Robótica grupo2
Clase 5

Facultad de Ingeniería UNAM

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Derechos reservados

Conceptos básicos/Elemento base

- Resumen de conceptos

- Elemento base (eslabón)
  - Planteamiento de su modelado cinemático.
    - Modelo cinemático de la posición.
    - Modelo cinemático de las velocidades.
    - Modelos cinemático de sus aceleraciones.

Introducción

Modelo cinemático de la postura

$$^{0}\boldsymbol{\xi}_P = {}^{0}\boldsymbol{\xi}_P(\mathbf{q})$$

$$\mathbf{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$$

$$C = \{q_1, \ldots, q_n\}$$

$$\Downarrow$$

$$\mathbf{F} = {}^{0}\boldsymbol{\xi}_P - {}^{0}\boldsymbol{\xi}_P(\mathbf{q}) = 0$$

Introducción

Modelo cinemático directo de las velocidades

$$ {}^{0}\dot{\boldsymbol{\xi}}_P = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} $$

$$ \dot{\mathbf{q}} = \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix} $$

Modelo cinemático directo de las velocidades

$$ \dot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{+}\, {}^{0}\dot{\boldsymbol{\xi}}_P $$

Introducción

Modelo cinemático directo de las aceleraciones

$$ {}^{0}\ddot{\boldsymbol{\xi}}_{P} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} $$

Modelo cinemático inverso de las aceleraciones

$$ \ddot{\mathbf{q}} = \mathbf{J}(\mathbf{q})^{+}\,{}^{0}\ddot{\boldsymbol{\xi}}_{P} + \dot{\mathbf{J}}(\mathbf{q})^{+}\,{}^{0}\dot{\boldsymbol{\xi}}_{P} $$

Introducción

Cálculo del par

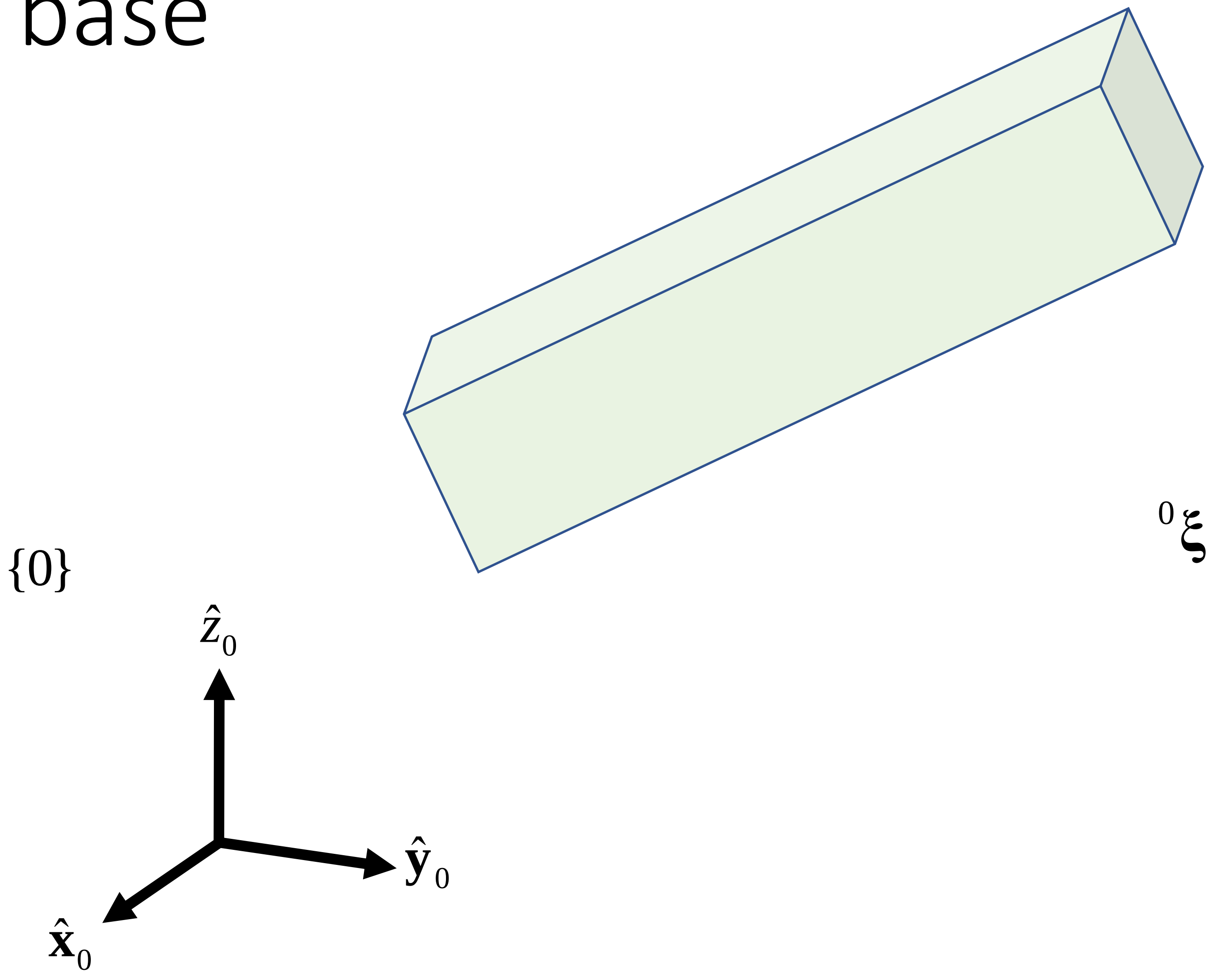
$$\Gamma = k - u = \sum_{i=1}^{n} k_i - \sum_{i=1}^{n} u_i$$

$$\tau_i = \frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_i}\Gamma\right) - \frac{\partial}{\partial q_i}\Gamma$$

Modelo dinámico

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_{ex} + \boldsymbol{\tau}_{\text{int}}$$

Elemento base



$$^{0}\boldsymbol{\xi}_P = \begin{pmatrix} ^{0}\mathbf{p}_P \\ ^{0}\boldsymbol{\theta}_P \end{pmatrix}$$

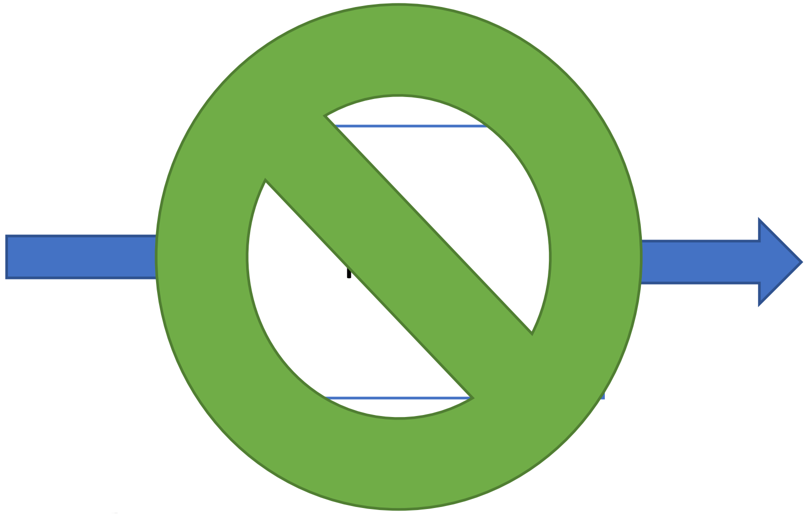
Elemento base

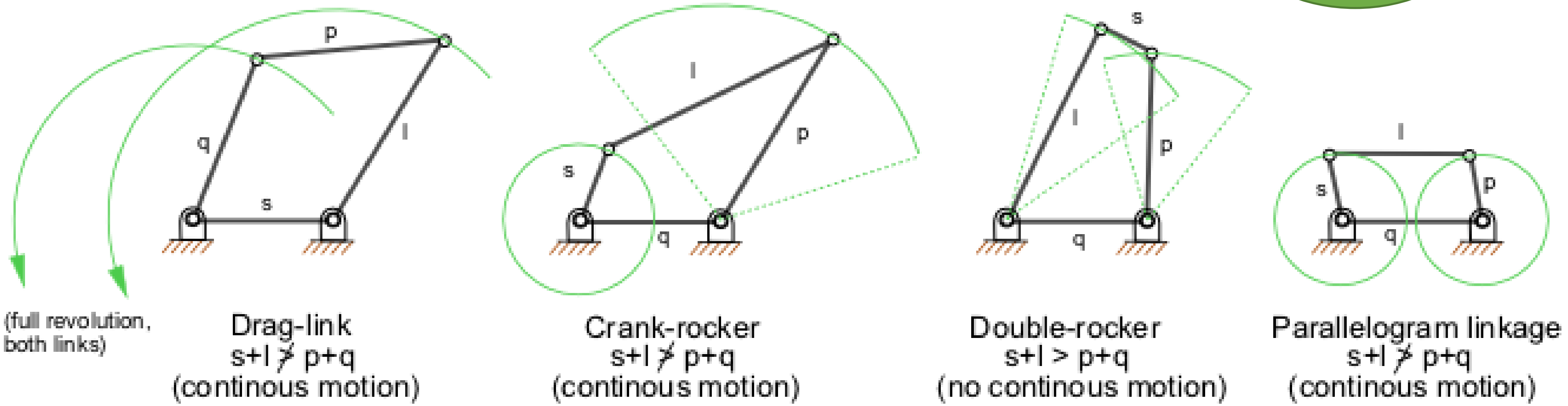
Elemento base

Eslabón

Un eslabón es un cuerpo rígido ideal el cual se relaciona con otros elementos (eslabones) por medio de un arreglo mecánico denominado junta.

