Robótica grupo2
Clase 10

Facultad de Ingeniería UNAM

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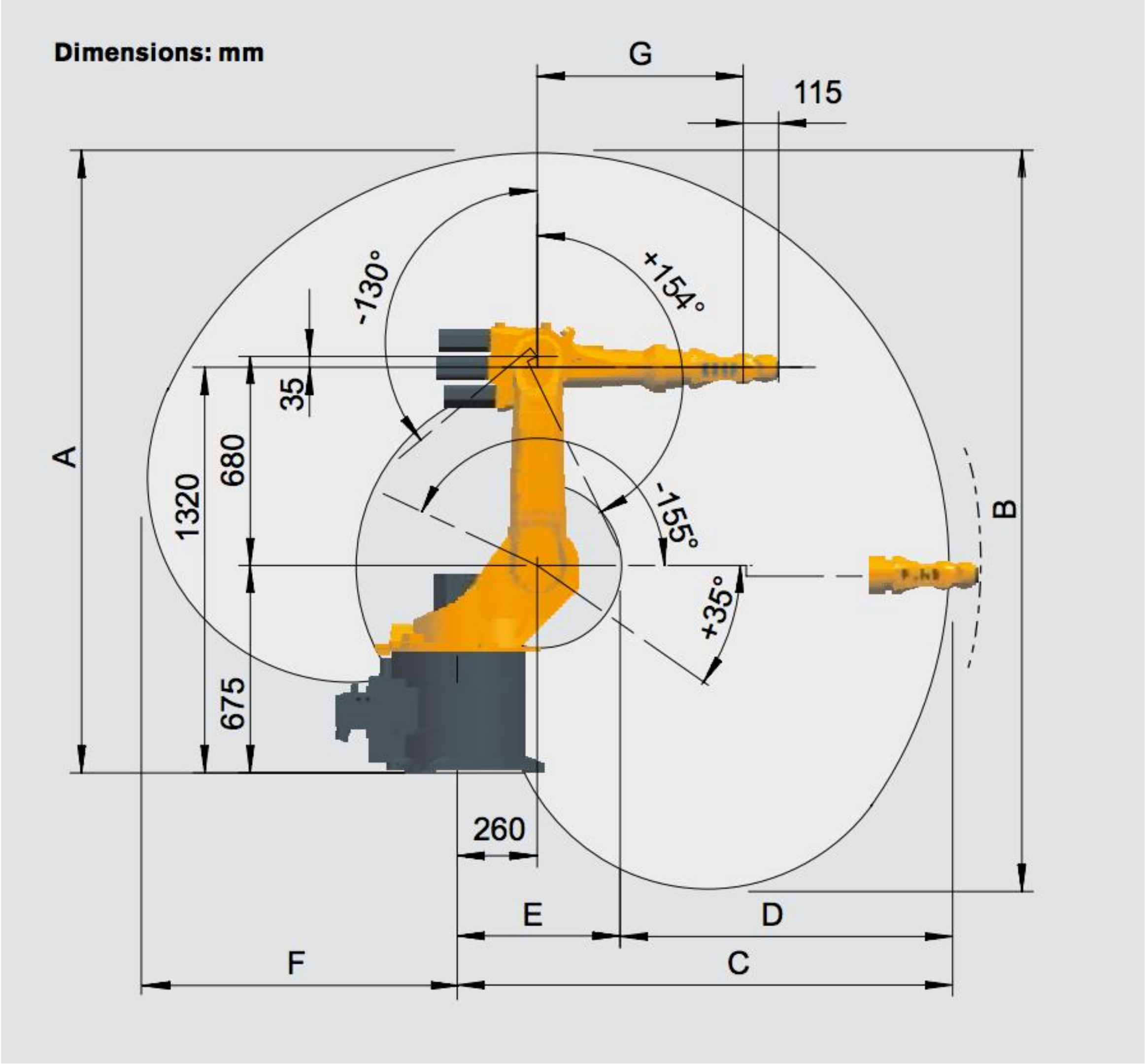
Derechos reservados

Repaso/Elemento base (caso de estudio)

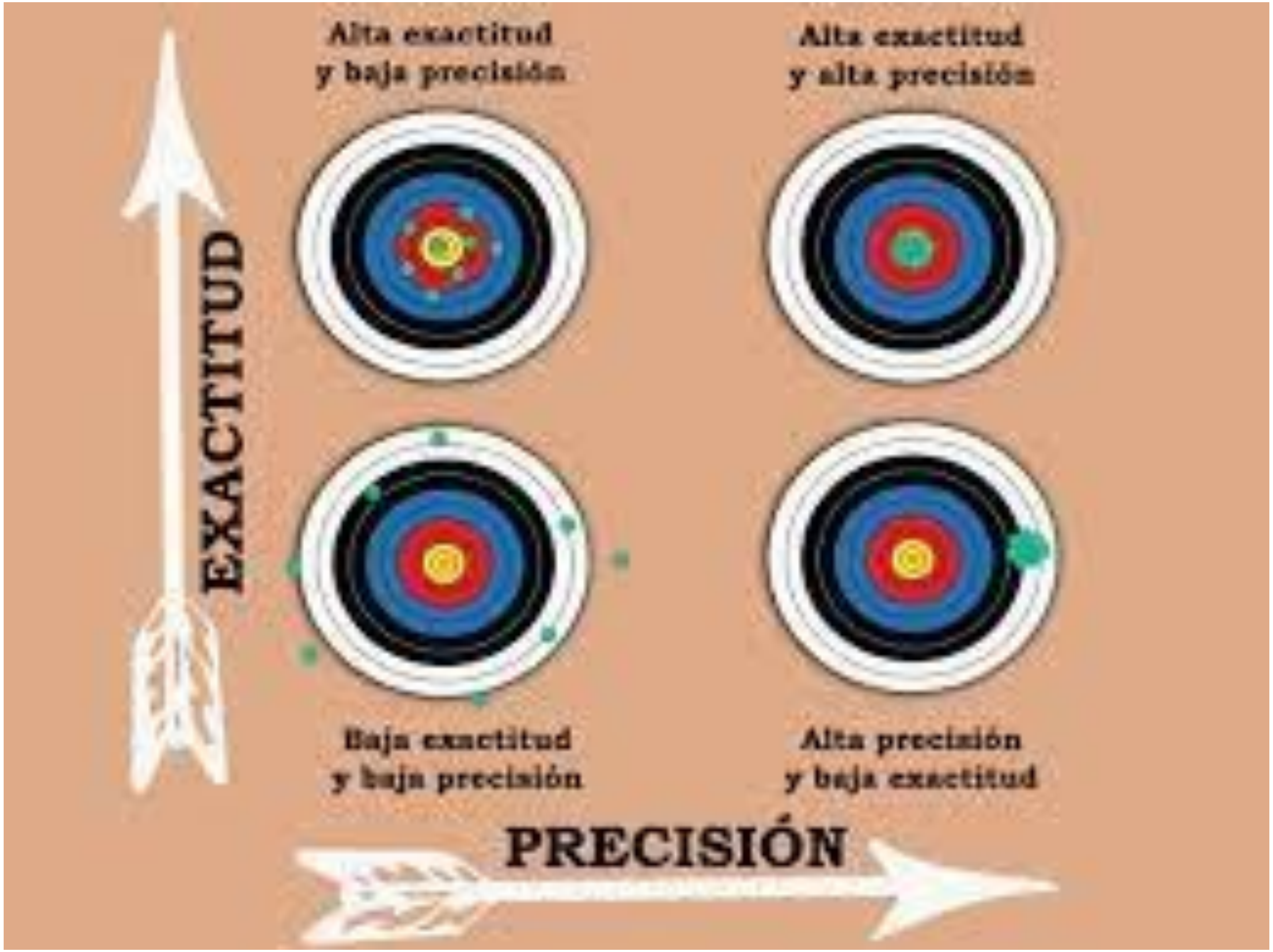
- Elemento base de la robótica (robot RRR)
  - Planteamiento del modelo cinemático
    - Planteamiento del modelo cinemático del postura
      - Cinemático inverso de la postura
    - Planteamiento del modelo cinemático de las velocidades
      - Modelo cinemático directo de las velocidades
      - Modelo cinemático inverso de las velocidades
    - Planteamiento del modelo cinemático de las aceleraciones
  - Plantemiento dinámico

