Robótica grupo2
Clase 7

Facultad de Ingeniería UNAM

M.I. Erik Peña Medina

Derechos reservados

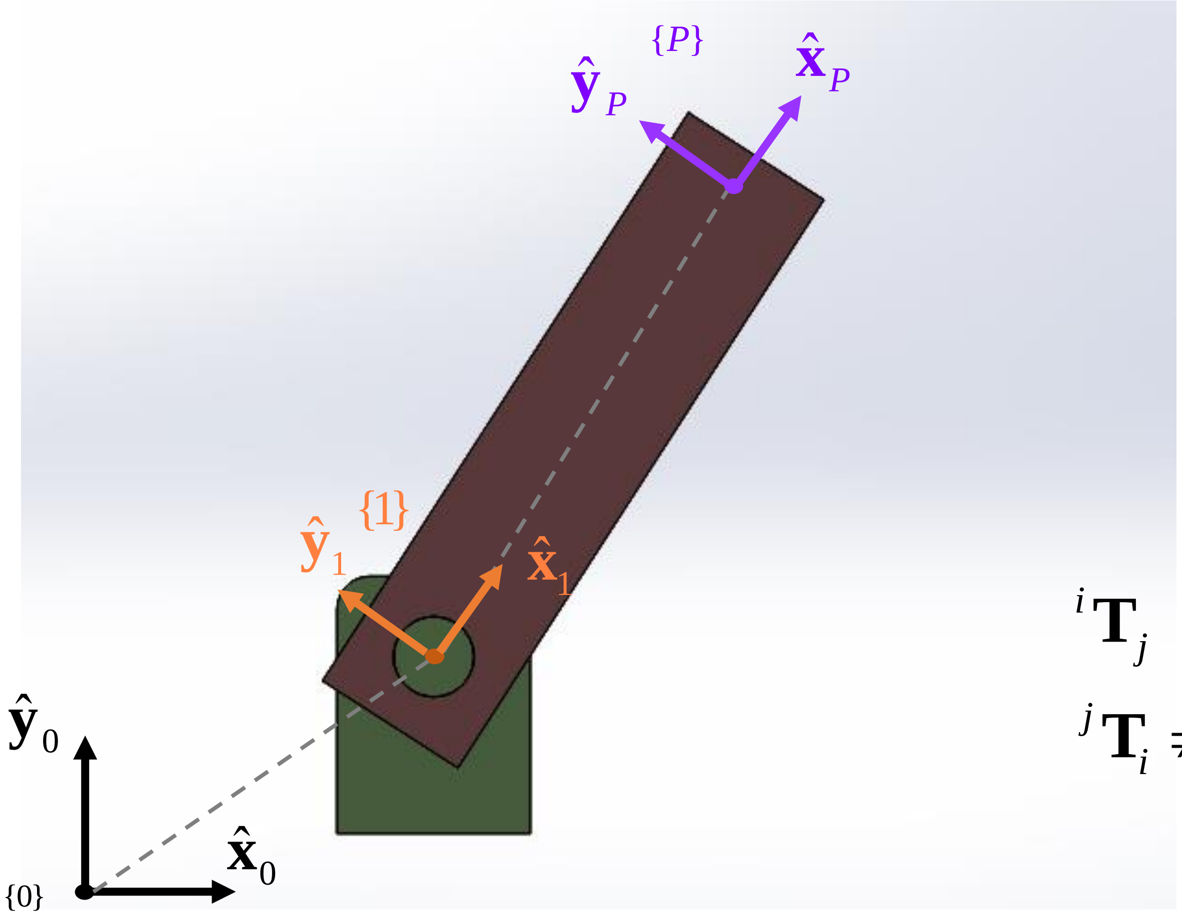
Conceptos básicos/Elemento base

- Resumen de conceptos

- Elemento base (eslabón)
  - Planteamiento de su modelado cinemático.
    - Modelo cinemático de la posición.
    - Modelo cinemático de las velocidades.
    - Modelos cinemático de sus aceleraciones.

Modelo cinemático de la posición

Junta rotacional



$$^{i}\mathbf{T}_j$$

$$^{j}\mathbf{T}_i \neq {}^{i}\mathbf{T}_j^{-1}$$

$$^{i}\mathbf{T}_j = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} =$$

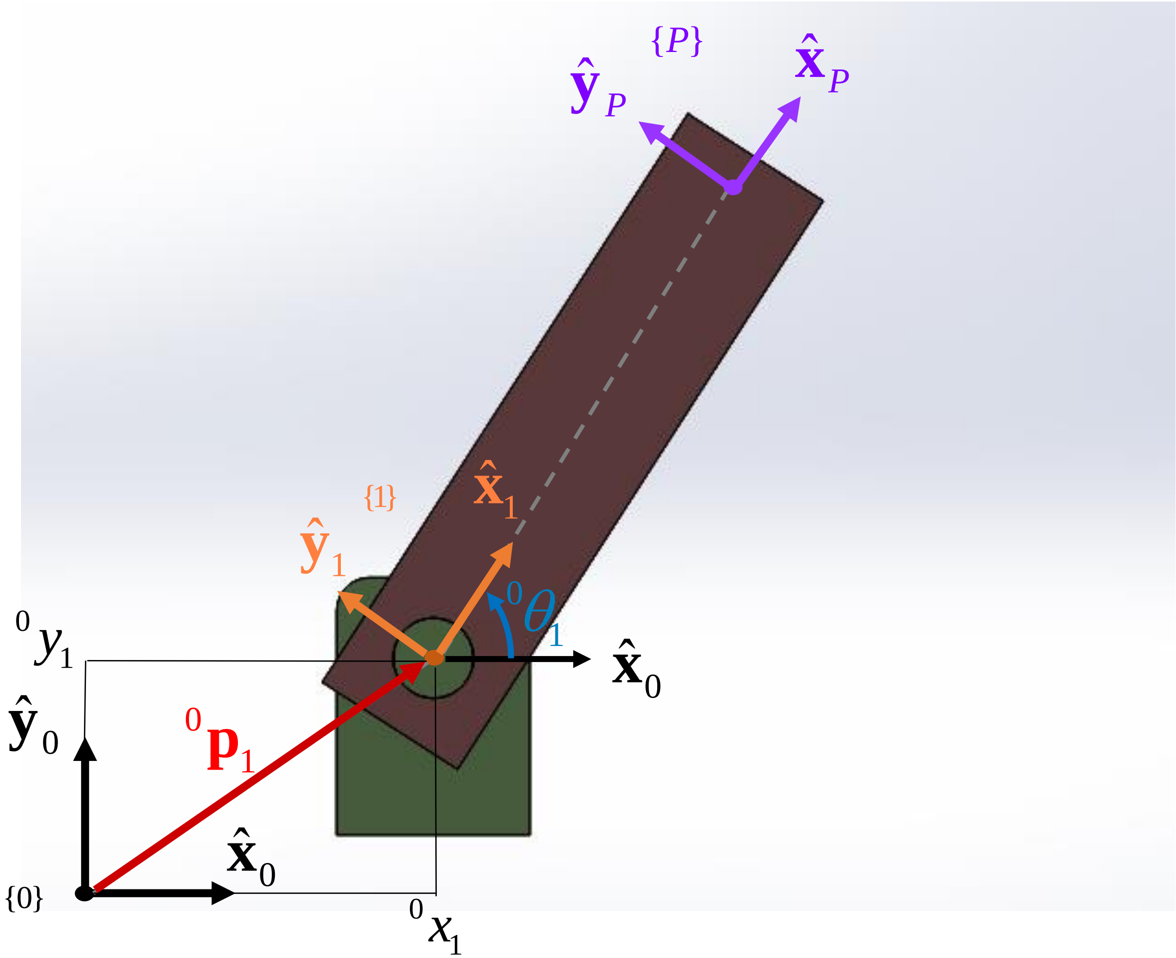
$$^{i}\mathbf{R}_i = \mathbf{R}_z({}^{i}\theta_j)\mathbf{R}_y(0)\mathbf{R}_x(0) =$$

$$= \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & 0 \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$^{i}\mathbf{p}_j = \begin{pmatrix} {}^{i}x_j \\ {}^{i}y_j \\ 0 \end{pmatrix}$$

