Robótica grupo2
Clase 6

Facultad de Ingeniería UNAM

M.I. Erik Peña Medina

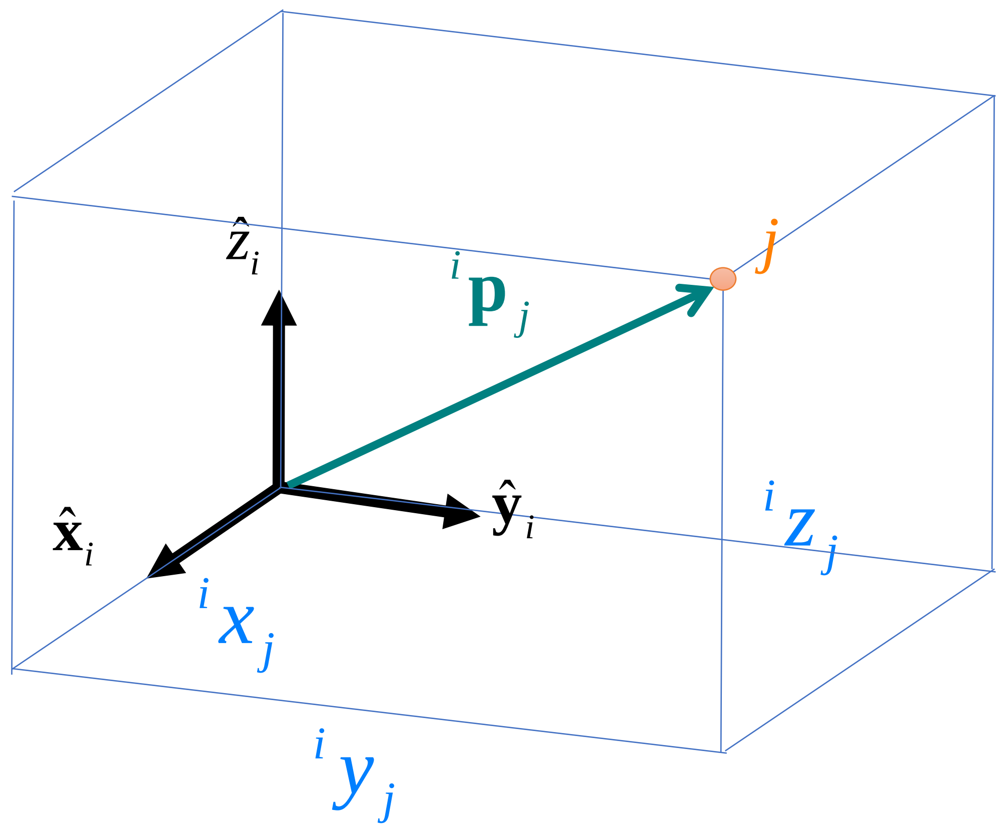
Derechos reservados

Conceptos básicos/Elemento base

- Resumen de conceptos

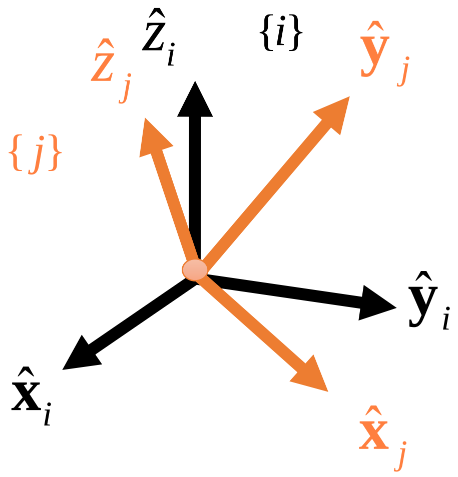
- Elemento base (eslabón)
  - Planteamiento de su modelado cinemático.
    - Modelo cinemático de la posición.
    - Modelo cinemático de las velocidades.
    - Modelos cinemático de sus aceleraciones.

Posición

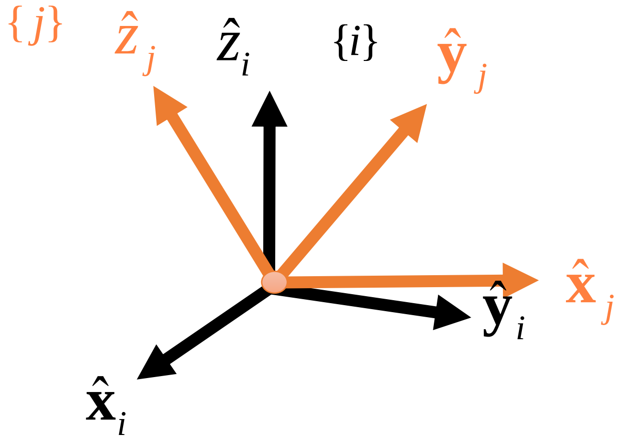


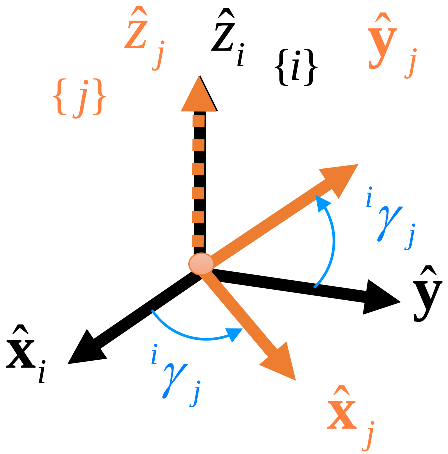
$${}^{i}\mathbf{p}_j = \begin{pmatrix} {}^{i}x_j \\ {}^{i}y_j \\ {}^{i}z_j \end{pmatrix}$$

Orientación

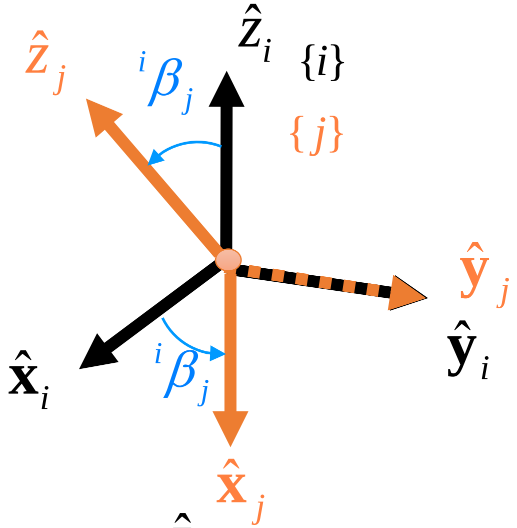


Orientación

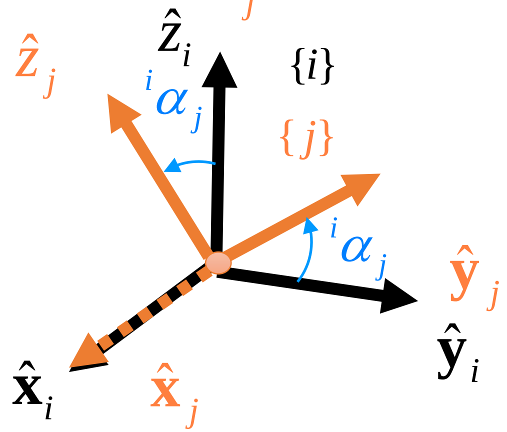




Con respecto al eje z

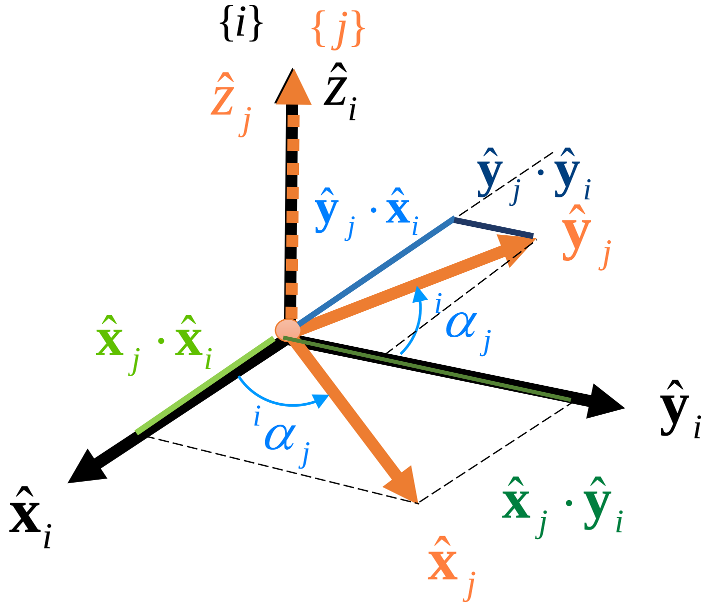


Con respecto al eje y



Con respecto al eje x

Orientación



$$\mathbf{R}_z({}^i\alpha_j) = \begin{pmatrix} \hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i \end{pmatrix}$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{x}}_i\right\| \cos({}^i\alpha_j) = \cos({}^i\alpha_j)$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{y}}_i\right\| \sin({}^i\alpha_j) = \sin({}^i\alpha_j)$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i = \left\|\hat{\mathbf{y}}_j\right\| \left\|\hat{\mathbf{x}}_i\right\| \sin({}^i\alpha_j) = -\sin({}^i\alpha_j)$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i = \left\|\hat{\mathbf{y}}_j\right\| \left\|\hat{\mathbf{y}}_i\right\| \cos({}^i\alpha_j) = \cos({}^i\alpha_j)$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i = \left\|\hat{\mathbf{z}}_j\right\| \left\|\hat{\mathbf{z}}_j\right\| \cos(0) = 1$$

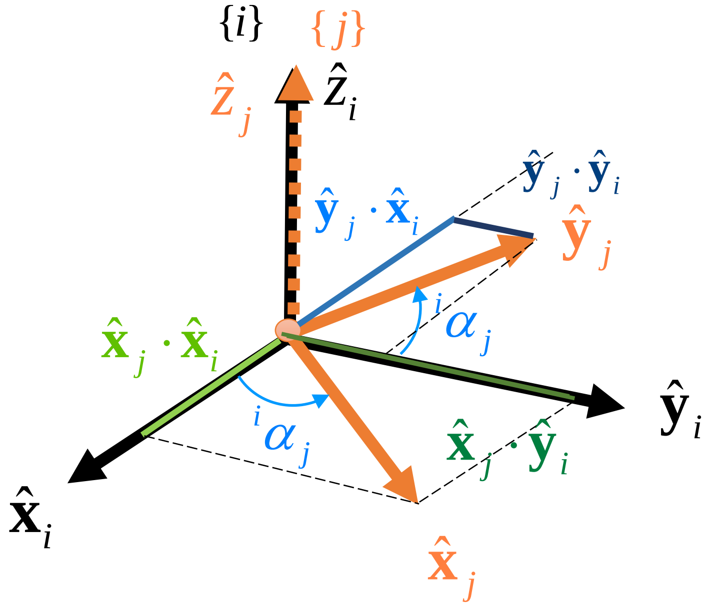
$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i = 0$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i = 0$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i = 0$$

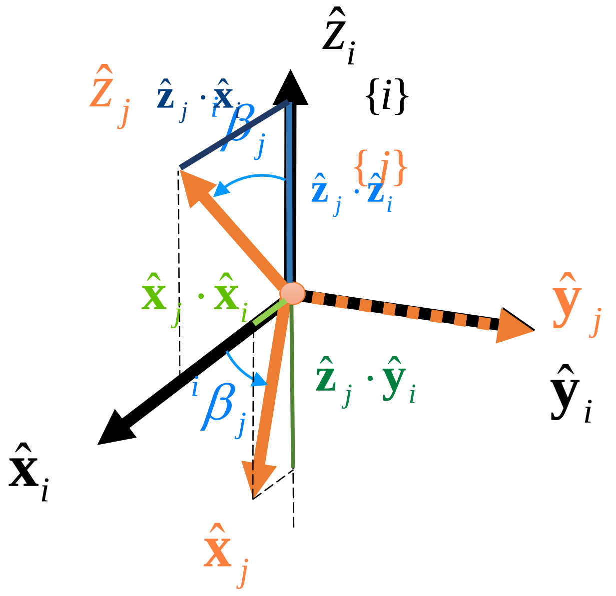
$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i = 0$$

Orientación



$$\mathbf{R}_z({}^{i}\alpha_j)=\begin{pmatrix} \cos({}^{i}\alpha_j) & -\sin({}^{i}\alpha_j) & 0 \\ \sin({}^{i}\alpha_j) & \cos({}^{i}\alpha_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Orientación



$$\mathbf{R}_y({}^i\beta_j) = \begin{pmatrix} \hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i \end{pmatrix}$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{x}}_i\right\| \cos({}^i\beta_j) = \cos({}^i\beta_j)$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i = \left\|\hat{\mathbf{y}}_j\right\| \left\|\hat{\mathbf{y}}_i\right\| \cos(0) = 1$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i = \left\|\hat{\mathbf{z}}_j\right\| \left\|\hat{\mathbf{x}}_i\right\| \sin({}^i\beta_j) = \sin({}^i\beta_j)$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{z}}_i\right\| \sin({}^i\beta_j) = -\sin({}^i\beta_j)$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i = \left\|\hat{\mathbf{z}}_j\right\| \left\|\hat{\mathbf{z}}_j\right\| \cos({}^i\beta_j) = \cos({}^i\beta_j)$$