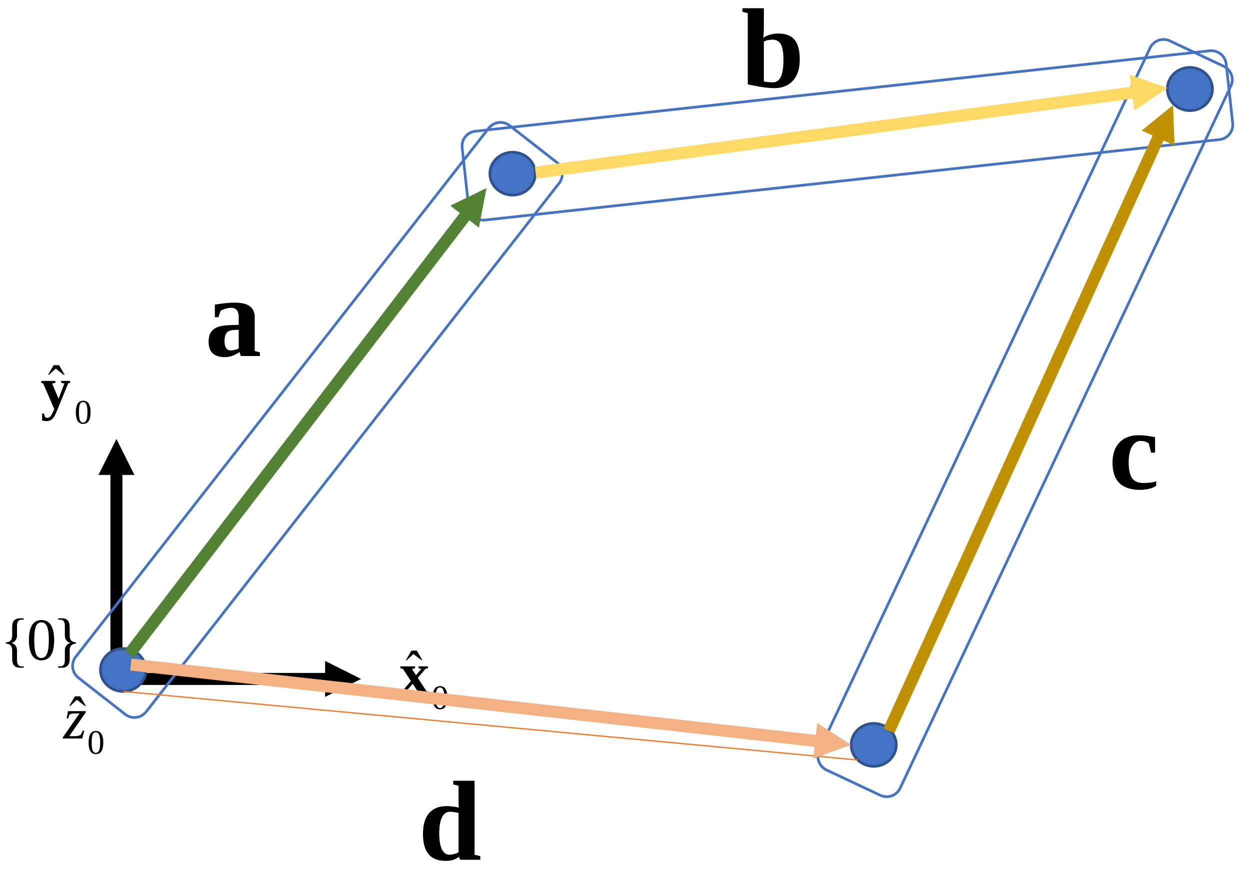
Elemento base

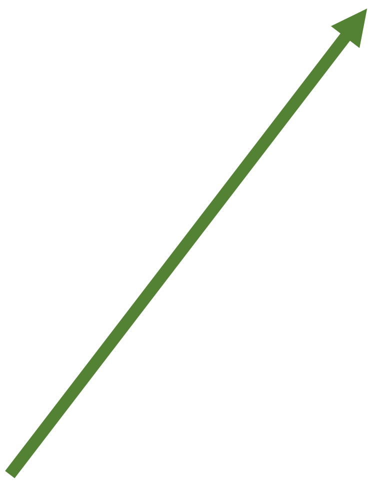




Elemento base

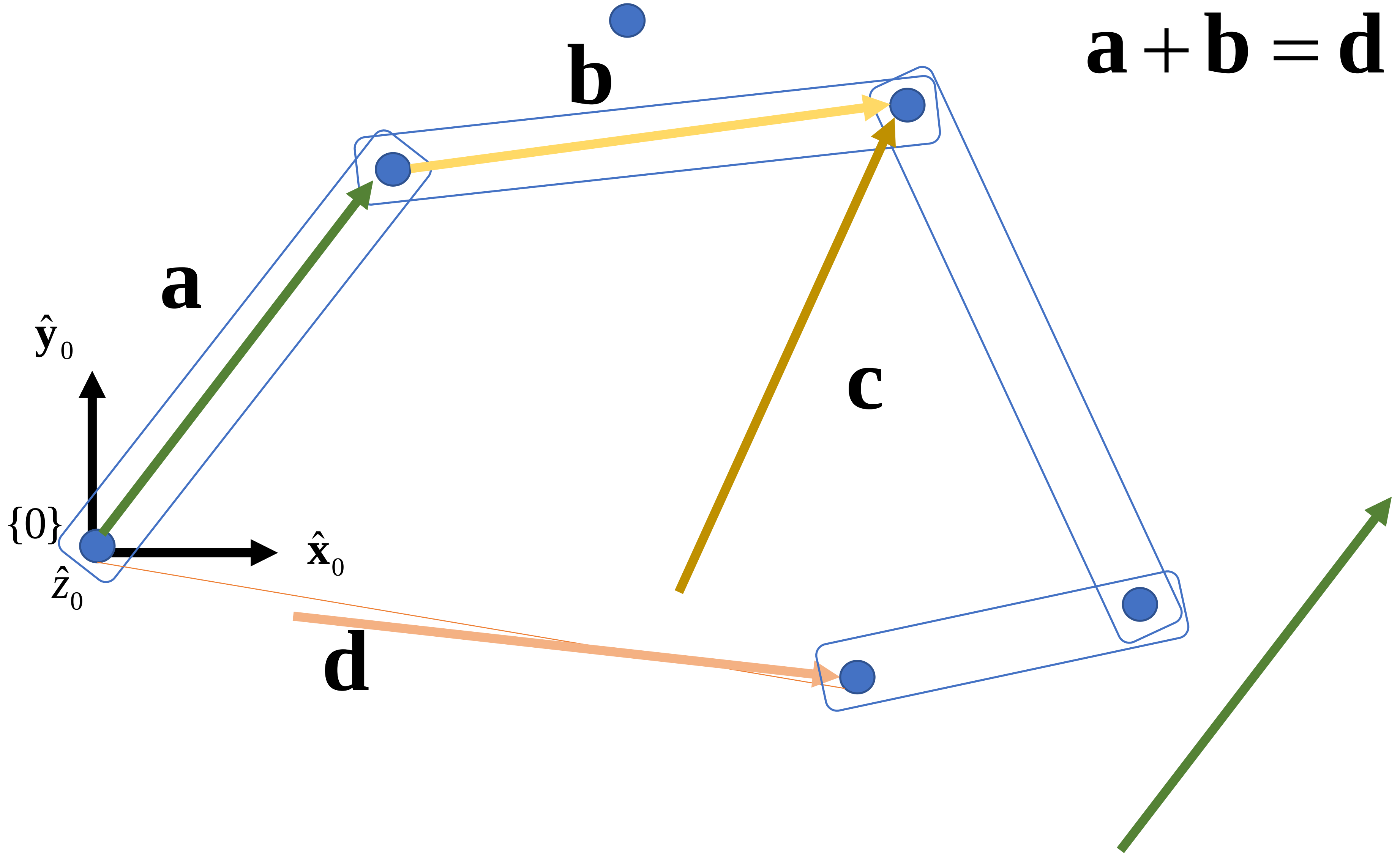
$$\mathbf{a}+\mathbf{b}=\mathbf{d}+\mathbf{c}$$