Modelo cinemático de la postura



Fabricante
-Capacidad de carga
-Alcance
-Resolución

Evaluación de un robot
-Repetibilidad
-Manipulación


Alta exactitud
y baja precisión
Alta exactitud
y alta precisión
EXACTITUD
Baja exactitud
y baja precisión
Alta precisión
y baja exactitud
PRECISIÓN

Elemento base de la robótica (robot RRR)

$$^{0}\boldsymbol{\xi}_P = \begin{pmatrix} x_P \\ y_P \\ \theta_P \end{pmatrix}$$

n grados de libertad de un robot

m grados de libertad del espacio de trabajo

n < m , robot subactuado

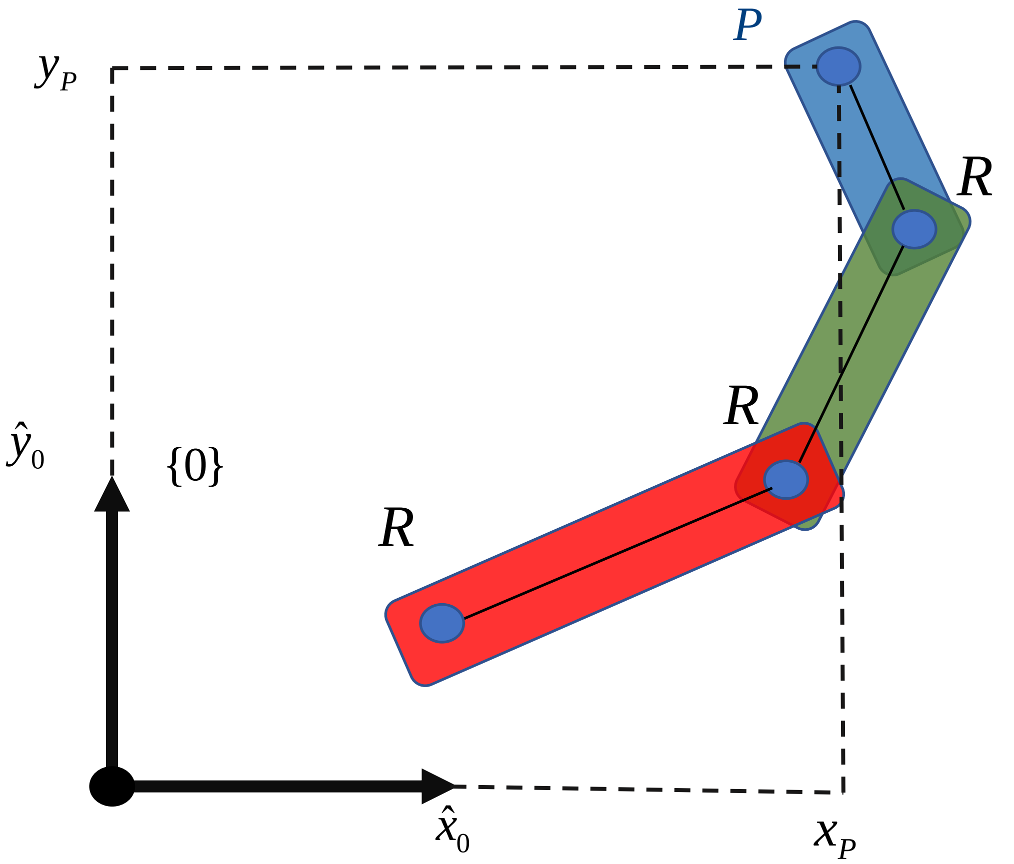
n = m , robot definido

n > m , robot sobreactuado o redundante

Modelo cinemático de la postura

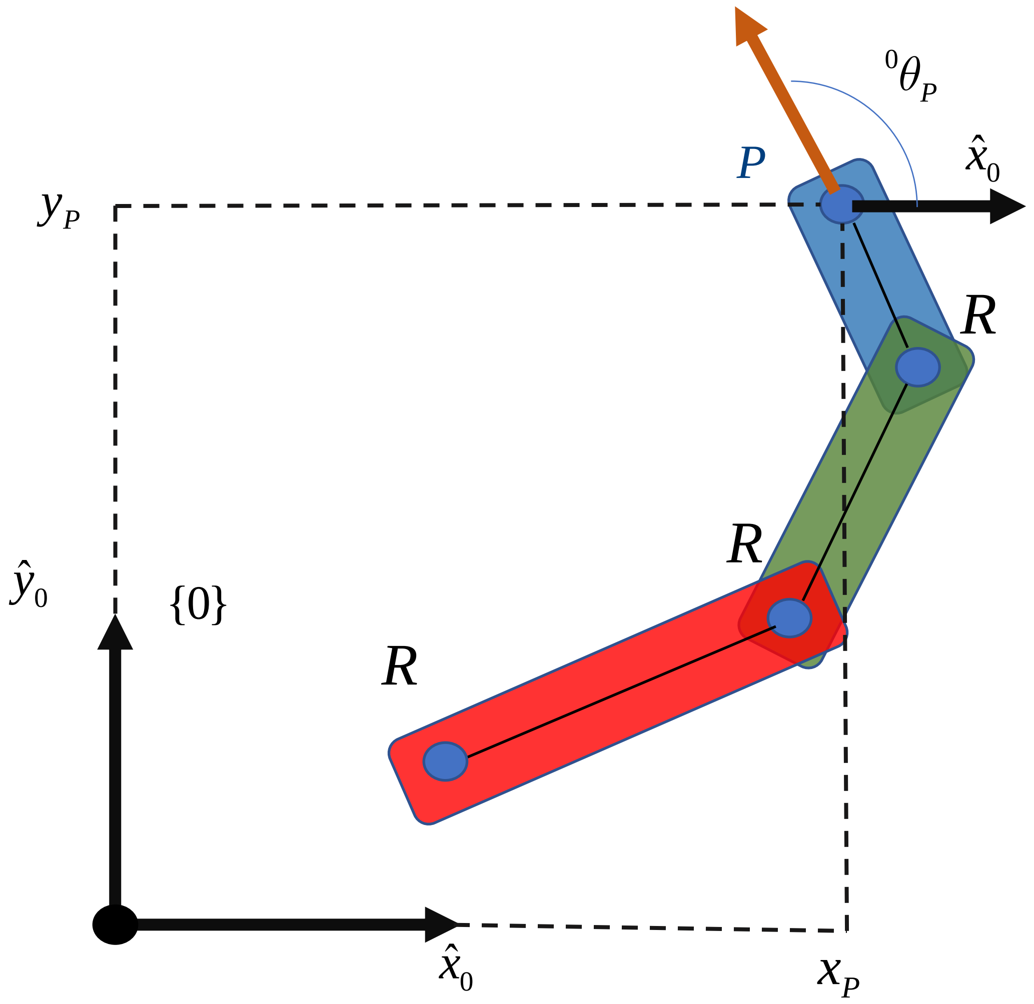
RRR

$3R$



$RPRR$

Modelo cinemático de la postura



$$ {}^{0}\xi_P = \begin{pmatrix} x_P \\ y_P \\ \theta_P \end{pmatrix} $$

n grados de libertad de un robot

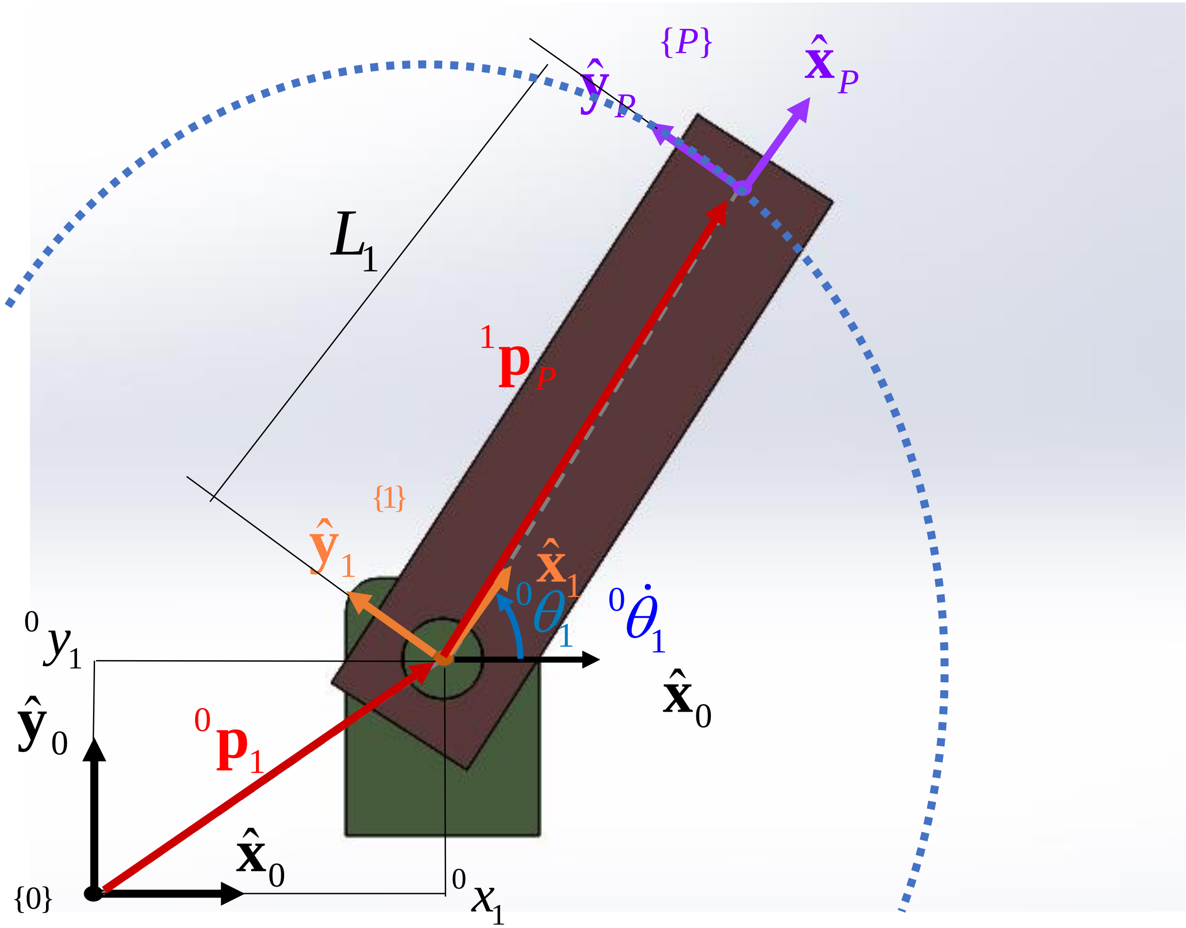
m la cantidad de grados de libertad que describen actuador

n < m , robot subactuado

n = m , robot definido

n > m , robot sobreactuado o redundante

Junta rotacional



$$
{}^0\boldsymbol{\xi}_P = \begin{pmatrix} x_P \\ y_P \\ \theta_P \end{pmatrix}