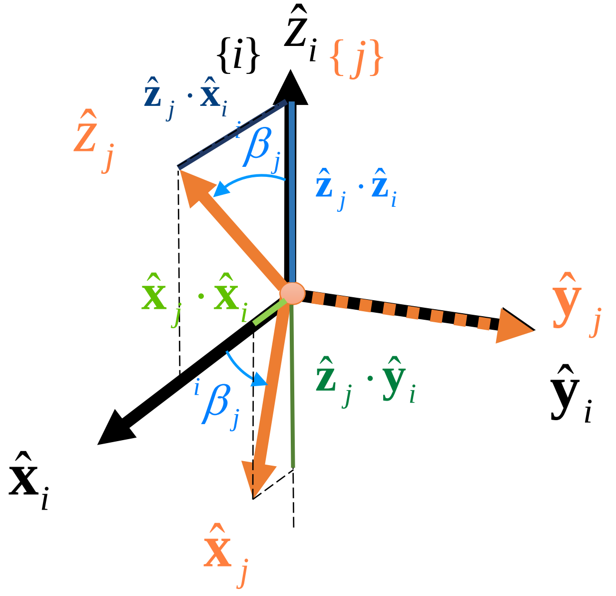
$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i = 0$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i = 0$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i = 0$$

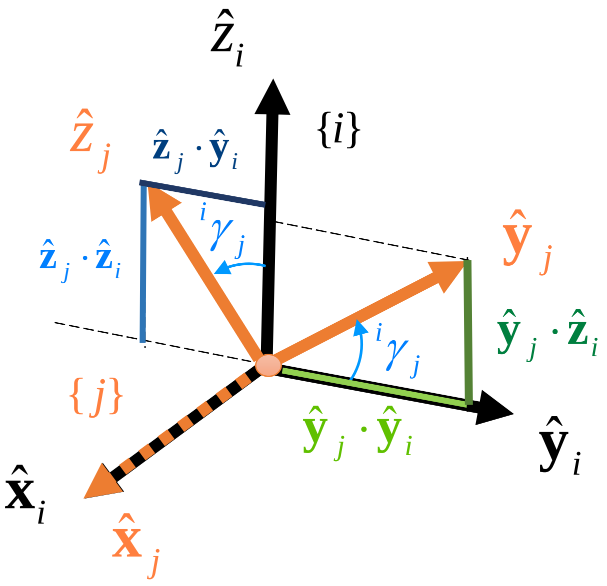
$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i = 0$$

Orientación



$$\mathbf{R}_y({}^i\beta_j) = \begin{pmatrix} \cos({}^i\beta_j) & 0 & \sin({}^i\beta_j) \\ 0 & 1 & 0 \\ -\sin({}^i\beta_j) & 0 & \cos({}^i\beta_j) \end{pmatrix}$$

Orientación



$$\mathbf{R}_x({}^i\gamma_j) = \begin{pmatrix} \hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{x}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i \\ \hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i & \hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i \end{pmatrix}$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{y}}_i = \left\|\hat{\mathbf{y}}_j\right\| \left\|\hat{\mathbf{y}}_i\right\| \cos({}^i\alpha_j) = \cos({}^i\alpha_j)$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{x}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{x}}_i\right\| \cos(0) = 1$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i = \left\|\hat{\mathbf{z}}_j\right\| \left\|\hat{\mathbf{y}}_i\right\| \sin({}^i\alpha_j) = -\sin({}^i\beta_j)$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{x}}_i = 0$$

$$\hat{\mathbf{y}}_j \cdot \hat{\mathbf{z}}_i = \left\|\hat{\mathbf{x}}_j\right\| \left\|\hat{\mathbf{z}}_i\right\| \sin({}^i\gamma_j) = \sin({}^i\gamma_j)$$

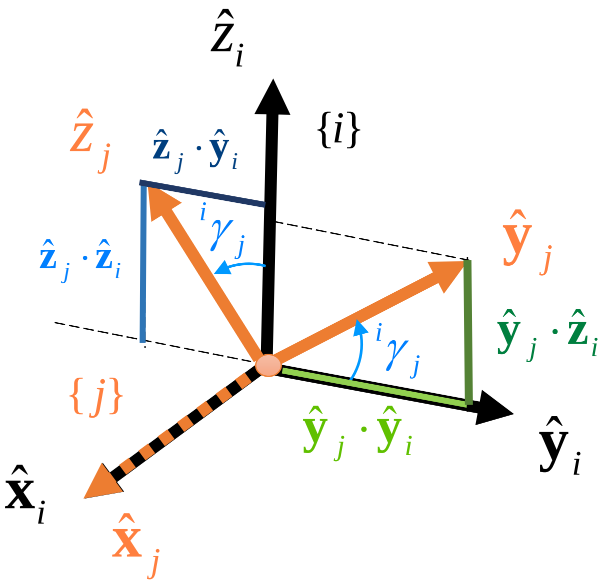
$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{y}}_i = 0$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{y}}_i = 0$$

$$\hat{\mathbf{z}}_j \cdot \hat{\mathbf{z}}_i = \left\|\hat{\mathbf{z}}_j\right\| \left\|\hat{\mathbf{z}}_j\right\| \cos({}^i\gamma_j) = \cos({}^i\gamma_j)$$

$$\hat{\mathbf{x}}_j \cdot \hat{\mathbf{z}}_i = 0$$

Orientación



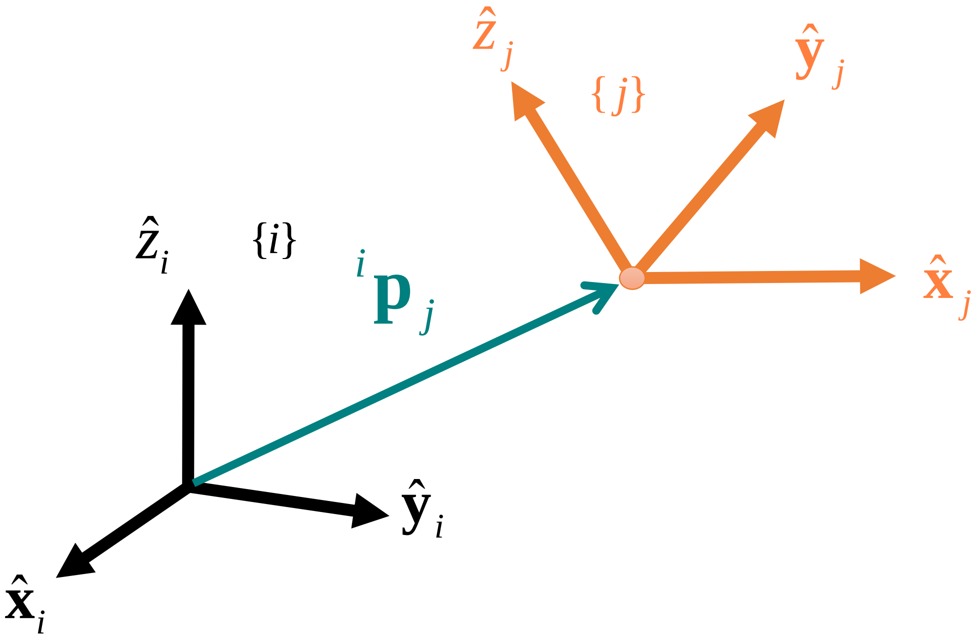
$$\mathbf{R}_x({}^i\gamma_j) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos({}^i\alpha_j) & -\sin({}^i\alpha_j) \\ 0 & \sin({}^i\alpha_j) & \cos({}^i\alpha_j) \end{pmatrix}$$

Orientación

$$\mathbf{R}_{\mathrm{z}}({}^{i}\alpha_{j})=\begin{pmatrix}\cos({}^{i}\alpha_{j}) & -\sin({}^{i}\alpha_{j}) & 0\\ \sin({}^{i}\alpha_{j}) & \cos({}^{i}\alpha_{j}) & 0\\ 0 & 0 & 1\end{pmatrix}\qquad \mathbf{R}_{\mathrm{y}}({}^{i}\beta_{j})=\begin{pmatrix}\cos({}^{i}\beta_{j}) & 0 & \sin({}^{i}\beta_{j})\\ 0 & 1 & 0\\ -\sin({}^{i}\beta_{j}) & 0 & \cos({}^{i}\beta_{j})\end{pmatrix}\qquad \mathbf{R}_{\mathrm{x}}({}^{i}\gamma_{j})=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos({}^{i}\gamma_{j}) & -\sin({}^{i}\gamma_{j})\\ 0 & \sin({}^{i}\gamma_{j}) & \cos({}^{i}\gamma_{j})\end{pmatrix}$$

$${}^{i}\mathbf{R}_{i}=\mathbf{R}_{\mathrm{z}}({}^{i}\alpha_{j})\mathbf{R}_{\mathrm{y}}({}^{i}\beta_{j})\mathbf{R}_{\mathrm{x}}({}^{i}\gamma_{j})=\begin{pmatrix}\cos({}^{i}\alpha_{j})\cos({}^{i}\beta_{j}) & \cos({}^{i}\alpha_{j})\sin({}^{i}\beta_{j})\sin({}^{i}\gamma_{j})-\cos({}^{i}\gamma_{j})\sin({}^{i}\alpha_{j}) & \sin({}^{i}\alpha_{j})\sin({}^{i}\gamma_{j})+\cos({}^{i}\alpha_{j})\cos({}^{i}\gamma_{j})\sin({}^{i}\beta_{j})\\ \cos({}^{i}\beta_{j})\sin({}^{i}\alpha_{j}) & \cos({}^{i}\alpha_{j})\cos({}^{i}\gamma_{j})+\sin({}^{i}\alpha_{j})\sin({}^{i}\beta_{j})\sin({}^{i}\gamma_{j}) & \cos({}^{i}\gamma_{j})\sin({}^{i}\alpha_{j})\sin({}^{i}\beta_{j})-\cos({}^{i}\alpha_{j})\sin({}^{i}\gamma_{j})\\ -\sin({}^{i}\beta_{j}) & \cos({}^{i}\beta_{j})\sin({}^{i}\gamma_{j}) & \cos({}^{i}\beta_{j})\cos({}^{i}\gamma_{j})\end{pmatrix}$$

