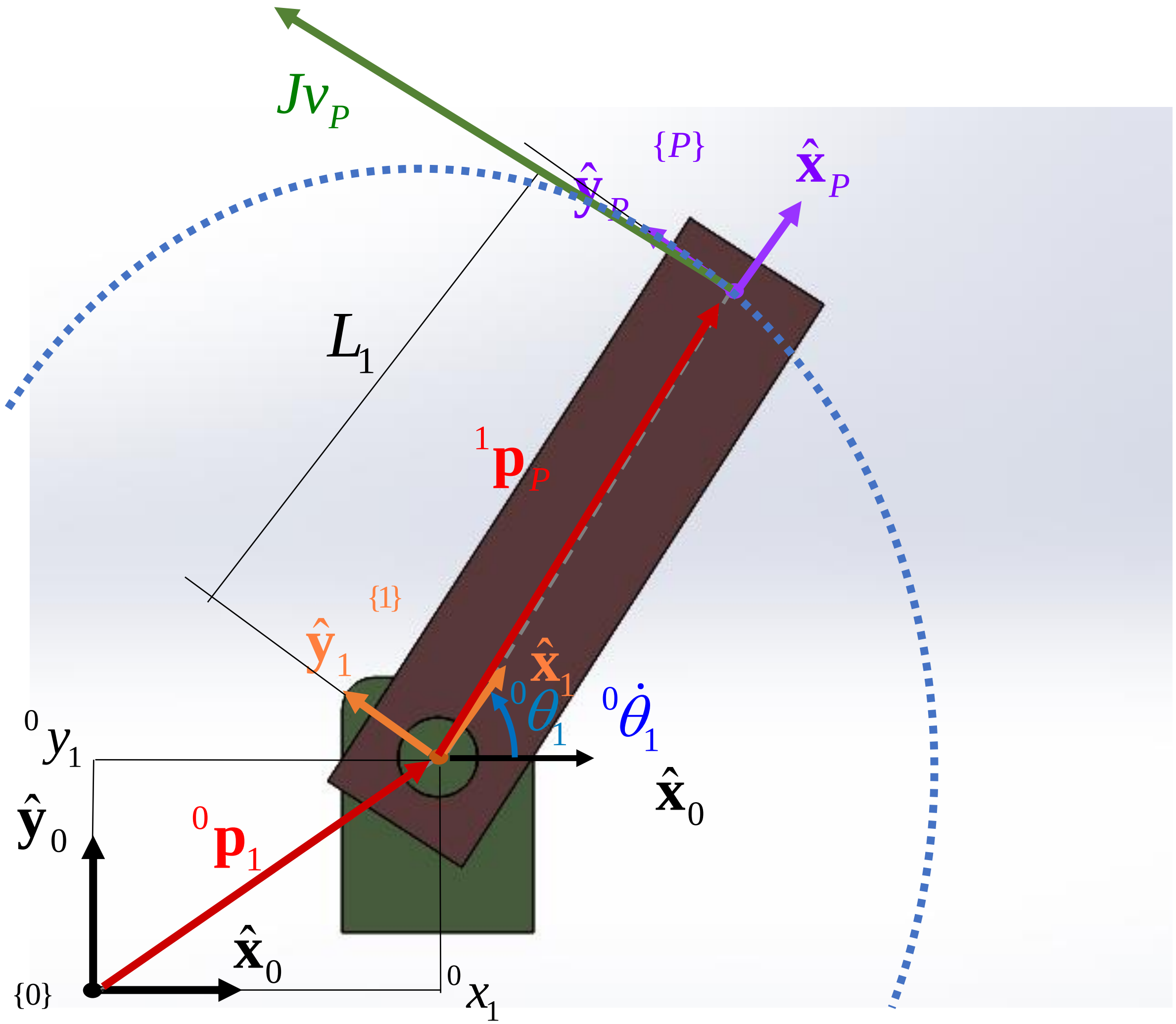
$$\frac{d}{dt}{}^{0}\boldsymbol{\xi}_P(q) = \frac{\partial}{\partial {}^{0}\theta_1}{}^{0}\boldsymbol{\xi}_P(q)\,{}^{0}\dot{\theta}_1 + \frac{\partial}{\partial {}^{1}\theta_2}{}^{0}\boldsymbol{\xi}_P(q)\,{}^{1}\dot{\theta}_2 + \frac{\partial}{\partial {}^{2}\theta_3}{}^{0}\boldsymbol{\xi}_P(q)\,{}^{2}\dot{\theta}_3 =$$

$$^{0}\dot{\boldsymbol{\xi}}_P = \mathbf{J}_\theta(q)\dot{\mathbf{q}}$$

$$= \begin{pmatrix} \frac{\partial}{\partial {}^{0}\theta_1}{}^{0}\boldsymbol{\xi}_P(q) & \frac{\partial}{\partial {}^{1}\theta_2}{}^{0}\boldsymbol{\xi}_P(q) & \frac{\partial}{\partial {}^{2}\theta_3}{}^{0}\boldsymbol{\xi}_P(q) \end{pmatrix}\begin{pmatrix} {}^{0}\dot{\theta}_1 \\ {}^{1}\dot{\theta}_2 \\ {}^{2}\dot{\theta}_3 \end{pmatrix} = \mathbf{J}_\theta(q)\dot{\mathbf{q}}$$

