Robótica grupo2
Clase 20

Facultad de Ingeniería UNAM

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Planeación de movimientos en el espacio de las juntas

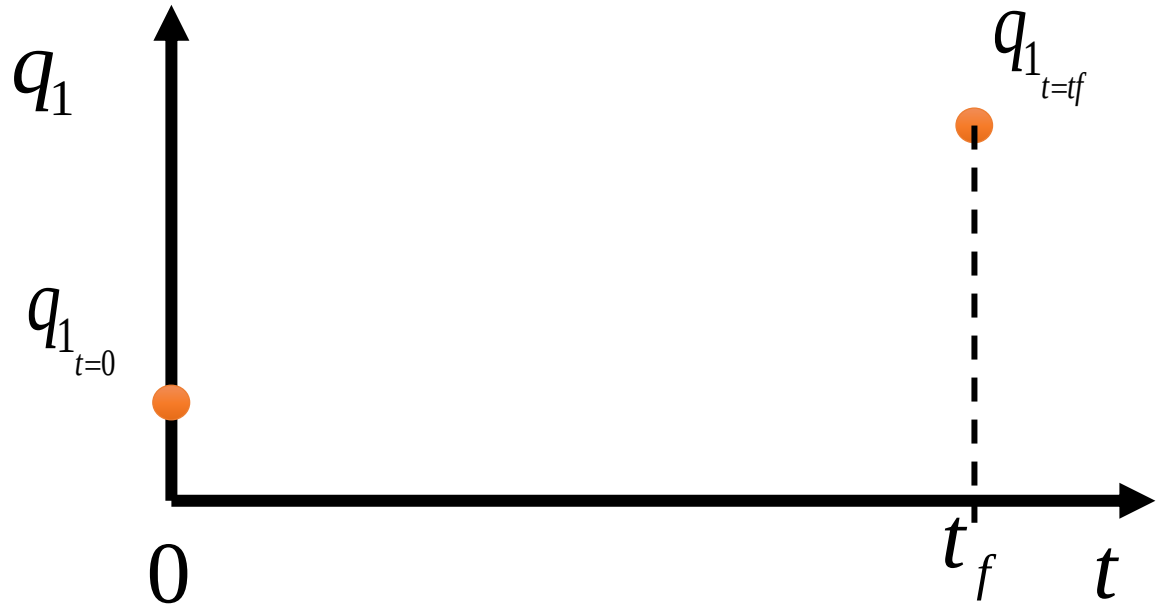
Planeación de movimientos en el espacio de las juntas del robot

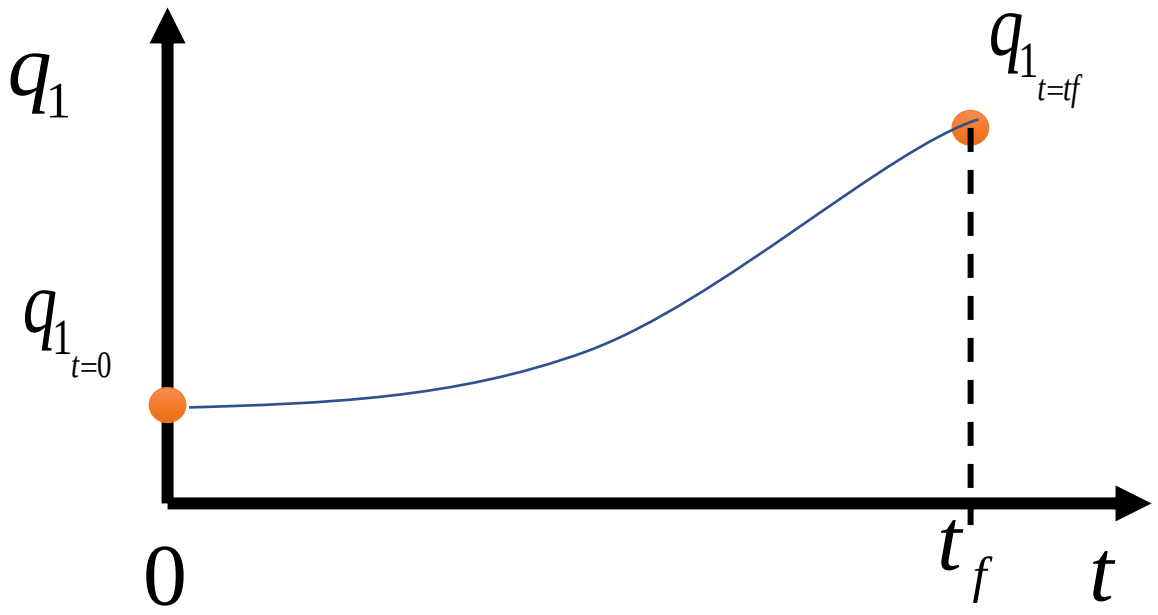
https://www.youtube.com/watch?reload=9&v=t_UAyEIpKks

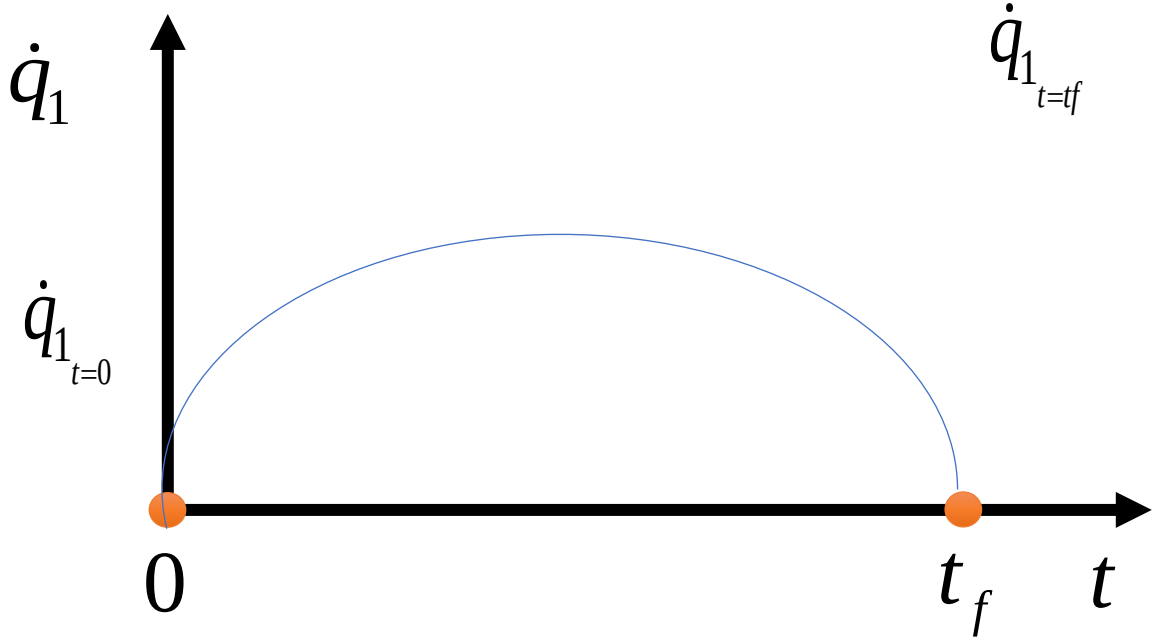
$$\mathbf{q_0} = \begin{pmatrix} q_{1_0} \\ \vdots \\ q_{n_0} \end{pmatrix} \rightarrow \mathbf{q}_1 = \begin{pmatrix} q_{1_1} \\ \vdots \\ q_{n_1} \end{pmatrix} \rightarrow \mathbf{q}_2 = \begin{pmatrix} q_{1_2} \\ \vdots \\ q_{n_2} \end{pmatrix}$$