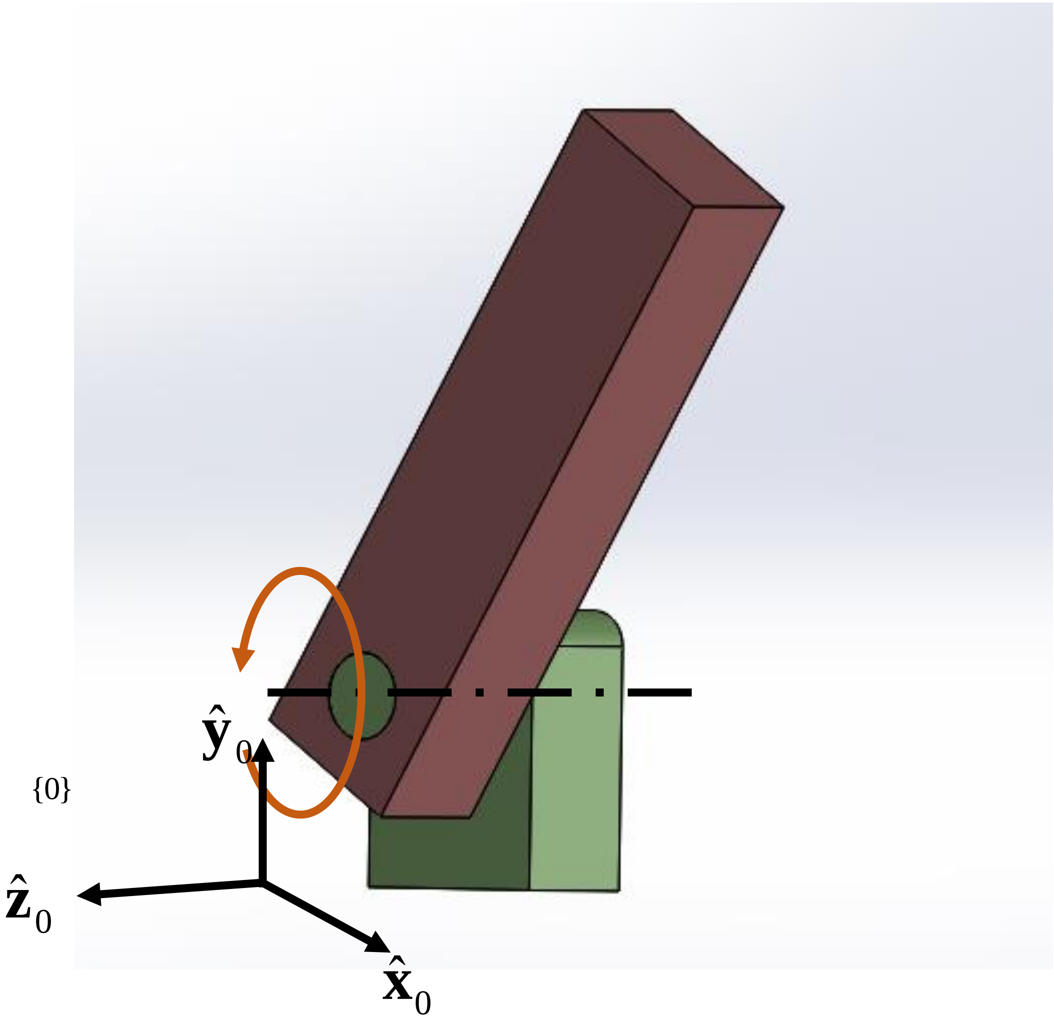
Posición y orientación



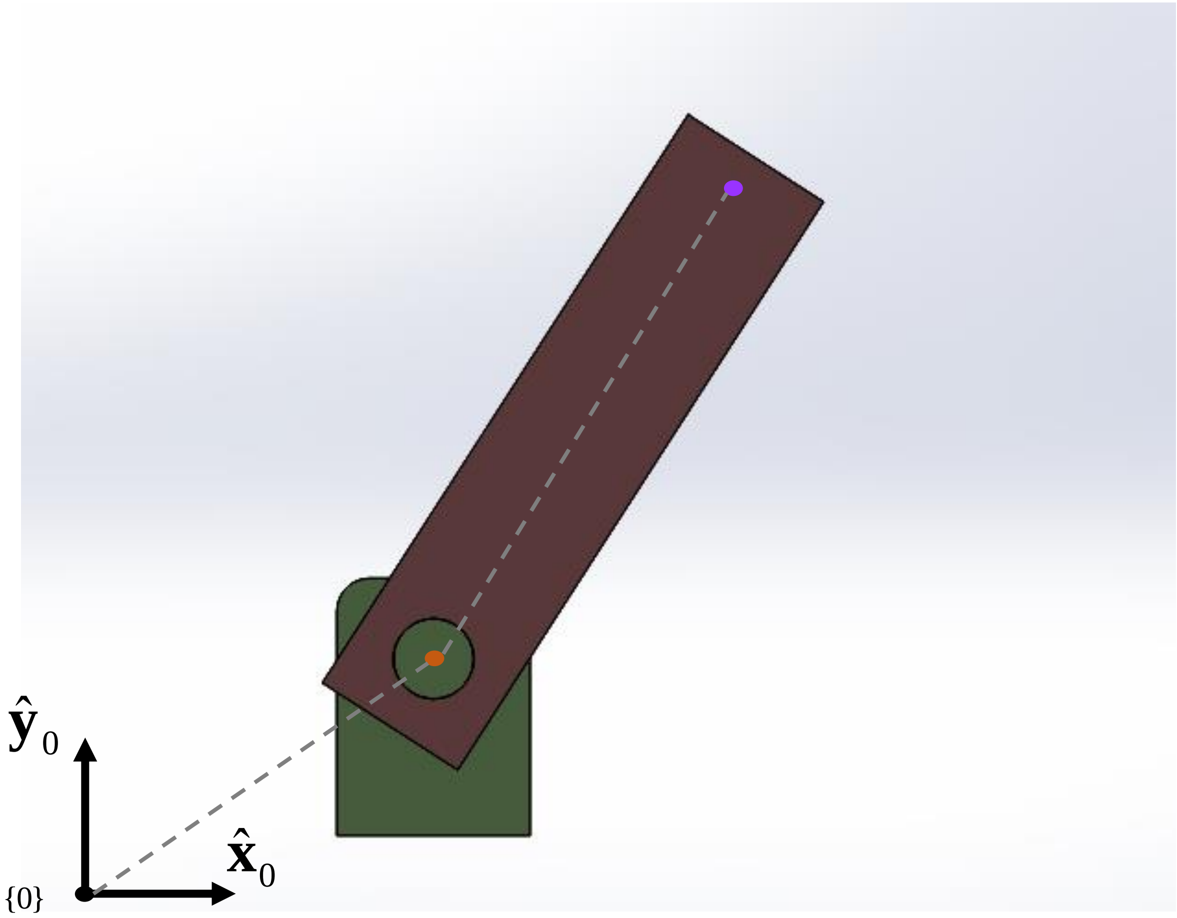
$$
{}^{i}\mathbf{T}_j = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix}
\cos({}^{i}\alpha_j)\cos({}^{i}\beta_j) & \cos({}^{i}\alpha_j)\sin({}^{i}\beta_j)\sin({}^{i}\gamma_j) - \cos({}^{i}\gamma_j)\sin({}^{i}\alpha_j) & \sin({}^{i}\alpha_j)\sin({}^{i}\gamma_j) + \cos({}^{i}\alpha_j)\cos({}^{i}\gamma_j)\sin({}^{i}\beta_j) & {}^{i}x_j \\
\cos({}^{i}\beta_j)\sin({}^{i}\alpha_j) & \cos({}^{i}\alpha_j)\cos({}^{i}\gamma_j) + \sin({}^{i}\alpha_j)\sin({}^{i}\beta_j)\sin({}^{i}\gamma_j) & \cos({}^{i}\gamma_j)\sin({}^{i}\alpha_j)\sin({}^{i}\beta_j) - \cos({}^{i}\alpha_j)\sin({}^{i}\gamma_j) & {}^{i}y_j \\
-\sin({}^{i}\beta_j) & \cos({}^{i}\beta_j)\sin({}^{i}\gamma_j) & \cos({}^{i}\beta_j)\cos({}^{i}\gamma_j) & {}^{i}z_j \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

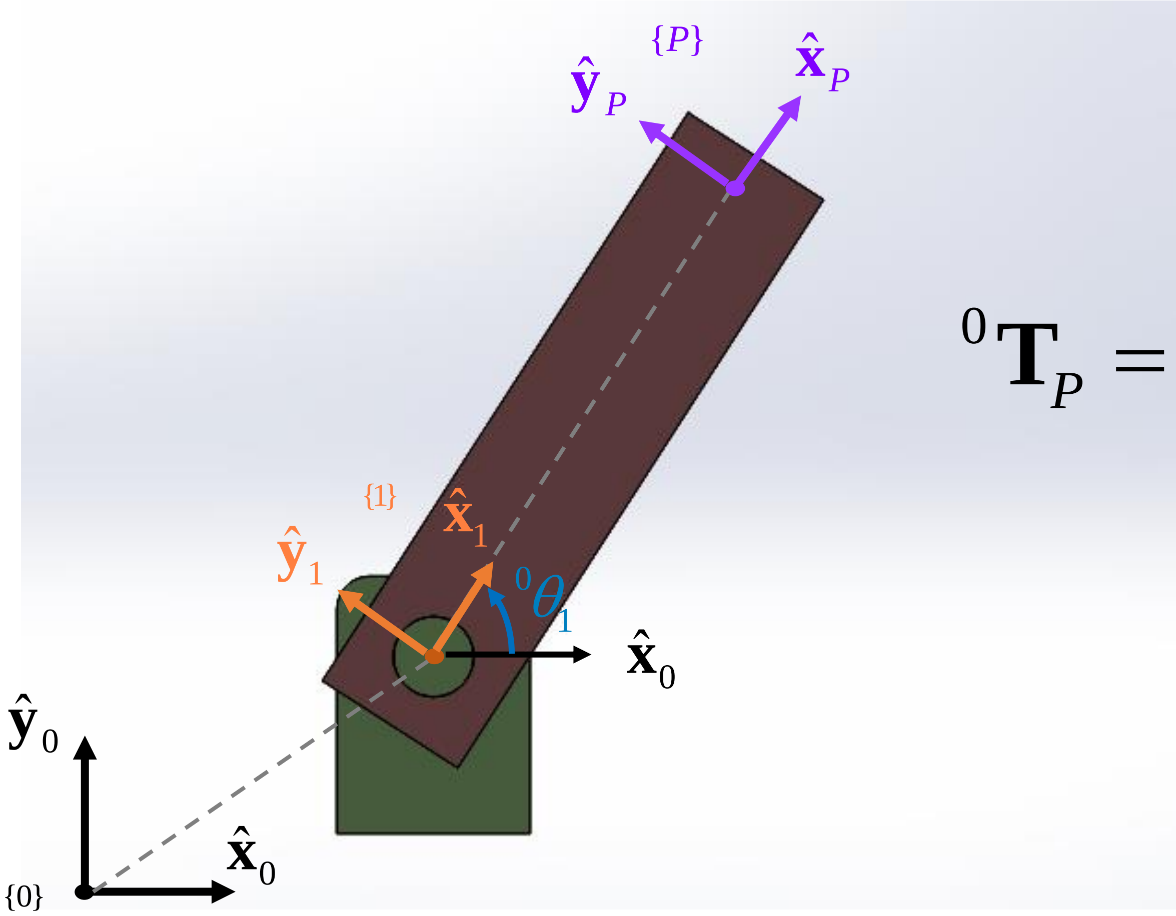
Junta rotacional



Junta rotacional

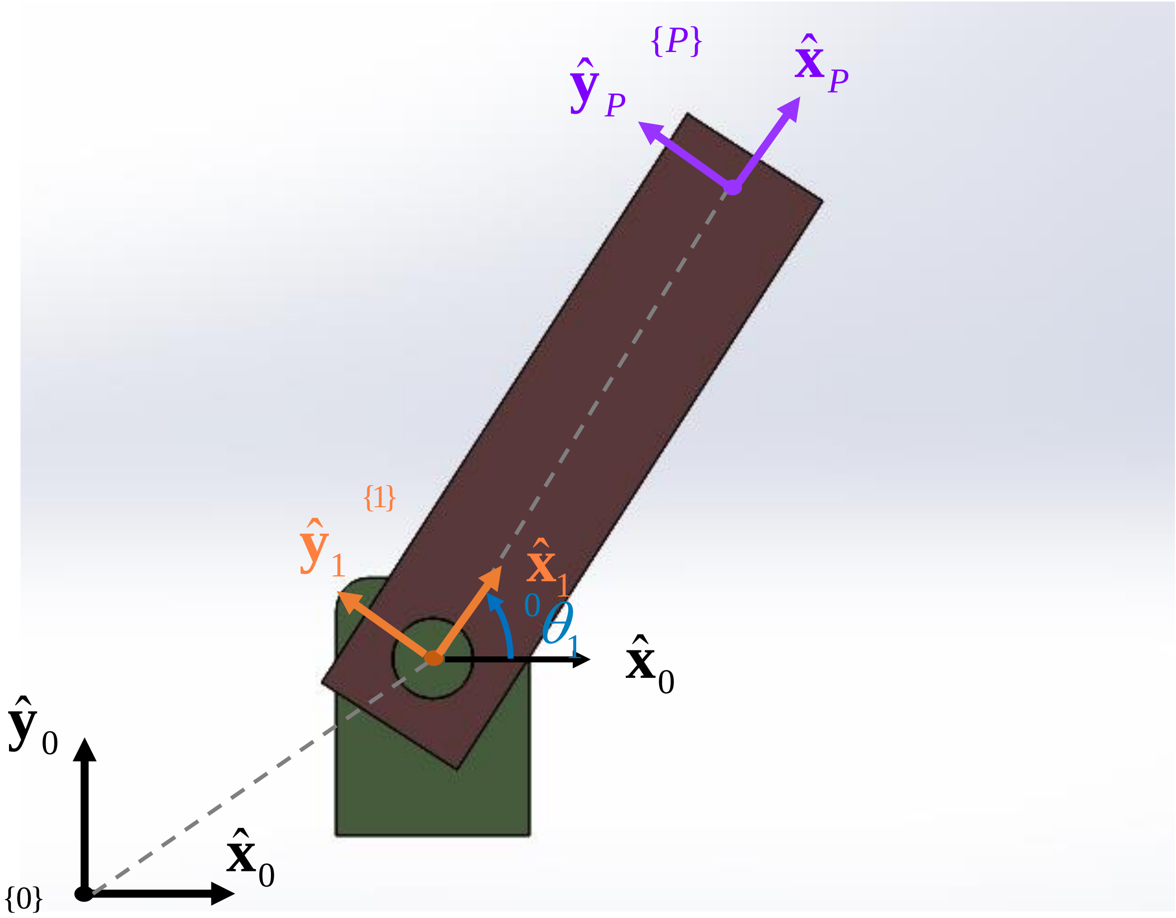


Junta rotacional

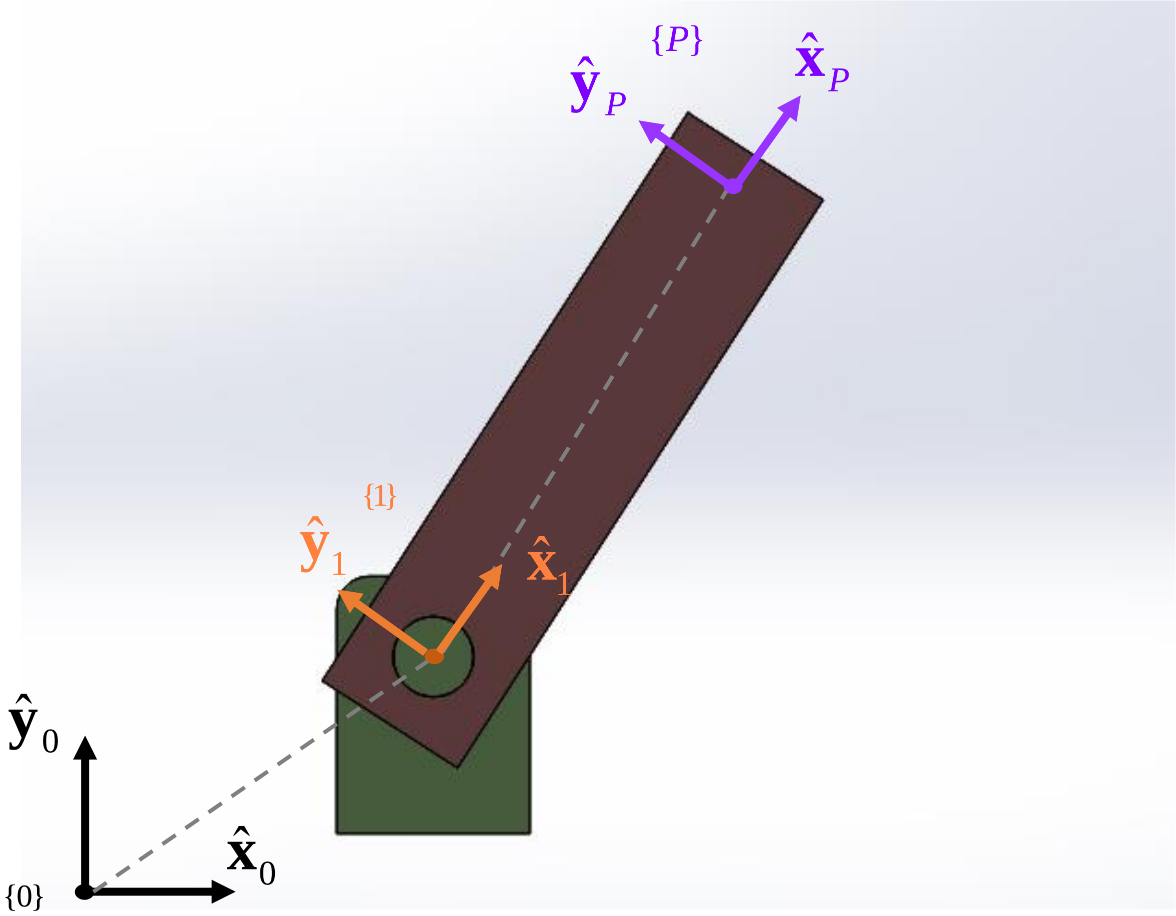


$$ {}^{0}\mathbf{T}_{P} = {}^{0}\mathbf{T}_{\cancel{1}}\,{}^{\cancel{1}}\mathbf{T}_{P} $$