$$

n grados de libertad de un robot

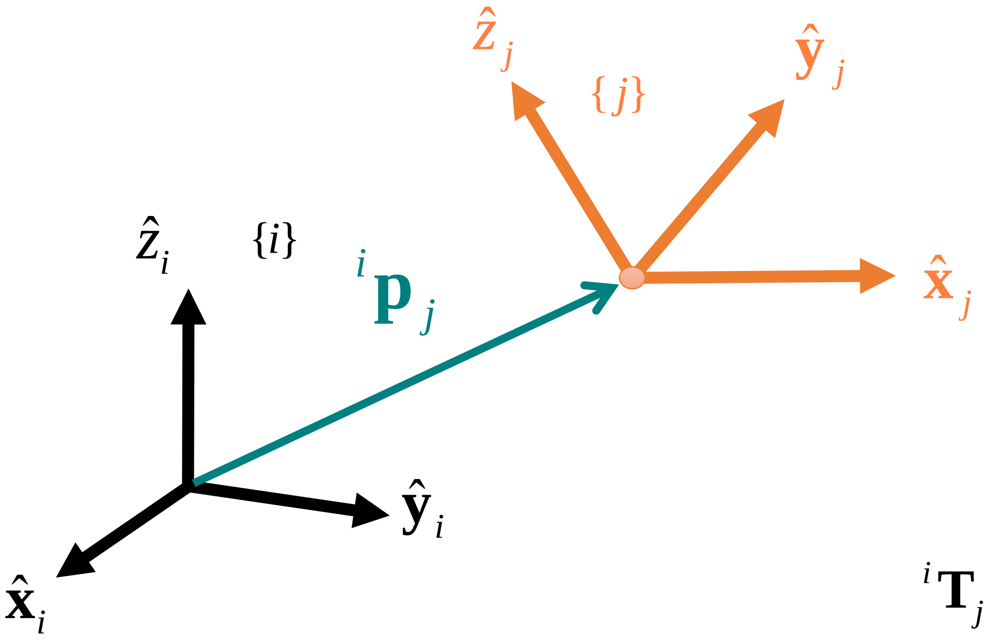
m grados de libertad del espacio de trabajo

n < m , robot subactuado

n = m , robot definido

n > m , robot sobreactuado o redundante

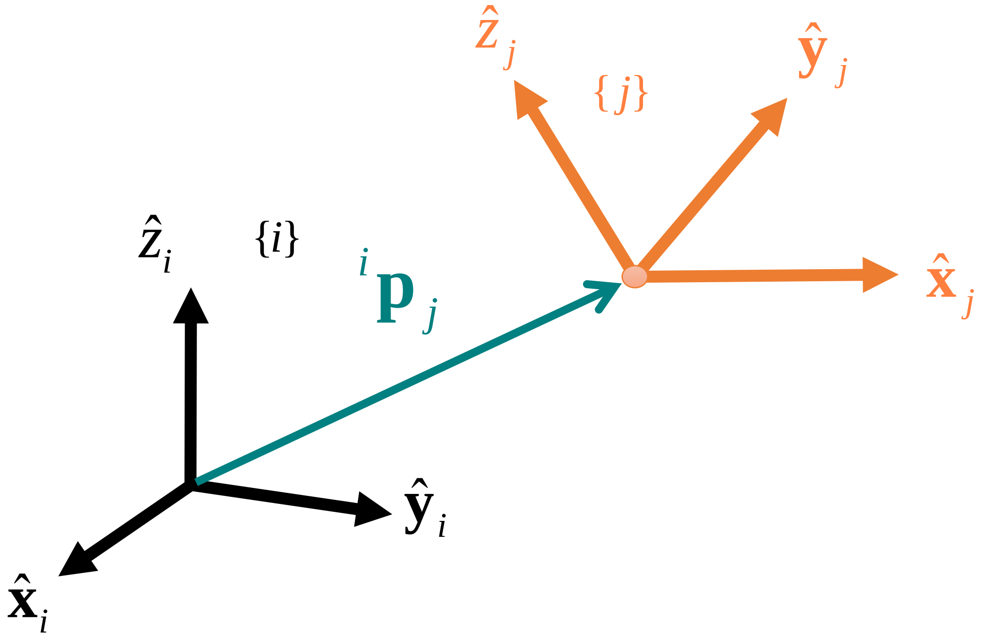
Posición y orientación



$${}^{i}\mathbf{T}_j({}^{i}\alpha_j, {}^{i}\beta_j, {}^{i}\gamma_j, {}^{i}x_j, {}^{i}y_j, {}^{i}z_j) = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos({}^{i}\alpha_j)\cos({}^{i}\beta_j) & \cos({}^{i}\alpha_j)\sin({}^{i}\beta_j)\sin({}^{i}\gamma_j) - \cos({}^{i}\gamma_j)\sin({}^{i}\alpha_j) & \sin({}^{i}\alpha_j)\sin({}^{i}\gamma_j) + \cos({}^{i}\alpha_j)\cos({}^{i}\gamma_j)\sin({}^{i}\beta_j) & {}^{i}x_j \\ \cos({}^{i}\beta_j)\sin({}^{i}\alpha_j) & \cos({}^{i}\alpha_j)\cos({}^{i}\gamma_j) + \sin({}^{i}\alpha_j)\sin({}^{i}\beta_j)\sin({}^{i}\gamma_j) & \cos({}^{i}\gamma_j)\sin({}^{i}\alpha_j)\sin({}^{i}\beta_j) - \cos({}^{i}\alpha_j)\sin({}^{i}\gamma_j) & {}^{i}y_j \\ -\sin({}^{i}\beta_j) & \cos({}^{i}\beta_j)\sin({}^{i}\gamma_j) & \cos({}^{i}\beta_j)\cos({}^{i}\gamma_j) & {}^{i}z_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Posición y orientación