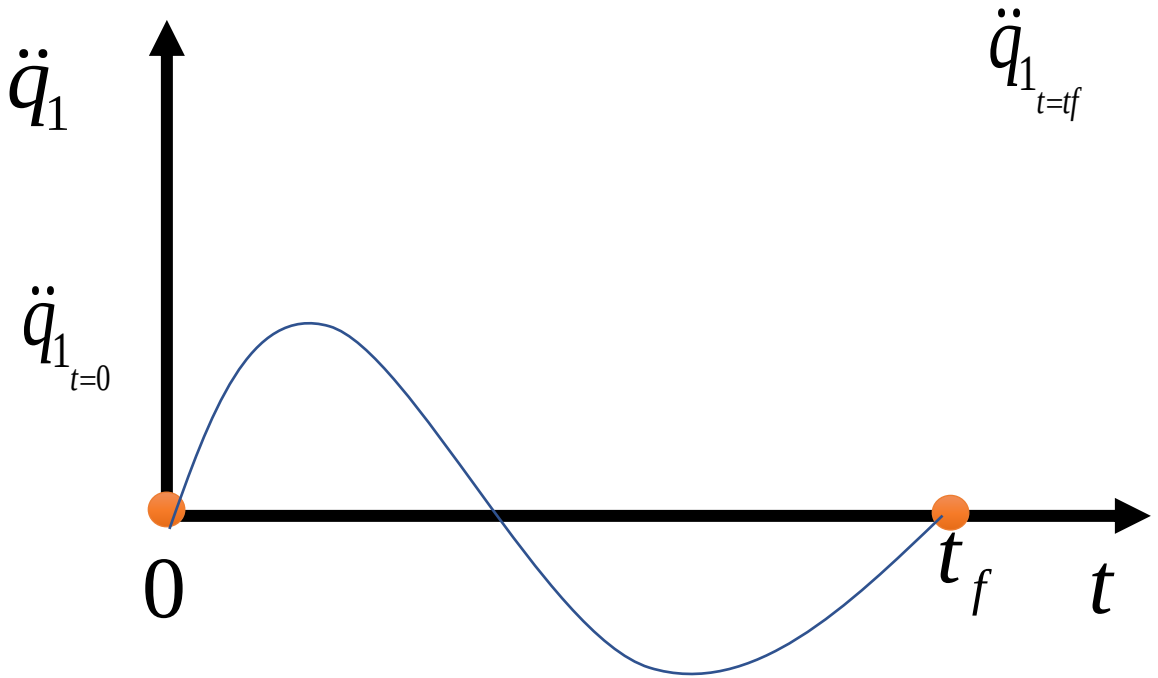
Planeación de movimientos en el espacio de las juntas

$$\mathbf{q_0} = \begin{pmatrix} q_{1_0} \\ \vdots \\ q_{n_0} \end{pmatrix} \Rightarrow \mathbf{q_1} = \begin{pmatrix} q_{1_1} \\ \vdots \\ q_{n_1} \end{pmatrix}$$




q_1
$q_{1_{t=tf}}$
$q_{1_{t=0}}$
0
t_f
t


$\dot{q}_1$
$\dot{q}_{1_{t=tf}}$
$\dot{q}_{1_{t=0}}$
0
t_f
t


$\ddot{q}_1$
$\ddot{q}_{1_{t=tf}}$
$\ddot{q}_{1_{t=0}}$
0
t_f
t

Planeación de movimientos

Planeación de movimientos en el espacio de las juntas del robot

$$\mathbf{q_0} = \begin{pmatrix} q_{1_0} \\ \vdots \\ q_{n_0} \end{pmatrix} \quad \Rightarrow \quad \mathbf{q_1} = \begin{pmatrix} q_{1_1} \\ \vdots \\ q_{n_1} \end{pmatrix}$$

$$\mathbf{q} = \begin{pmatrix} q_{1_0} + \lambda(t) \cdot (q_{1_1} - q_{1_0}) \\ \vdots \\ q_{n_0} + \lambda(t) \cdot (q_{n_1} - q_{n_0}) \end{pmatrix}$$

Planeación de movimientos en el espacio de las juntas

Planeación de movimientos en el espacio de las juntas del robot

https://www.youtube.com/watch?reload=9&v=t_UAyEIpKks

$$\mathbf{q_0} = \begin{pmatrix} q_{1_0} \\ \vdots \\ q_{n_0} \end{pmatrix} \rightarrow \mathbf{q}_1 = \begin{pmatrix} q_{1_1} \\ \vdots \\ q_{n_1} \end{pmatrix} \rightarrow \mathbf{q}_2 = \begin{pmatrix} q_{1_2} \\ \vdots \\ q_{n_2} \end{pmatrix}$$

Planeación de movimientos en el espacio de las juntas

Planeación de movimientos en el espacio de las juntas del robot

$$\mathbf{q_0} = \begin{pmatrix} q_{1_0} \\ \vdots \\ q_{n_0} \end{pmatrix} \quad \Rightarrow \quad \mathbf{q_1} = \begin{pmatrix} q_{1_1} \\ \vdots \\ q_{n_1} \end{pmatrix}$$

$$\mathbf{q} = \begin{pmatrix} q_{1_0} + \lambda(t) \cdot (q_{1_1} - q_{1_0}) \\ \vdots \\ q_{n_0} + \lambda(t) \cdot (q_{n_1} - q_{n_0}) \end{pmatrix}$$

$$\mathbf{q} = \begin{pmatrix} q_{1_0} + \lambda(t) \cdot (q_{1_1} - q_{1_0}) \\ \vdots \\ q_{n_0} + \lambda(t) \cdot (q_{n_1} - q_{n_0}) \end{pmatrix}$$

$\lambda(t)$

$0 \le \lambda(t) \le 1$ $\qquad \lambda(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

$t = 0$	$t = t_f$
$\lambda(0) = 0$	$\lambda(t_f) = 1$
$\dot{\lambda}(0) = 0$	$\dot{\lambda}(t_f) = 0$
$\ddot{\lambda}(0) = 0$	$\ddot{\lambda}(t_f) = 0$

$$t = t_f$$

$$\lambda(t_f) = 1 = a_3 t_f^{\,3} + a_4 t_f^{\,4} + a_5 t_f^{\,5}$$

$$\dot{\lambda}(t_f) = 0 = 3a_3 t_f^{\,2} + 4a_4 t_f^{\,3} + 5a_5 t_f^{\,4}$$

$$\ddot{\lambda}(t_f) = 0 = 6a_3 t_f + 12a_4 t_f^{\,2} + 20a_5 t_f^{\,3}$$

$$\mathbf{Ax} = \mathbf{c} \Rightarrow \mathbf{x} = \mathbf{A}^{-1}\mathbf{c}$$

$$\begin{pmatrix} t_f^{\,3} & t_f^{\,4} & t_f^{\,5} \\ 3t_f^{\,2} & 4t_f^{\,3} & 5t_f^{\,4} \\ 6t_f & 12t_f^{\,2} & 20t_f^{\,3} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} t_f^{\,3} & t_f^{\,4} & t_f^{\,5} \\ 3t_f^{\,2} & 4t_f^{\,3} & 5t_f^{\,4} \\ 6t_f & 12t_f^{\,2} & 20t_f^{\,3} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