Junta rotacional



Junta rotacional



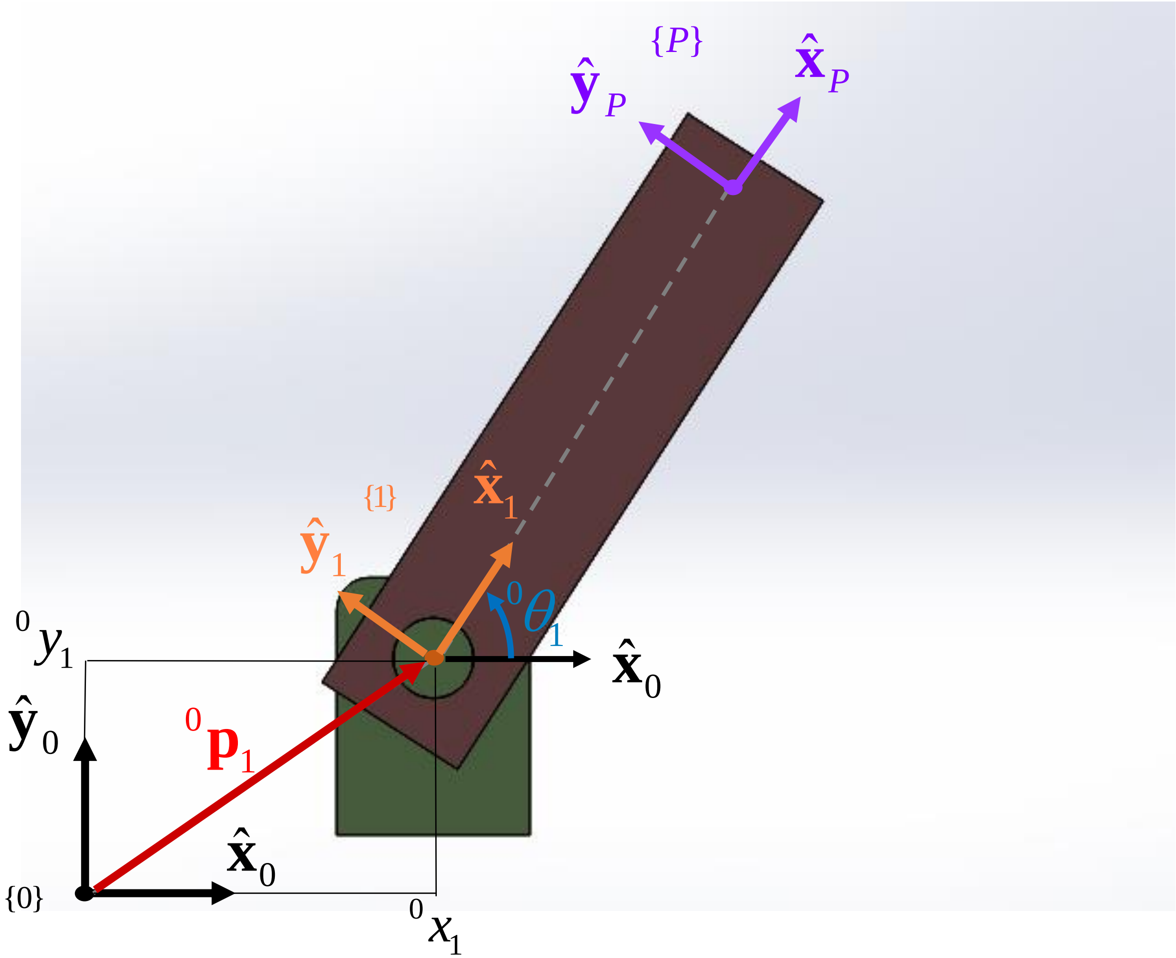
$$ {}^{i}\mathbf{T}_{j} = \begin{pmatrix} {}^{i}\mathbf{R}_{j} & {}^{i}\mathbf{p}_{j} \\ \mathbf{0}^{T} & 1 \end{pmatrix} = $$

$$ {}^{i}\mathbf{R}_{i} = \mathbf{R}_{z}({}^{i}\theta_{j})\mathbf{R}_{y}(0)\mathbf{R}_{x}(0) = $$

$$ = \begin{pmatrix} \cos({}^{i}\theta_{j}) & -\sin({}^{i}\theta_{j}) & 0 \\ \sin({}^{i}\theta_{j}) & \cos({}^{i}\theta_{j}) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{i}\mathbf{p}_{j} = \begin{pmatrix} {}^{i}x_{j} \\ {}^{i}y_{j} \\ 0 \end{pmatrix} $$

Junta rotacional

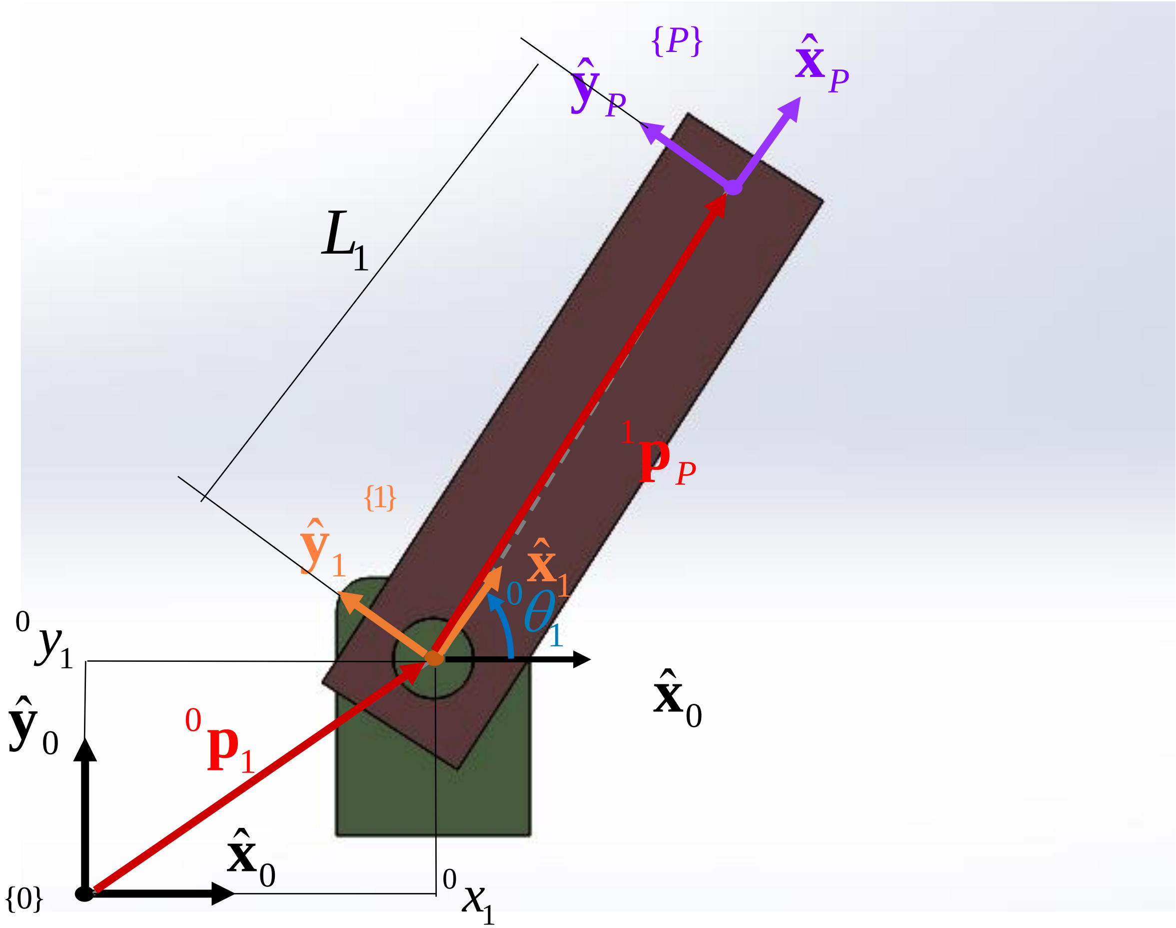


$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1 = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la posición

Junta rotacional

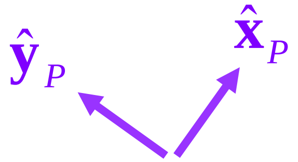


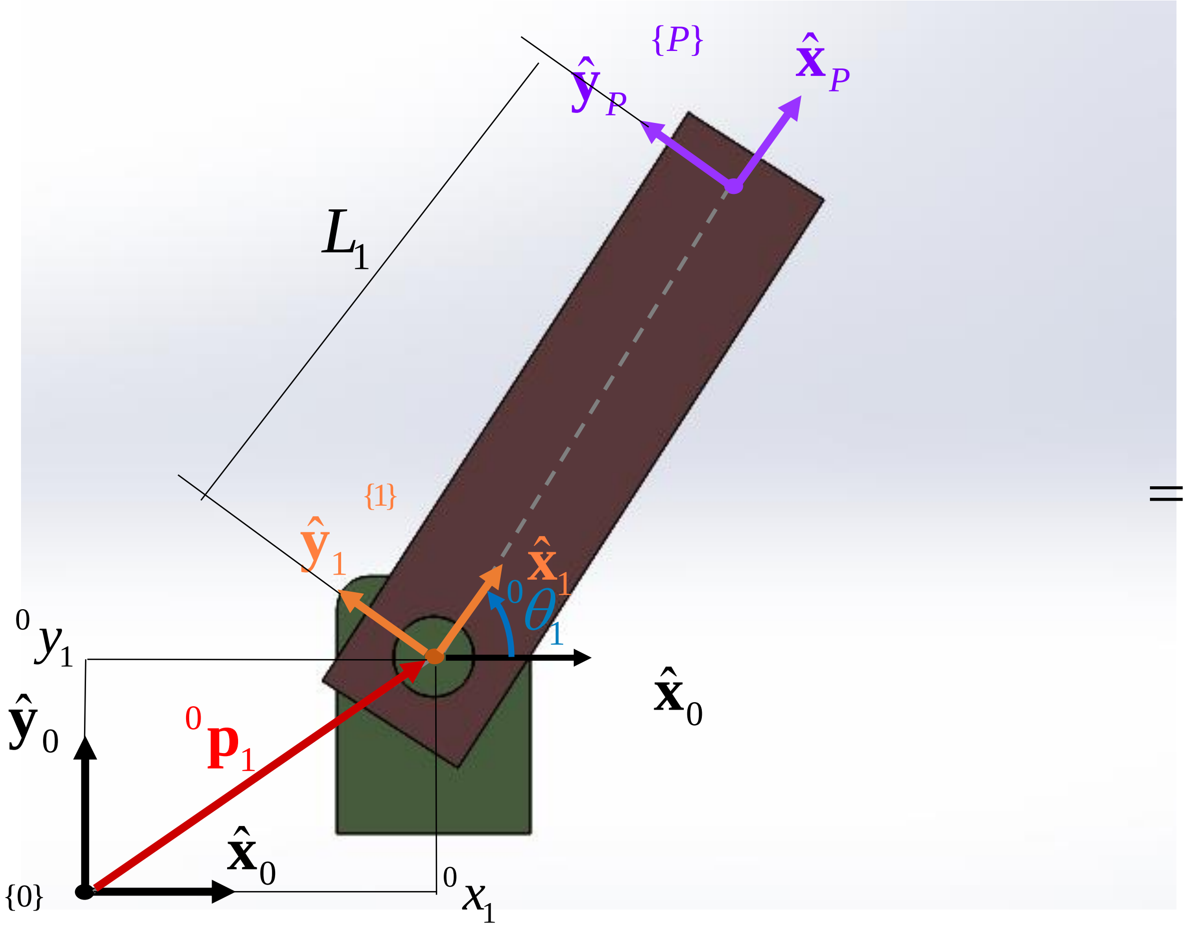
$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la posición