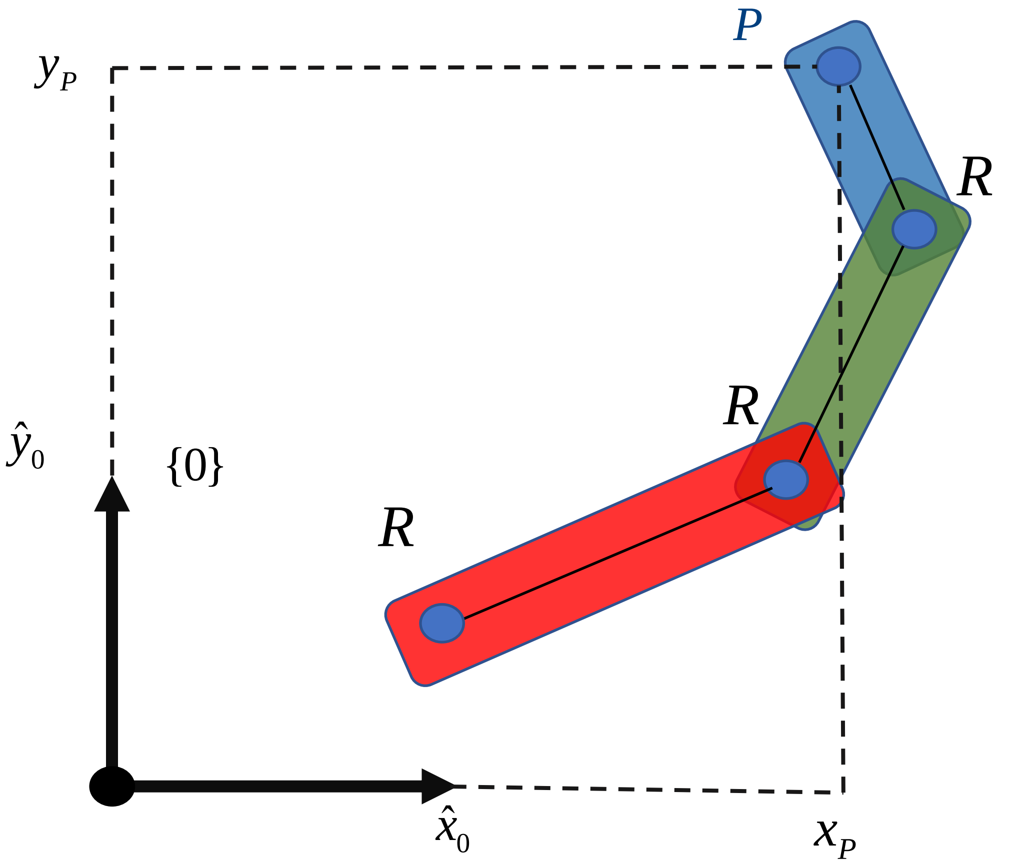
$$
{}^{i}\mathbf{T}_j({}^{i}\alpha_j, 0, 0, {}^{i}x_j, {}^{i}y_j, 0) = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix} \cos({}^{i}\alpha_j) & -\sin({}^{i}\alpha_j) & 0 & {}^{i}x_j \\ \sin({}^{i}\alpha_j) & \cos({}^{i}\alpha_j) & 0 & {}^{i}y_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