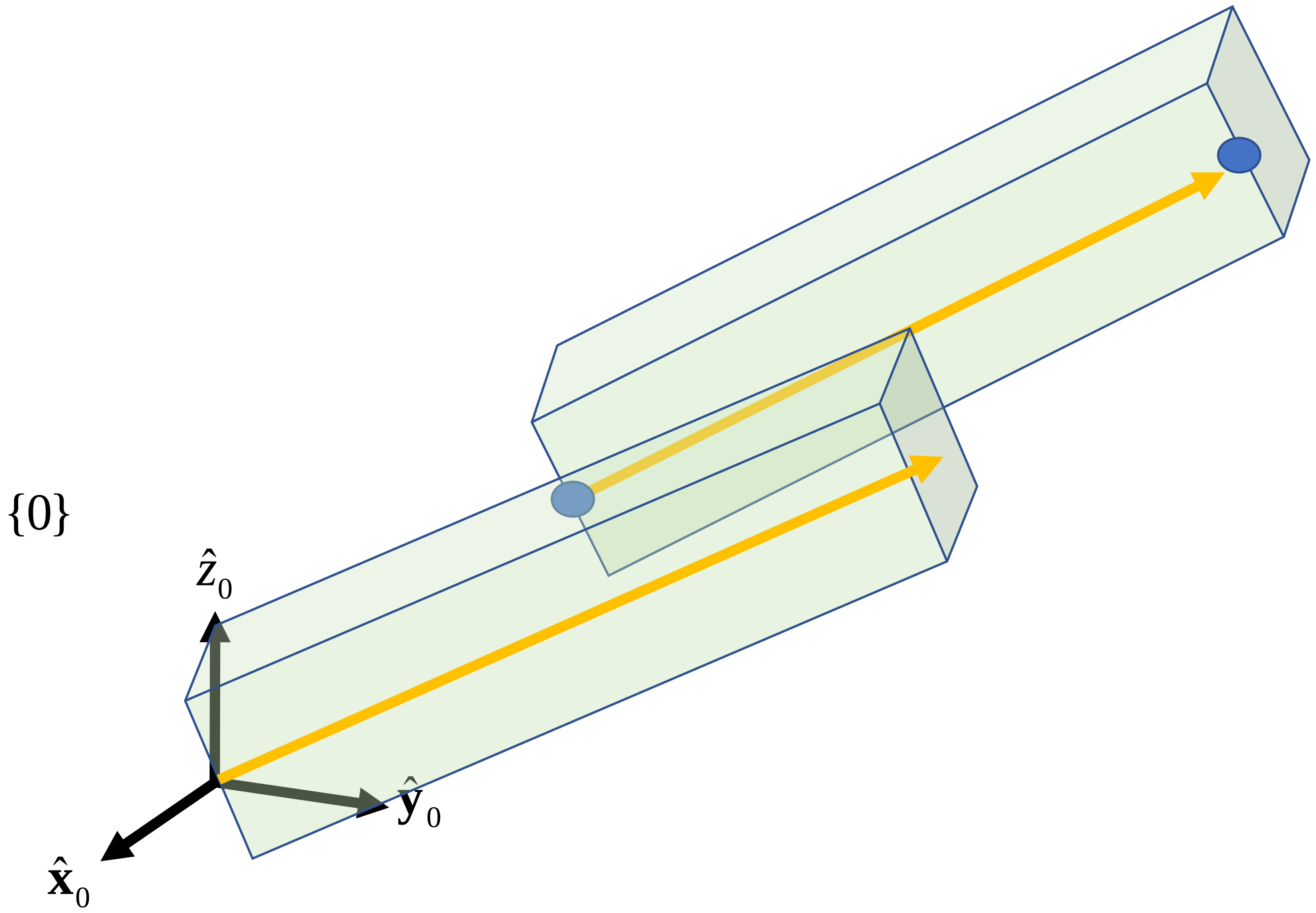


Elemento base

$$\mathbf{a}+\mathbf{b}=\mathbf{d}+\mathbf{c}$$

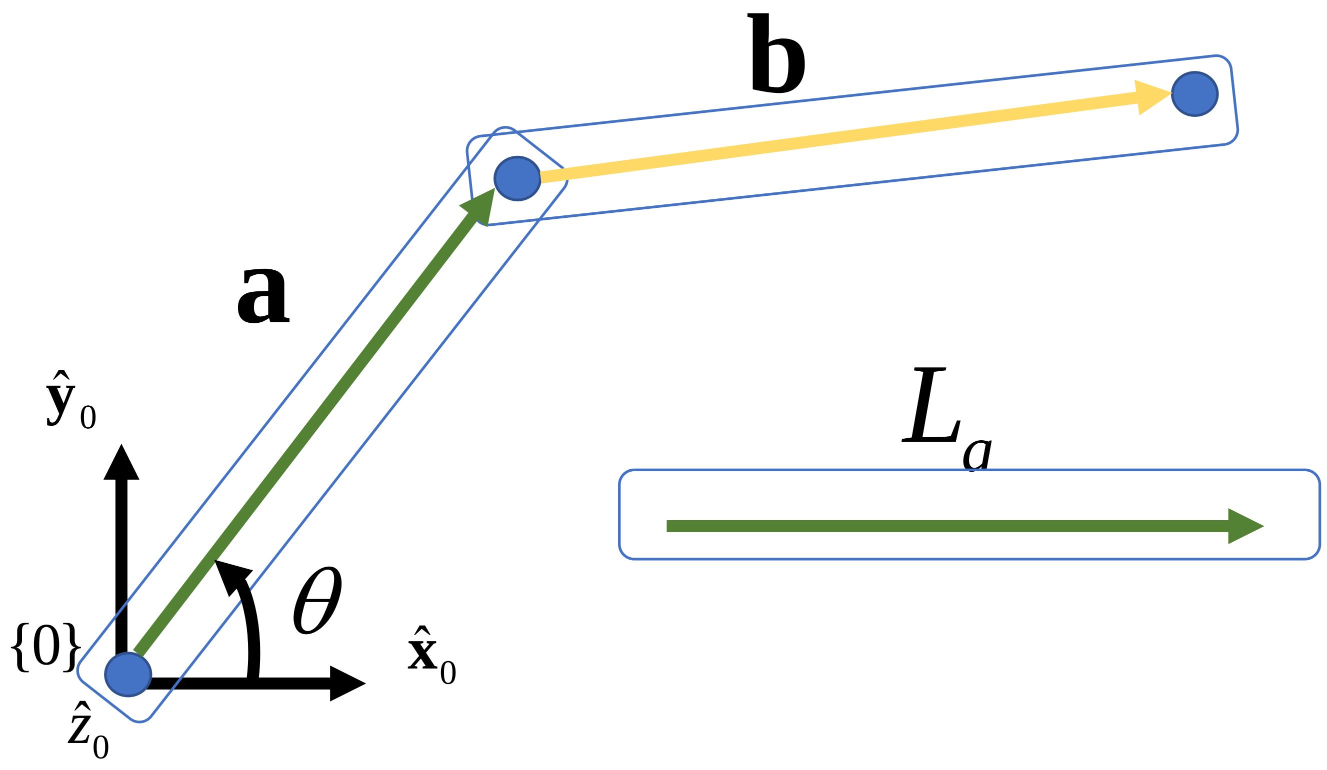


Elemento base



Elemento base

$$\mathbf{a} = \begin{pmatrix} x_a \\ y_a \\ 0 \end{pmatrix}$$



$$\mathbf{a} + \mathbf{b} = \mathbf{d} + \mathbf{c}$$

$$\mathbf{a} = \mathbf{R}(\theta)\mathbf{x}_a$$