Junta rotacional



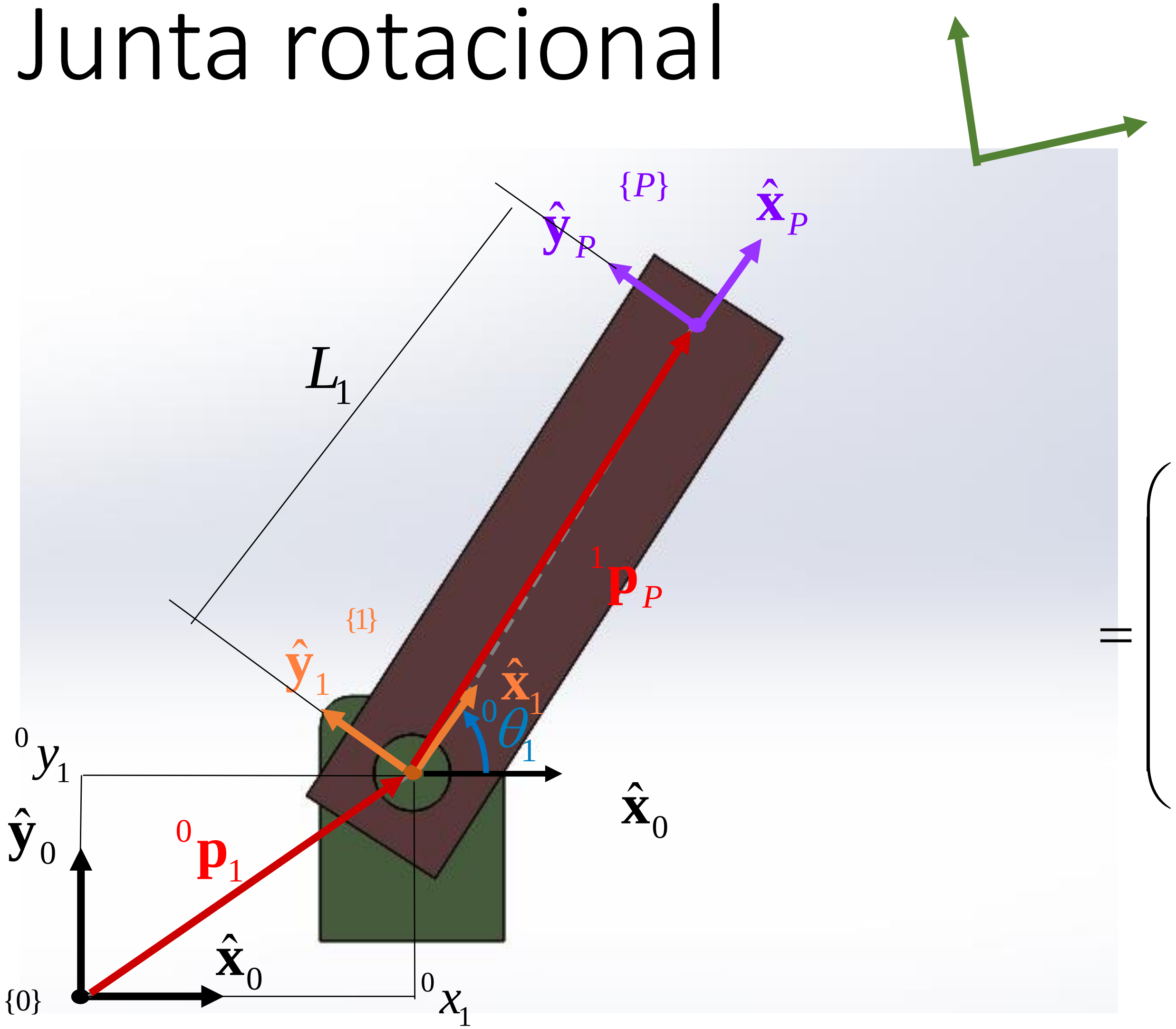


$${}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P =$$

$$= \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la posición

Junta rotacional

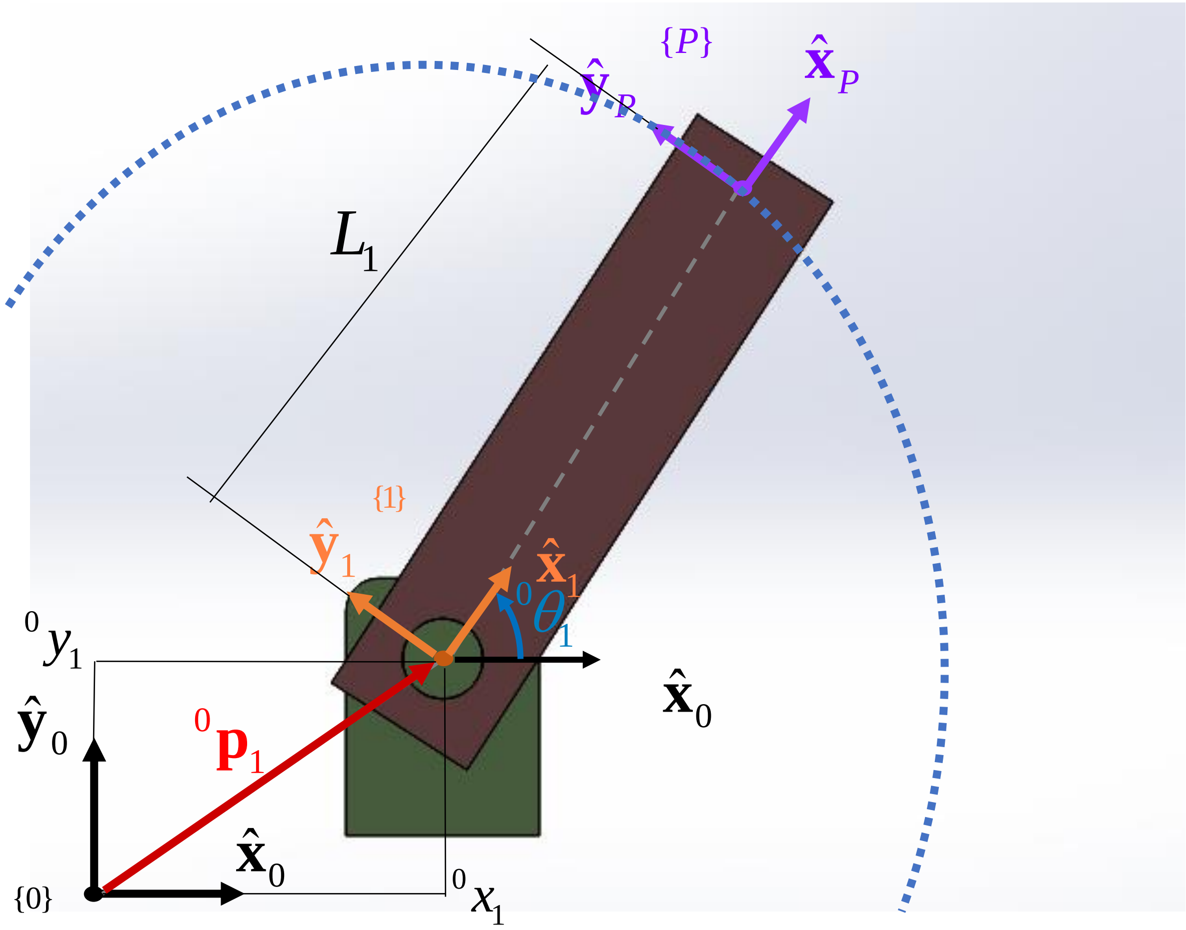


$$
{}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P =
$$

$$
= \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Junta rotacional

Modelo cinemático de la posición



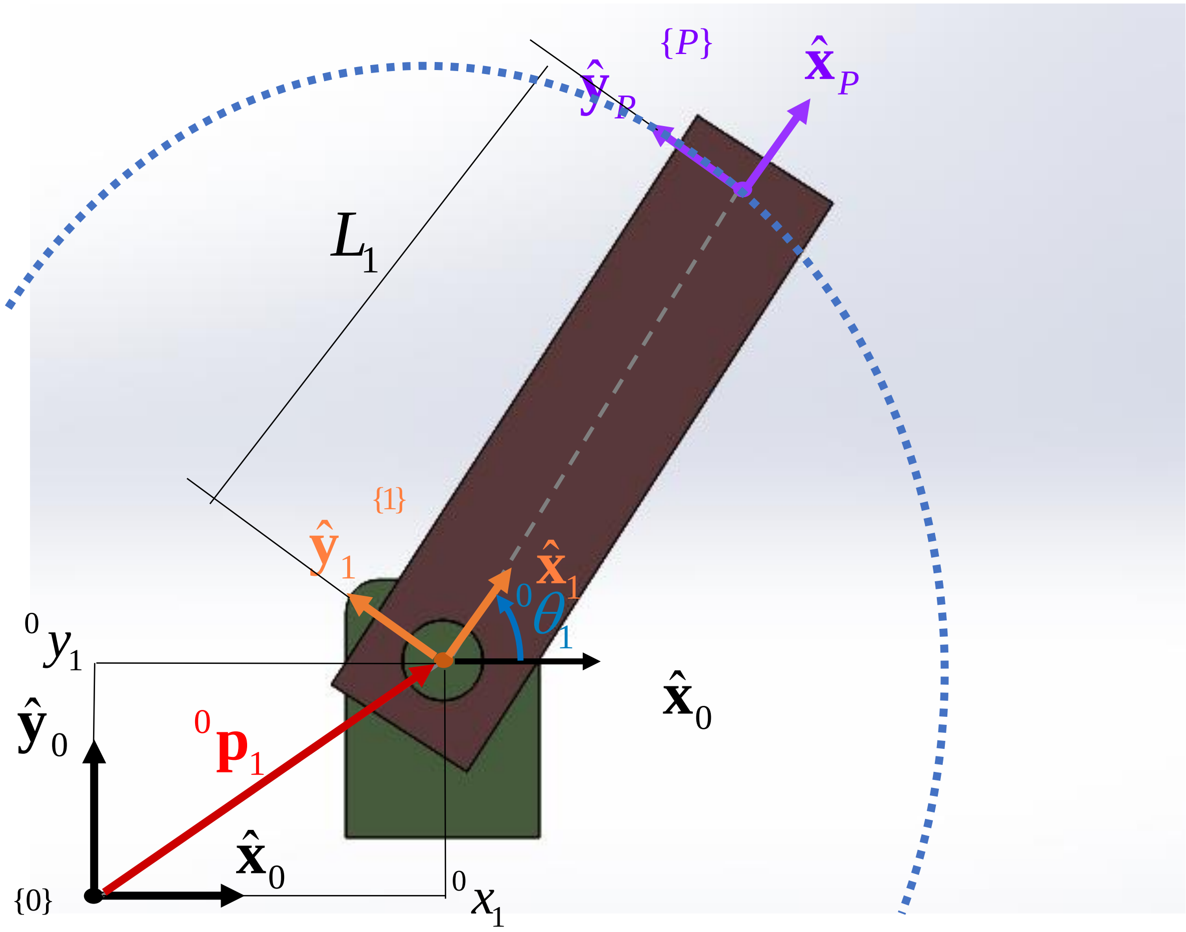
$${}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P =$$

$$= \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1\right)$$

Modelo cinemático de la posición

Junta rotacional



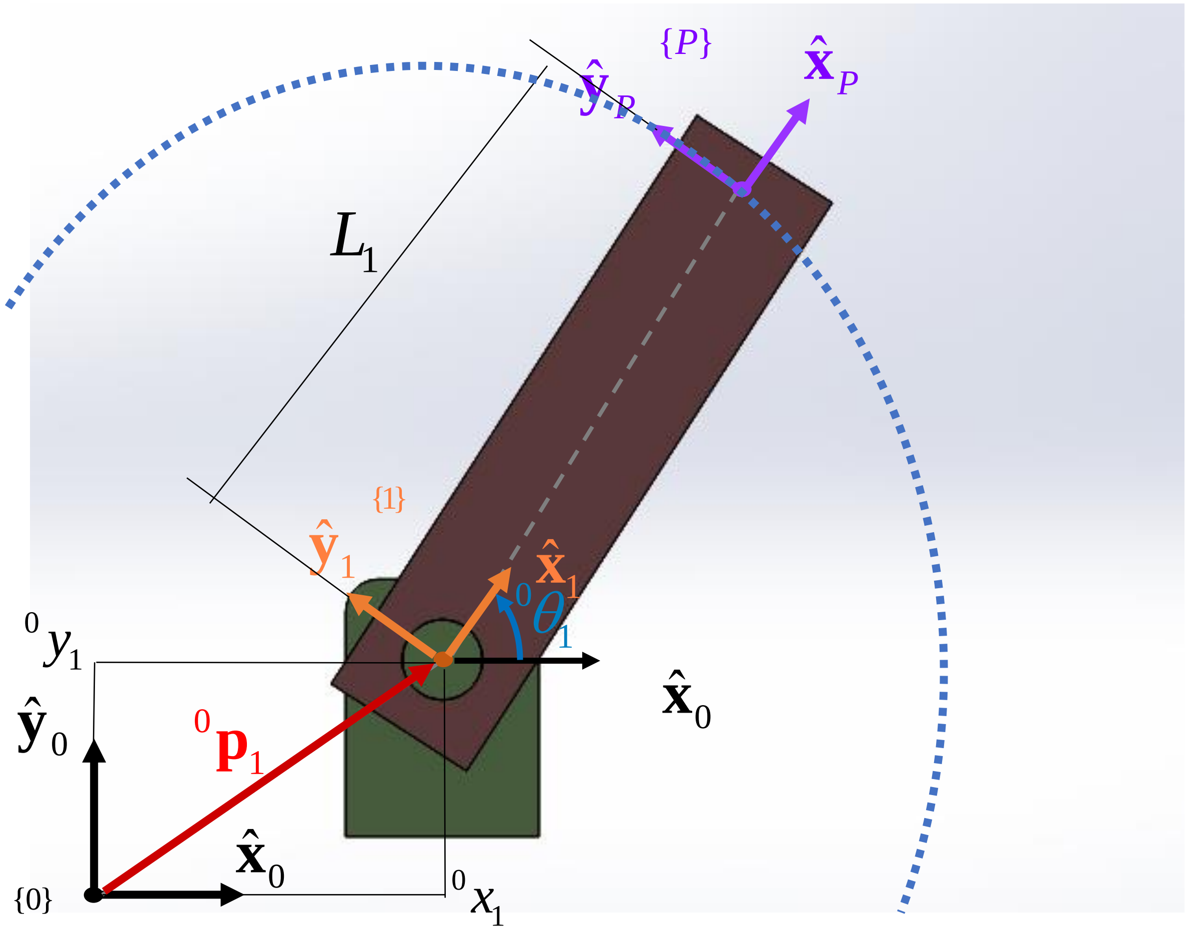
$$
{}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1\right)
$$

Vector de la postura de un eslabón

$$
{}^0\boldsymbol{\xi}_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix}
$$