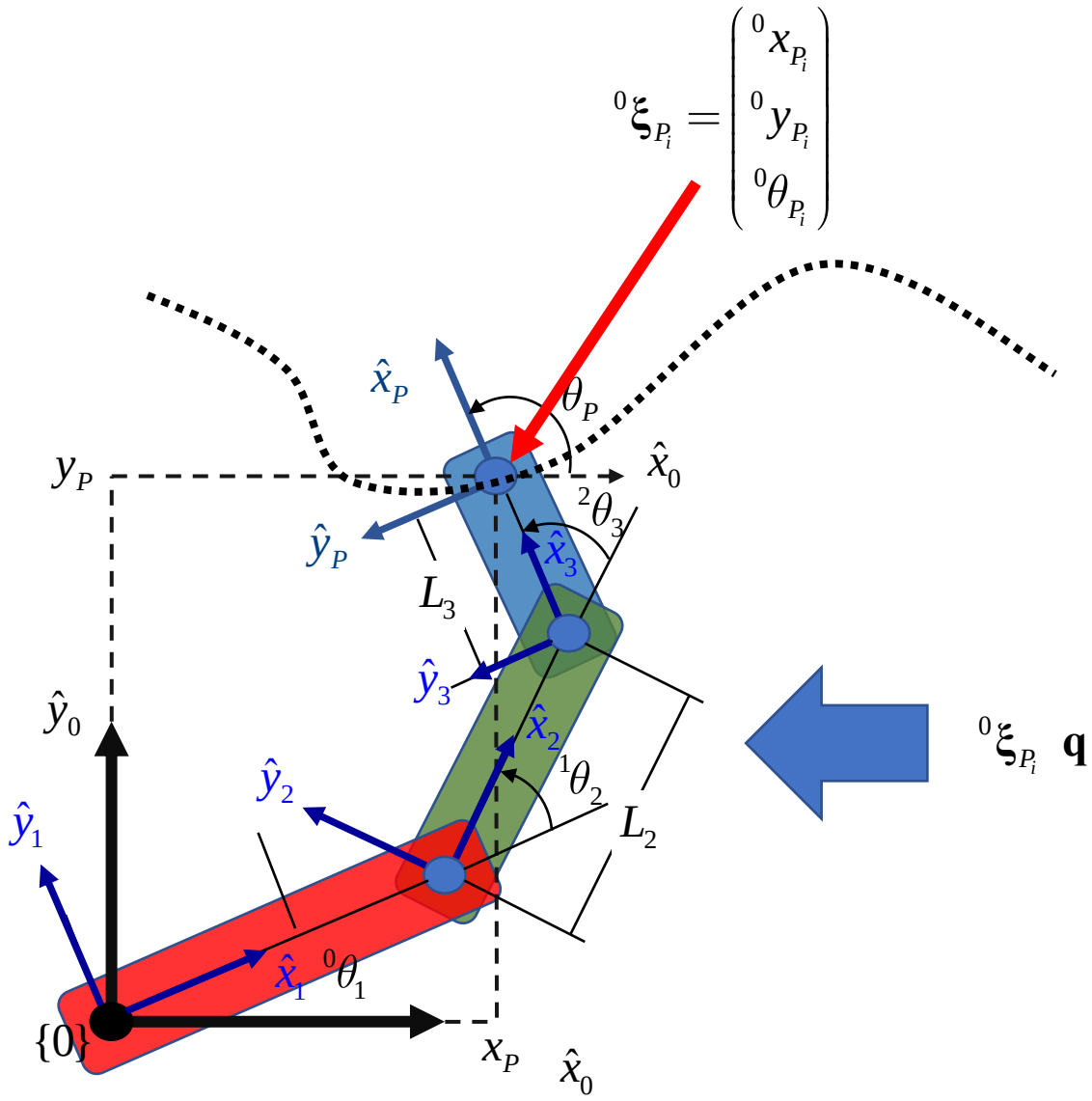
$$t = t_f$$

$$\lambda(t) = \frac{10}{t_f^{\,3}} t^3 - \frac{15}{t_f^{\,4}} t^4 + \frac{6}{t_f^{\,5}} t^5$$

$$\dot{\lambda}(t) = \frac{30}{t_f^{\,3}} t^2 - \frac{60}{t_f^{\,4}} t^3 + \frac{30}{t_f^{\,5}} t^4$$

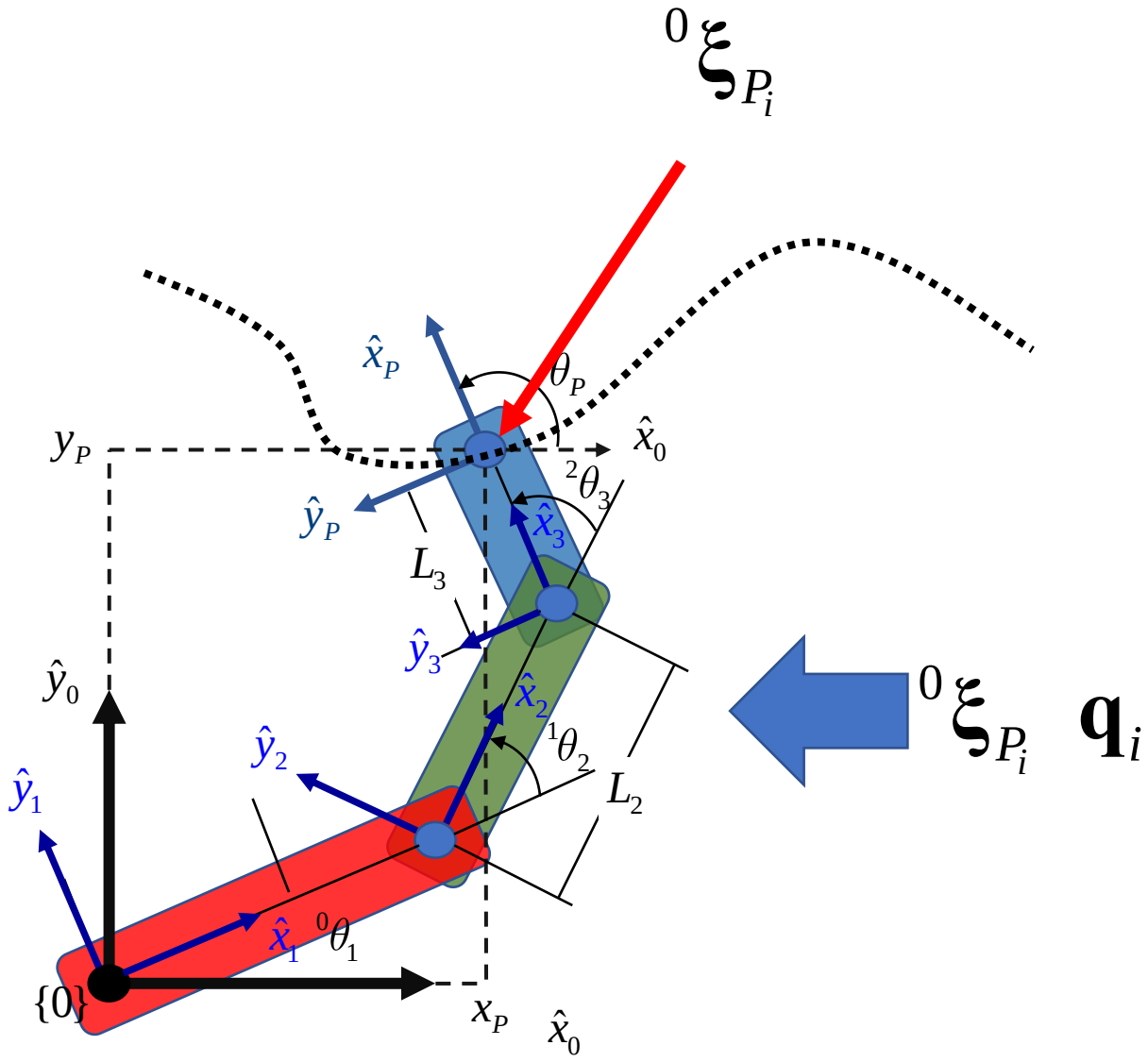
$$\ddot{\lambda}(t) = \frac{60}{t_f^{\,3}} t - \frac{180}{t_f^{\,4}} t^2 + \frac{120}{t_f^{\,5}} t^3$$