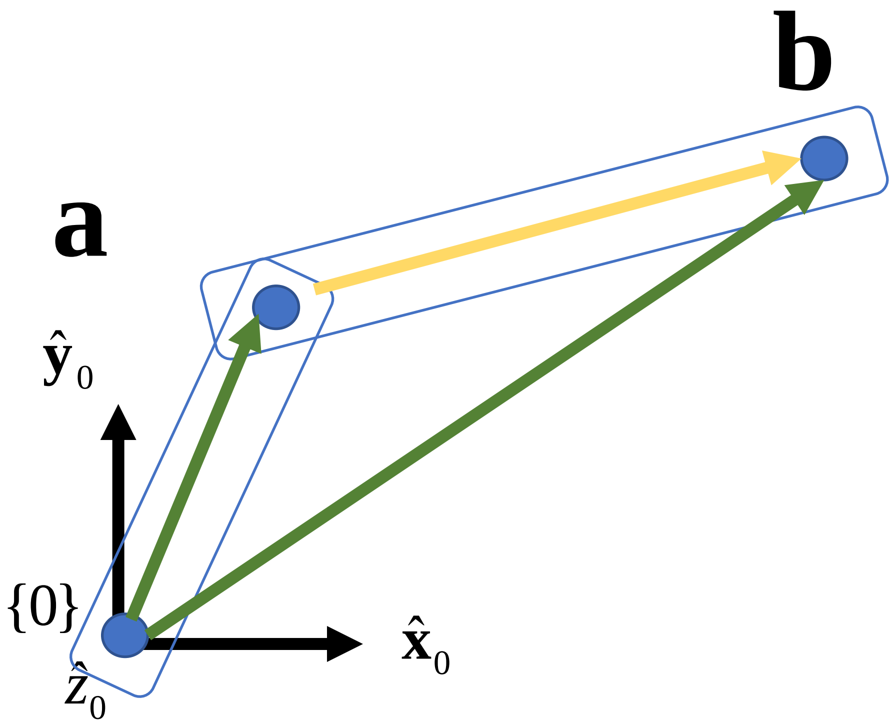
$$\mathbf{x}_a = \begin{pmatrix} L_a \\ 0 \\ 0 \end{pmatrix}$$

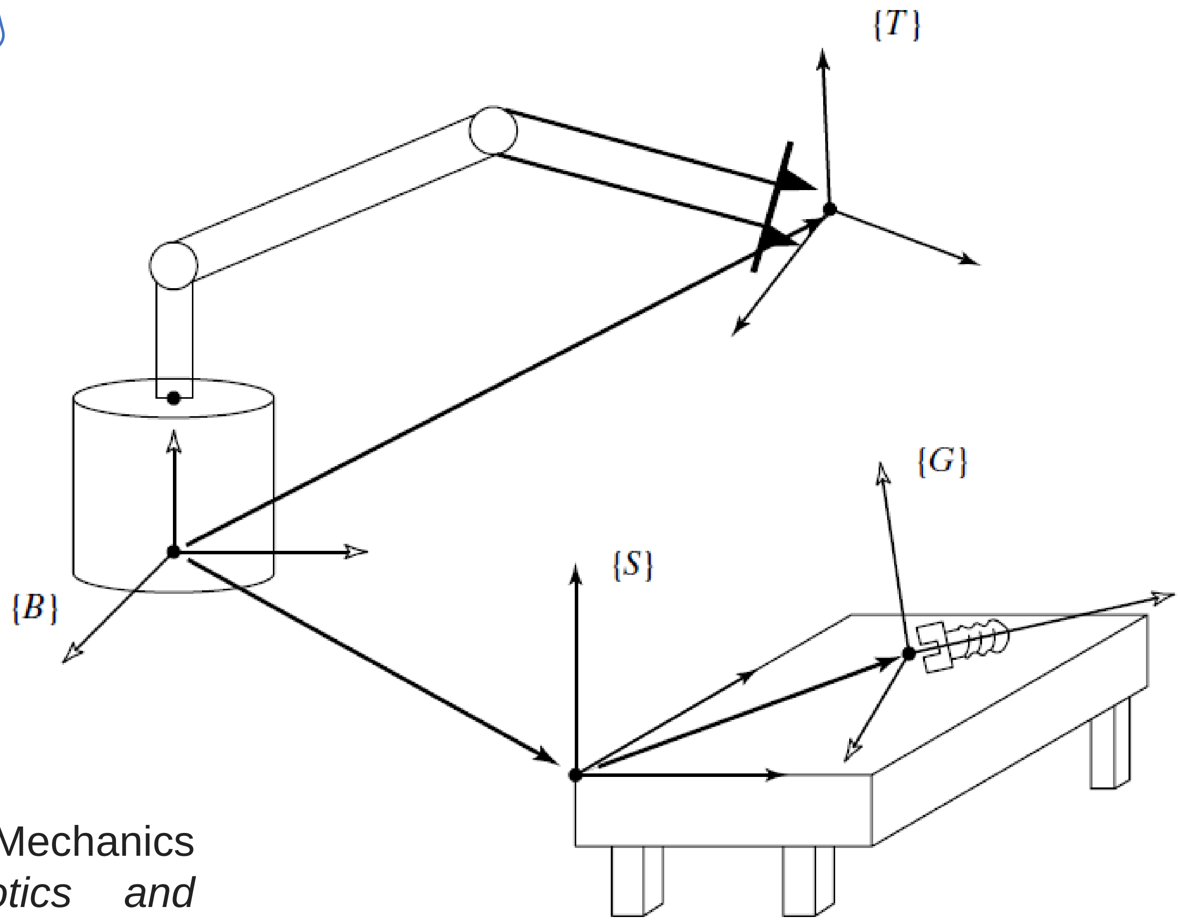
$$\mathbf{a} = \mathbf{R}(\theta)\mathbf{x}_a = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L_a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