Modelo cinemático de la posición

Junta rotacional

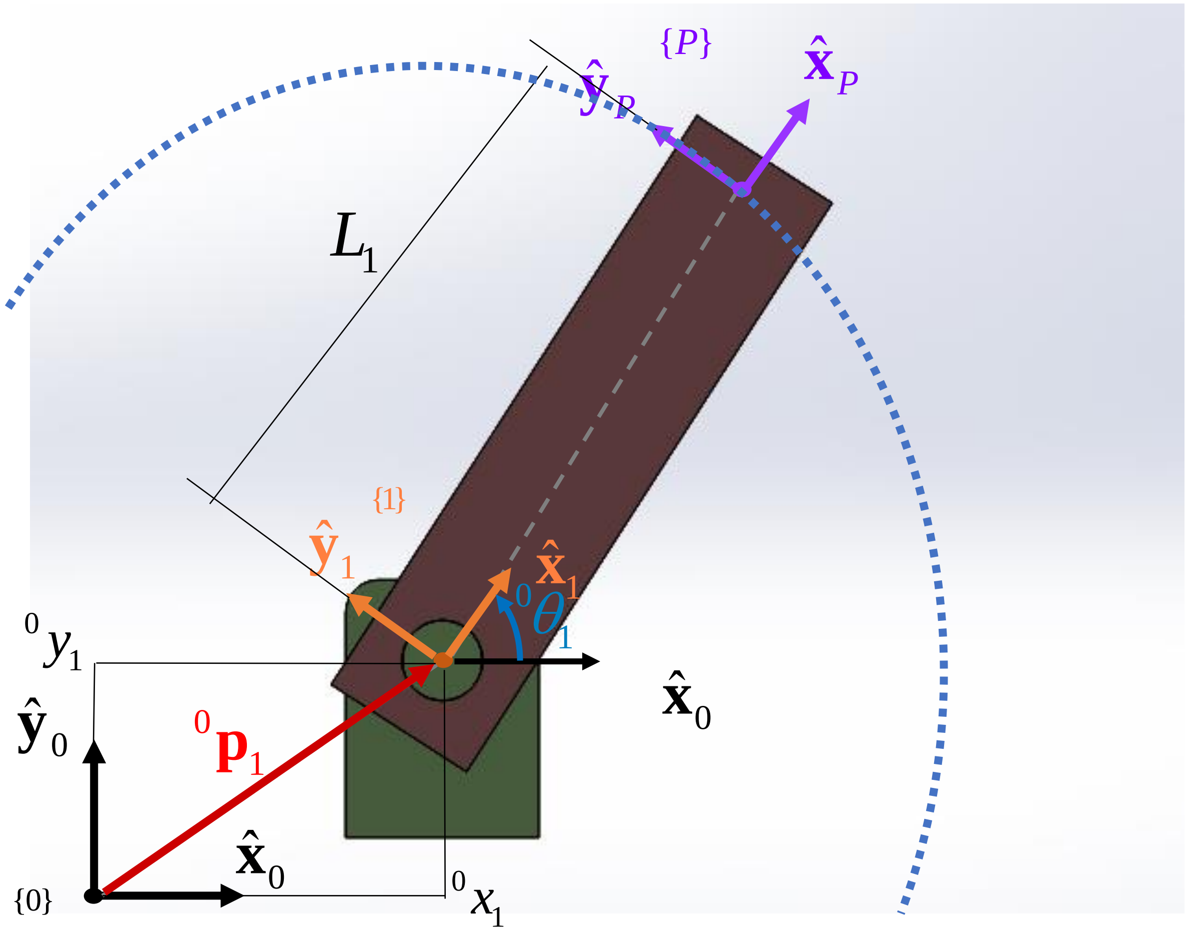


$$
{}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 \\ {}^0y_1 \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1 \right)
$$

Vector de la postura de un eslabón

$$
{}^0\boldsymbol{\xi}_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix}
$$

Junta rotacional



Vector de la postura de un eslabón

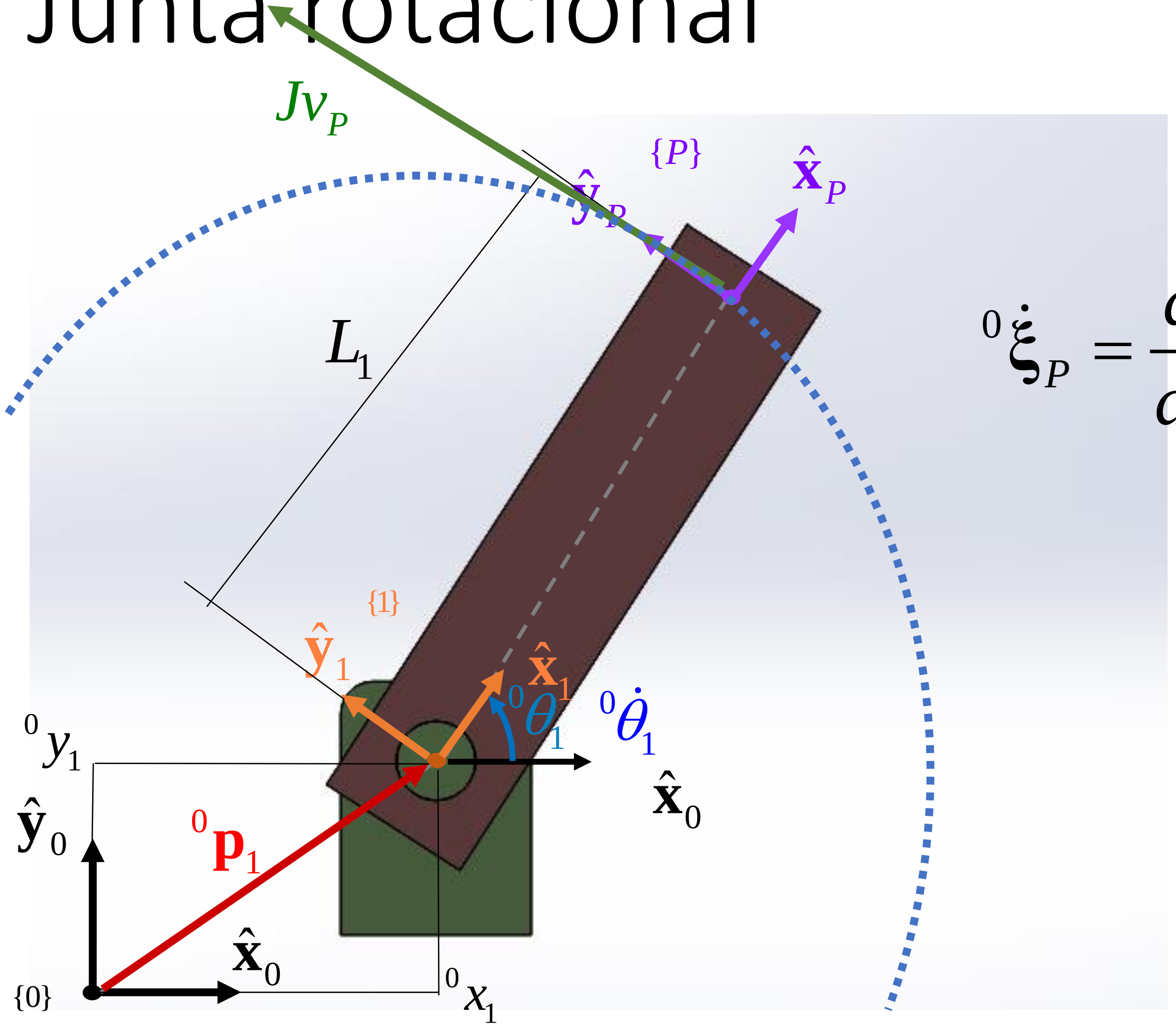
$$
{}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix}
$$

Vector de velocidades del eslabón

$$
{}^0\dot{\xi}_P = \frac{d}{dt}\,{}^0\xi_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\xi_P\,{}^0\dot{\theta}_1
$$

Modelo cinemático de la velocidad

Junta rotacional



Vector de velocidades del eslabón

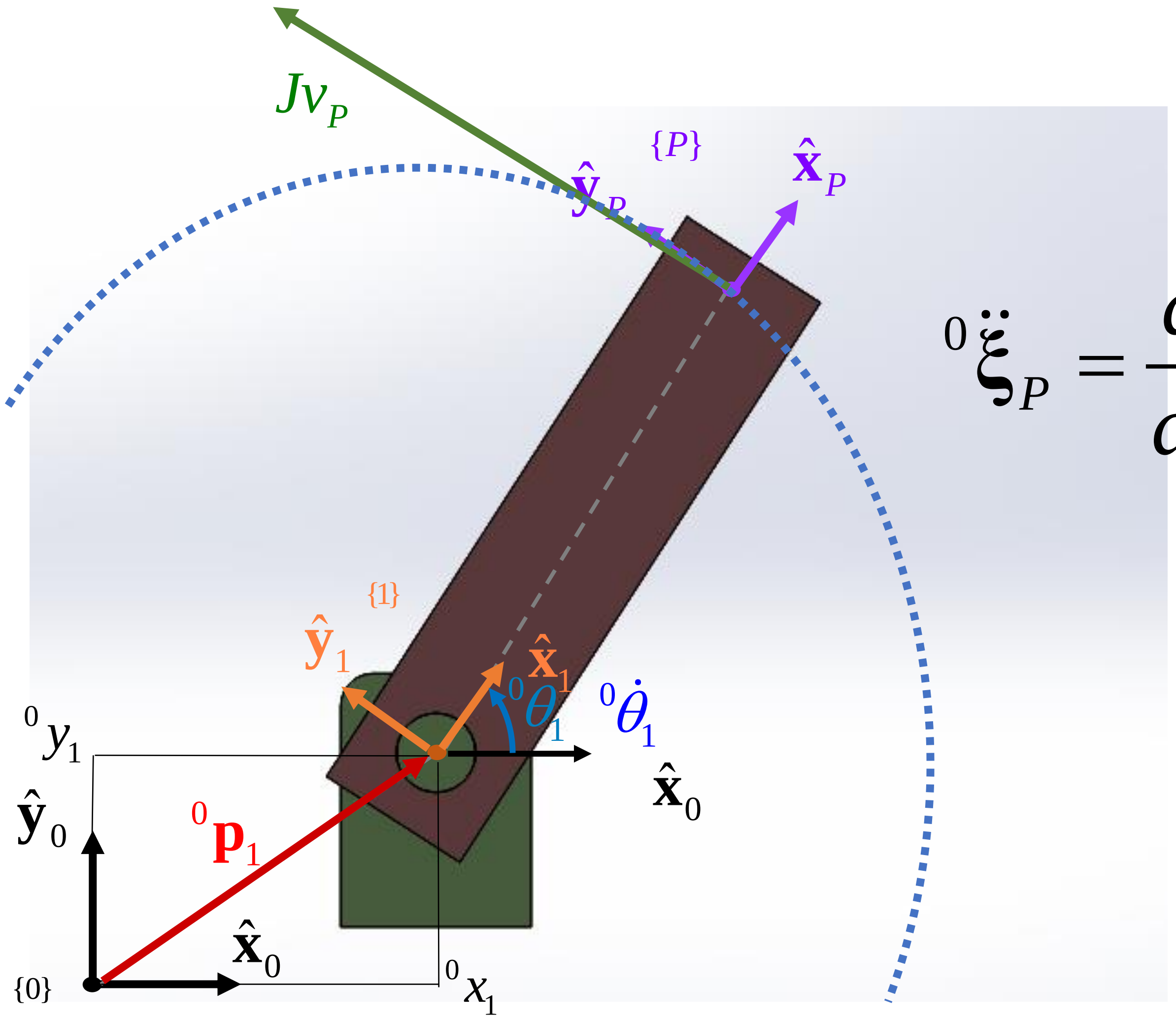
$$
{}^{0}\dot{\boldsymbol{\xi}}_P = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\boldsymbol{\xi}_P\,{}^0\dot{\theta}_1 = \begin{pmatrix} \boxed{\begin{matrix}-L_1\sin({}^0\theta_1)\\ L_1\cos({}^0\theta_1)\end{matrix}} \\ \boxed{1}\end{pmatrix} {}^0\dot{\theta}_1
$$

Vector de aceleraciones del eslabón

$$
{}^{0}\ddot{\boldsymbol{\xi}}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial\,{}^0\dot{\theta}_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\ddot{\theta}_1
$$