Junta rotacional

Modelo cinemático de la velocidad



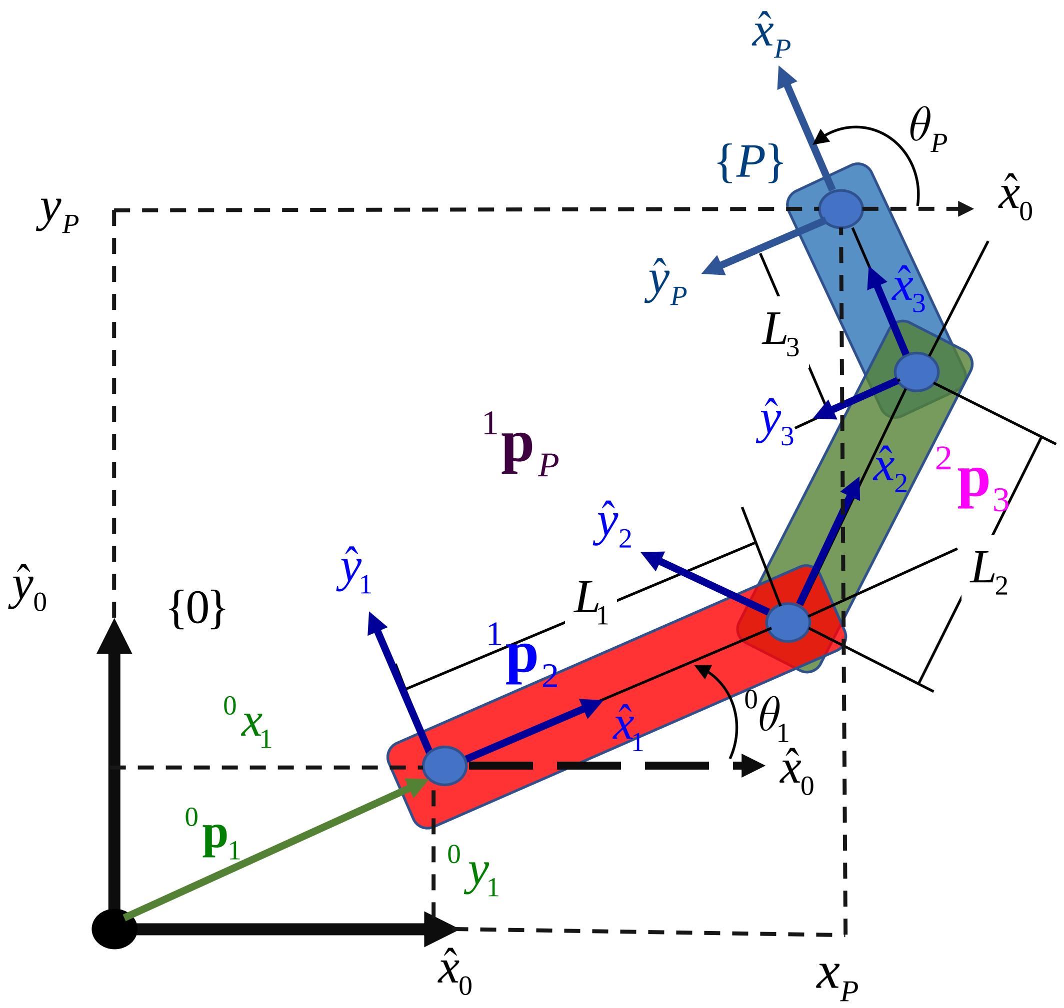
Vector de la postura de un eslabón

$$ {}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \end{matrix}} \\ \boxed{{}^0\theta_1} \end{pmatrix} $$