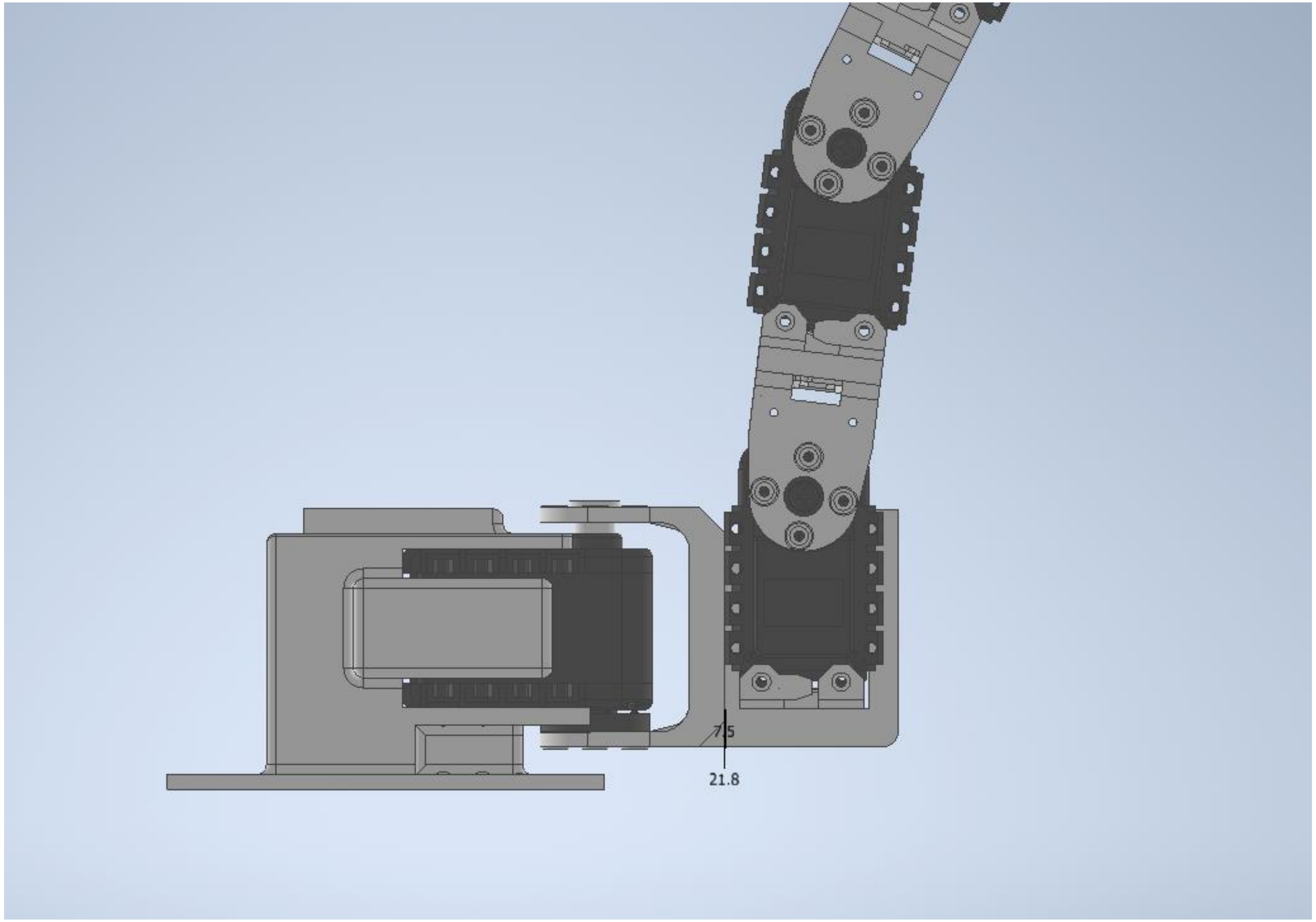
RRR

$3R$

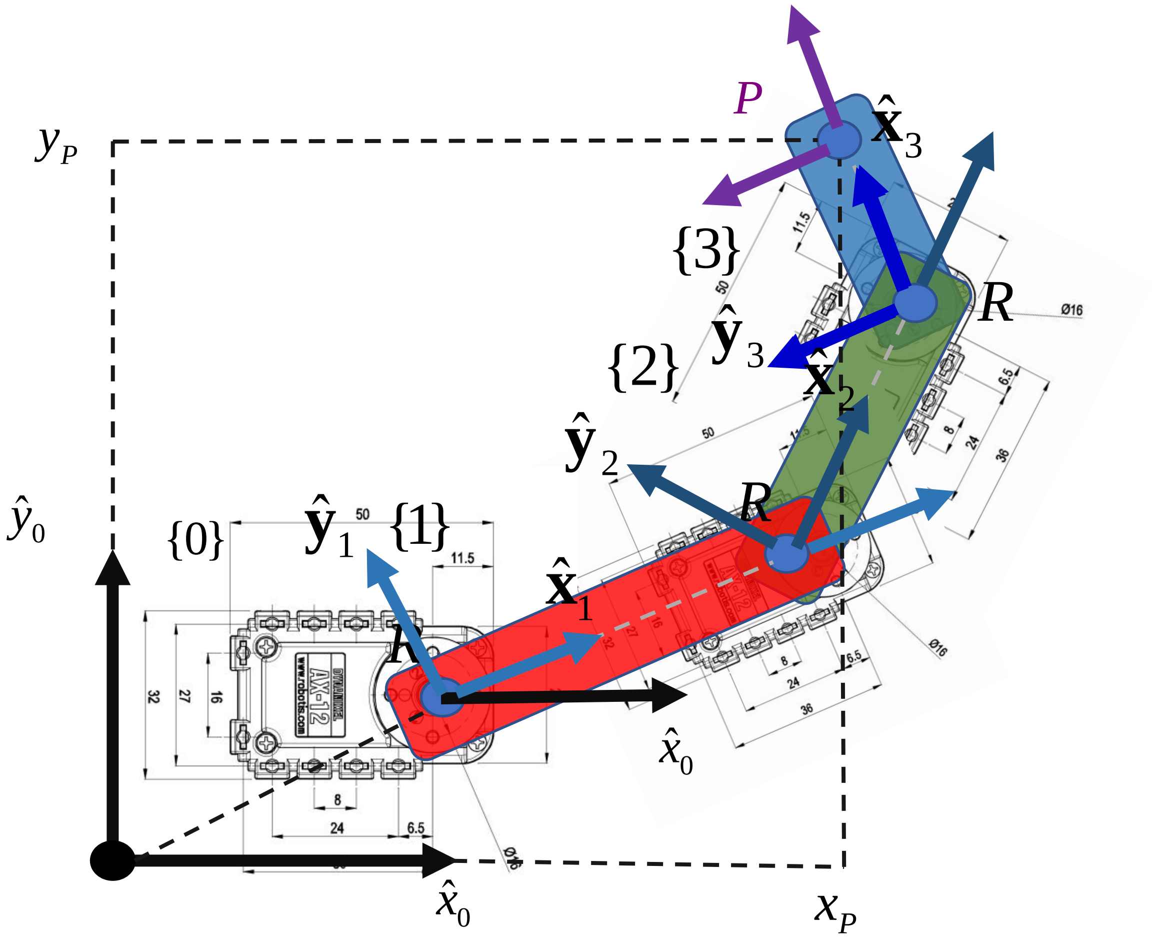


$RPRR$

Modelo cinemático de la postura



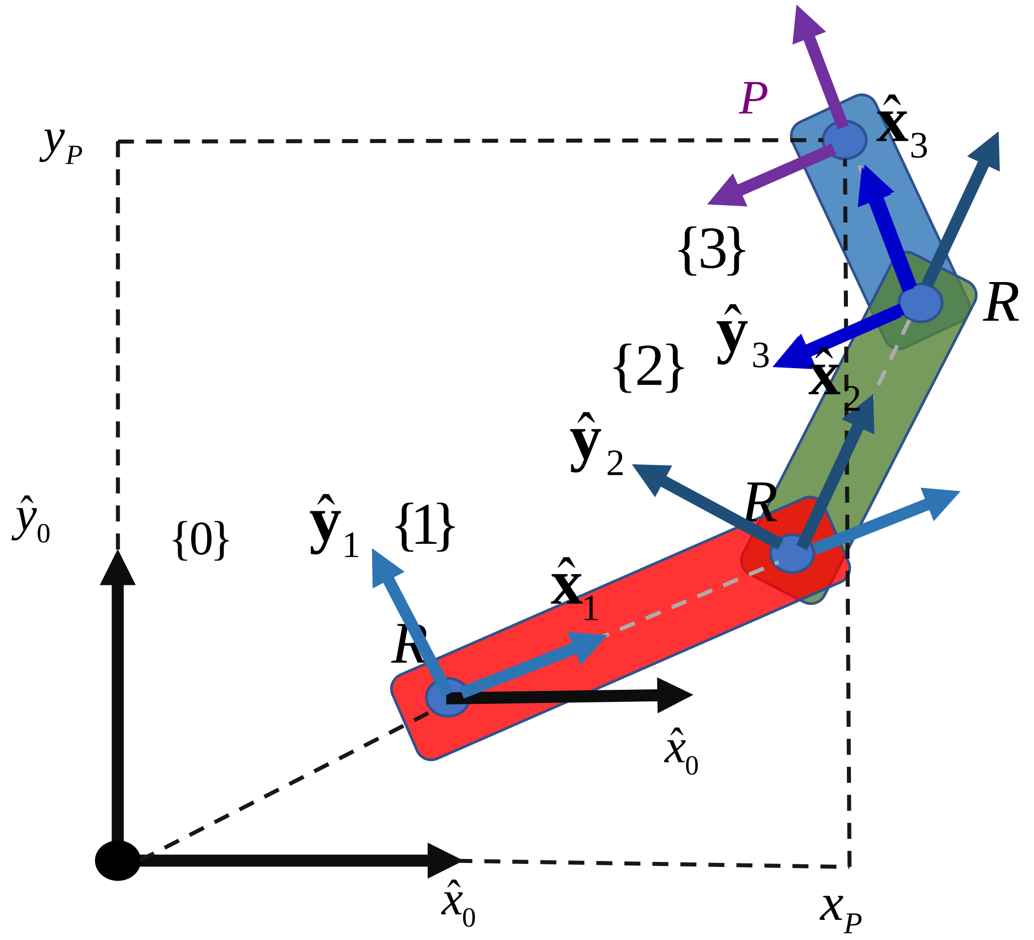
Modelo cinemático de la postura



$$
{}^{i}\mathbf{T}_{j}({}^{i}\alpha_{j}, {}^{i}x_{j}, {}^{i}y_{j}) = \begin{pmatrix} {}^{i}\mathbf{R}_{j} & {}^{i}\mathbf{p}_{j} \\ \mathbf{0}^{T} & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix} \cos({}^{i}\theta_{j}) & -\sin({}^{i}\theta_{j}) & 0 & {}^{i}x_{j} \\ \sin({}^{i}\theta_{j}) & \cos({}^{i}\theta_{j}) & 0 & {}^{i}y_{j} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

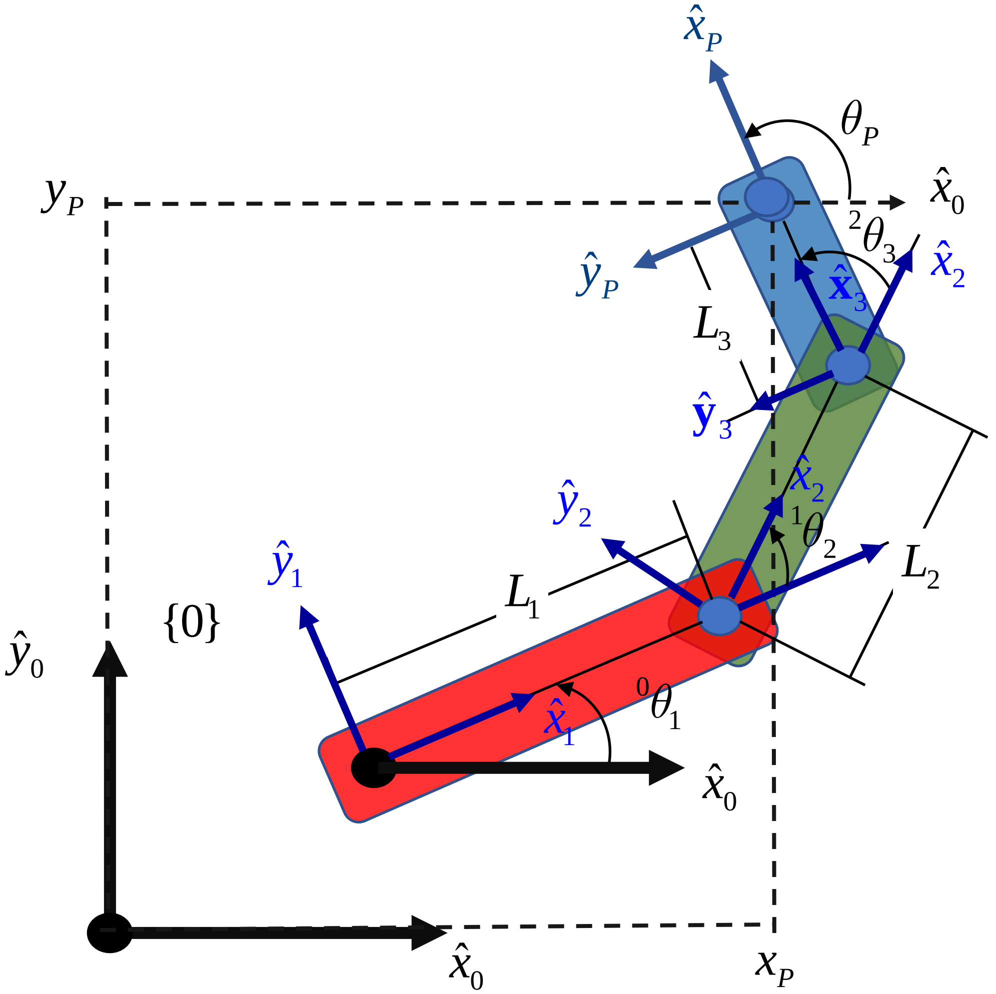
Modelo cinemático de la postura



$$
{}^{i}\mathbf{T}_j({}^{i}\alpha_j, {}^{i}x_j, {}^{i}y_j) = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & 0 & {}^{i}x_j \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & 0 & {}^{i}y_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Elemento base de la robótica (robot RRR)