Modelo cinemático de la posición

Junta rotacional

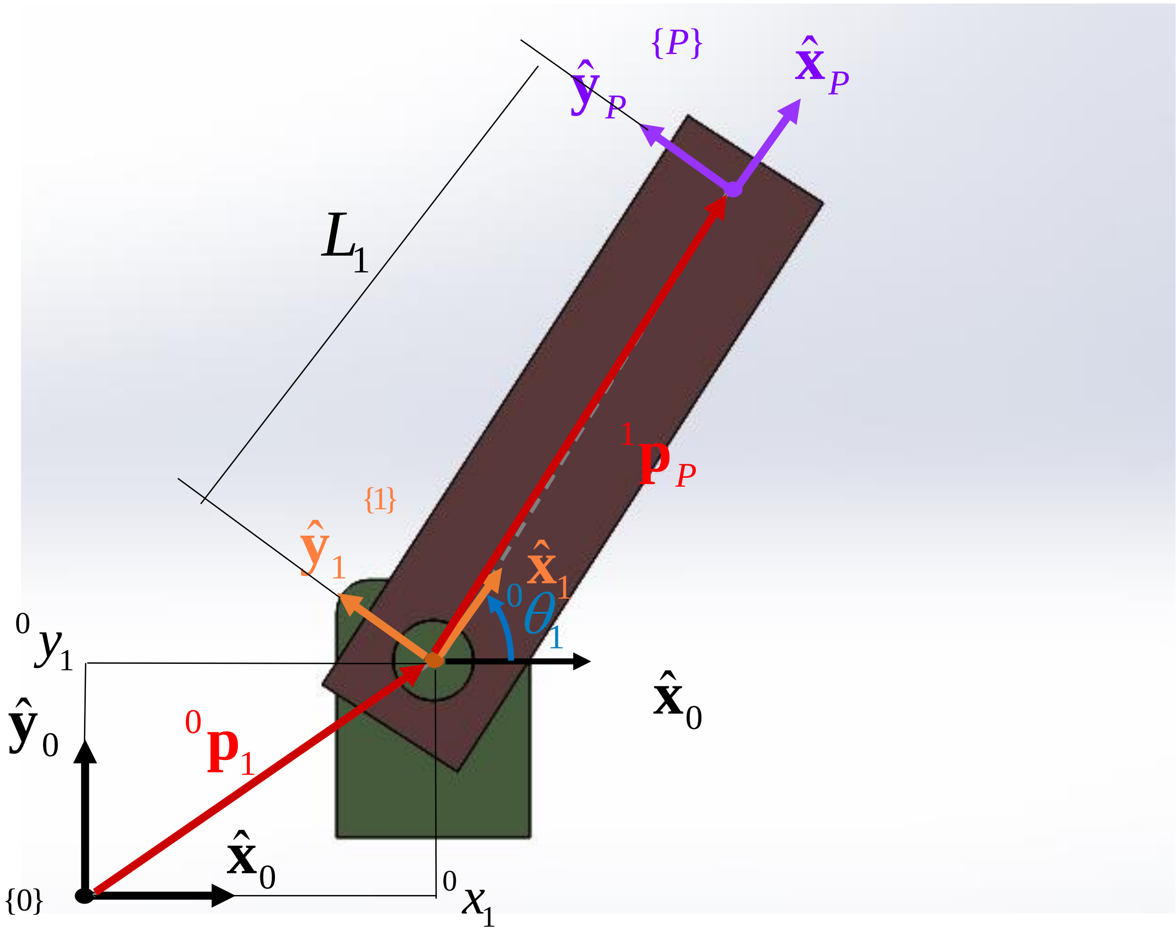


$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1 = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la posición

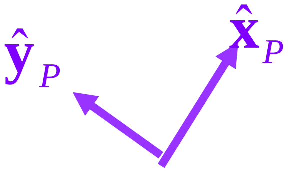
Junta rotacional

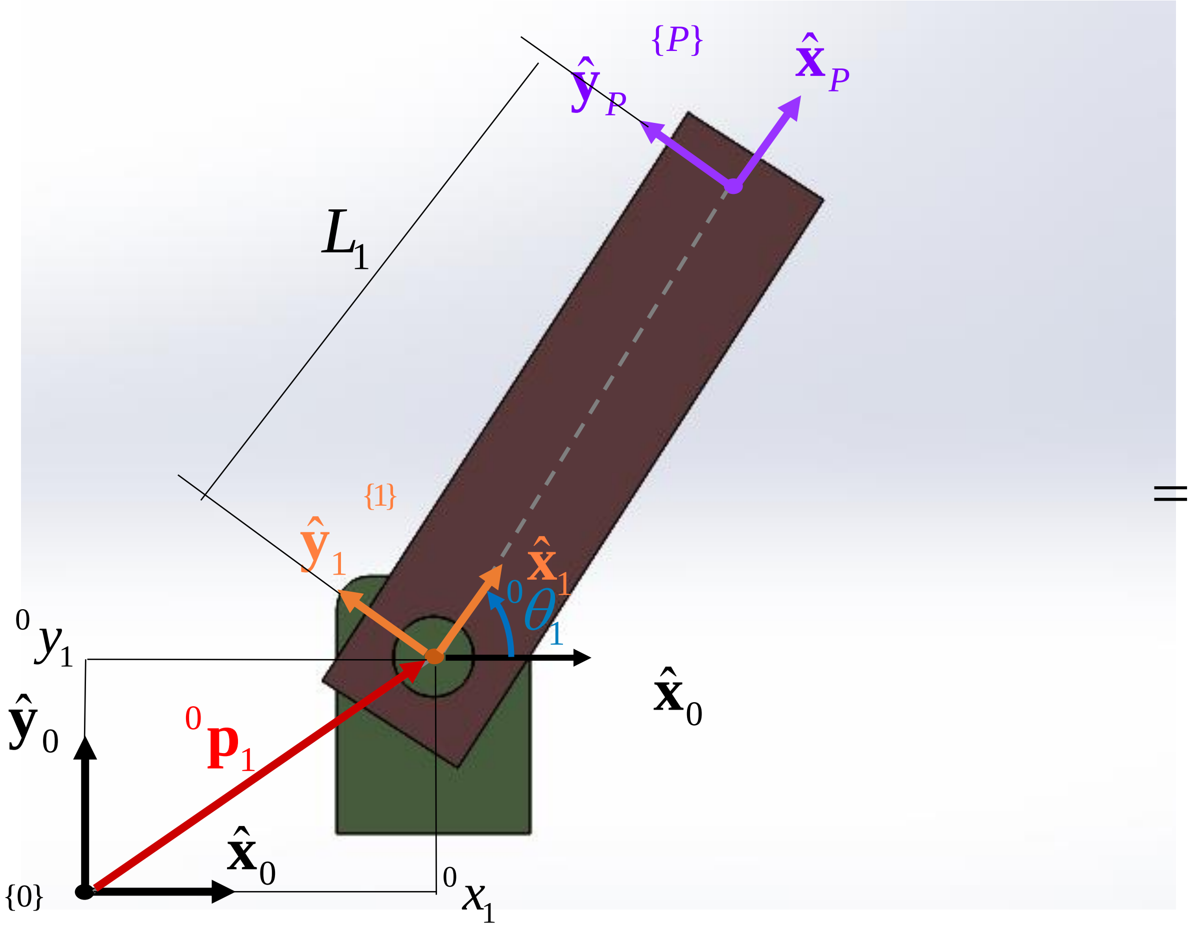


$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Junta rotacional



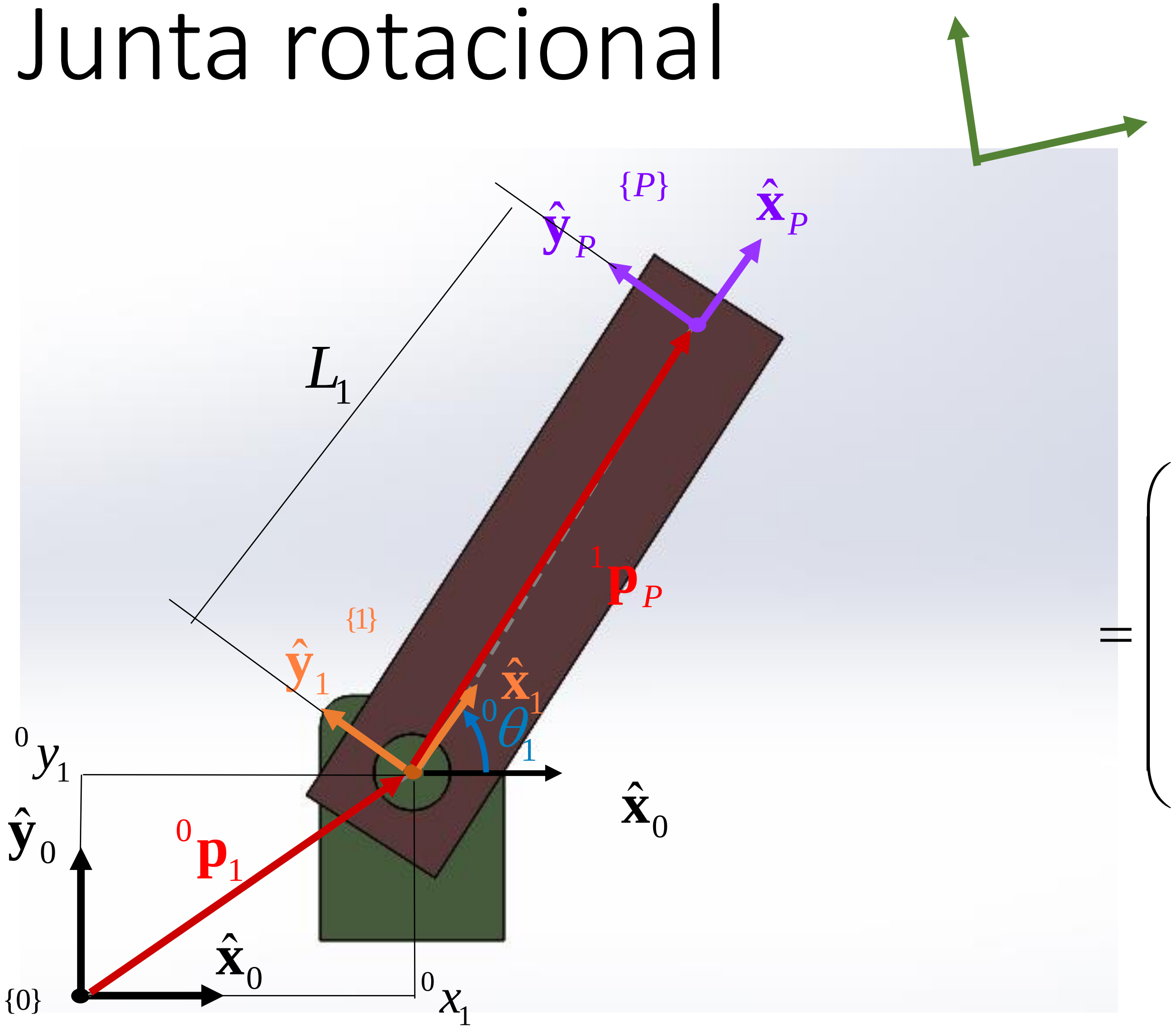


$$
{}^{0}\mathbf{T}_P = {}^{0}\mathbf{T}_1 \, {}^{1}\mathbf{T}_P =
$$

$$
= \begin{pmatrix}
\cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\
\sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Modelo cinemático de la posición