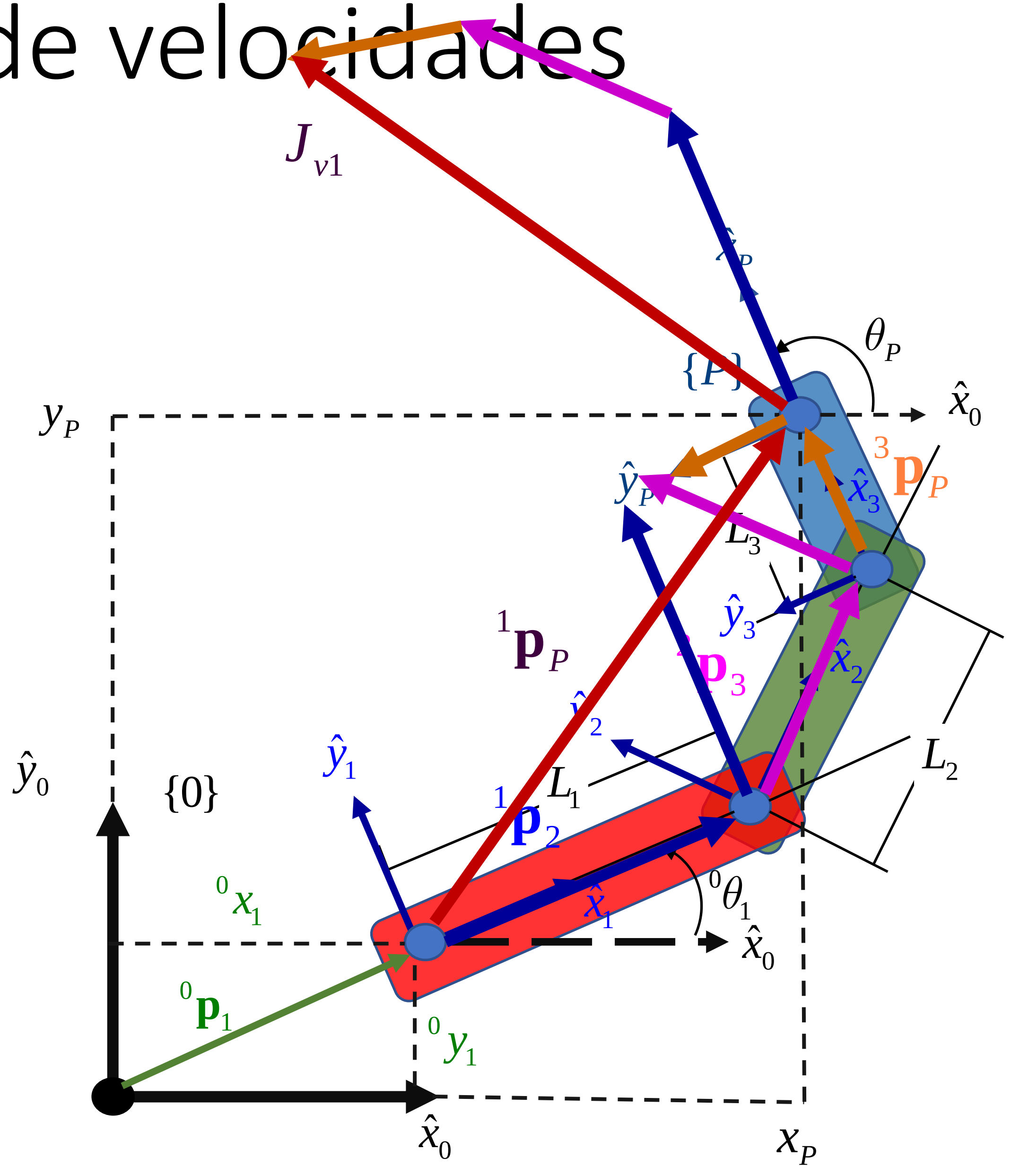
Vector de velocidades del eslabón

$$ {}^0\dot{\xi}_P = \frac{d}{d\,{}^0\theta_1}\,{}^0\xi_P\,{}^0\dot{\theta}_1 = \begin{pmatrix} \boxed{\begin{matrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \end{matrix}} \\ \boxed{1} \end{pmatrix} {}^0\dot{\theta}_1 $$