Elemento base

$$\mathbf{a} + \mathbf{b} = \mathbf{x}$$

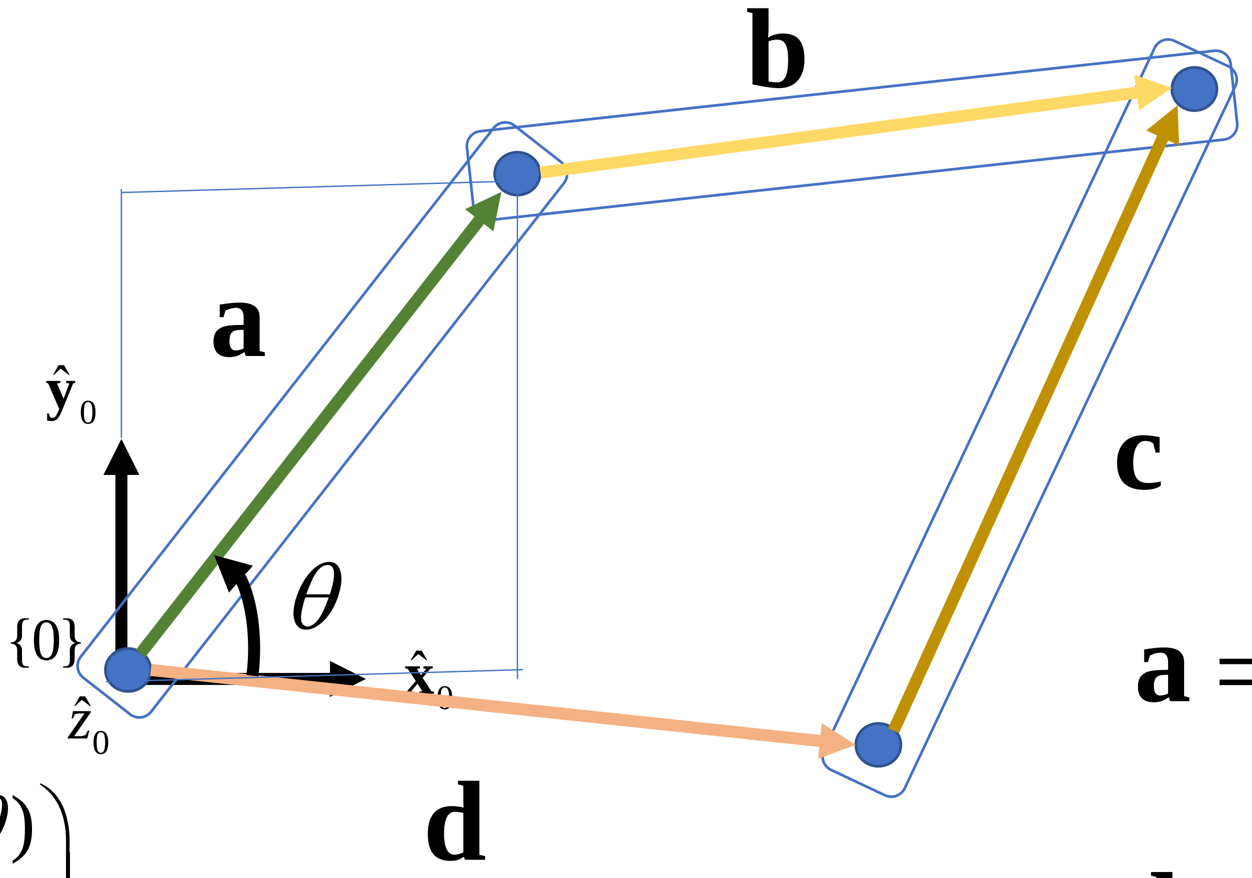




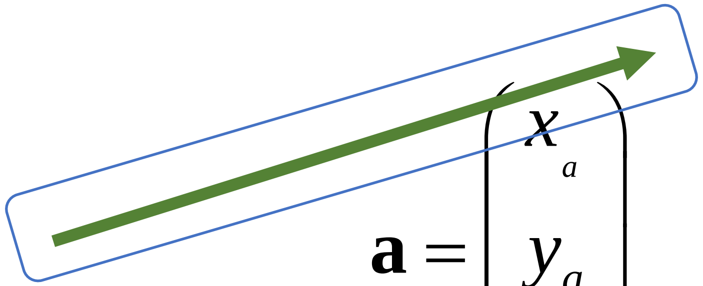
Merat, F. (1987). Introduction to robotics: Mechanics and control. *IEEE Journal on Robotics and Automation*, *3*(2), 166-166.

FIGURE 2.16: Manipulator reaching for a bolt.

Elemento base



$$\mathbf{a} + \mathbf{b} = \mathbf{d} + \mathbf{c}$$



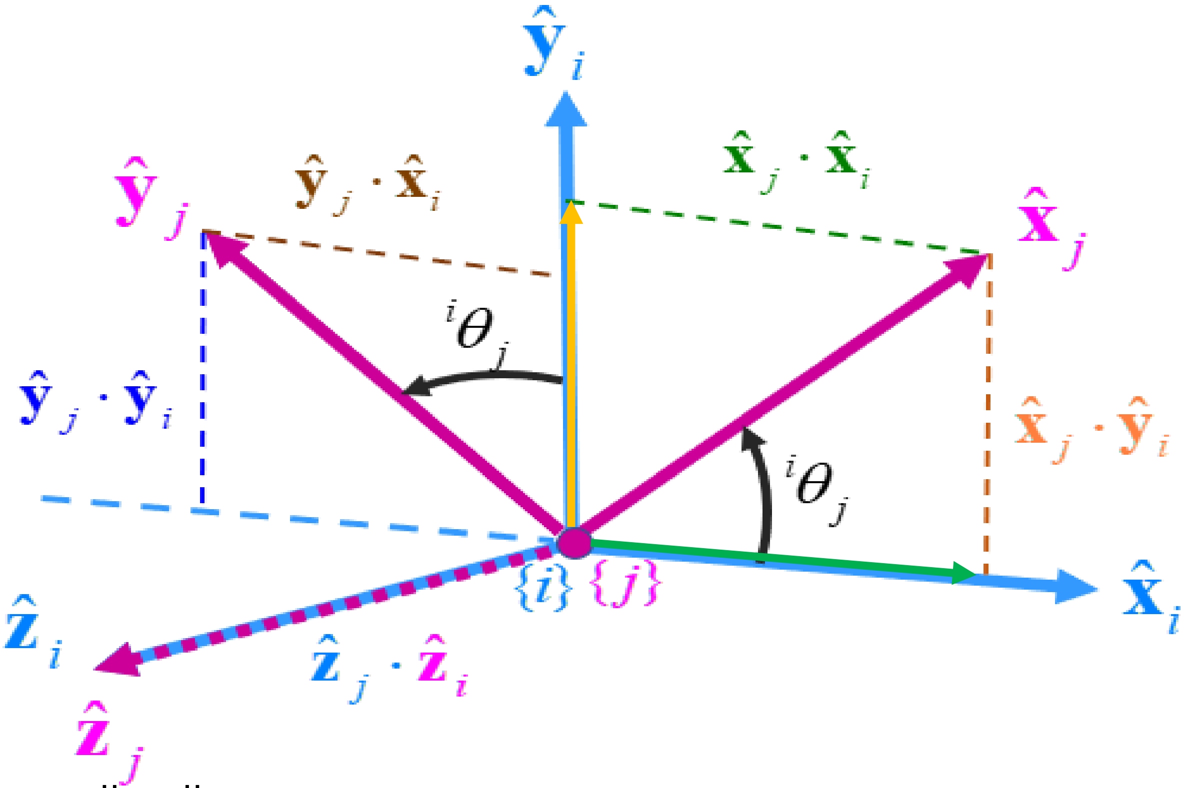
$$\mathbf{a} = \begin{pmatrix} x_a \\ y_a \\ 0 \end{pmatrix}$$

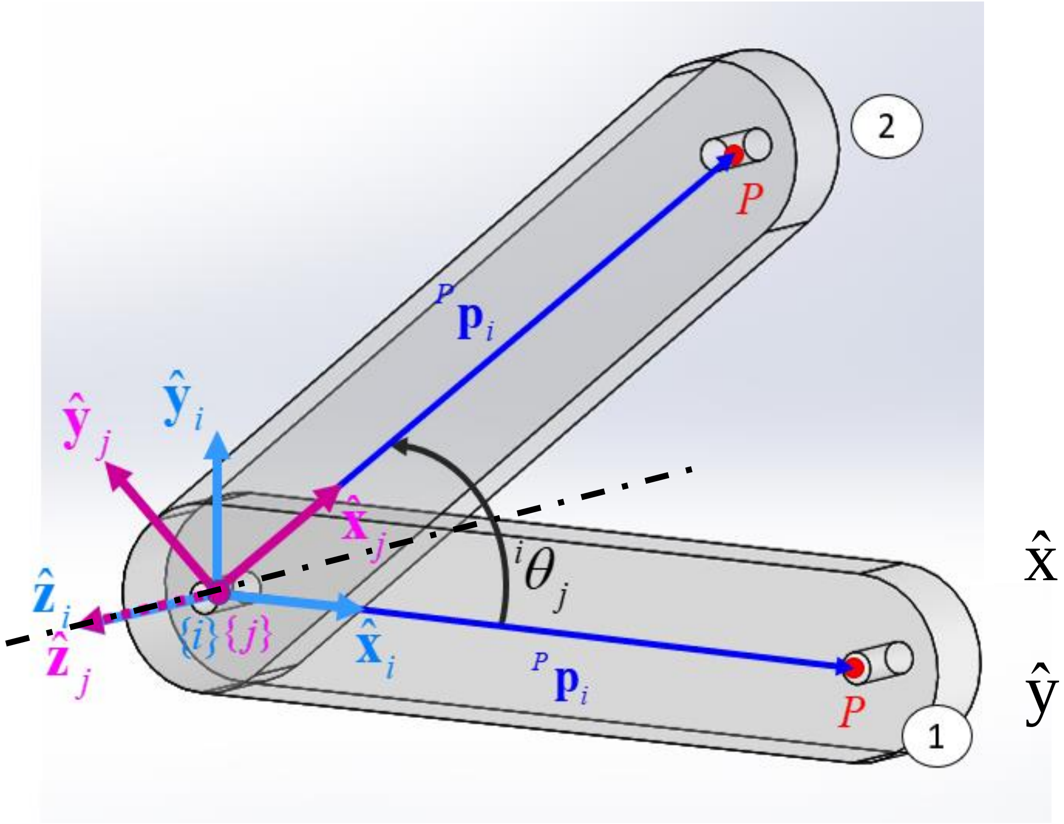
$$\mathbf{a} = \mathbf{R}(\theta)\mathbf{x}_a$$

$$\mathbf{b} = \mathbf{R}(\theta)\mathbf{x}_b$$

$$\mathbf{a} = \begin{pmatrix} L_a \cos(\theta) \\ L_a \sin(\theta) \\ 0 \end{pmatrix}$$

Elemento base





$$\hat{x}_j \cdot \hat{x}_i = \|\hat{x}_j\| \|\hat{x}_i\| \cos({}^{i}\theta_j) = \cos({}^{i}\theta_j)$$