Junta rotacional

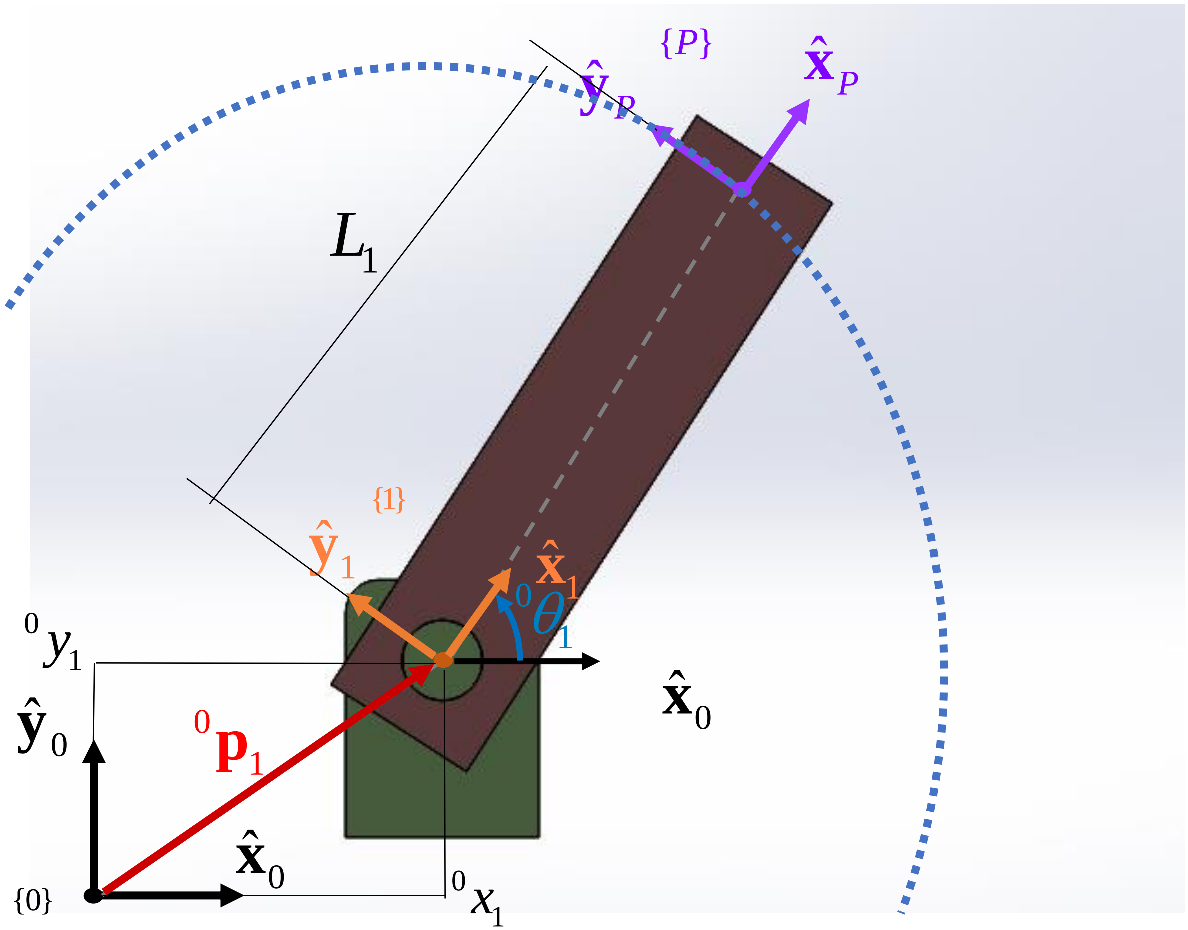


$${}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P =$$

$$= \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Junta rotacional

Modelo cinemático de la posición



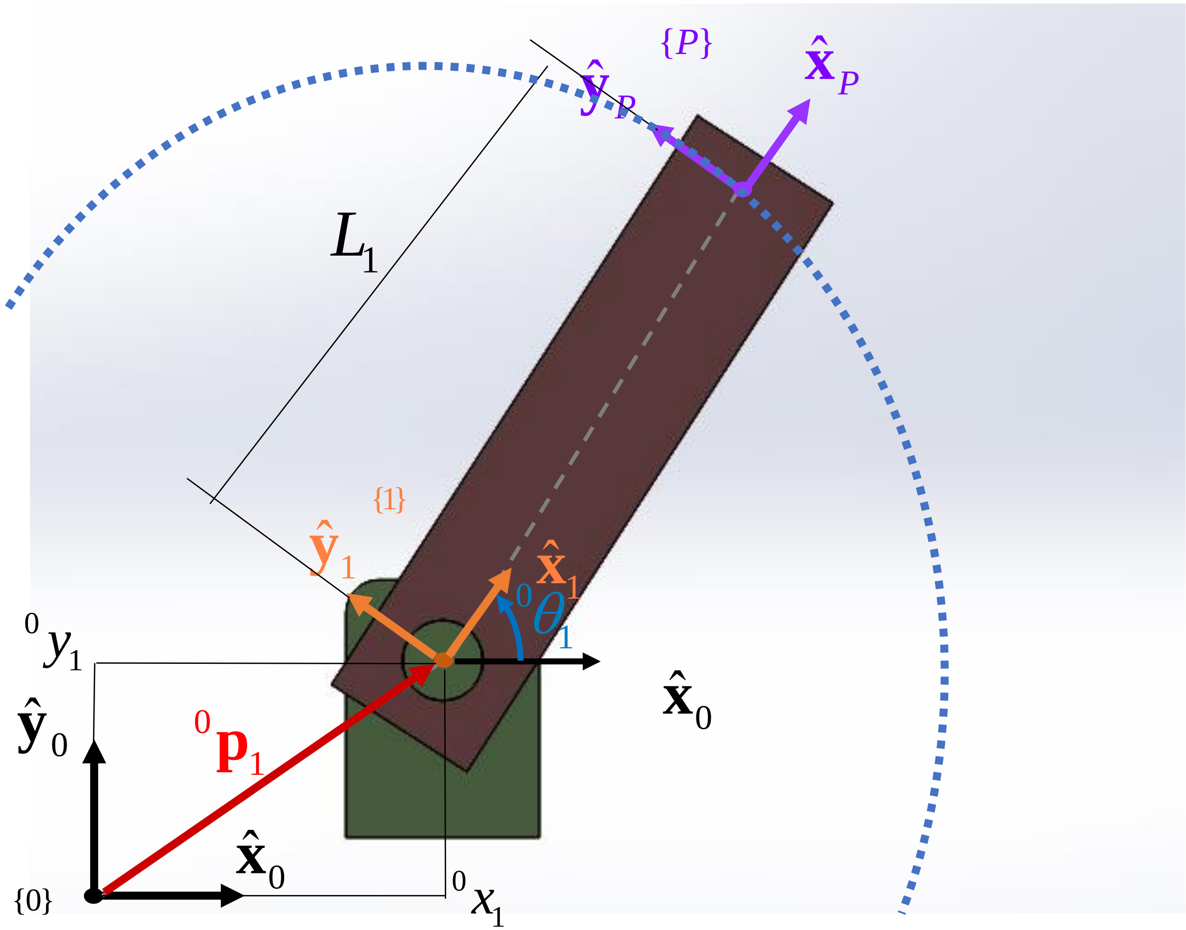
$${}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P =$$

$$= \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 + L_1\cos({}^0\theta_1) \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1\right)$$

Modelo cinemático de la posición

Junta rotacional



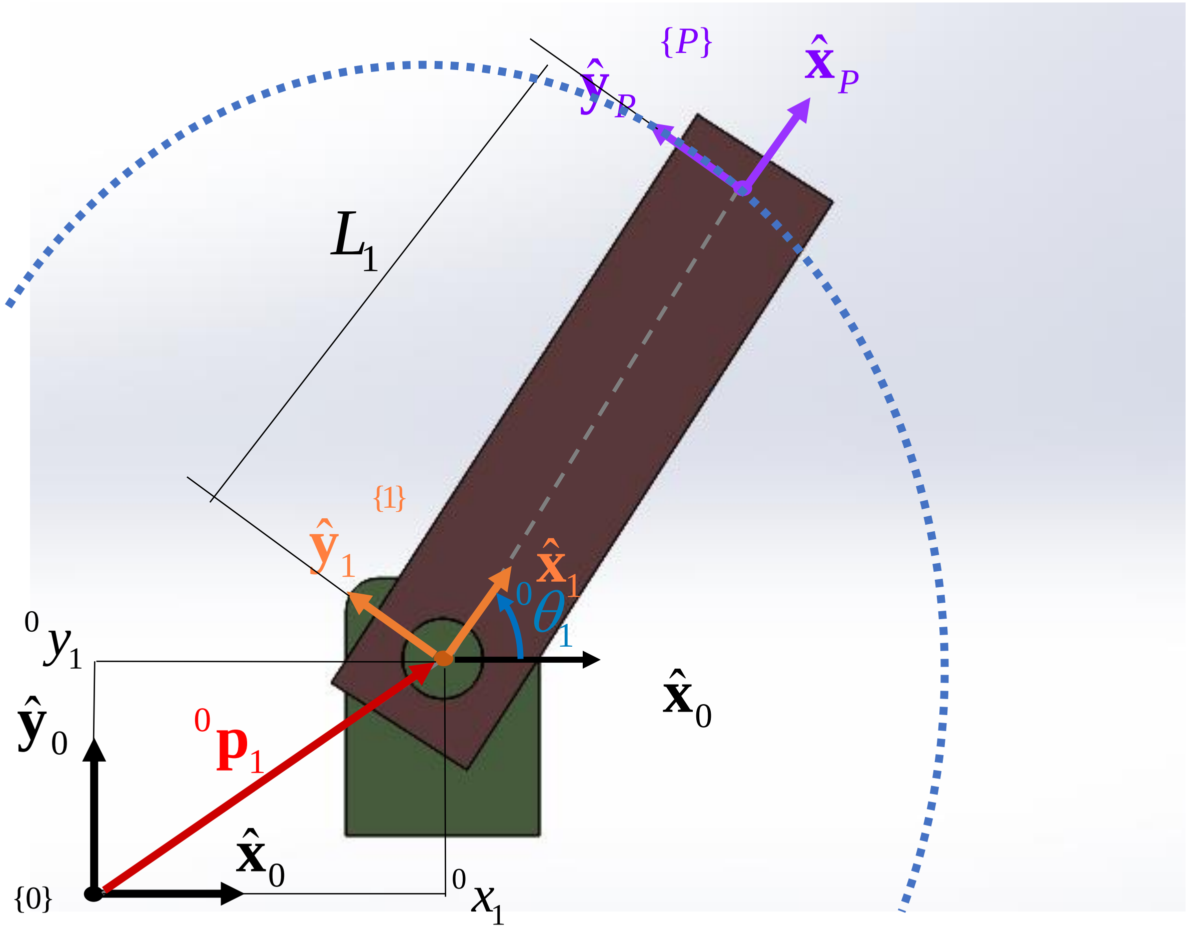
$$ {}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1\right) $$