Planeación de movimientos en el espacio de trabajo



$$^0\xi_{P_i}\ \mathbf{q} = \begin{pmatrix} L_1\cos(^0\theta_{1_i}) + L_2\cos(^0\theta_{1_i} + ^1\theta_{2_i}) + L_3\cos(^0\theta_{1_i} + ^1\theta_{2_i} + ^2\theta_{3_i}) \\ L_1\sin(^0\theta_{1_i}) + L_2\sin(^0\theta_{1_i} + ^1\theta_{2_i}) + L_3\sin(^0\theta_{1_i} + ^1\theta_{2_i} + ^2\theta_{3_i}) \\ ^0\theta_{1_i} + ^1\theta_{2_i} + ^2\theta_{3_i} \end{pmatrix}$$

Planeación de movimientos en el espacio de trabajo



$$ {}^0\xi_{P_i} = {}^0\xi_{P_i}\ \mathbf{q}_i $$

$$ \mathbf{F} = {}^0\xi_{P_i} - {}^0\xi_{P_i}\ \mathbf{q}_i = \mathbf{0} $$

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$$ \mathbf{F} = {}^0\xi_{P_i}\ \mathbf{q}_i - {}^0\xi_{P_i} = \mathbf{0} $$

Planeación de movimientos en el espacio de trabajo

$$\mathbf{F} = {}^{0}\boldsymbol{\xi}_{P_i} - {}^{0}\boldsymbol{\xi}_{P_i}\ \mathbf{q}_i = \begin{pmatrix} {}^{0}x_{P_i} - L_1\cos({}^{0}\theta_{1_i}) - L_2\cos({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i}) - L_3\cos({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i} + {}^{2}\theta_{3_i}) \\ {}^{0}y_{P_i} - L_1\sin({}^{0}\theta_{1_i}) - L_2\sin({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i}) - L_3\sin({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i} + {}^{2}\theta_{3_i}) \\ {}^{0}\theta_{P_i} - {}^{0}\theta_{1_i} - {}^{1}\theta_{2_i} - {}^{2}\theta_{3_i} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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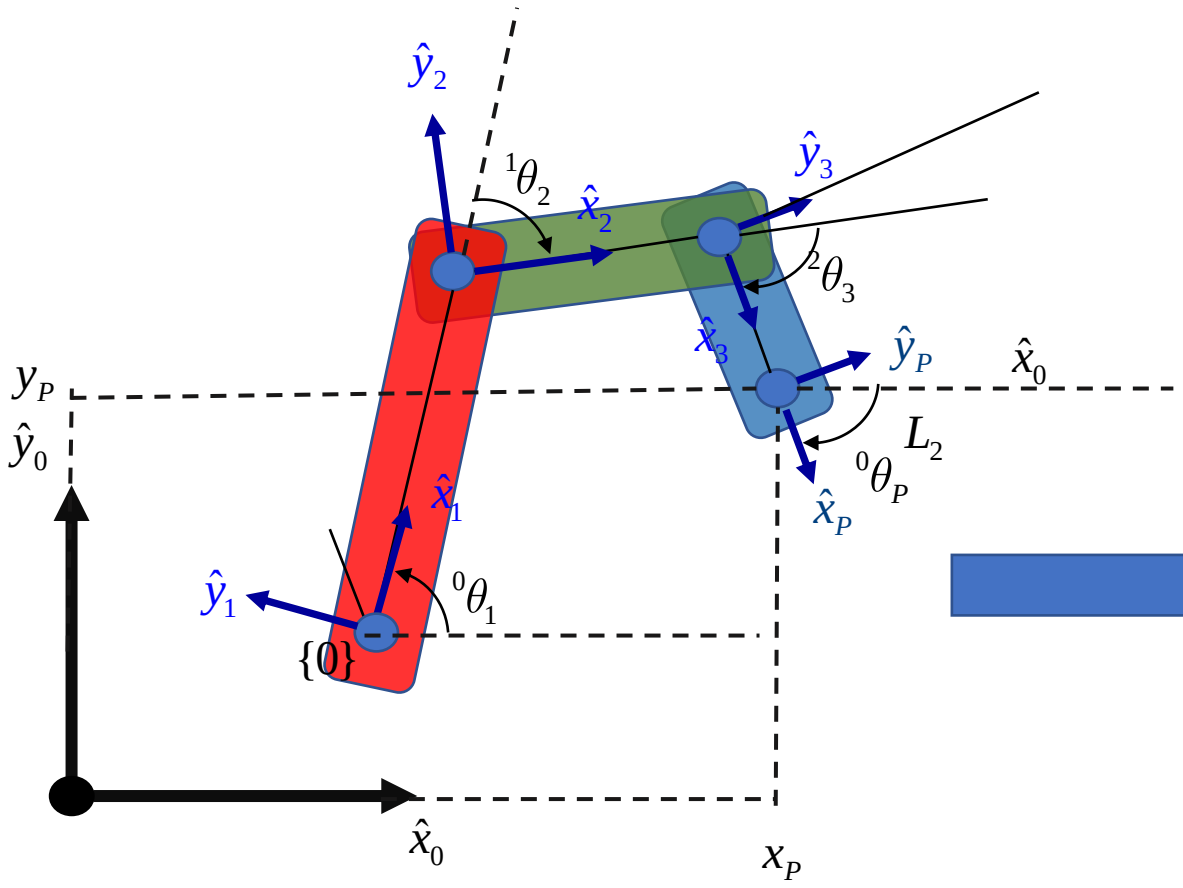
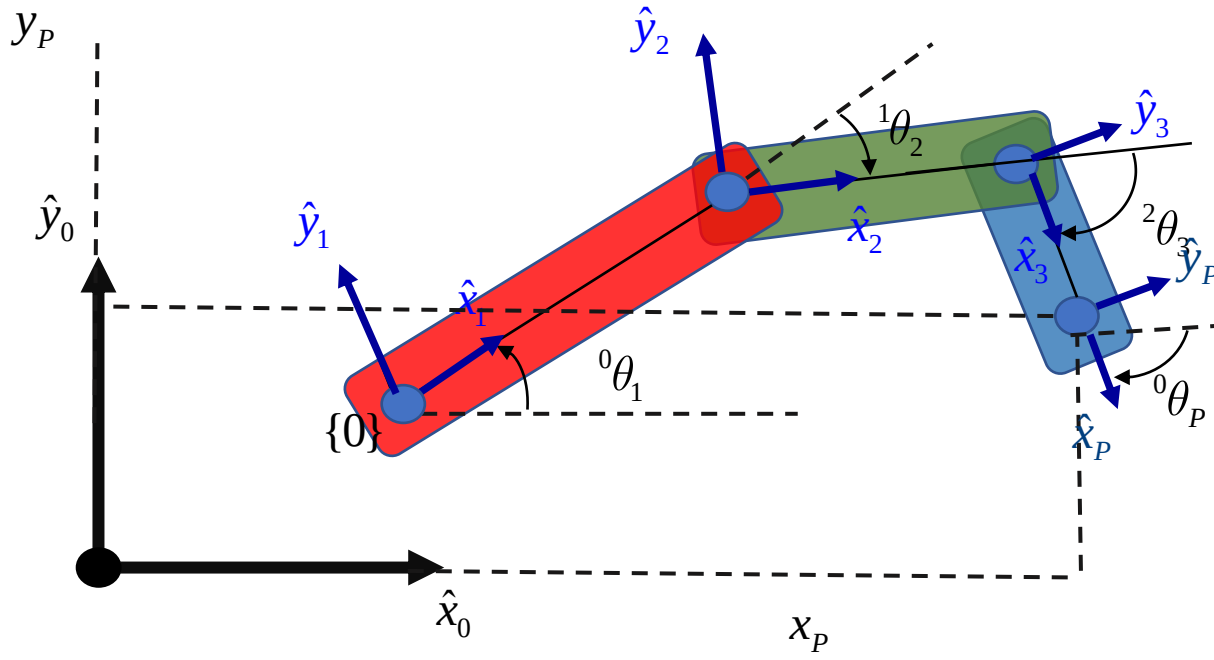
$$\mathbf{F} = {}^{0}\boldsymbol{\xi}_{P_i}\ \mathbf{q}_i\ - {}^{0}\boldsymbol{\xi}_{P_i} = \begin{pmatrix} L_1\cos({}^{0}\theta_{1_i}) + L_2\cos({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i}) + L_3\cos({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i} + {}^{2}\theta_{3_i}) - {}^{0}x_{P_i} \\ L_1\sin({}^{0}\theta_{1_i}) + L_2\sin({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i}) + L_3\sin({}^{0}\theta_{1_i} + {}^{1}\theta_{2_i} + {}^{2}\theta_{3_i}) - {}^{0}y_{P_i} \\ {}^{0}\theta_{1_i} + {}^{1}\theta_{2_i} + {}^{2}\theta_{3_i} - {}^{0}\theta_{P_i} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Planteamiento de la solución en el espacio de trabajo