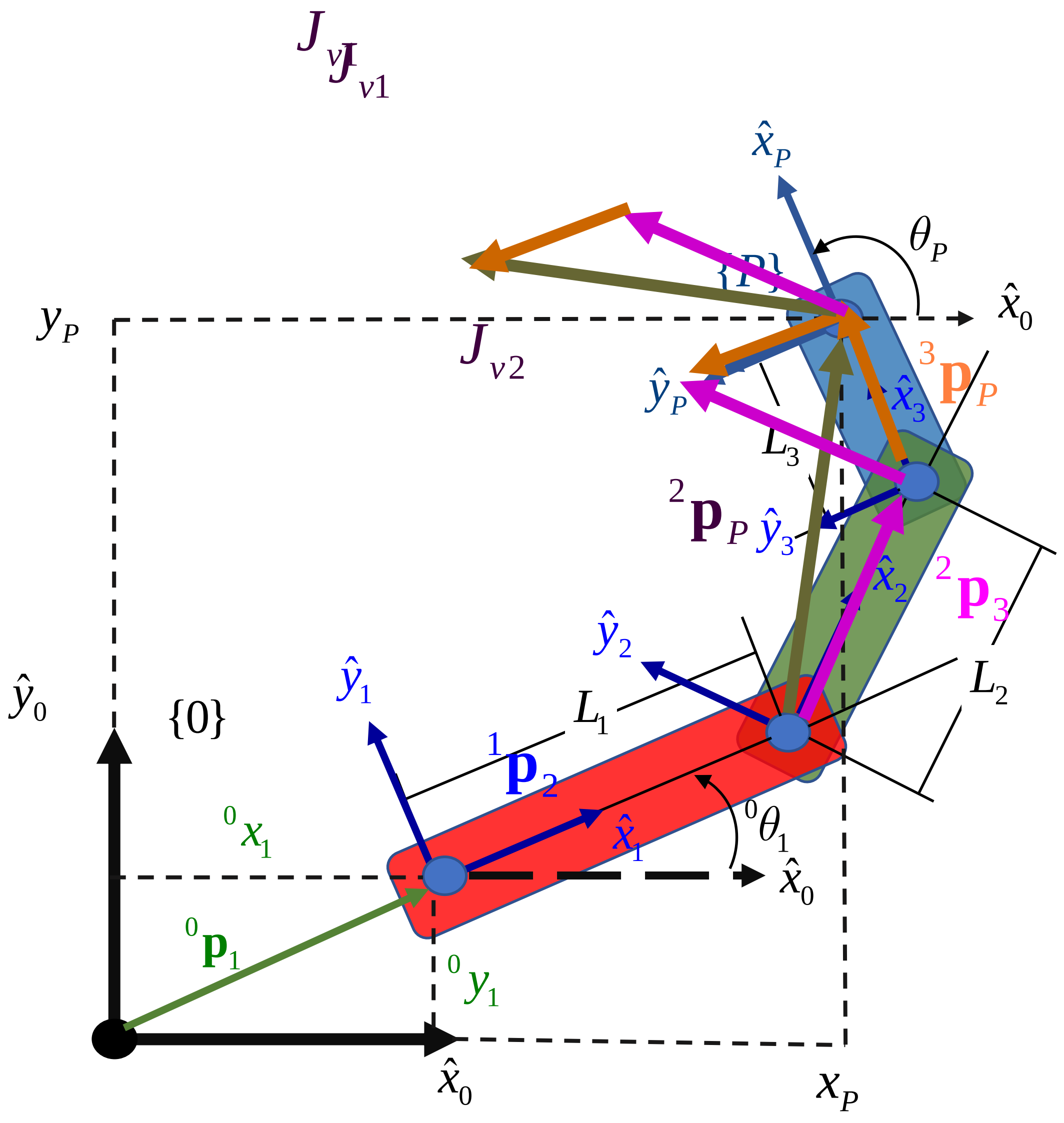
Propagación de velocidades



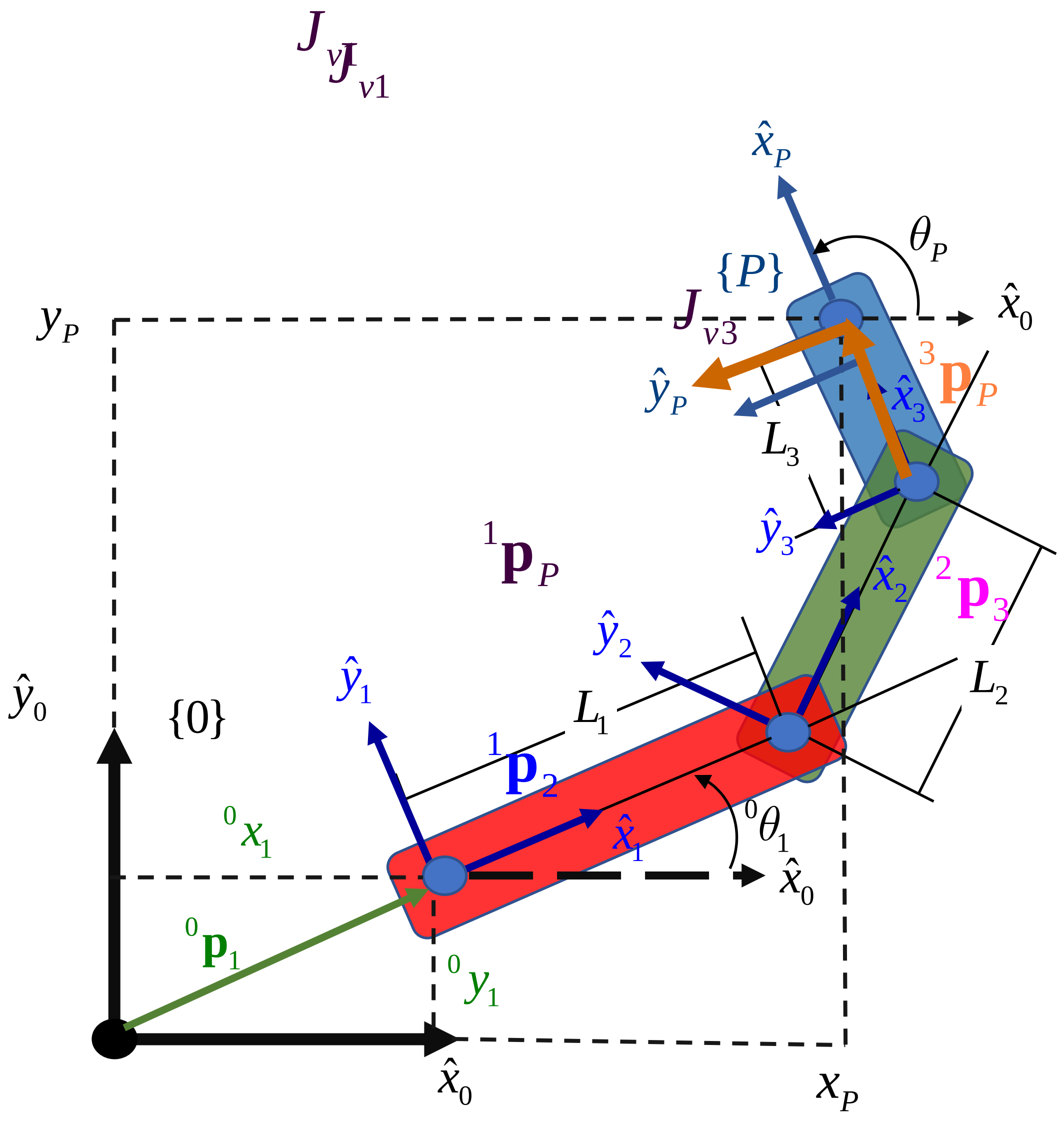
Propagación de velocidades



Propagación de velocidades



Propagación de velocidades



Modelo cinemático directo de las velocidades

Modelo cinemático directo de las velocidades

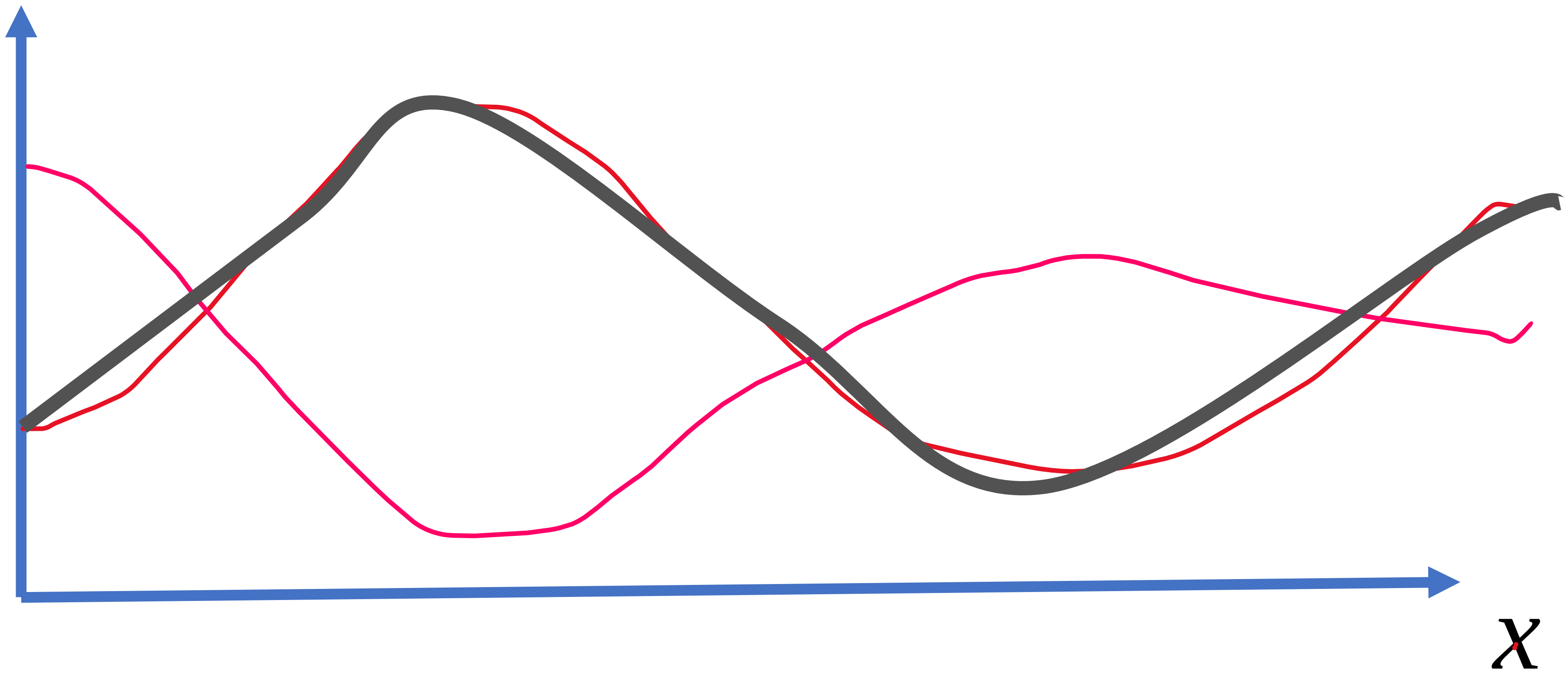
$$
{}^0\dot{\boldsymbol{\xi}}_P = \begin{pmatrix} {}^0\dot{x}_P \\ {}^0\dot{y}_P \\ {}^0\dot{\theta}_P \end{pmatrix}
$$

$$
{}^0\dot{\boldsymbol{\xi}}_P = \mathbf{J}_\theta(q)\dot{\mathbf{q}}
$$

$$
\mathbf{J}_\theta(q) =
\begin{array}{c}
\begin{array}{ccc} \quad {}^0\dot{\theta}_1 \quad & \quad {}^1\dot{\theta}_2 \quad & \quad {}^2\dot{\theta}_3 \quad \end{array} \\
\begin{pmatrix}
-L_1\sin({}^0\theta_1) - L_2\sin({}^0\theta_1 + {}^1\theta_2) - L_3\sin({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) & -L_2\sin({}^0\theta_1 + {}^1\theta_2) - L_3\sin({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) & -L_3\sin({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) \\
L_1\cos({}^0\theta_1) + L_2\cos({}^0\theta_1 + {}^1\theta_2) + L_3\cos({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) & L_2\cos({}^1\theta_1 + {}^1\theta_2) + L_3\cos({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) & L_3\cos({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) \\
1 & 1 & 1
\end{pmatrix}
\end{array}
$$

$$
\dot{\mathbf{q}} = \begin{pmatrix} {}^0\dot{\theta}_1 \\ {}^1\dot{\theta}_2 \\ {}^2\dot{\theta}_3 \end{pmatrix}
$$

$$f(x) = a_0 + a_1 x + \ldots + a_n x$$



x_1

Modelo cinemático directo de las velocidades

Modelo cinemático directo de las velocidades

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}} $$

Modelo cinemático inverso de las velocidades

$$ \dot{\mathbf{q}} = \mathbf{J}_{\theta}^{-1}(q)\,{}^{0}\dot{\boldsymbol{\xi}}_{P} $$