Vector de la postura de un eslabón

$$ {}^0\boldsymbol{\xi}_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix} $$

Modelo cinemático de la posición

Junta rotacional



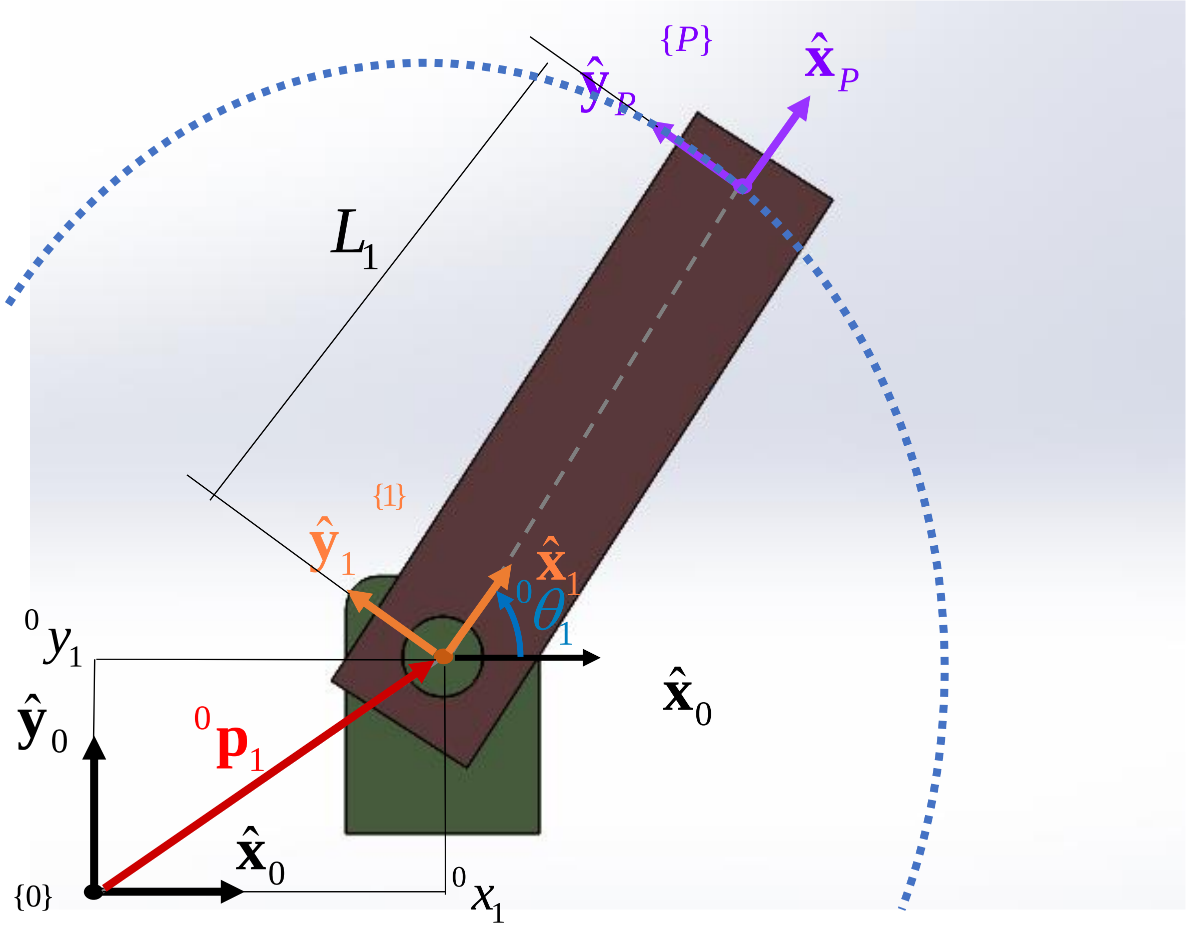
$$ {}^0\mathbf{p}_P = \begin{pmatrix} {}^0x_1 \\ {}^0y_1 \\ 0 \end{pmatrix} \qquad {}^0\boldsymbol{\theta}_P = \left({}^0\theta_1 \right) $$

Vector de la postura de un eslabón

$$ {}^0\boldsymbol{\xi}_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1 \cos({}^0\theta_1) \\ {}^0y_1 + L_1 \sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix} $$

Modelo cinemático de la velocidad

Junta rotacional



Vector de la postura de un eslabón

$$ {}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + L_1\cos({}^0\theta_1) \\ {}^0y_1 + L_1\sin({}^0\theta_1) \\ {}^0\theta_1 \end{pmatrix} $$

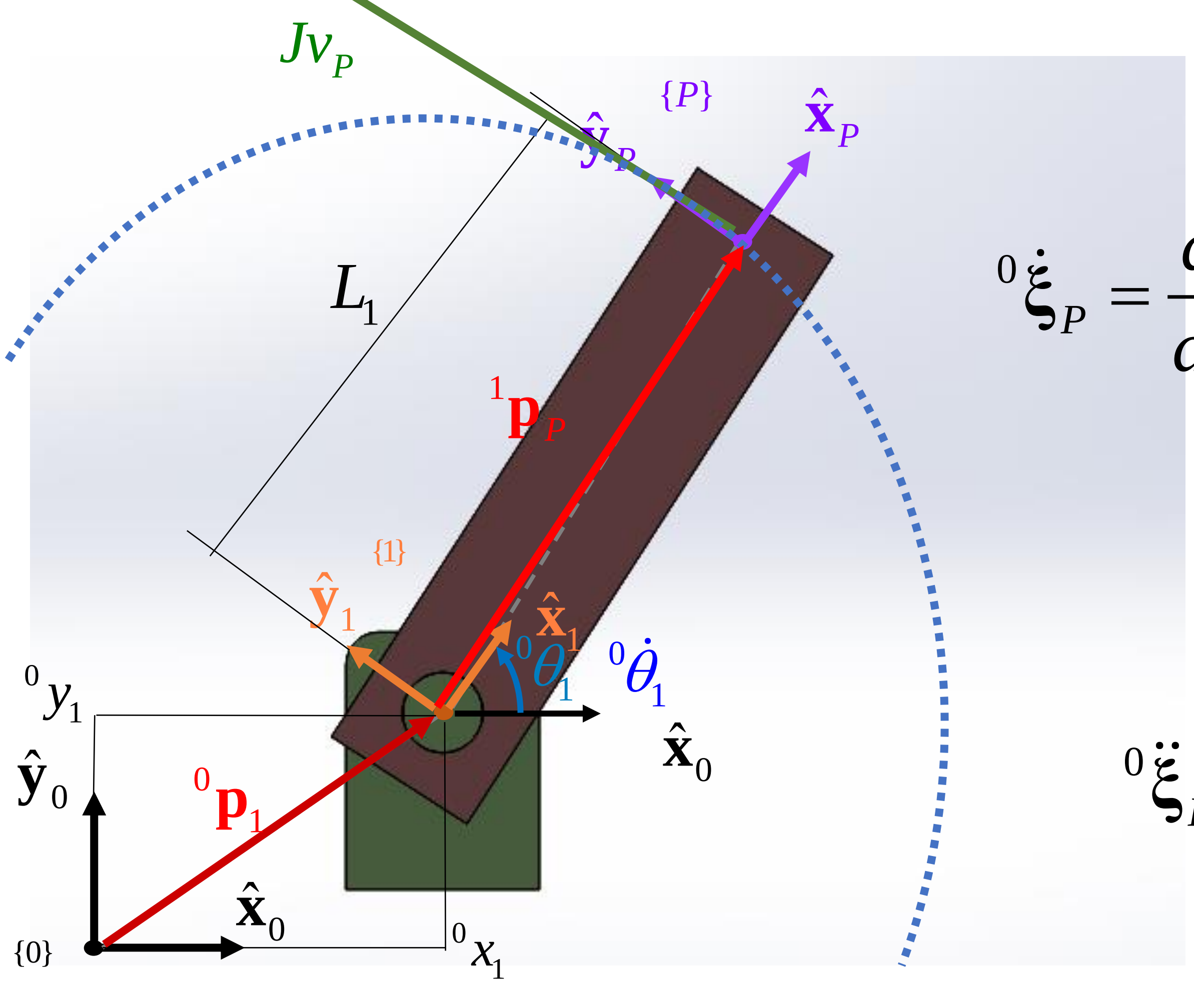
Vector de velocidades del eslabón

$$ {}^0\dot{\xi}_P = \frac{d}{dt}\,{}^0\xi_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\xi_P\,{}^0\theta_1 $$