Junta rotacional

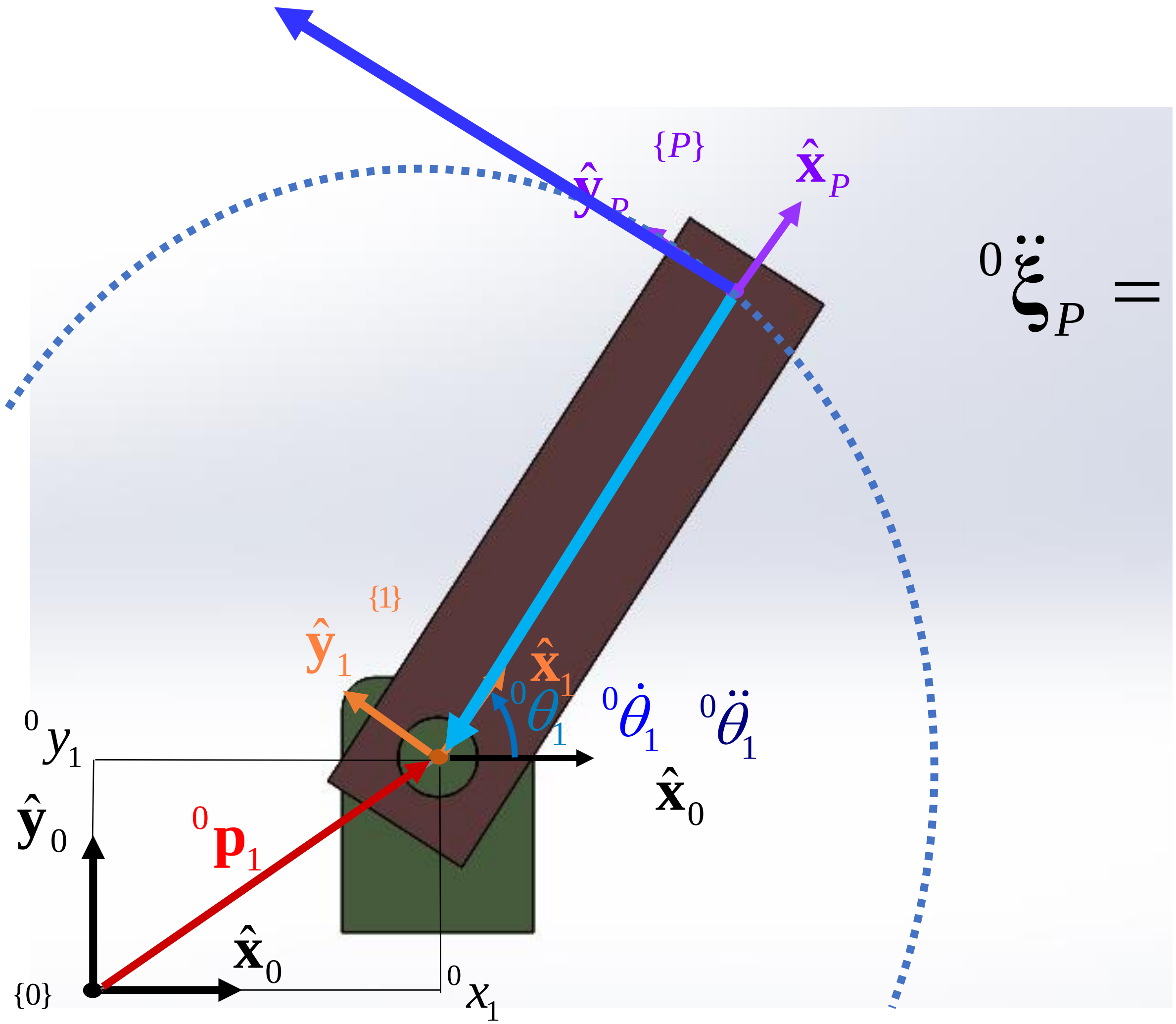
Modelo cinemático de la aceleración



$$
{}^0\ddot{\xi}_P = \frac{d}{dt}\,{}^0\dot{\xi}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\dot{\xi}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial\,{}^0\dot{\theta}_1}\,{}^0\dot{\xi}_P\,{}^0\ddot{\theta}_1
$$

Junta rotacional

Modelo cinemático de la aceleración

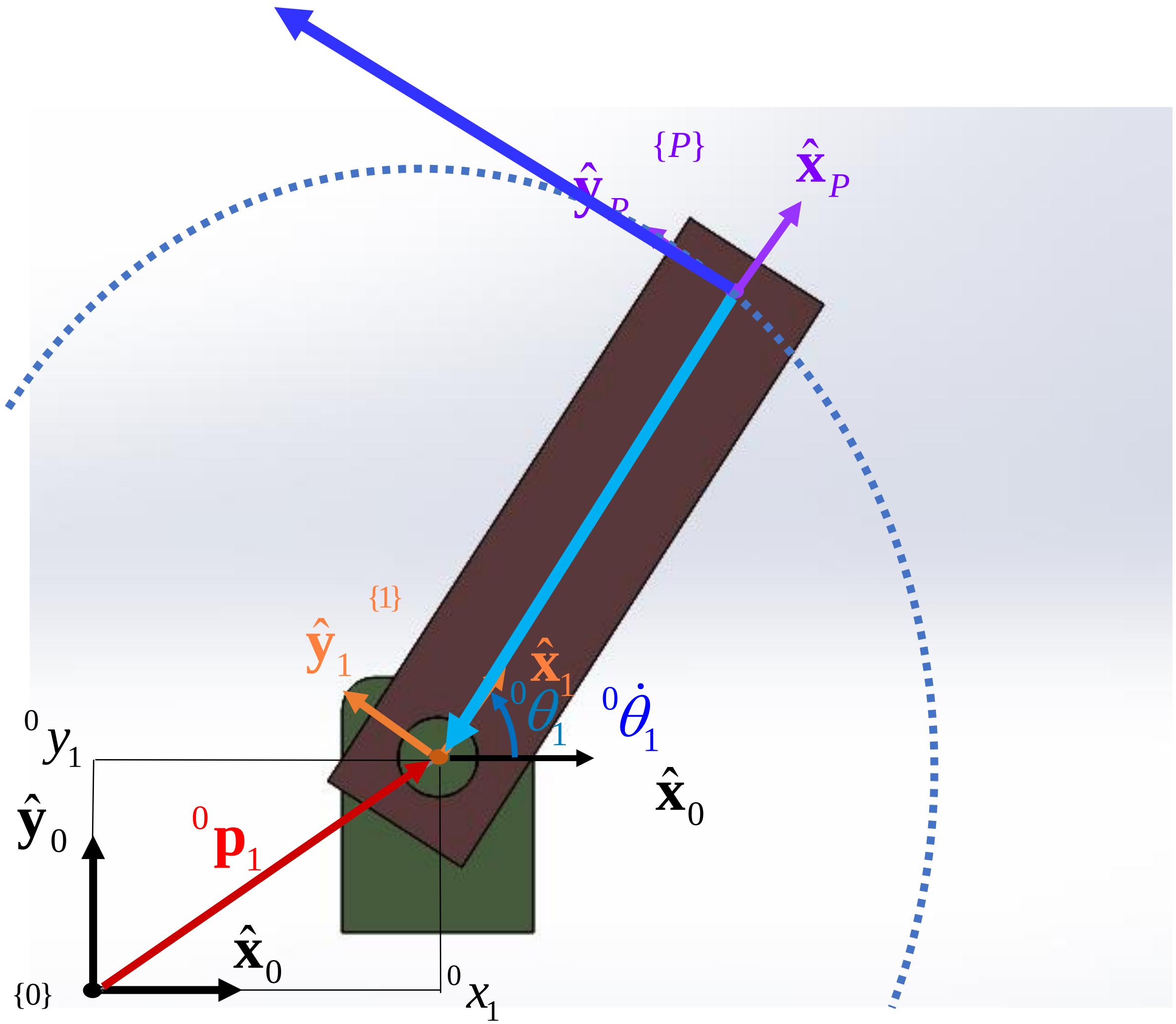


$$
{}^0\ddot{\xi}_P = \frac{d}{dt}\,{}^0\dot{\xi}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\dot{\xi}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial\,{}^0\dot{\theta}_1}\,{}^0\dot{\xi}_P\,{}^0\ddot{\theta}_1
$$

$$
= \begin{pmatrix} -L_1\cos({}^0\theta_1) \\ -L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix} {}^0\dot{\theta}_1^{\,2} + \begin{pmatrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \\ 1 \end{pmatrix} {}^0\ddot{\theta}_1
$$

Junta rotacional

Modelo cinemático de la aceleración

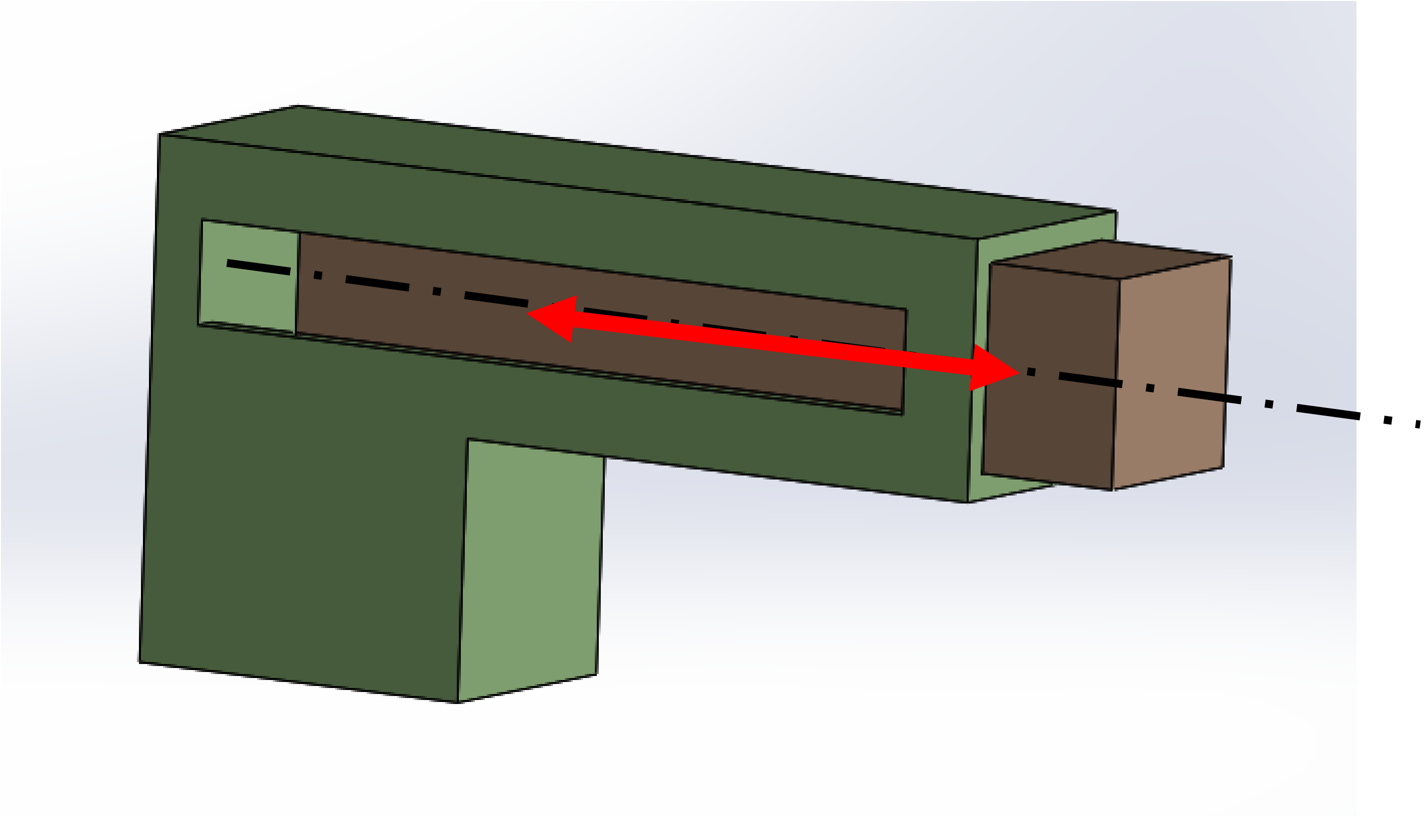


Vector de velocidades del eslabón

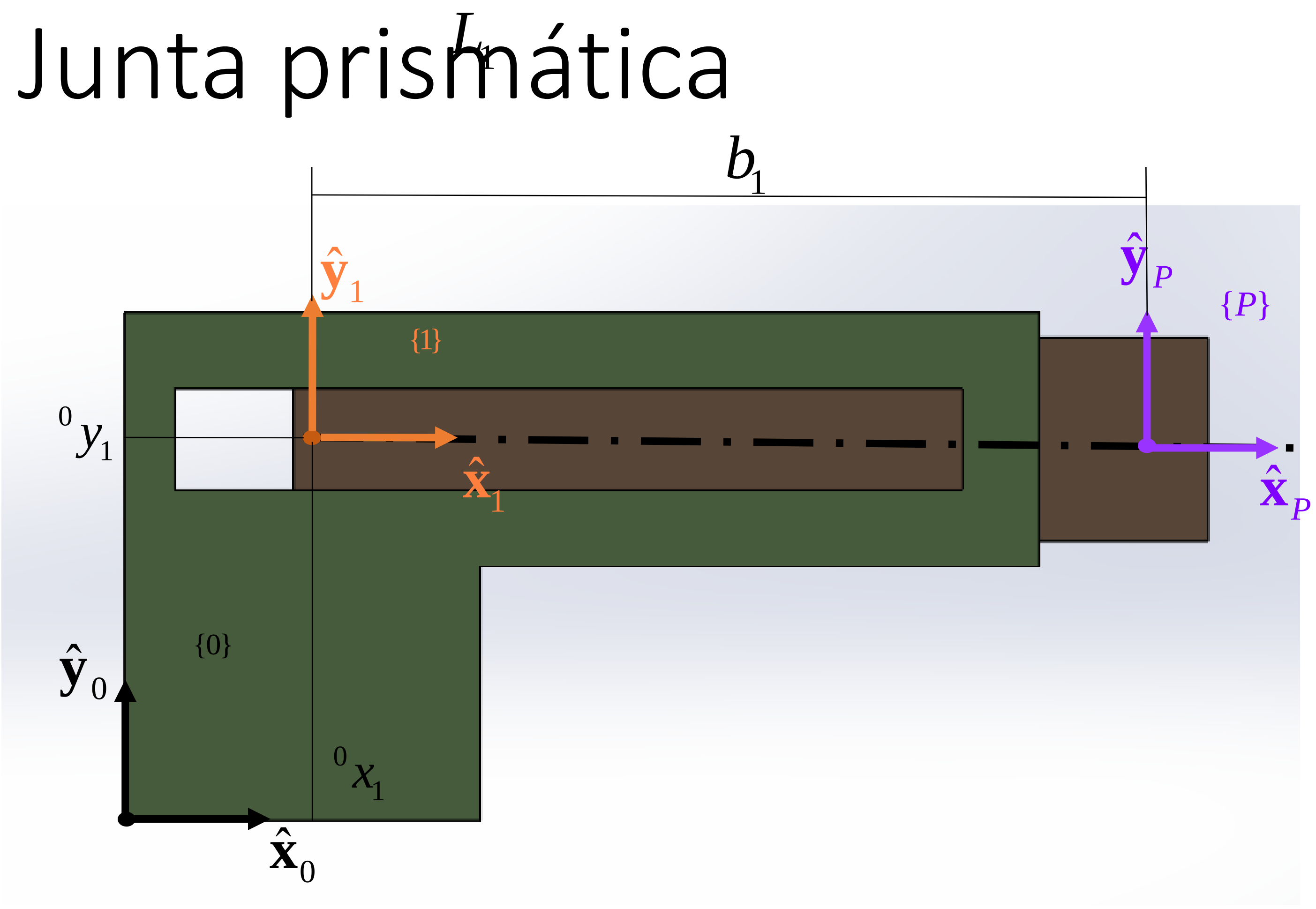
$$
{}^0\ddot{\xi}_P = \frac{\partial}{\partial\, {}^0\theta_1}\, {}^0\dot{\xi}_P\, {}^0\dot{\theta}_1 + \frac{\partial}{\partial\, {}^0\dot{\theta}_1}\, {}^0\dot{\xi}_P\, {}^0\ddot{\theta}_1 =
$$

$$
= \boxed{\begin{pmatrix} -L_1\cos({}^0\theta_1) \\ -L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix}}\, {}^0\dot{\theta}_1^{\,2} + \boxed{\begin{pmatrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \\ 1 \end{pmatrix}}\, {}^0\ddot{\theta}_1
$$