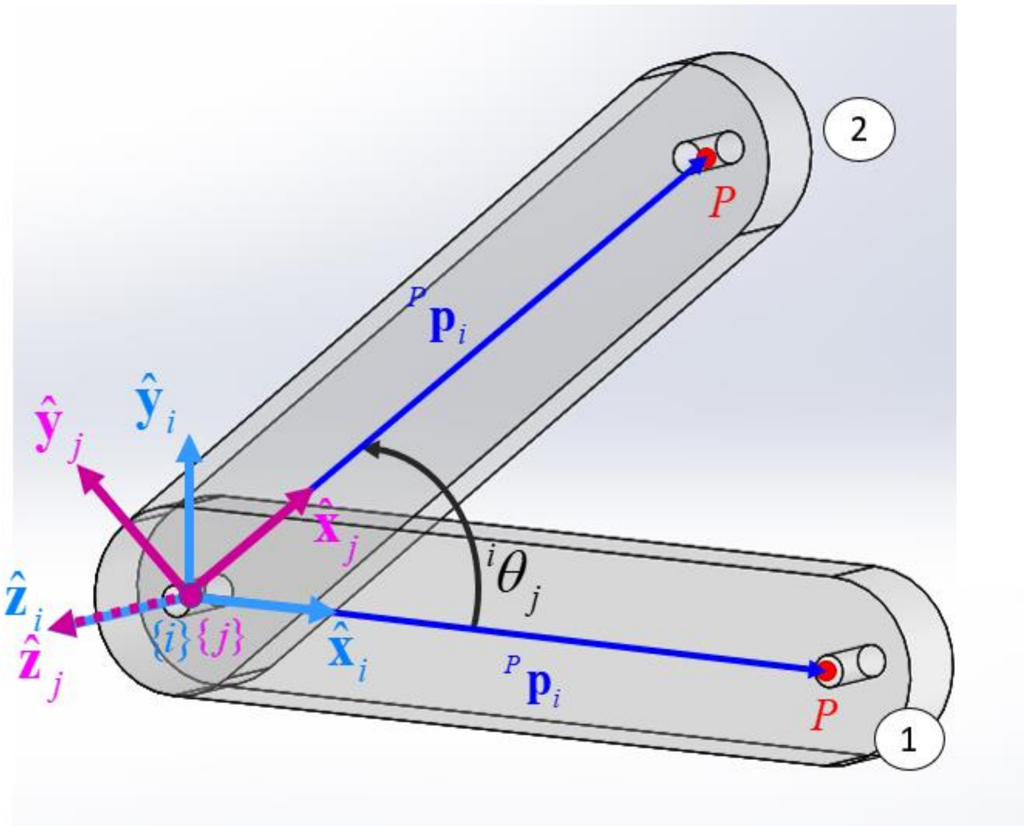
$$\hat{y}_j \cdot \hat{x}_i = \|\hat{y}_j\| \|\hat{x}_i\| \sin({}^{i}\theta_j) = -\sin({}^{i}\theta_j)$$

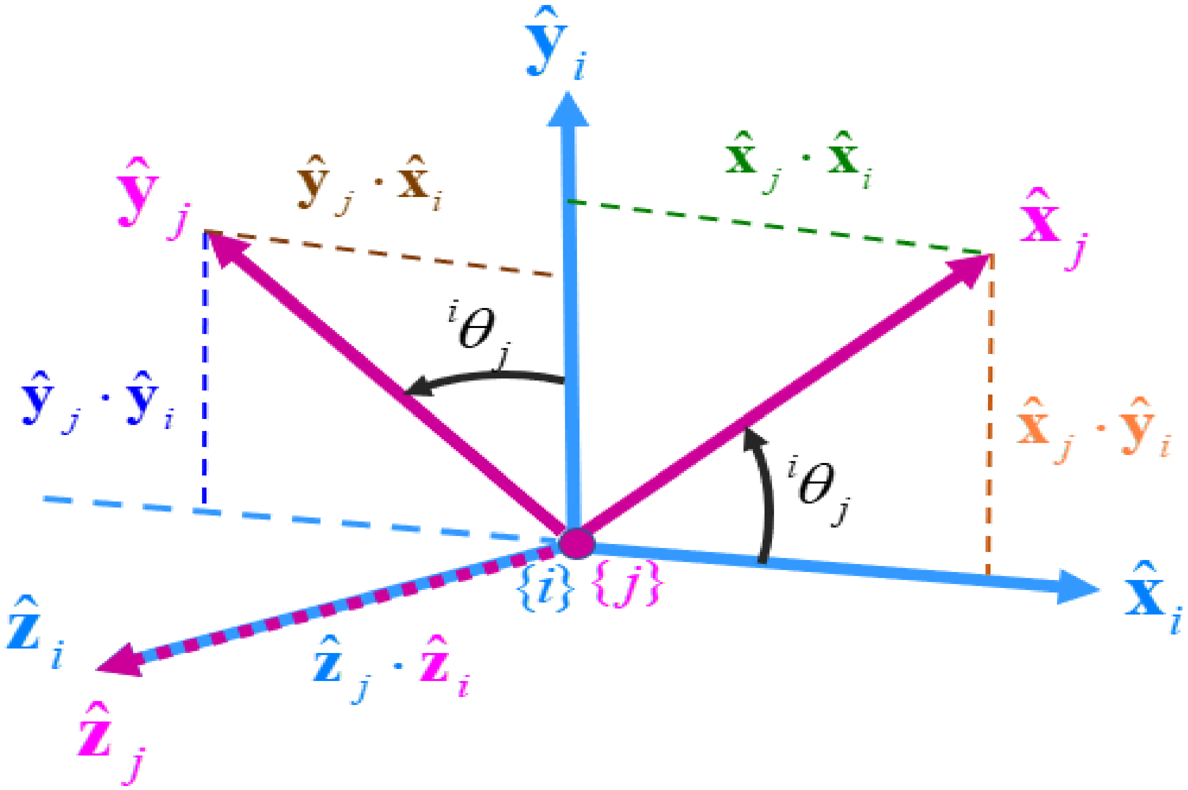
$$\hat{x}_j \cdot \hat{y}_i = \|\hat{x}_j\| \|\hat{y}_i\| \cos(\frac{\pi}{2} - {}^{i}\theta_j) = \|\hat{x}_j\| \|\hat{y}_i\| \sin({}^{i}\theta_j) = \sin({}^{i}\theta_j)$$

$$\hat{y}_j \cdot \hat{y}_i = \|\hat{y}_j\| \|\hat{y}_i\| \cos({}^{i}\theta_j) = \cos({}^{i}\theta_j)$$