Modelo cinemático de la postura

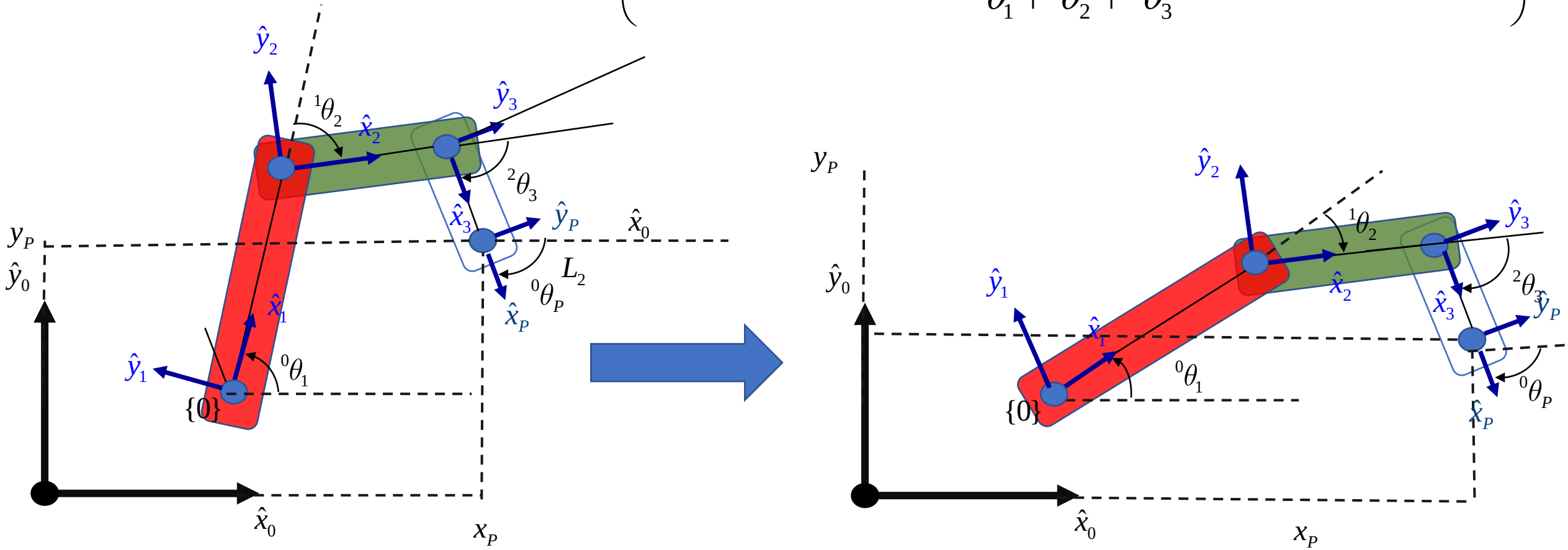
$$ {}^{0}\boldsymbol{\xi}_P = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P(q) = \begin{pmatrix} {}^{0}\mathbf{p}_P \\ {}^{0}\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3 \end{pmatrix} $$

$$ \mathbf{F} = {}^{0}\boldsymbol{\xi}_P - {}^{0}\boldsymbol{\xi}_P(q) = \mathbf{0} = \begin{pmatrix} {}^{0}x_P - {}^{0}x_1 - L_1\cos({}^{0}\theta_1) - L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_P - {}^{0}y_1 - L_1\sin({}^{0}\theta_1) - L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_P - {}^{0}\theta_1 - {}^{1}\theta_2 - {}^{2}\theta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $$