Junta prismática



Junta prismática



Modelo cinemático de la posición

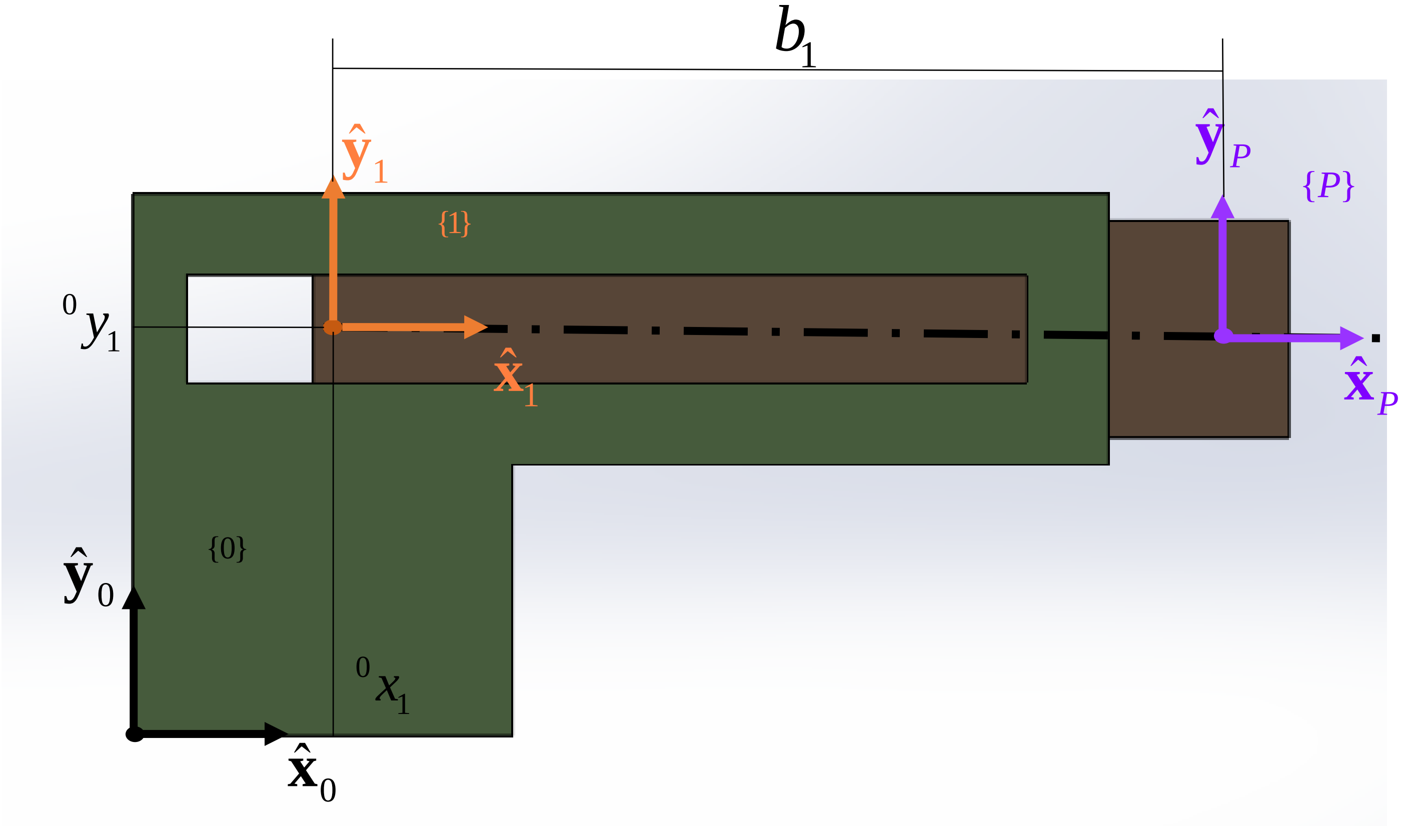
$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1 = \begin{pmatrix} 1 & 0 & 0 & {}^0x_1 \\ 0 & 1 & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la posición

Junta prismática

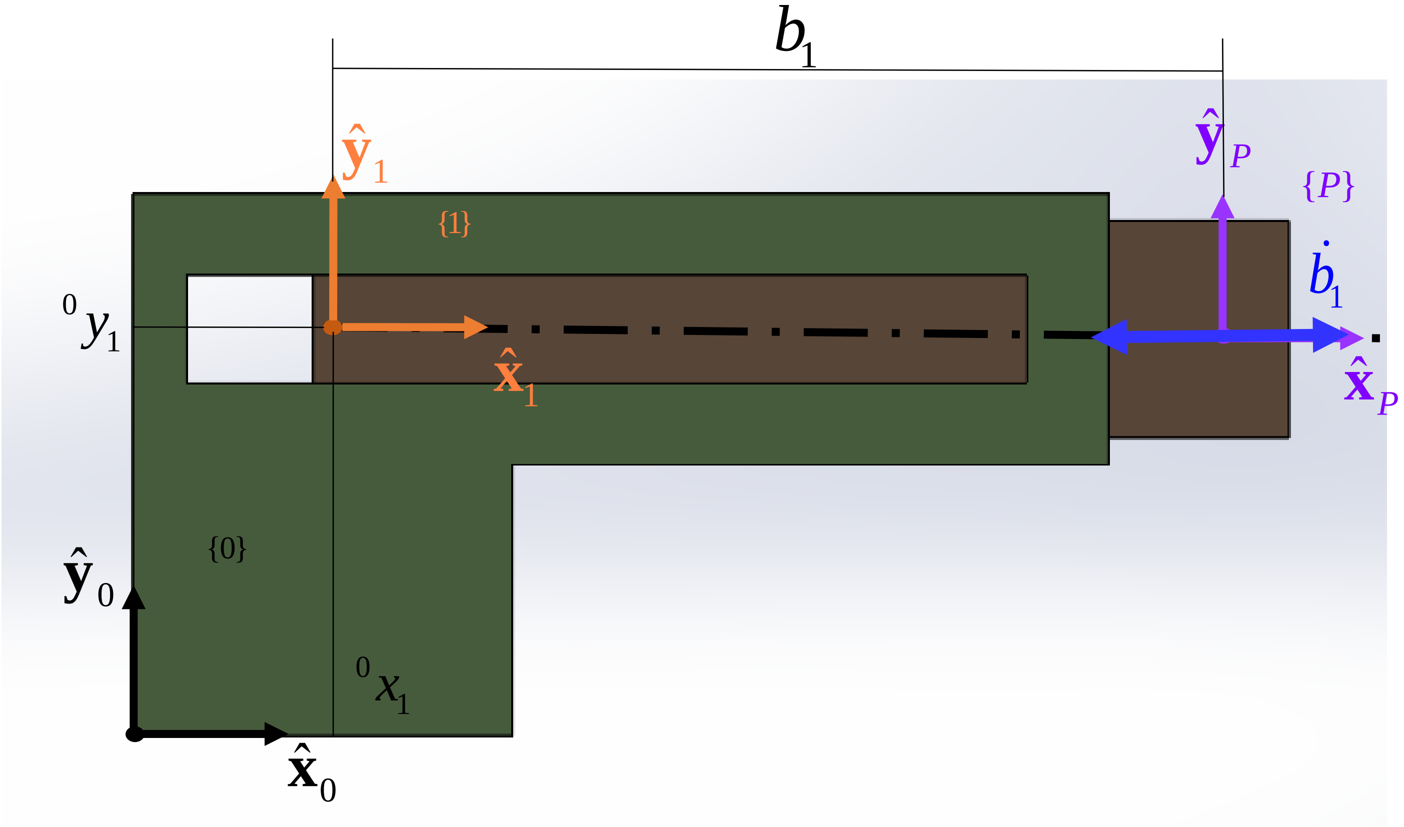


$$
{}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & {}^0x_1 + b_1 \\ 0 & 1 & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Vector de la postura de un eslabón

$$
{}^0\boldsymbol{\xi}_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + b_1 \\ {}^0y_1 \\ 0 \end{pmatrix}
$$

Junta prismática



Vector de la postura de un eslabón

$$^0\xi_P = \begin{pmatrix} ^0\mathbf{p}_P \\ ^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} ^0x_1 + b_1 \\ ^0y_1 \\ 0 \end{pmatrix}$$

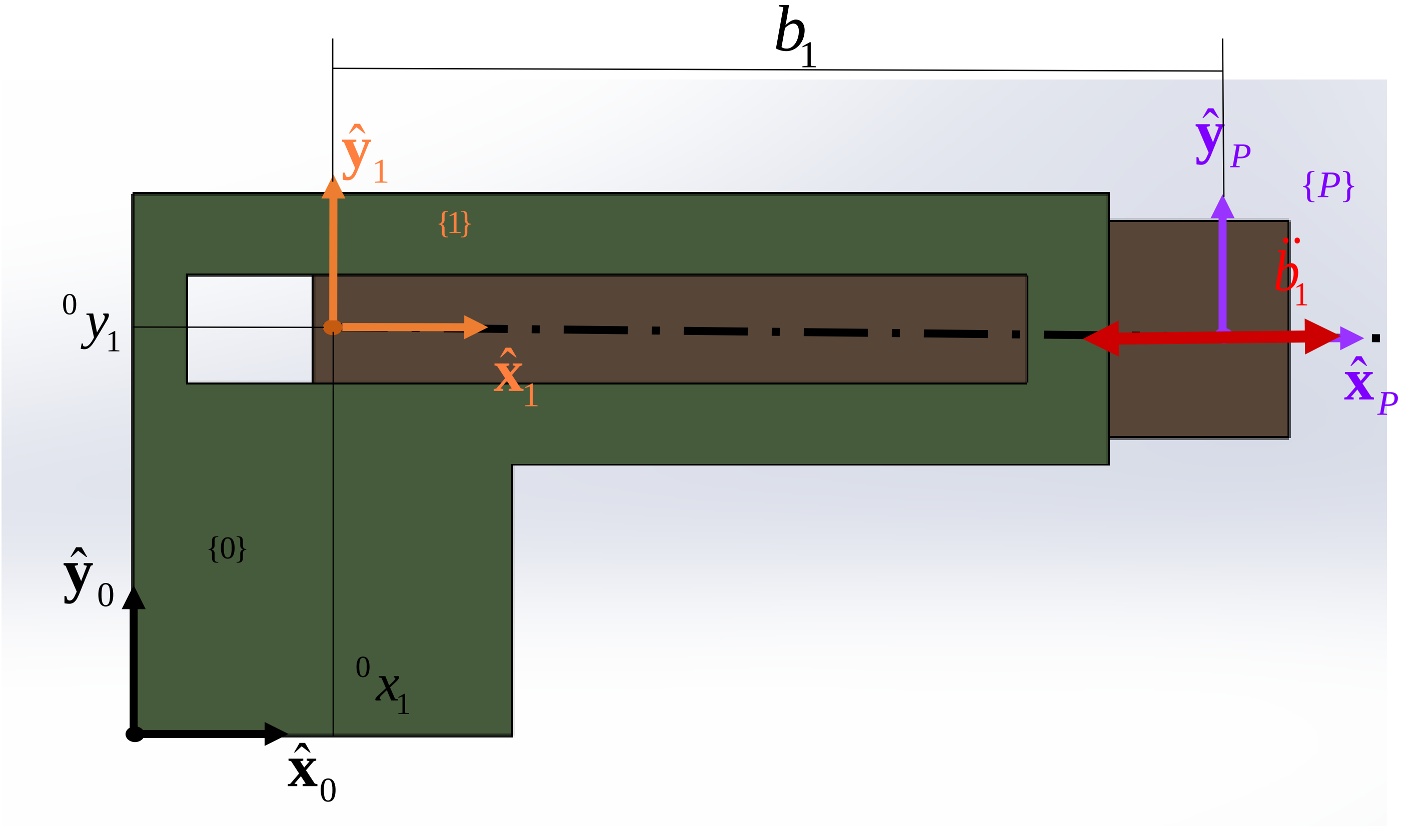
Vector de velocidades del eslabón

$$^0\dot{\xi}_P = \frac{d}{db_1}\,{}^0\xi_P \dot{b}_1 = \begin{pmatrix} \dot{b}_1 \\ 0 \\ 0 \end{pmatrix}$$

Modelo cinemático de la velocidad

aceleración

Junta prismática



Vector de velocidades del eslabón

$$ {}^0\dot{\xi}_P = \frac{d}{db_1}\,{}^0\xi_P \dot{b}_1 = \begin{pmatrix} \dot{b}_1 \\ 0 \\ 0 \end{pmatrix} $$

Vector de aceleraciones del eslabón

$$ {}^0\ddot{\xi}_P = \frac{d}{d\dot{b}_1}\,{}^0\xi_P \ddot{b}_1 = \begin{pmatrix} \ddot{b}_1 \\ 0 \\ 0 \end{pmatrix} $$