$$ {}^0\boldsymbol{\xi}_P = \begin{pmatrix} x_P \\ y_P \\ \theta_P \end{pmatrix} $$

n grados de libertad de un robot

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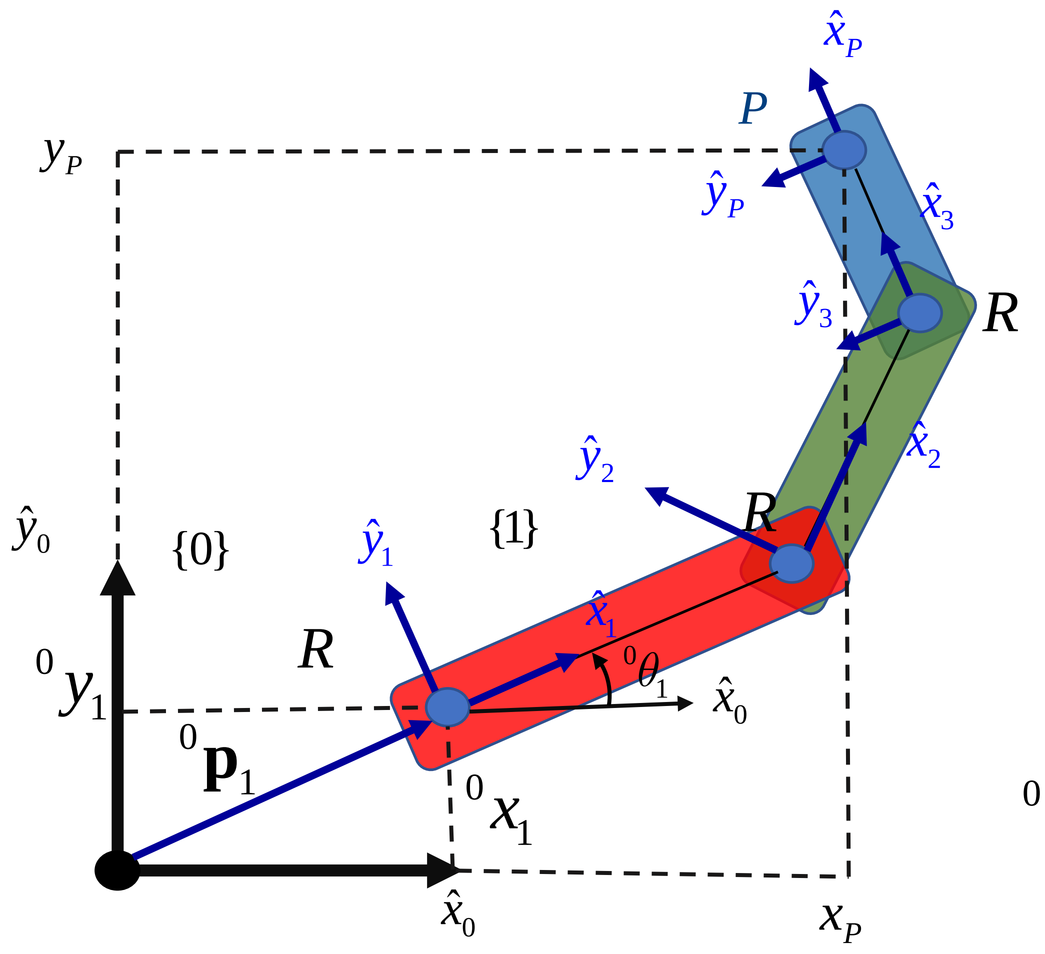
Modelo cinemático de la postura

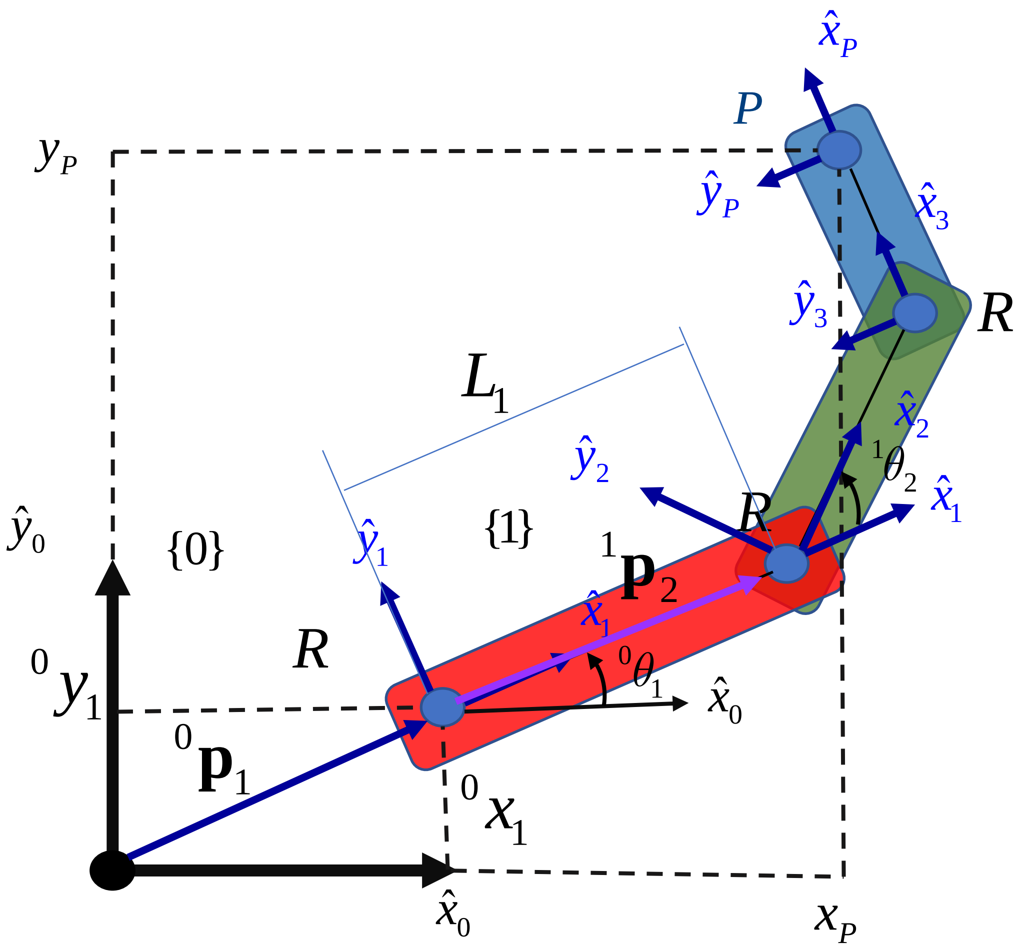


$$
{}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
{}^0\mathbf{p}_1 = \begin{pmatrix} {}^0x_1 \\ {}^0y_1 \\ 0 \end{pmatrix} \quad {}^0R_1 = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