$$ f(g(x)) $$

Modelo cinemático de la velocidad

Junta rotacional



Vector de velocidades del eslabón

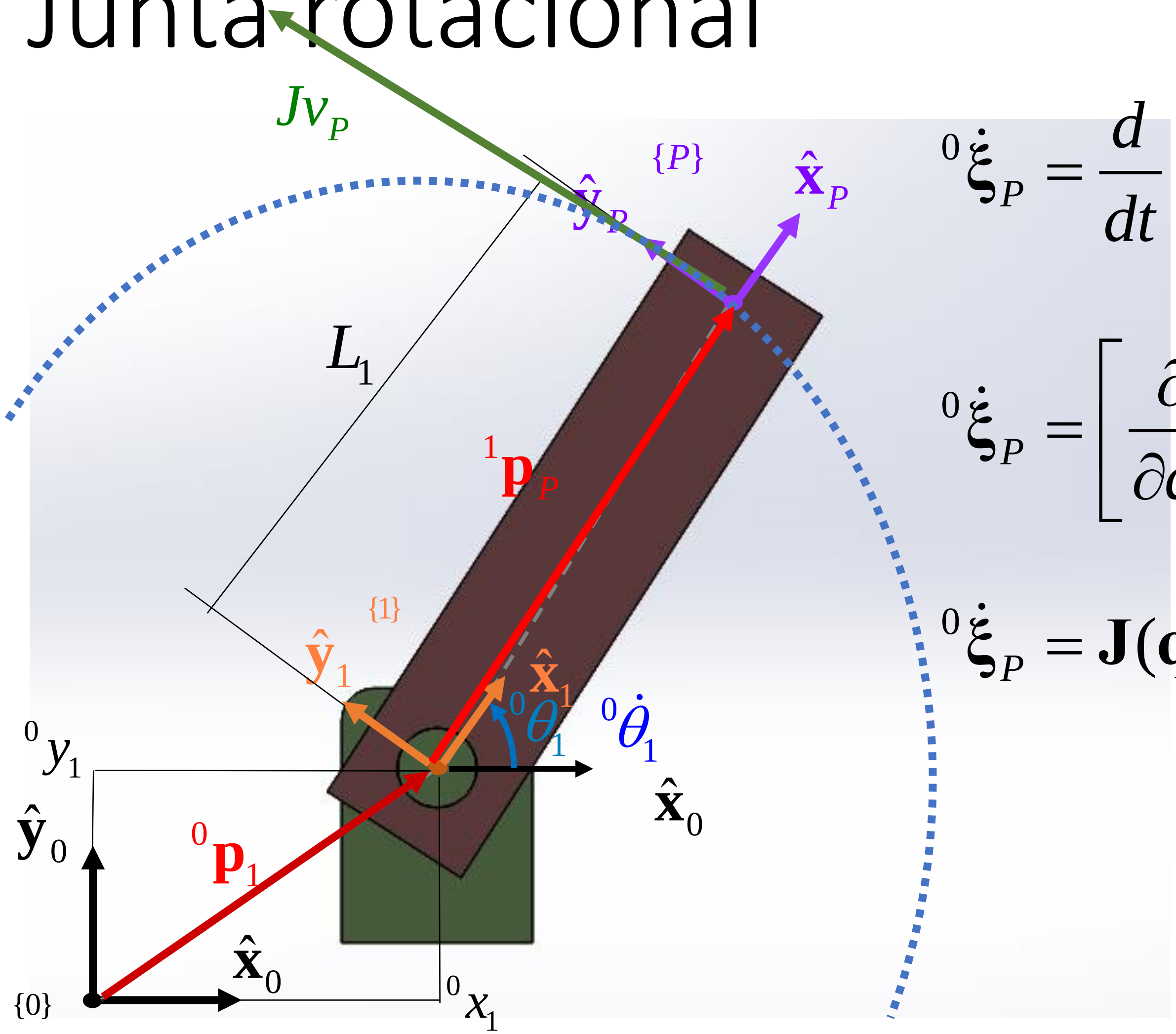
$$ {}^0\dot{\xi}_P = \frac{d}{dt}{}^0\xi_P = \frac{\partial}{\partial {}^0\theta_1}{}^0\xi_P\,{}^0\dot{\theta}_1 = \begin{pmatrix} \boxed{\begin{matrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \end{matrix}} \\ \boxed{1} \end{pmatrix} {}^0\dot{\theta}_1 $$

Vector de aceleraciones del eslabón

$$ {}^0\ddot{\xi}_P = \frac{d}{dt}{}^0\dot{\xi}_P = \frac{\partial}{\partial {}^0\theta_1}{}^0\dot{\xi}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial {}^0\dot{\theta}_1}{}^0\dot{\xi}_P\,{}^0\ddot{\theta}_1 $$

Modelo cinemático de la velocidad

Junta rotacional



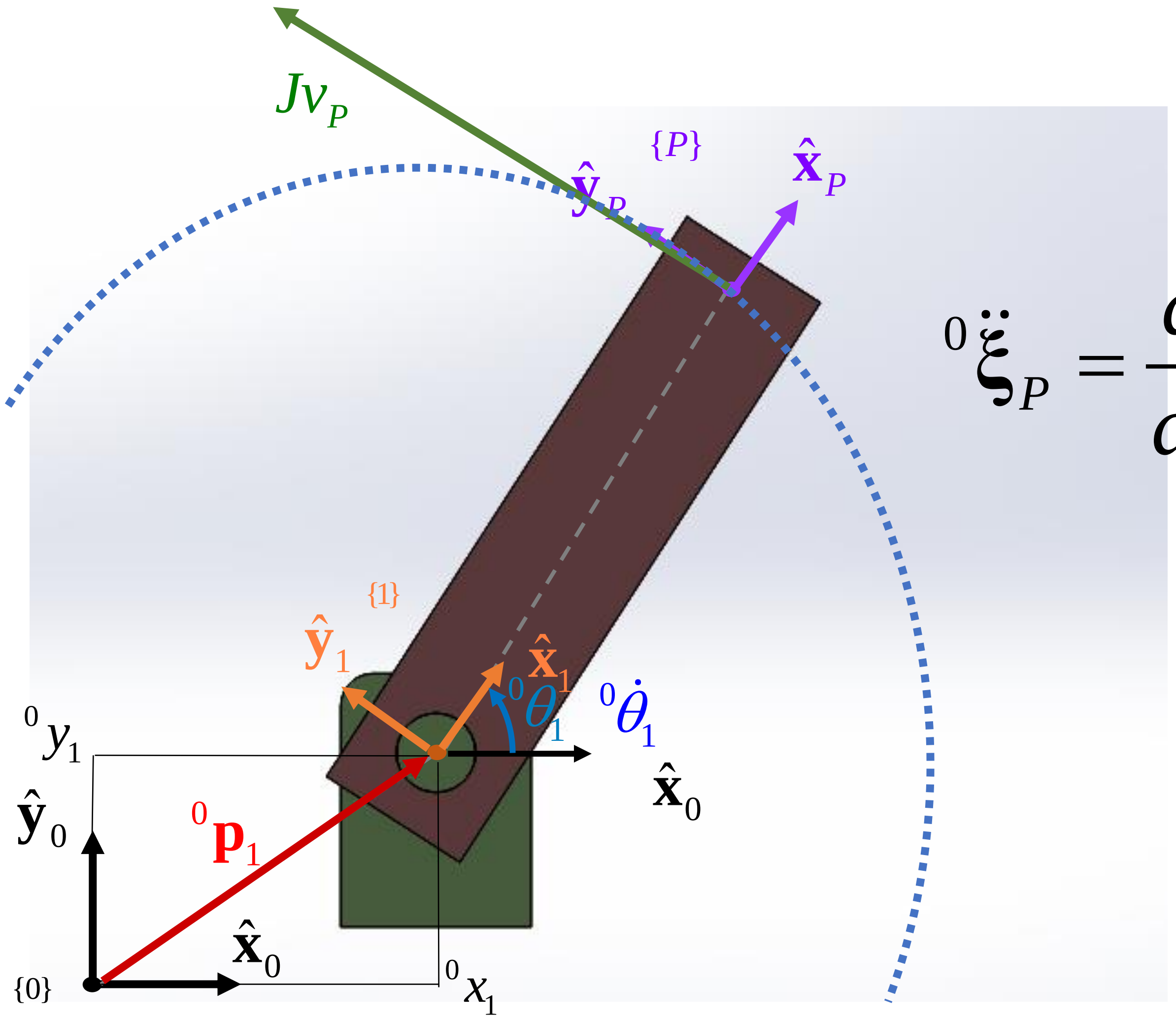
$${}^0\dot{\xi}_P = \frac{d}{dt}\,{}^0\xi_P = \frac{\partial}{\partial q_1}\,{}^0\xi_P\dot{q}_1 + \frac{\partial}{\partial q_2}\,{}^0\xi_P\dot{q}_2 + \ldots + \frac{\partial}{\partial q_n}\,{}^0\xi_P\dot{q}_n$$

$${}^0\dot{\xi}_P = \left[\frac{\partial}{\partial q_1}\,{}^0\xi_P \quad \cdots \quad \frac{\partial}{\partial q_n}\,{}^0\xi_P\right]\begin{pmatrix}\dot{q}_1\\ \vdots \\ \dot{q}_n\end{pmatrix}$$