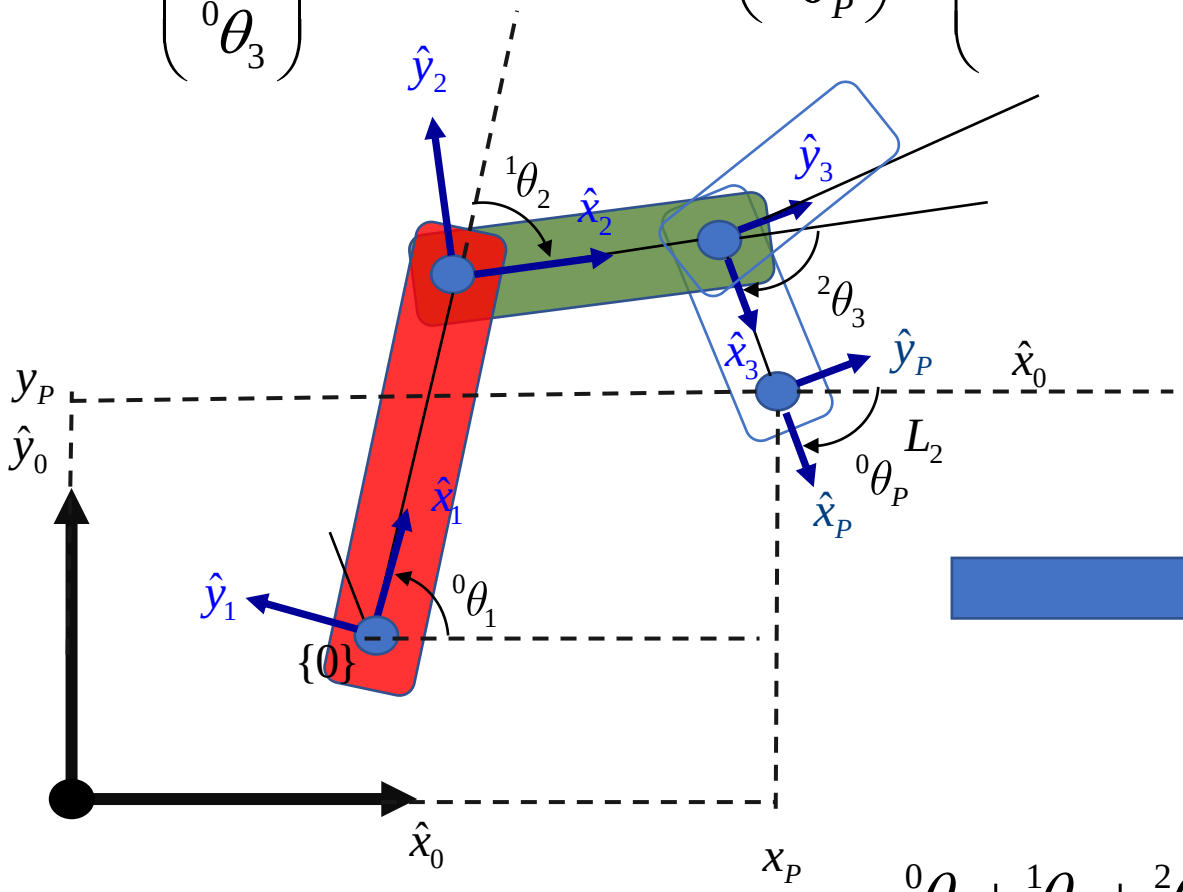
Planteamiento de la solución en el espacio de trabajo

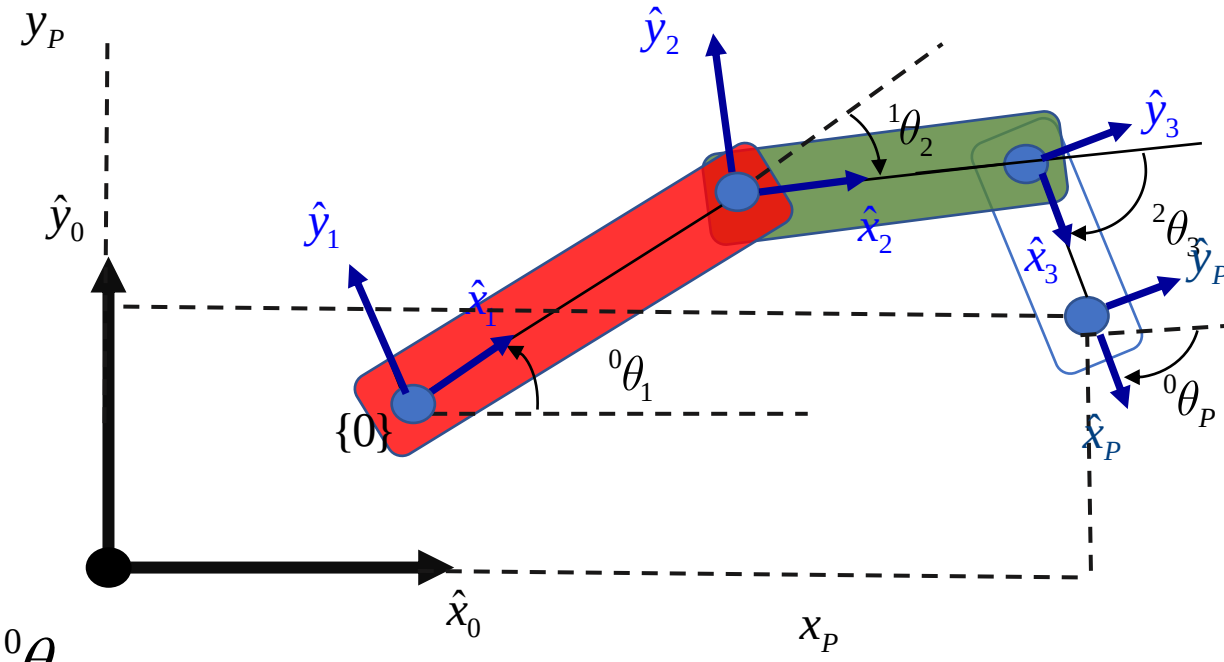
$$
{}^0\xi_P = \begin{pmatrix} {}^0x_P \\ {}^0y_P \\ {}^0\theta_P \end{pmatrix}
\qquad
{}^0\xi_P(q) = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) + L_2\cos({}^0\theta_1 + {}^1\theta_2) + L_3\cos({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) \\ {}^0y_1 + L_1\sin({}^0\theta_1) + L_2\sin({}^0\theta_1 + {}^1\theta_2) + L_3\sin({}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3) \\ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 \end{pmatrix}
$$



Planteamiento del modelo dinámico

$$ {}^0\xi_3 = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} \qquad {}^0\xi_P(q) = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) + L_2\cos({}^0\theta_1 + {}^1\theta_2) \\ {}^0y_1 + L_1\sin({}^0\theta_1) + L_2\sin({}^0\theta_1 + {}^1\theta_2) \\ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 \end{pmatrix} = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} $$