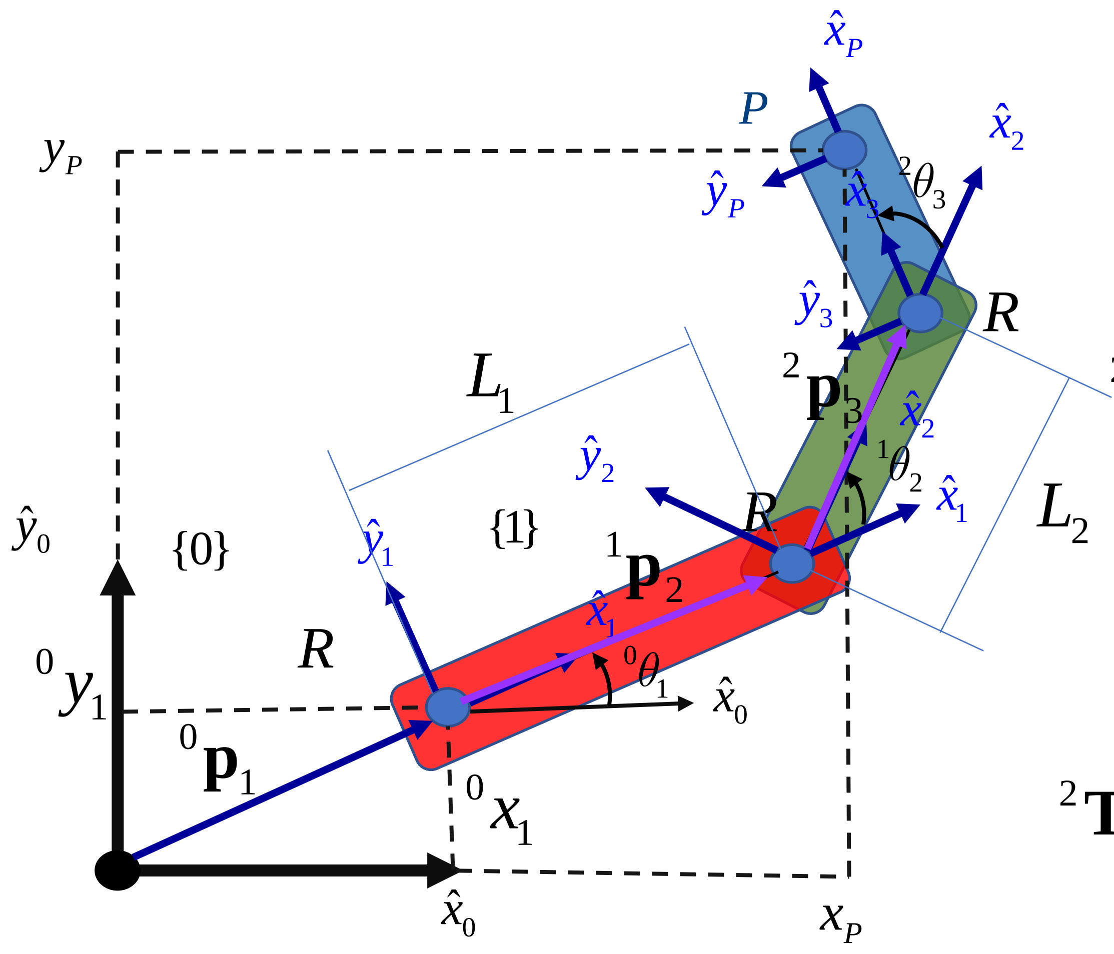


$${}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =$$

$${}^1\mathbf{p}_2 = \begin{pmatrix} L_1 \\ 0 \\ 0 \end{pmatrix} \qquad {}^1R_2 = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la postura

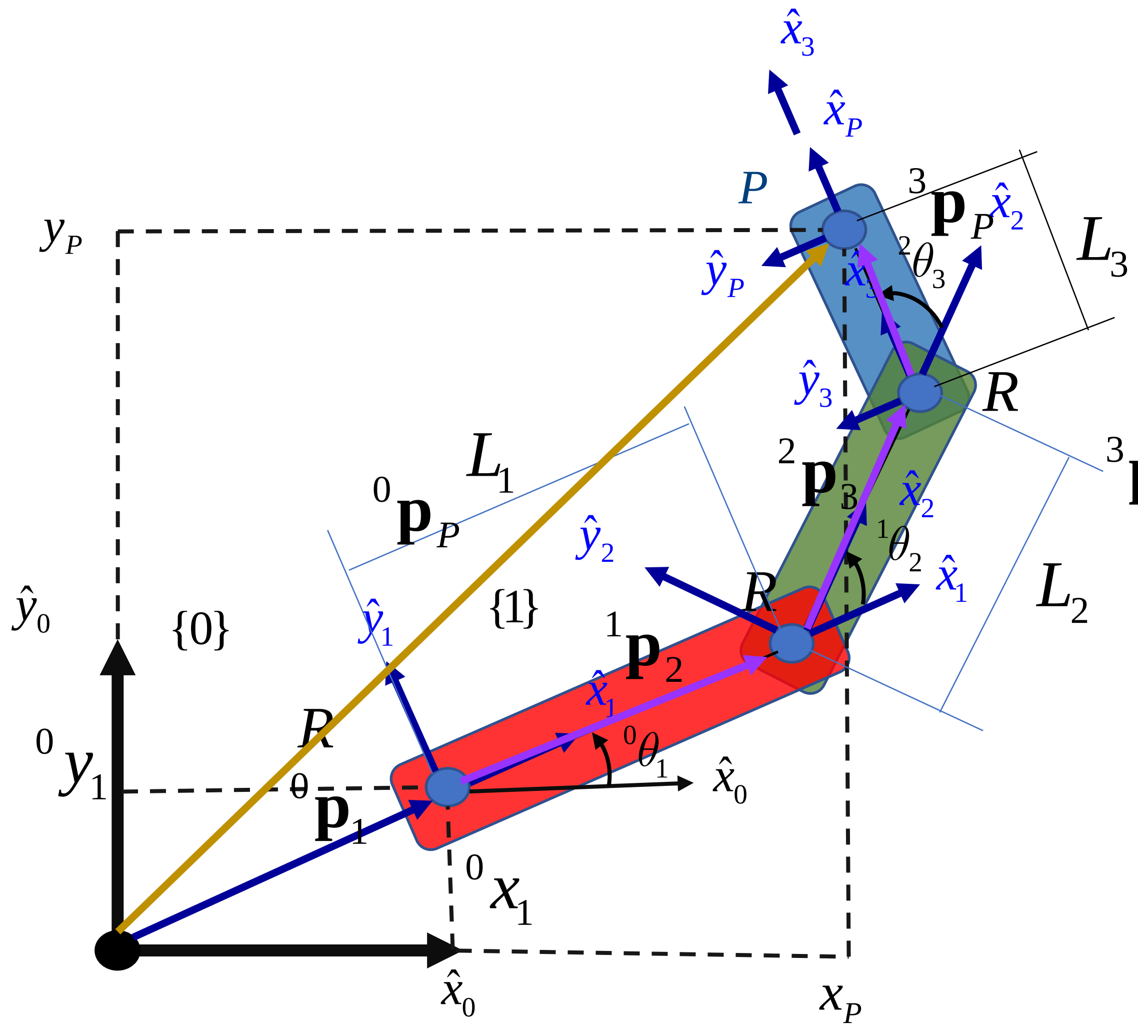


$$ {}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} = $$

$$ {}^2\mathbf{p}_3 = \begin{pmatrix} L_2 \\ 0 \\ 0 \end{pmatrix} \qquad {}^2R_3 = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura



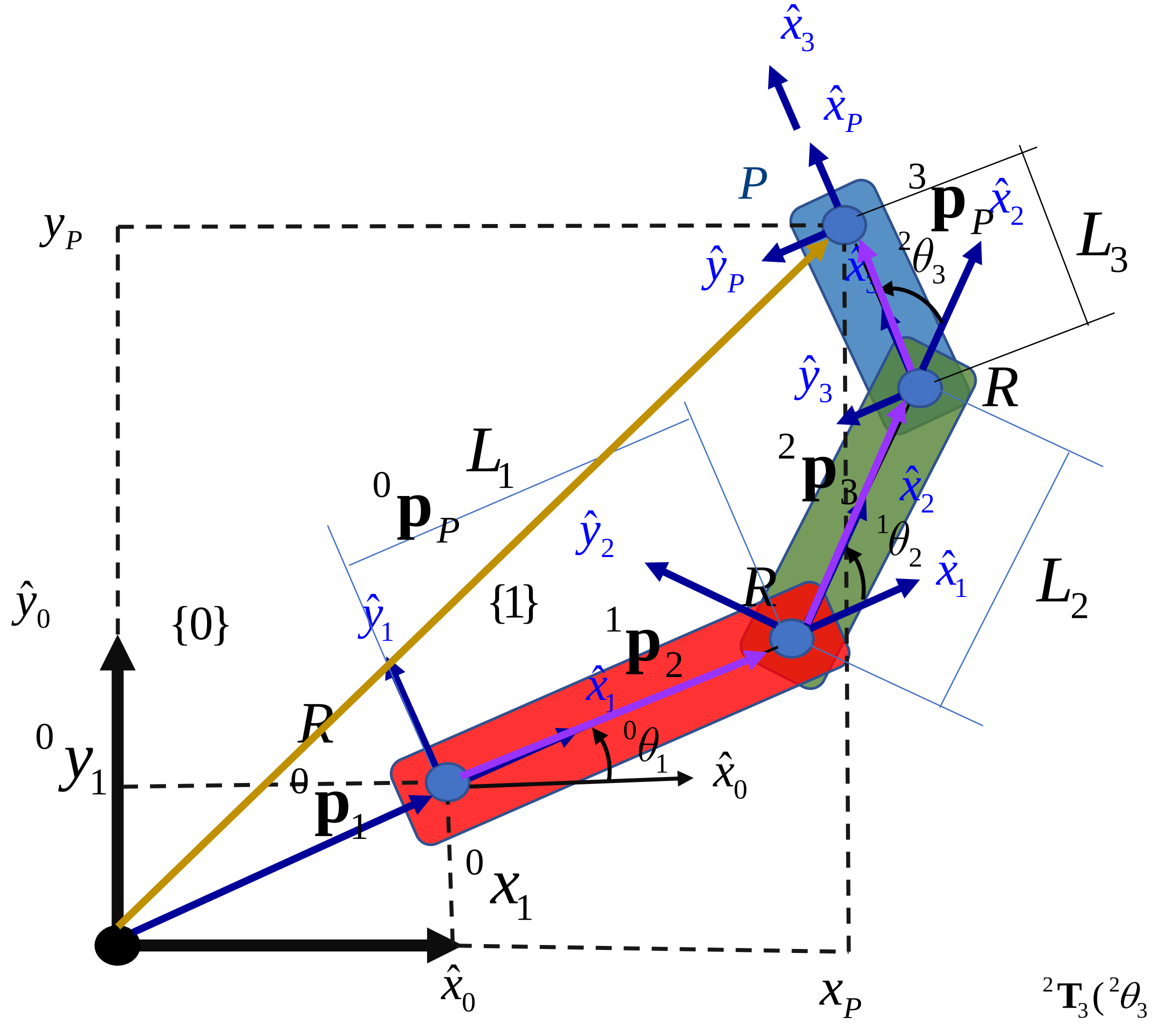
$$^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =$$

$$^3\mathbf{p}_P = \begin{pmatrix} L_3 \\ 0 \\ 0 \end{pmatrix}$$

$$^2R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la postura



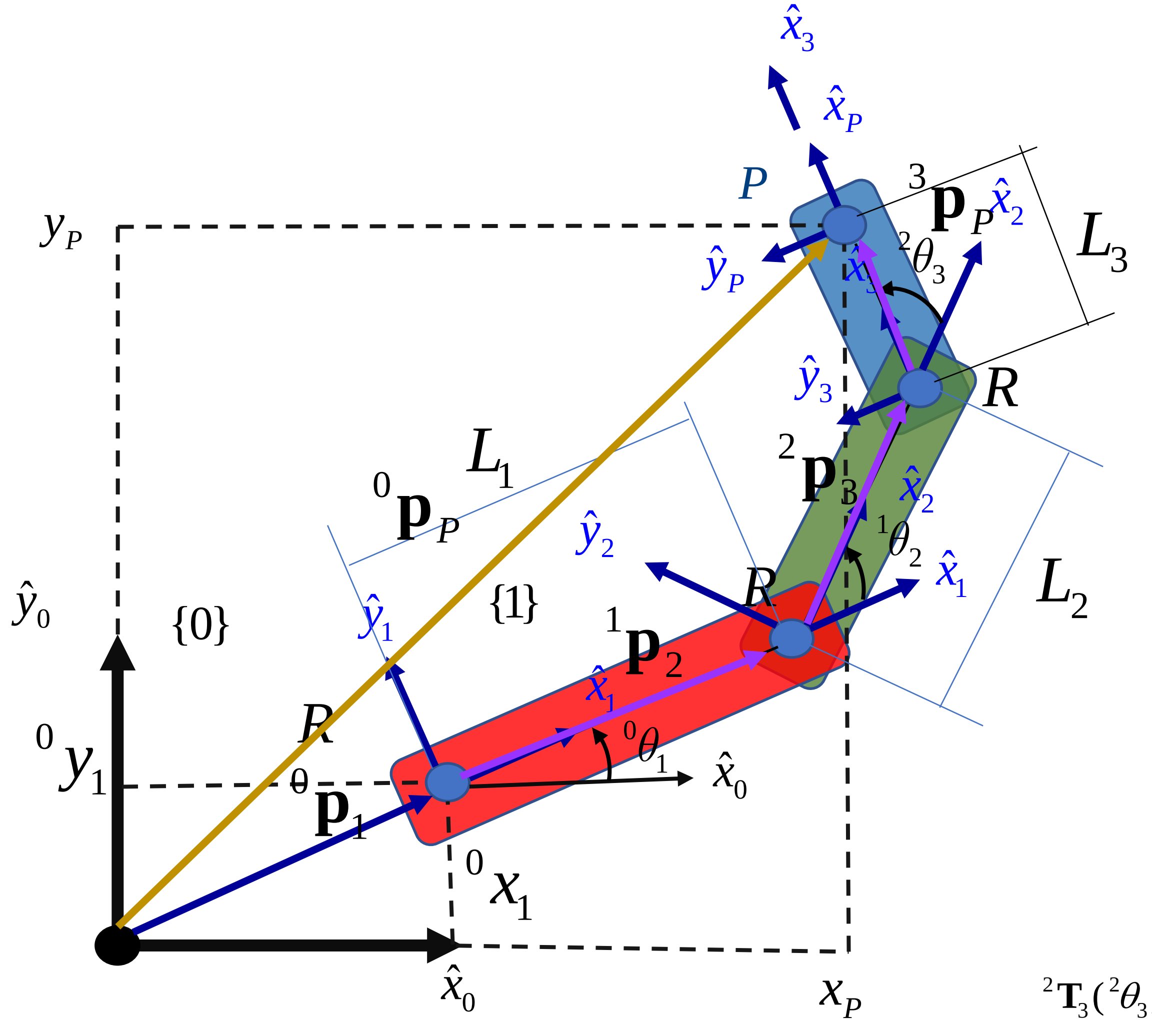
$$^0\mathbf{T}_P = \begin{pmatrix} ^0\mathbf{R}_P & ^0\mathbf{p}_P \\ \mathbf{0}^T & 1 \end{pmatrix} = {}^0\mathbf{T}_{\cancel{1}}\,{}^{\cancel{1}}\mathbf{T}_{\cancel{2}}\,{}^{\cancel{2}}\mathbf{T}_{\cancel{3}}\,{}^{\cancel{3}}\mathbf{T}_P$$

$$^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la postura



$$
{}^0\mathbf{T}_P = \begin{pmatrix} {}^0\mathbf{R}_P & {}^0\mathbf{p}_P \\ \mathbf{0}^T & 1 \end{pmatrix} = {}^0\mathbf{T}_{\cancel{1}}\,{}^{\cancel{1}}\mathbf{T}_{\cancel{2}}\,{}^{\cancel{2}}\mathbf{T}_{\cancel{3}}\,{}^{\cancel{3}}\mathbf{T}_P
$$

$$
{}^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\qquad
{}^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

$$ {}^{0}\mathbf{T}_{P} = {}^{0}\mathbf{T}_{1}\,{}^{1}\mathbf{T}_{2}\,{}^{2}\mathbf{T}_{3}\,{}^{3}\mathbf{T}_{P} = \begin{pmatrix} {}^{0}\mathbf{R}_{P} & {}^{0}\mathbf{p}_{P} \\ \mathbf{0}^{T} & 1 \end{pmatrix} = $$

$$ = \begin{pmatrix} \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & -\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$