$${}^0\dot{\xi}_P = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$$

Junta rotacional

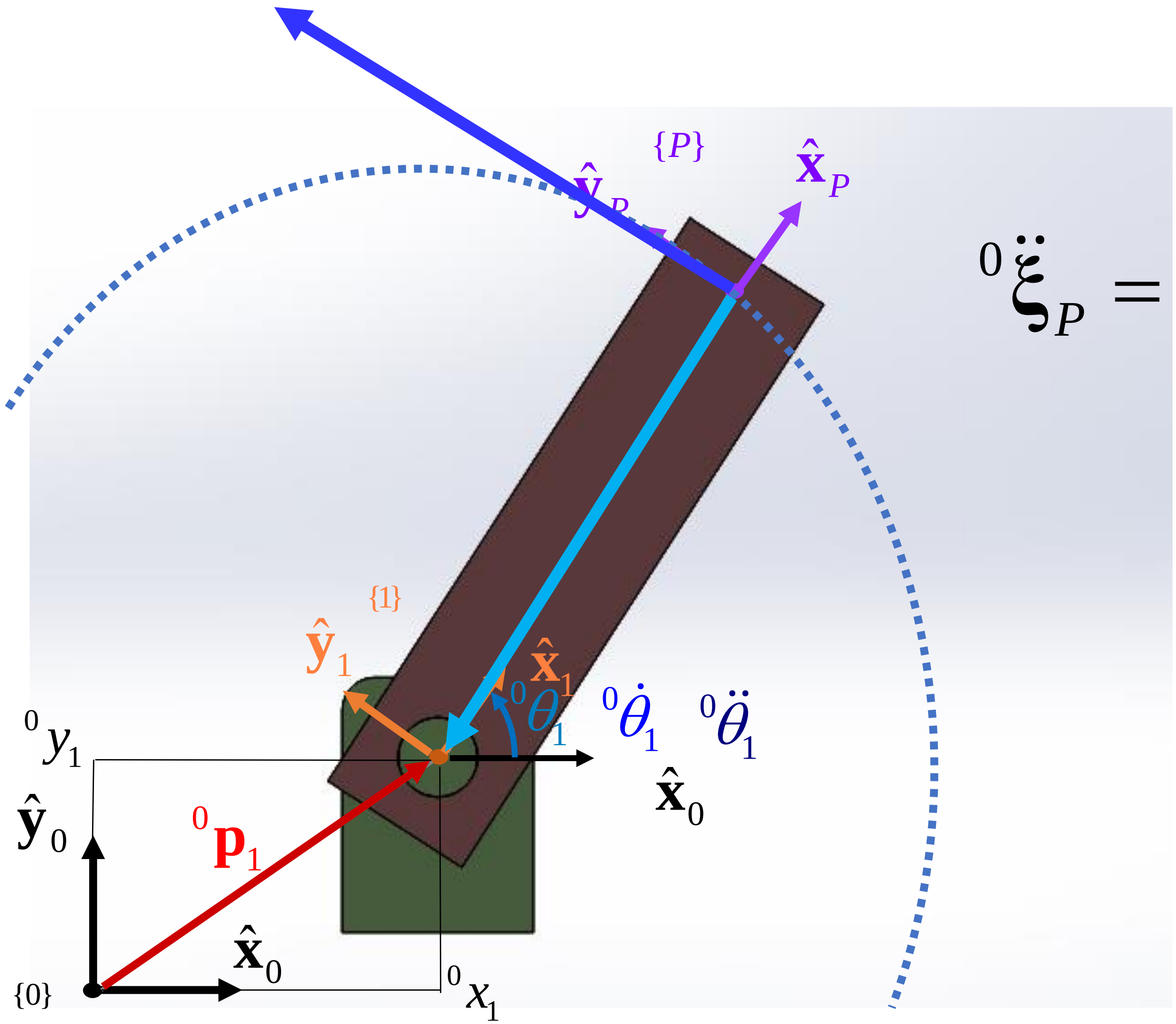
Modelo cinemático de la aceleración



$$
{}^0\ddot{\boldsymbol{\xi}}_P = \frac{d}{dt}\,{}^0\dot{\boldsymbol{\xi}}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial\,{}^0\dot{\theta}_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\ddot{\theta}_1
$$

Junta rotacional

Modelo cinemático de la aceleración

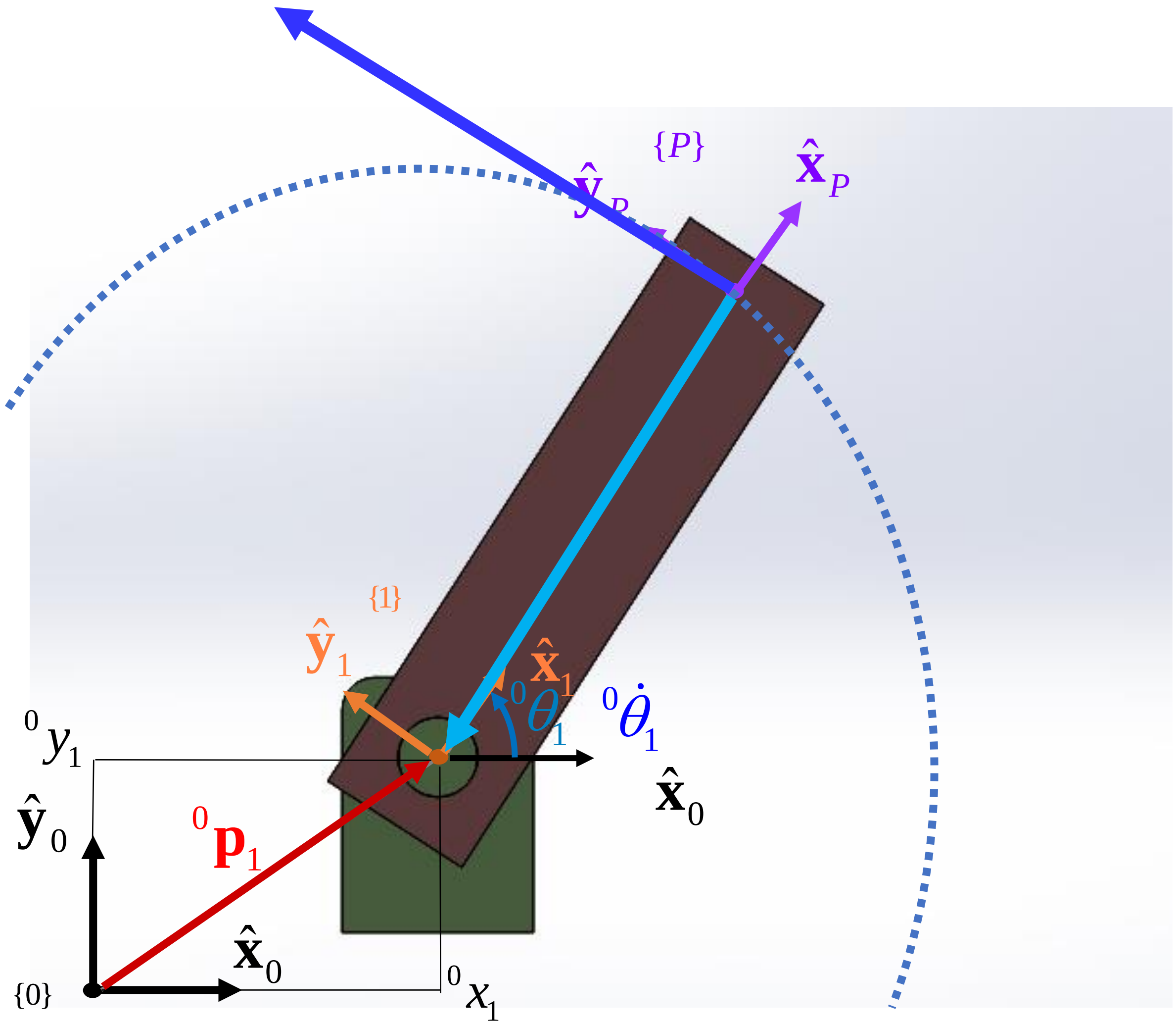


$$
{}^0\ddot{\boldsymbol{\xi}}_P = \frac{d}{dt}\,{}^0\dot{\boldsymbol{\xi}}_P = \frac{\partial}{\partial\,{}^0\theta_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\dot{\theta}_1 + \frac{\partial}{\partial\,{}^0\dot{\theta}_1}\,{}^0\dot{\boldsymbol{\xi}}_P\,{}^0\ddot{\theta}_1
$$

$$
= \begin{pmatrix} \boxed{\begin{matrix} -L_1\cos({}^0\theta_1) \\ -L_1\sin({}^0\theta_1) \end{matrix}} \\ 0 \end{pmatrix} {}^0\dot{\theta}_1^{\,2} + \begin{pmatrix} \boxed{\begin{matrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \end{matrix}} \\ 1 \end{pmatrix} {}^0\ddot{\theta}_1
$$