Modelo cinemático directo de las velocidades

$$x + 2y + z = 1$$

$$x - y + 2z = 3$$

$$2x - 3y = -1$$

$$\mathbf{A} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ b_{1,1} & b_{1,2} & b_{1,3} \\ c_{1,1} & c_{1,2} & c_{1,3} \end{pmatrix}$$

$$\mathbf{Ax} = \mathbf{c}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ 2 & 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{c}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ 2 & 3 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

Modelo cinemático directo de las velocidades

Modelo cinemático directo de las velocidades

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}} $$

$$ \mathbf{J}_{\theta}(q) = $$

$$ \begin{pmatrix} -L_1 \sin({}^{0}\theta_1) - L_2 \sin({}^{0}\theta_1 + {}^{1}\theta_2) - L_3 \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & -L_2 \sin({}^{0}\theta_1 + {}^{1}\theta_2) - L_3 \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & -L_3 \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ L_1 \cos({}^{0}\theta_1) + L_2 \cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3 \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & L_2 \cos({}^{1}\theta_1 + {}^{1}\theta_2) + L_3 \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & L_3 \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ 1 & 1 & 1 \end{pmatrix} $$

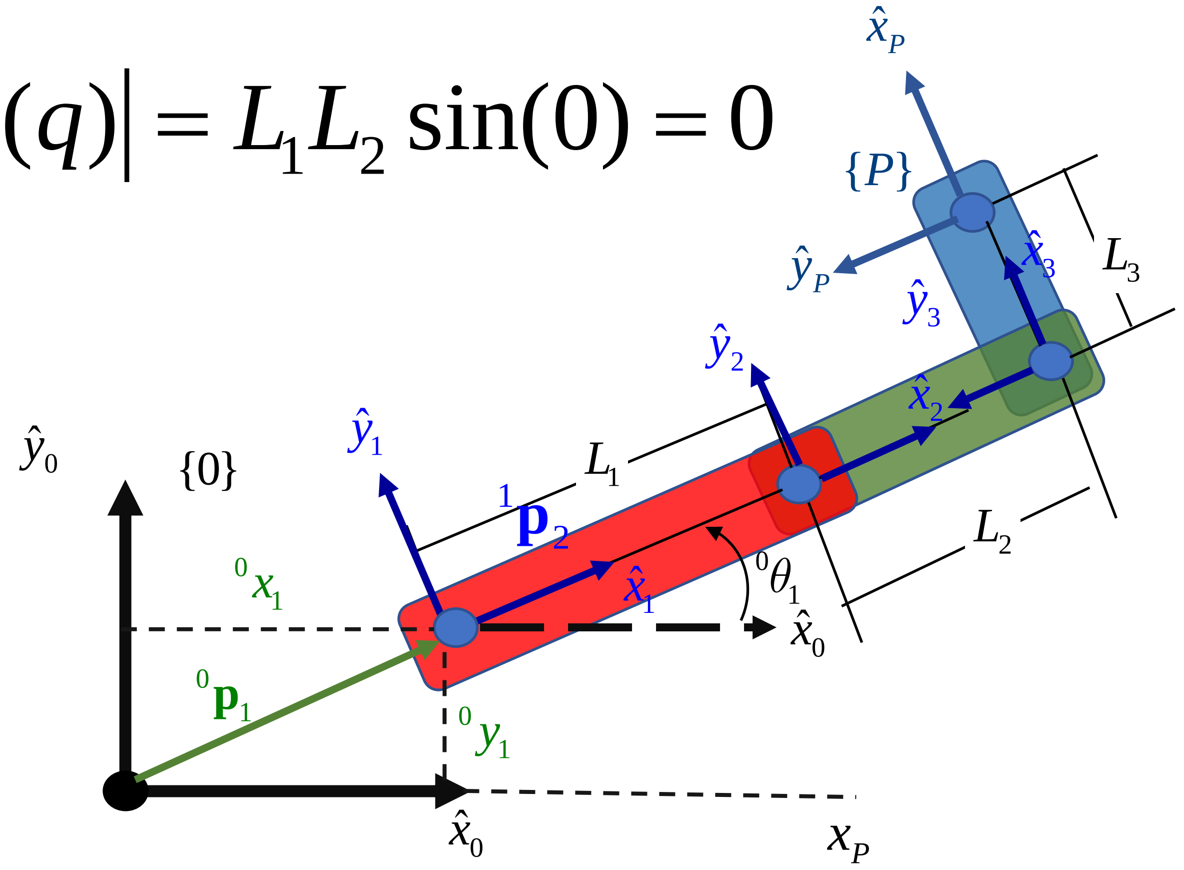
Índice de manipulabilidad

$$ w = \left| \mathbf{J}_{\theta}(q) \right| = L_1 L_2 \sin({}^{1}\theta_2) $$

Propagación de velocidades

Posturas singulares

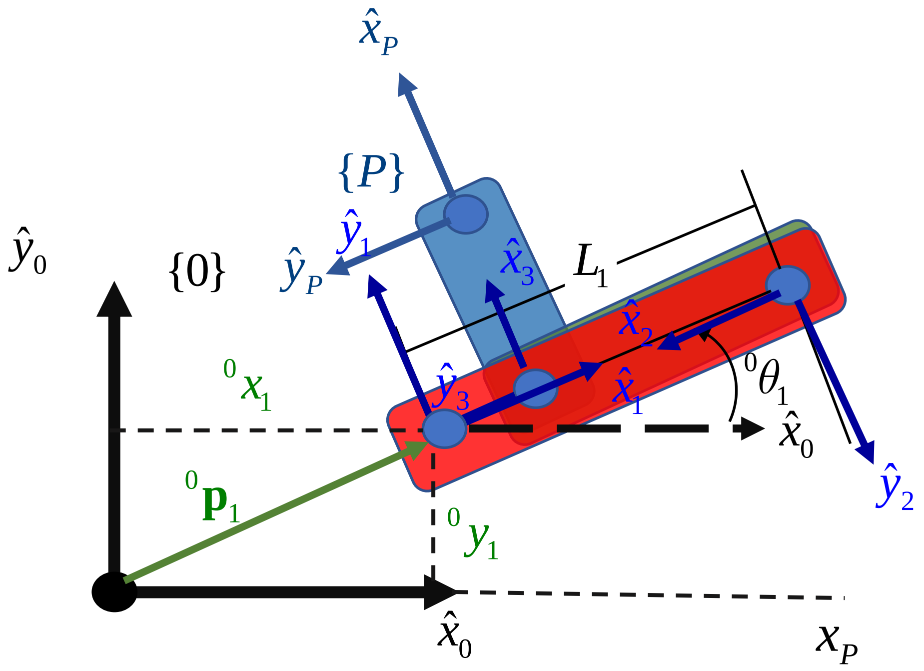
$$\left|\mathbf{J}_{\theta}(q)\right| = L_1 L_2 \sin(0) = 0$$