$$\hat{z}_j \cdot \hat{z}_i = \|\hat{z}_j\| \|\hat{z}_j\| \cos(0) = 1$$

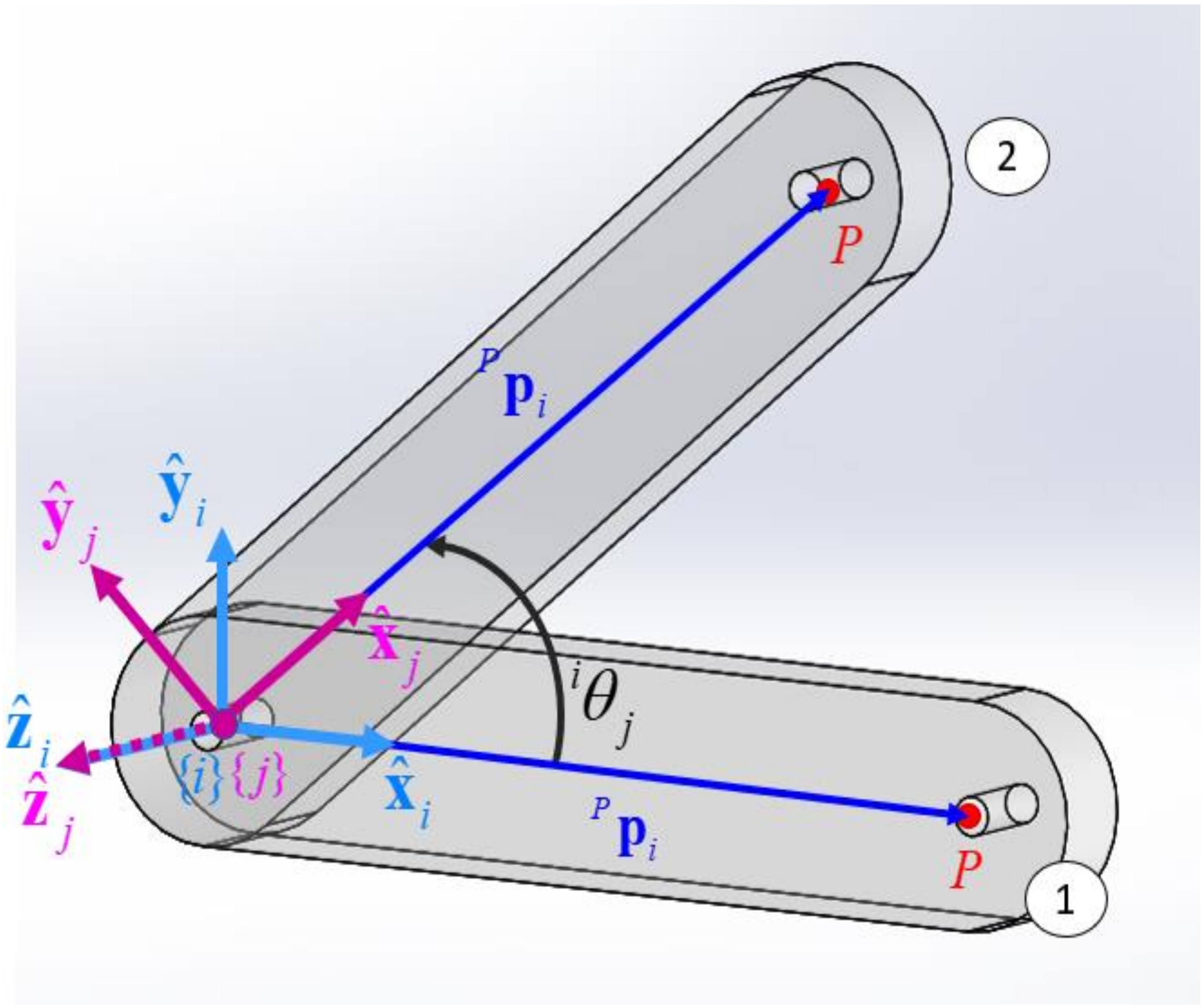
Elemento base





$$
{}^{i}\mathbf{R}_j = \begin{pmatrix} \hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i \\ \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i \\ \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i \end{pmatrix}
$$

Elemento base

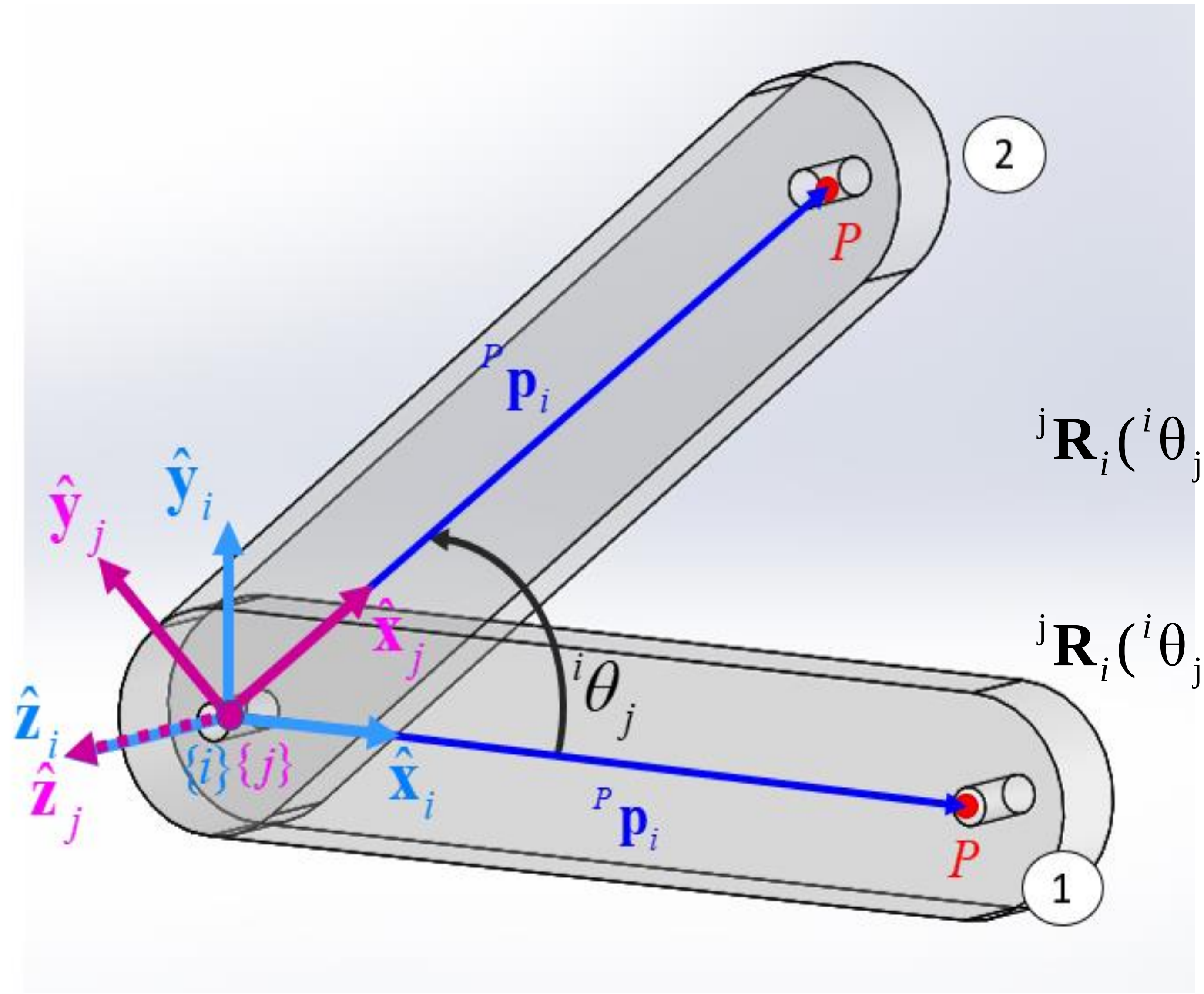


$$ {}^{\mathrm{i}}\mathbf{R}_{\mathrm{j}} = \begin{pmatrix} \hat{\mathbf{x}}_{\mathrm{j}} \cdot \hat{\mathbf{x}}_{\mathrm{i}} & \hat{\mathbf{y}}_{\mathrm{j}} \cdot \hat{\mathbf{x}}_{\mathrm{i}} & \hat{\mathbf{z}}_{\mathrm{j}} \cdot \hat{\mathbf{x}}_{\mathrm{i}} \\ \hat{\mathbf{x}}_{\mathrm{j}} \cdot \hat{\mathbf{y}}_{\mathrm{i}} & \hat{\mathbf{y}}_{\mathrm{j}} \cdot \hat{\mathbf{y}}_{\mathrm{i}} & \hat{\mathbf{z}}_{\mathrm{j}} \cdot \hat{\mathbf{y}}_{\mathrm{i}} \\ \hat{\mathbf{x}}_{\mathrm{j}} \cdot \hat{\mathbf{z}}_{\mathrm{i}} & \hat{\mathbf{y}}_{\mathrm{j}} \cdot \hat{\mathbf{z}}_{\mathrm{i}} & \hat{\mathbf{z}}_{\mathrm{j}} \cdot \hat{\mathbf{z}}_{\mathrm{i}} \end{pmatrix} $$

$$ {}^{\mathrm{i}}\mathbf{R}_{\mathrm{j}}({}^{i}\theta_{\mathrm{j}}) = \begin{pmatrix} \cos({}^{i}\theta_{\mathrm{j}}) & -\sin({}^{i}\theta_{\mathrm{j}}) & 0 \\ \sin({}^{i}\theta_{\mathrm{j}}) & \cos({}^{i}\theta_{\mathrm{j}}) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{j}\mathbf{p}_{P} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} $$