Junta rotacional

Modelo cinemático de la aceleración



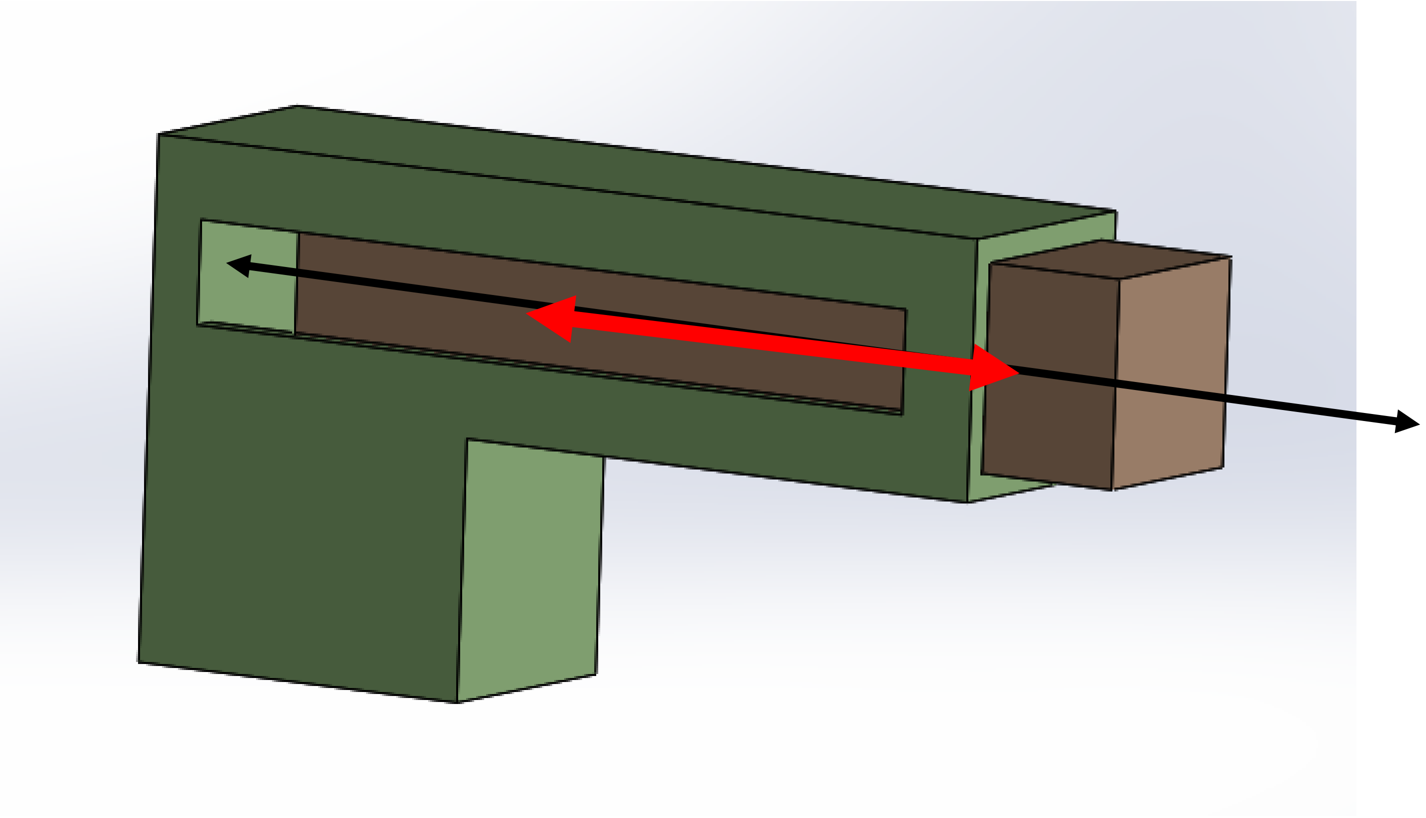
Vector de velocidades del eslabón

$$
{}^0\ddot{\boldsymbol{\xi}}_P = \frac{\partial}{\partial\, {}^0\theta_1}\, {}^0\dot{\boldsymbol{\xi}}_P\, {}^0\dot{\theta}_1 + \frac{\partial}{\partial\, {}^0\dot{\theta}_1}\, {}^0\dot{\boldsymbol{\xi}}_P\, {}^0\ddot{\theta}_1 =
$$

$$
= \boxed{\begin{pmatrix} -L_1\cos({}^0\theta_1) \\ -L_1\sin({}^0\theta_1) \\ 0 \end{pmatrix}} {}^0\dot{\theta}_1^{\,2} + \boxed{\begin{pmatrix} -L_1\sin({}^0\theta_1) \\ L_1\cos({}^0\theta_1) \\ 1 \end{pmatrix}} {}^0\ddot{\theta}_1
$$

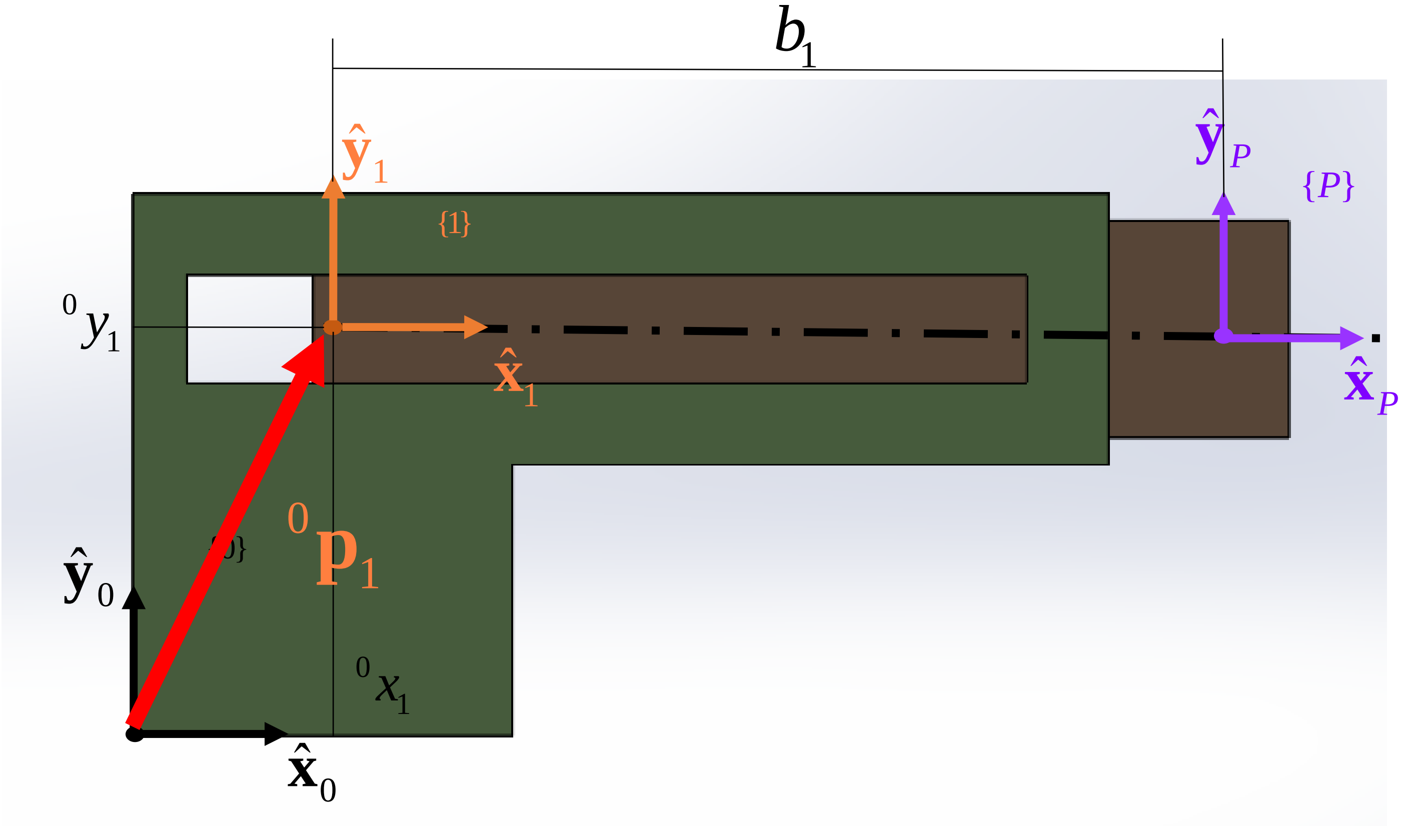
Modelo cinemático de la posición

Junta prismática



Modelo cinemático de la posición

Junta prismática



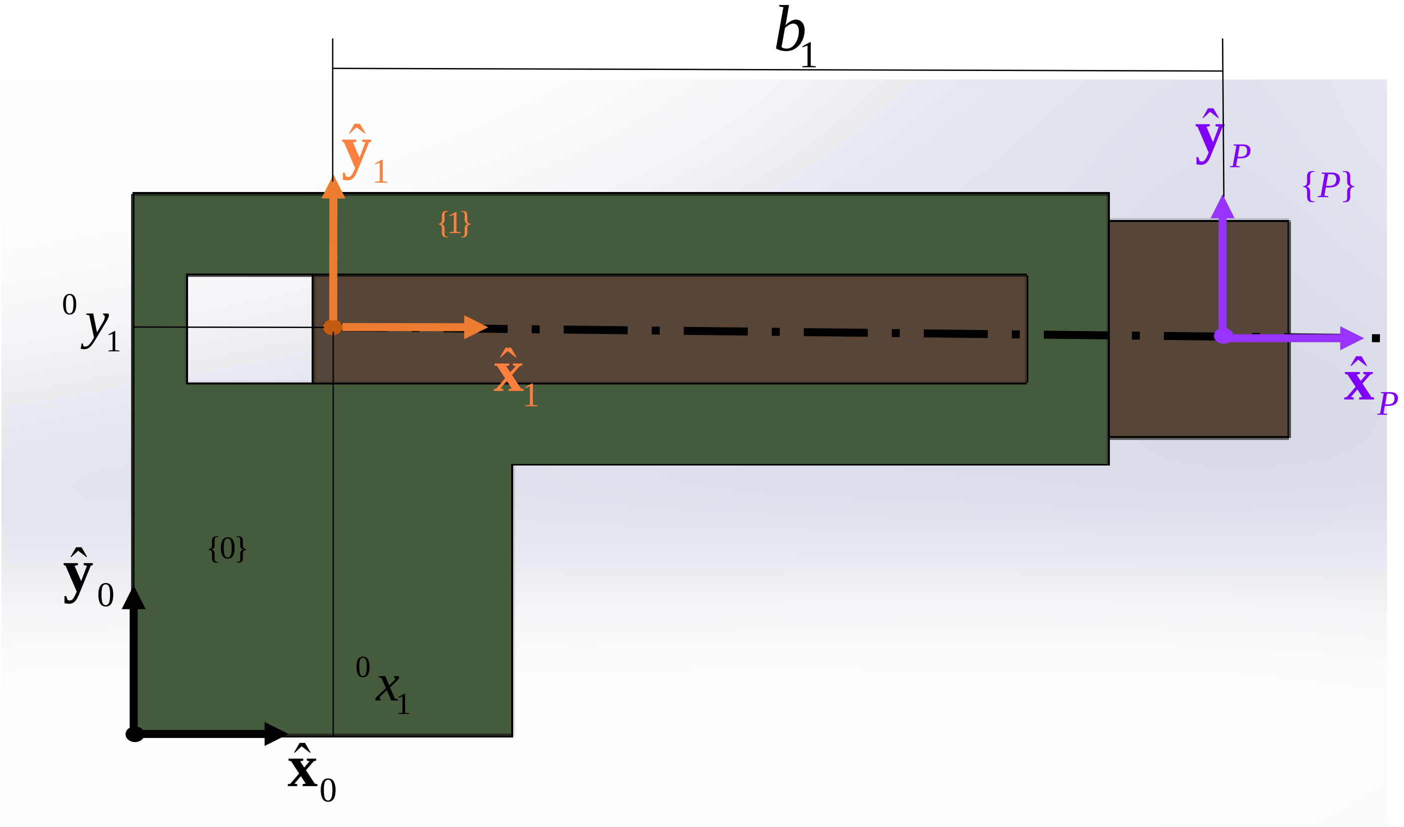
$$
{}^i\mathbf{T}_j = \begin{pmatrix} \cos({}^i\theta_j) & -\sin({}^i\theta_j) & 0 & {}^ix_j \\ \sin({}^i\theta_j) & \cos({}^i\theta_j) & 0 & {}^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1 = \begin{pmatrix} 1 & 0 & 0 & {}^0x_1 \\ 0 & 1 & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la posición