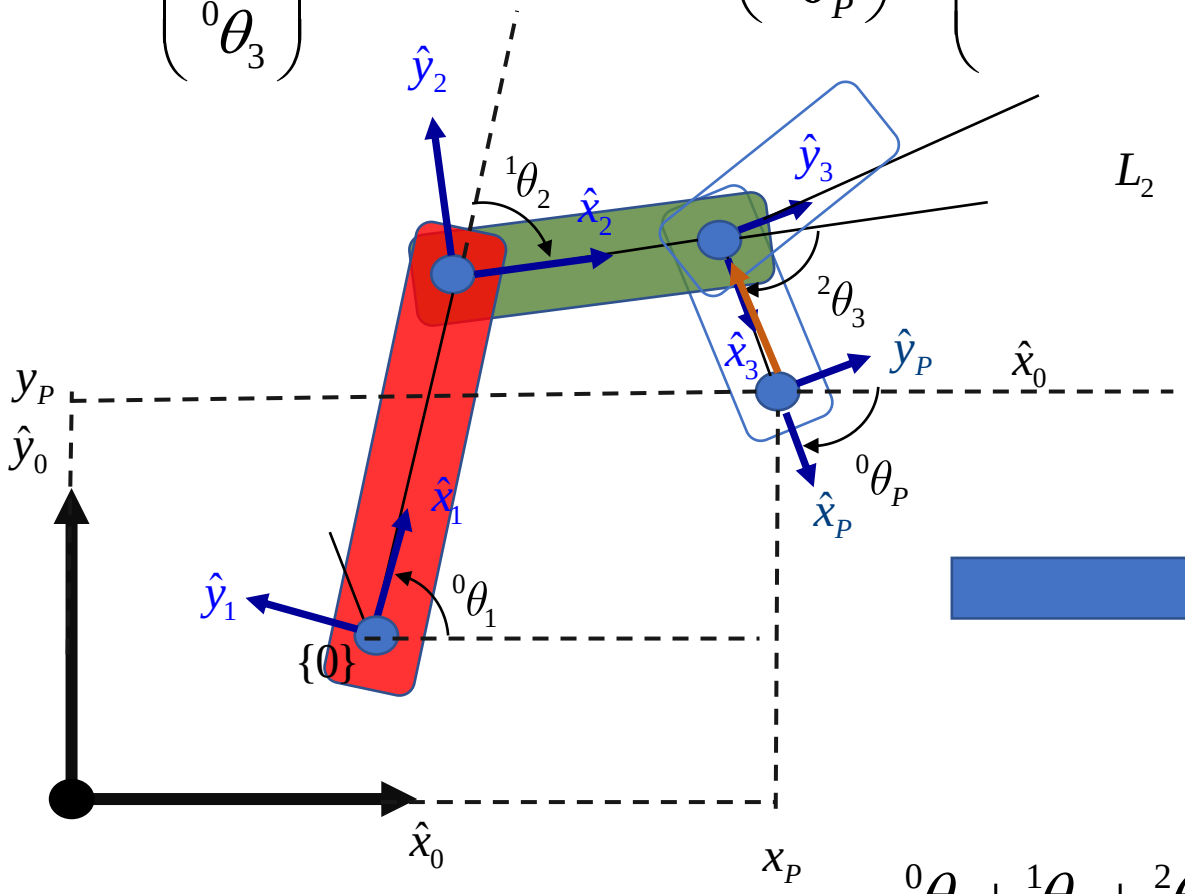


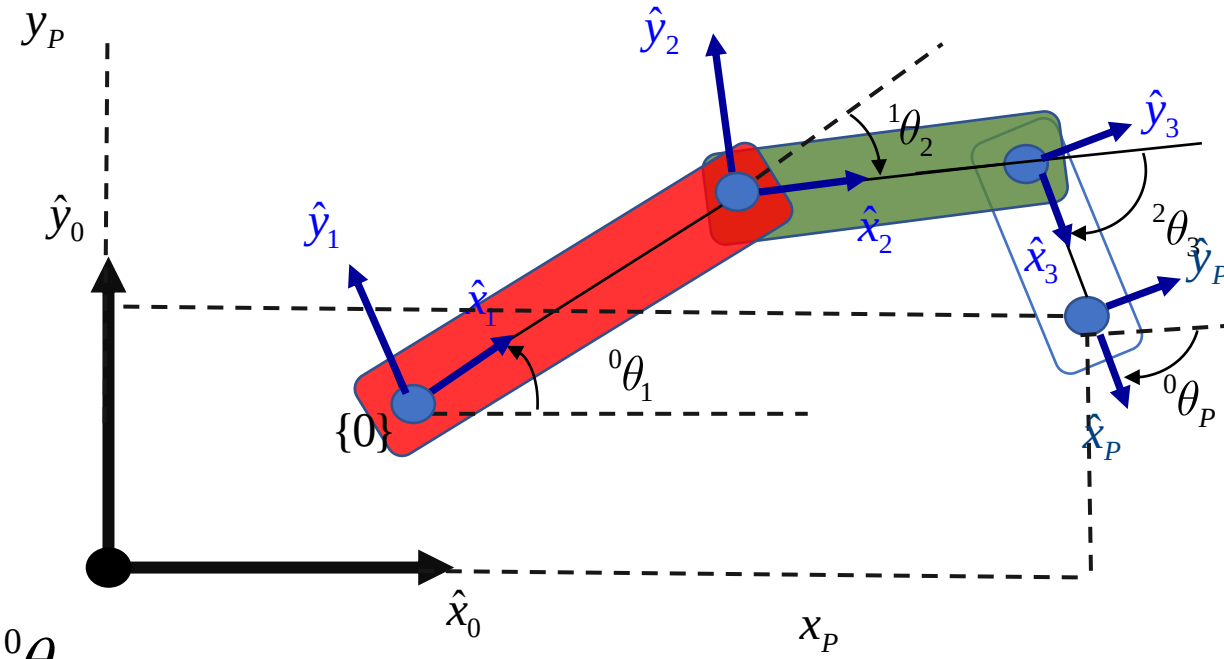
$$ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 = {}^0\theta_P $$

$$ {}^2\theta_3 = {}^0\theta_P - {}^0\theta_1 - {}^1\theta_2 $$

Planteamiento del modelo dinámico

$$ {}^0\xi_3 = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} \qquad {}^0\xi_P(q) = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) + L_2\cos({}^0\theta_1 + {}^1\theta_2) \\ {}^0y_1 + L_1\sin({}^0\theta_1) + L_2\sin({}^0\theta_1 + {}^1\theta_2) \\ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 \end{pmatrix} = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} $$