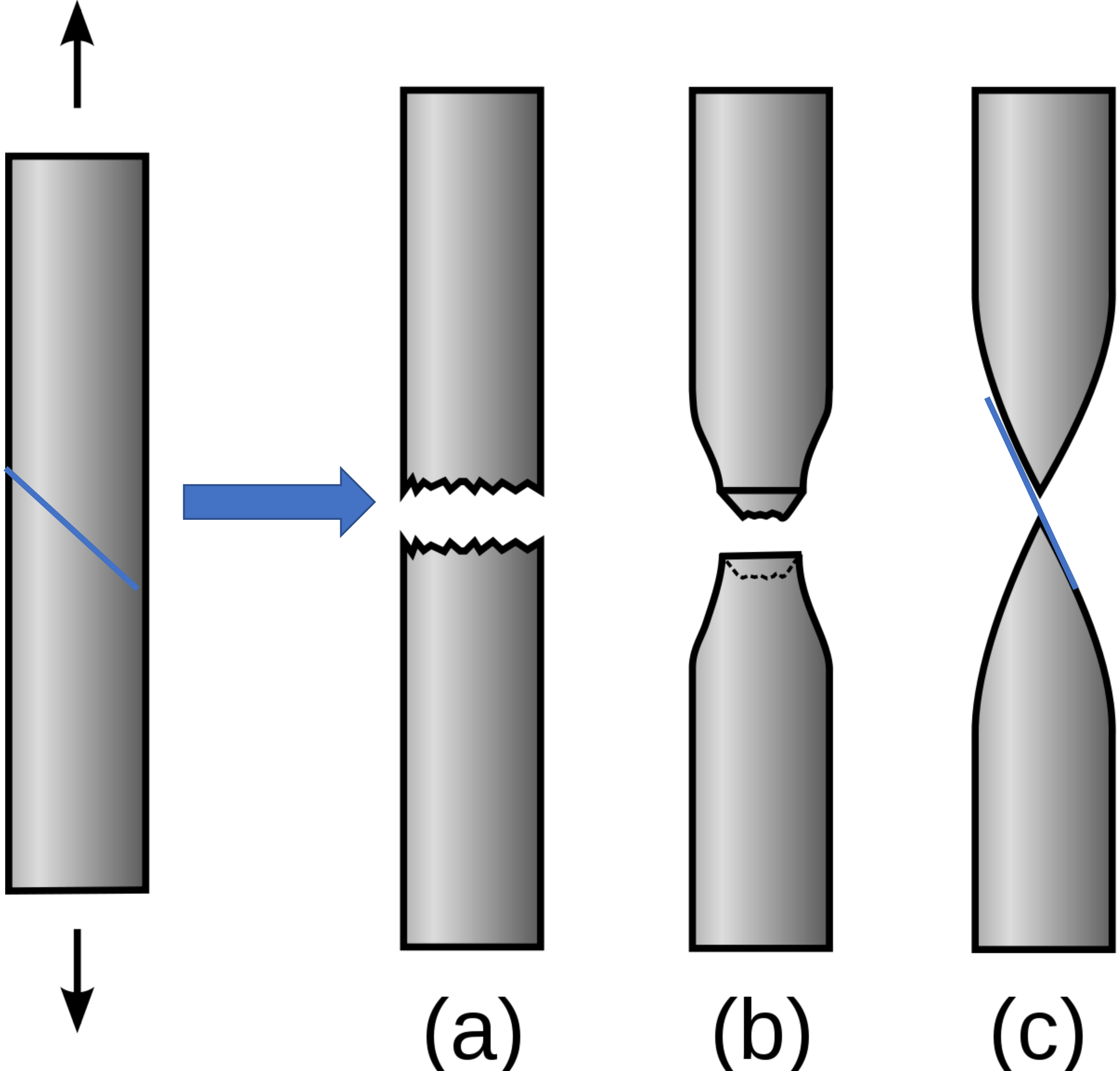


Propagación de velocidades

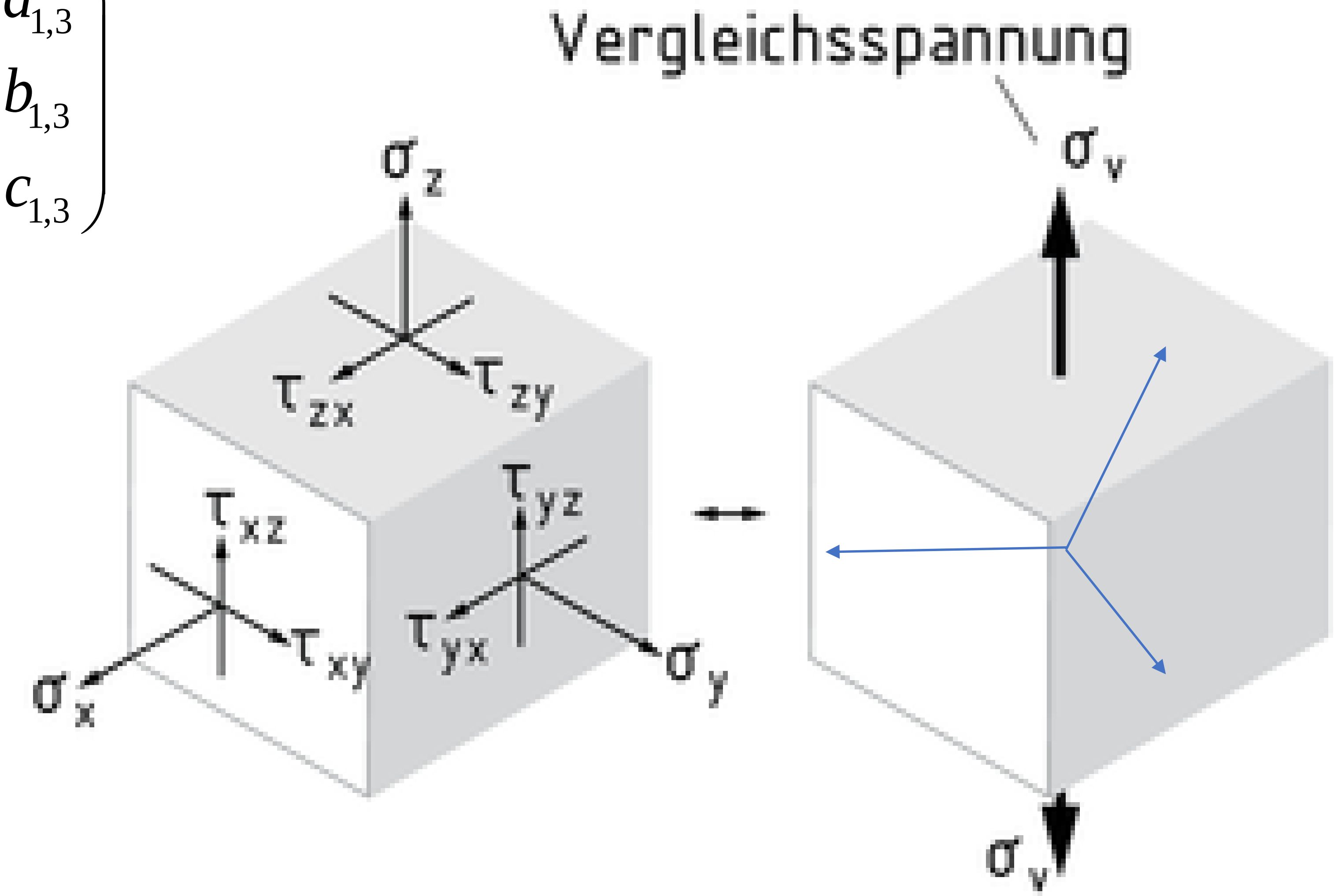
Posturas singulares

$$\left|\mathbf{J}_\theta(q)\right| = L_1 L_2 \sin(n\pi) = 0$$




(a)
(b)
(c)

$$\mathbf{A} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ b_{1,1} & b_{1,2} & b_{1,3} \\ c_{1,1} & c_{1,2} & c_{1,3} \end{pmatrix}$$



$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}} \Rightarrow w = \left|\mathbf{J}_{\theta}(q)\right| $$

$$ \dot{\mathbf{q}} = \mathbf{J}_{\theta}(q)^{+}\,{}^{0}\dot{\boldsymbol{\xi}}_{P} \Rightarrow \left|\mathbf{J}_{\theta}(q)^{+}\right| = 1/w $$