Elemento base



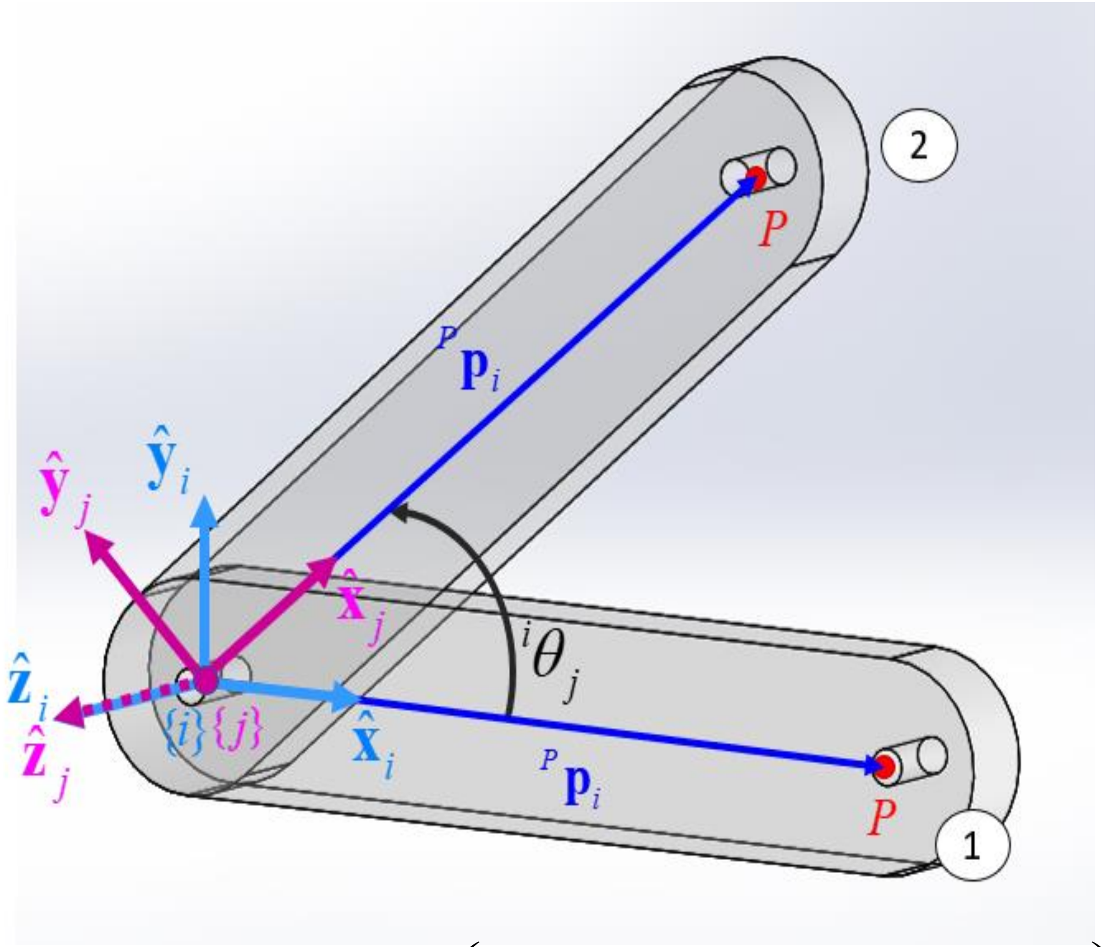
$${}^{\mathrm{i}}\mathbf{R}_{\mathrm{j}}({}^{i}\theta_{\mathrm{j}})=\begin{pmatrix} \cos({}^{i}\theta_{\mathrm{j}}) & -\sin({}^{i}\theta_{\mathrm{j}}) & 0 \\ \sin({}^{i}\theta_{\mathrm{j}}) & \cos({}^{i}\theta_{\mathrm{j}}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{\mathrm{j}}\mathbf{R}_{i}({}^{i}\theta_{\mathrm{j}}) = {}^{\mathrm{i}}\mathbf{R}_{j}({}^{i}\theta_{\mathrm{j}})^{-1}=\begin{pmatrix} \cos({}^{i}\theta_{\mathrm{j}}) & \sin({}^{i}\theta_{\mathrm{j}}) & 0 \\ -\sin({}^{i}\theta_{\mathrm{j}}) & \cos({}^{i}\theta_{\mathrm{j}}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{\mathrm{j}}\mathbf{R}_{i}({}^{i}\theta_{\mathrm{j}}) = {}^{\mathrm{i}}\mathbf{R}_{j}({}^{i}\theta_{\mathrm{j}})^{-1}= {}^{\mathrm{i}}\mathbf{R}_{j}({}^{i}\theta_{\mathrm{j}})^{T}$$

$${}^{j}\mathbf{p}_{P} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$$

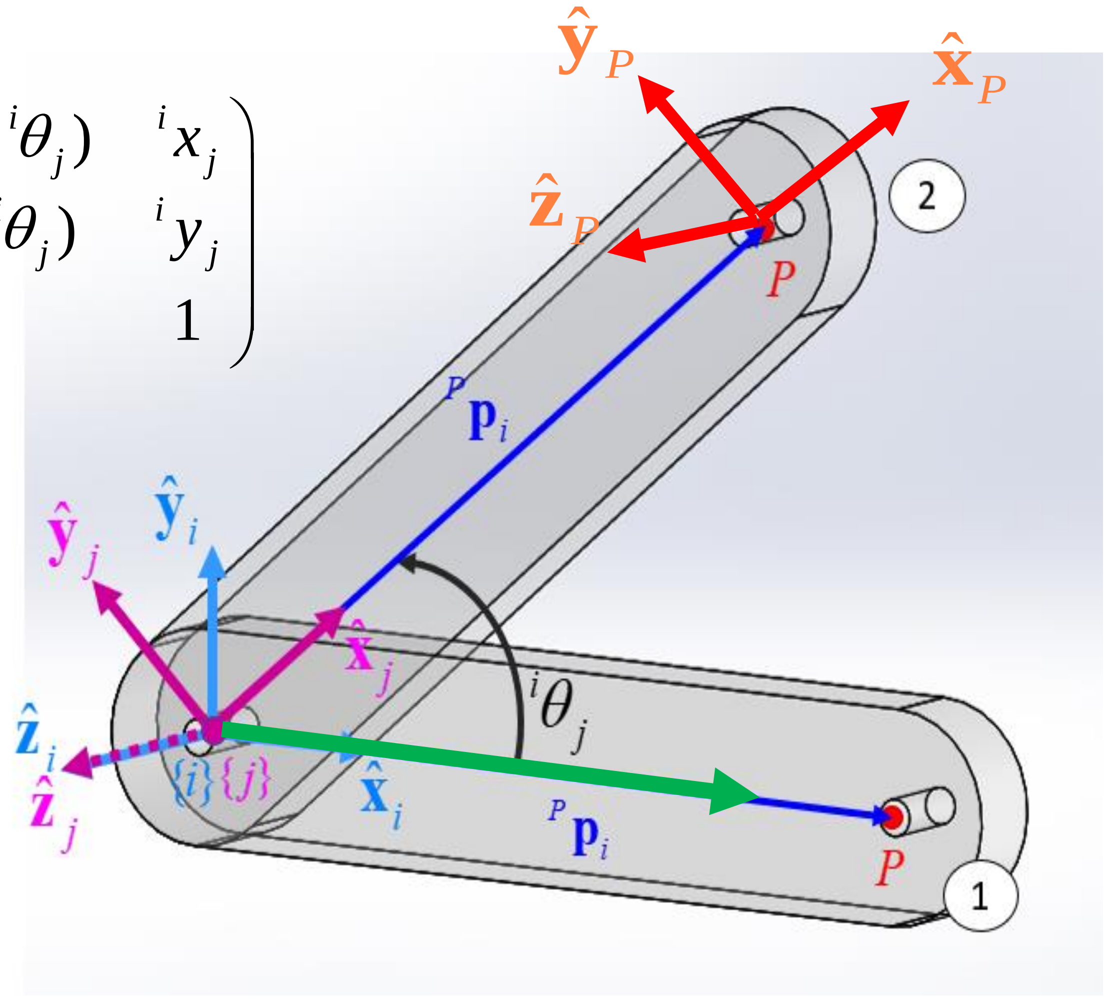
Elemento base



$$
{}^{i}\mathbf{T}_j({}^{i}\theta_j, {}^{i}x_j, {}^{i}y_j) = \begin{pmatrix} {}^{i}\mathbf{R}_j({}^{i}\theta_j) & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & {}^{i}x_j \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & {}^{i}y_j \\ 0 & 0 & 1 \end{pmatrix}
$$

Elemento base

$$ {}^{i}\mathbf{T}_j({}^{i}\theta_j, {}^{i}x_j, {}^{i}y_j) = \begin{pmatrix} {}^{i}\mathbf{R}_j({}^{i}\theta_j) & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & {}^{i}x_j \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & {}^{i}y_j \\ 0 & 0 & 1 \end{pmatrix} $$



$$ {}^{i}\mathbf{T}_P = {}^{i}\mathbf{T}_j \, {}^{j}\mathbf{T}_P $$

$$ {}^{i}\mathbf{T}_j = {}^{i}\mathbf{T}_j({}^{i}\theta_j, 0, 0) = \begin{pmatrix} {}^{i}\mathbf{R}_j({}^{i}\theta_j) & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & 0 \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{j}\mathbf{T}_P = {}^{i}\mathbf{T}_j(0, L, 0) = \begin{pmatrix} 1 & 0 & L_i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{i}\mathbf{T}_P = {}^{i}\mathbf{T}_j \, {}^{j}\mathbf{T}_P = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & 0 \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & L_i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & L_i\cos({}^{i}\theta_j) \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & L_i\sin({}^{i}\theta_j) \\ 0 & 0 & 1 \end{pmatrix} $$