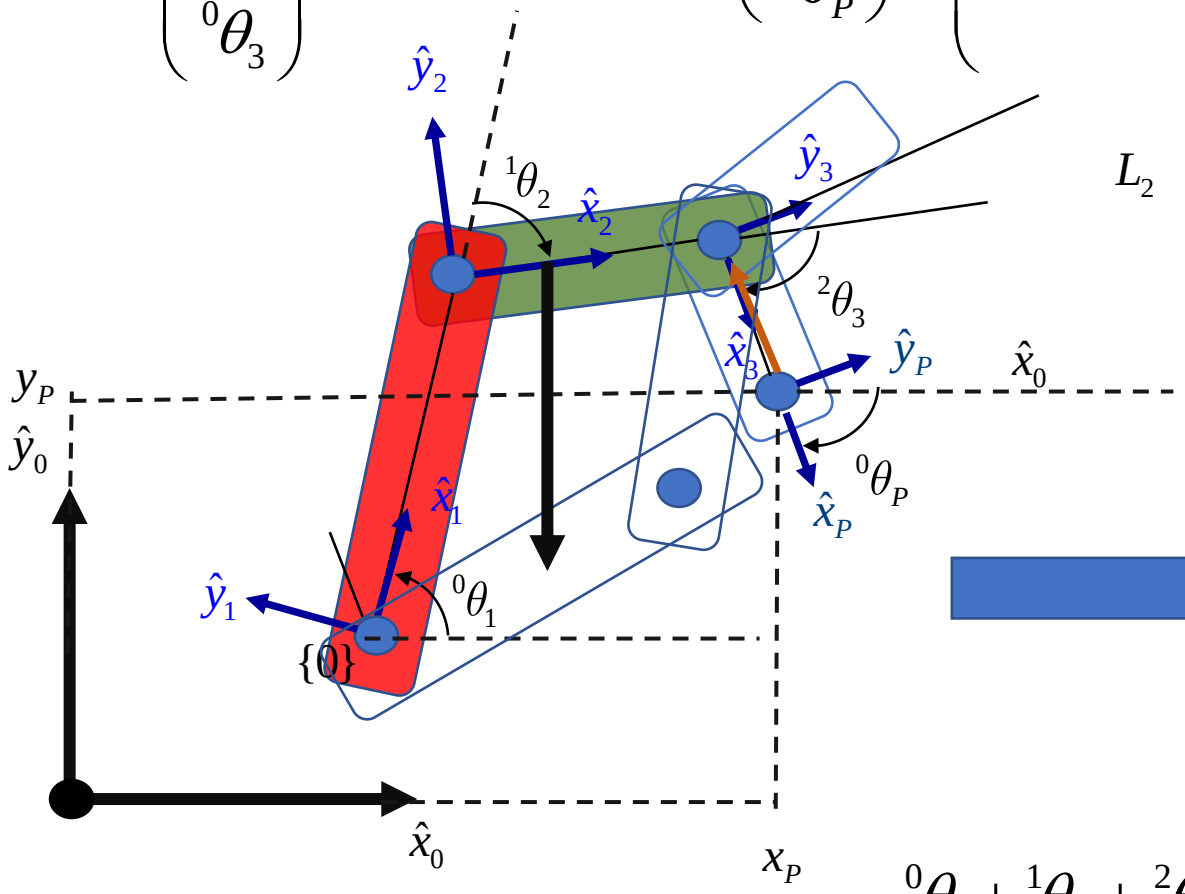


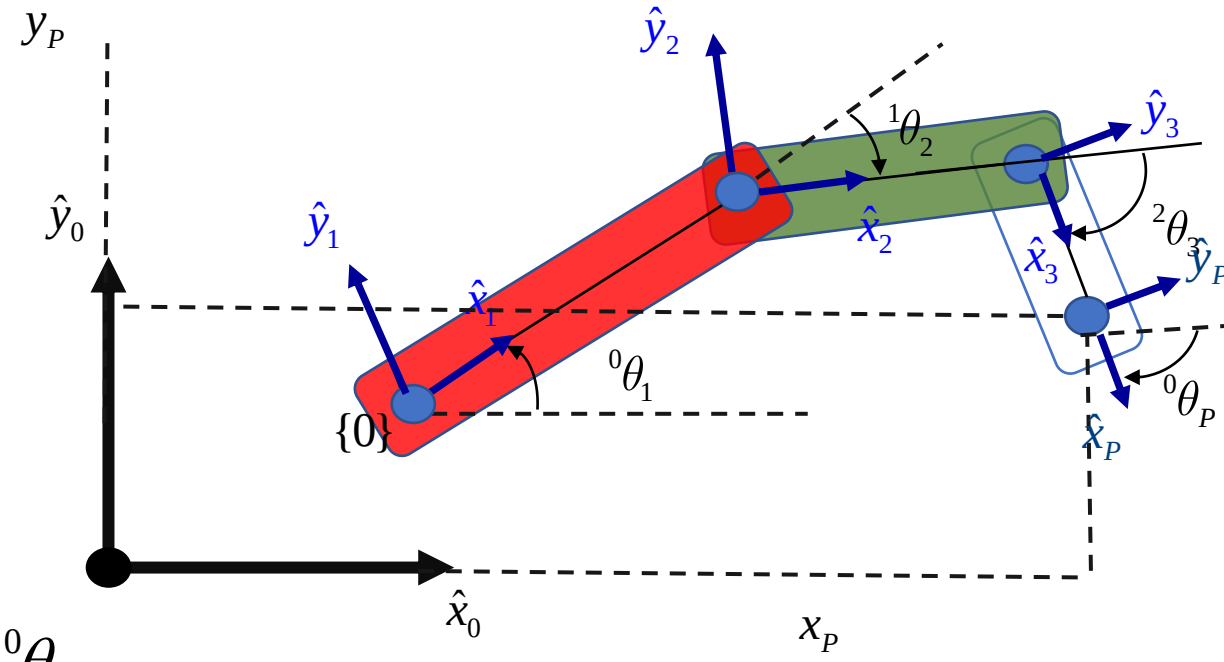
$$ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 = {}^0\theta_P $$

$$ {}^2\theta_3 = {}^0\theta_P - {}^0\theta_1 - {}^1\theta_2 $$

Planteamiento del modelo dinámico

$$ {}^0\xi_3 = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} \qquad {}^0\xi_P(q) = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) + L_2\cos({}^0\theta_1 + {}^1\theta_2) \\ {}^0y_1 + L_1\sin({}^0\theta_1) + L_2\sin({}^0\theta_1 + {}^1\theta_2) \\ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 \end{pmatrix} = \begin{pmatrix} {}^0x_3 \\ {}^0y_3 \\ {}^0\theta_3 \end{pmatrix} $$

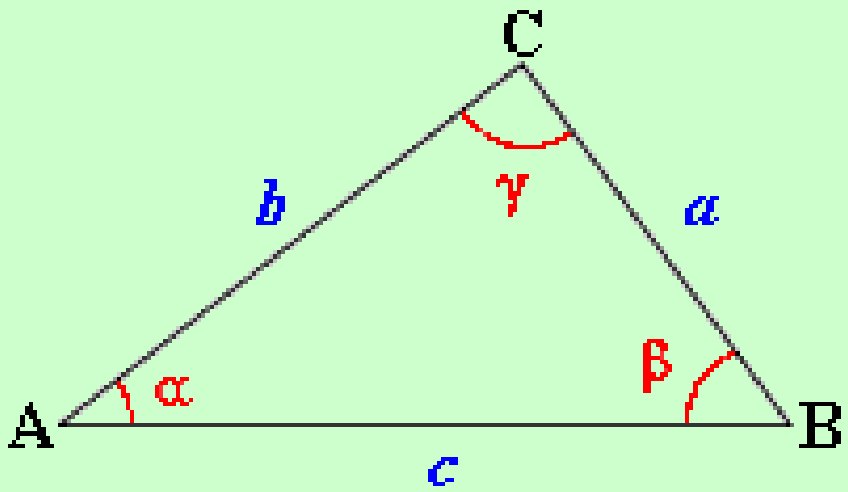




$$ {}^0\theta_1 + {}^1\theta_2 + {}^2\theta_3 = {}^0\theta_P $$

$$ {}^2\theta_3 = {}^0\theta_P - {}^0\theta_1 - {}^1\theta_2 $$

Ley de Cosenos



$$a^2 = b^2 + c^2 - 2bc\,sen\,\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ac\cos\gamma$$