Junta prismática

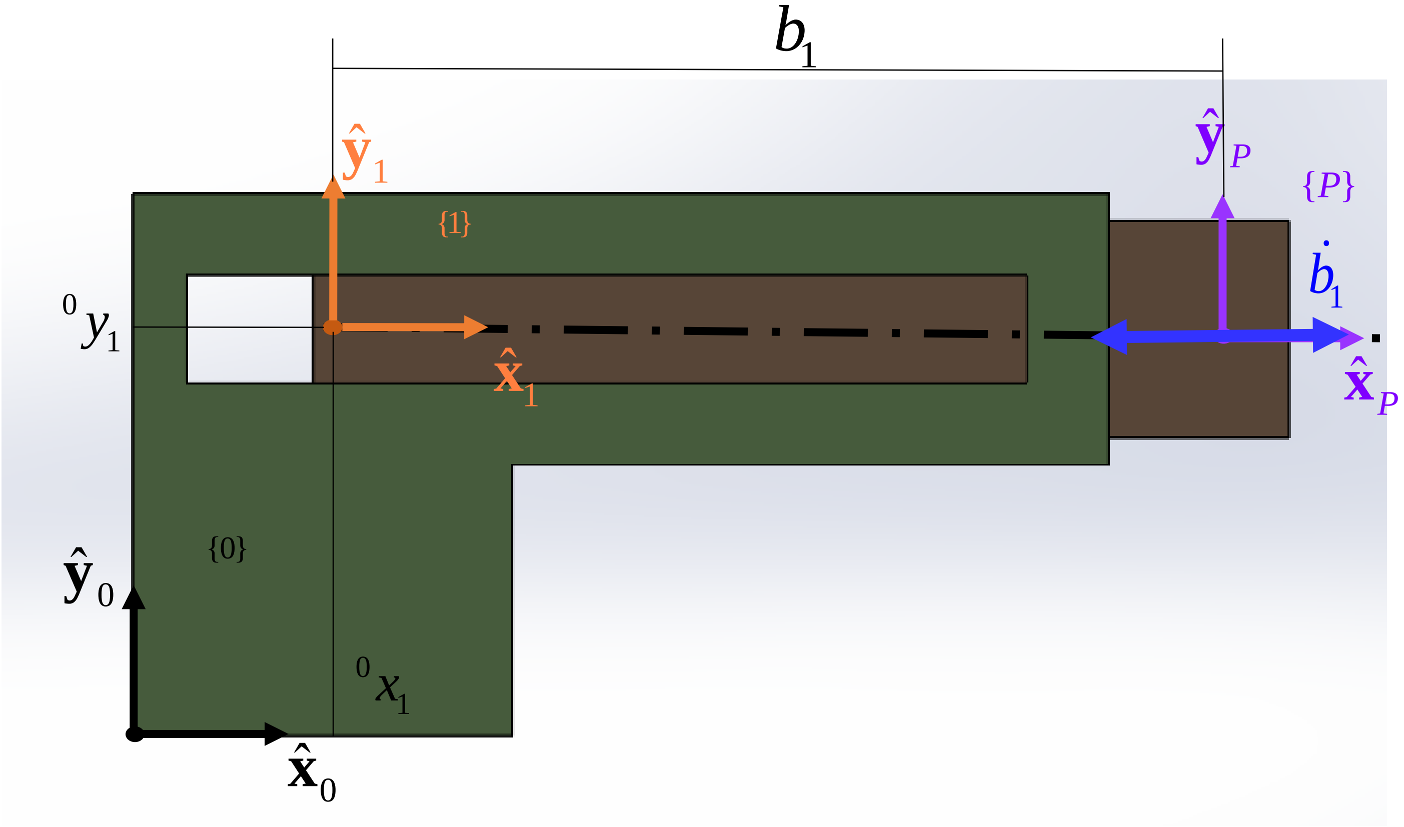


$$
{}^0\mathbf{T}_P = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_P = \begin{pmatrix} 1 & 0 & 0 & {}^0x_1 + b_1 \\ 0 & 1 & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Vector de la postura de un eslabón

$$
{}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + b_1 \\ {}^0y_1 \\ 0 \end{pmatrix}
$$

Junta prismática



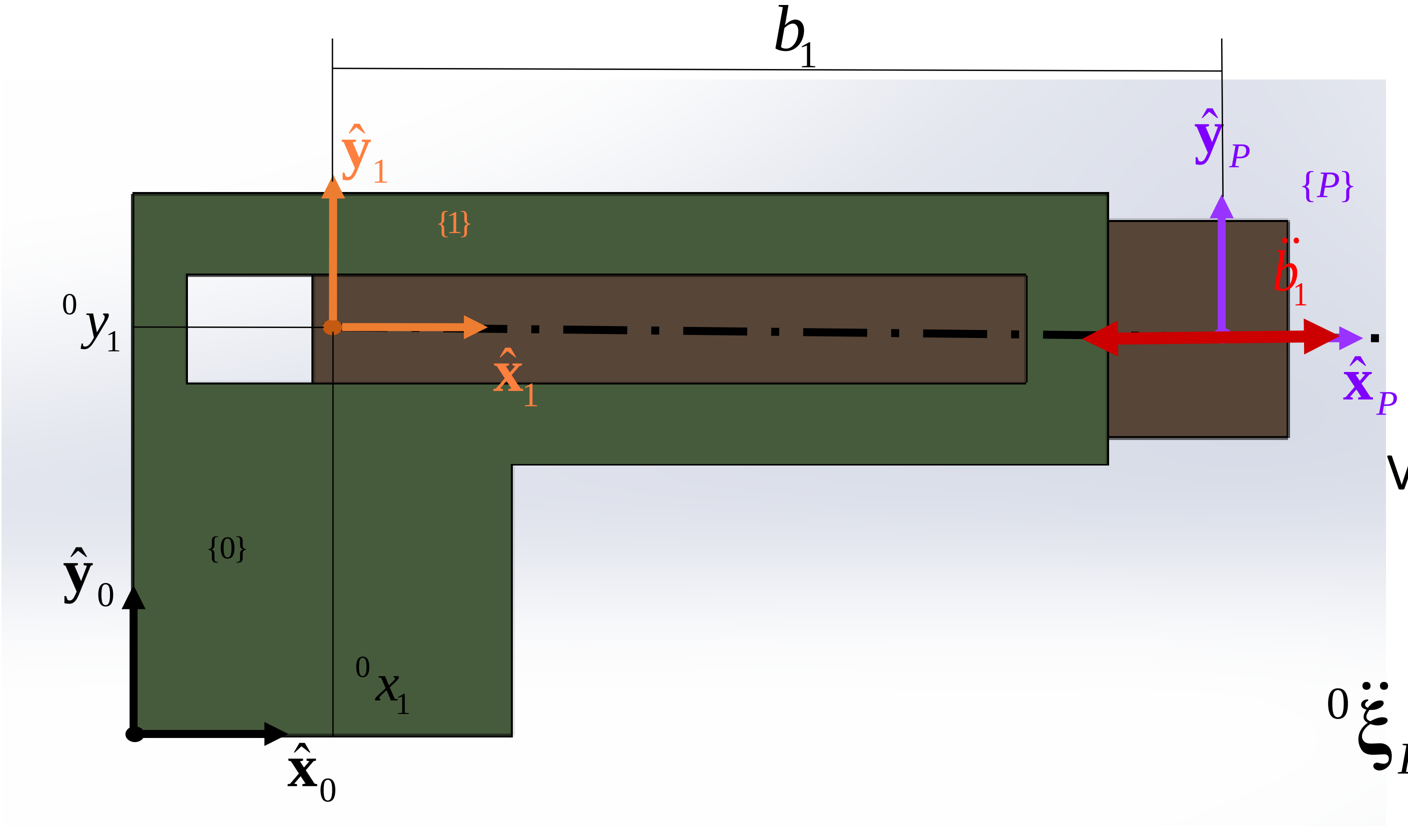
Vector de la postura de un eslabón

$$ {}^0\xi_P = \begin{pmatrix} {}^0\mathbf{p}_P \\ {}^0\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^0x_1 + b_1 \\ {}^0y_1 \\ 0 \end{pmatrix} $$

Vector de velocidades del eslabón

$$ {}^0\dot{\xi}_P = \frac{d}{dt}\,{}^0\xi_P = \frac{\partial}{\partial b_1}\,{}^0\xi_P \dot{b}_1 = \begin{pmatrix} \dot{b}_1 \\ 0 \\ 0 \end{pmatrix} $$

Junta prismática

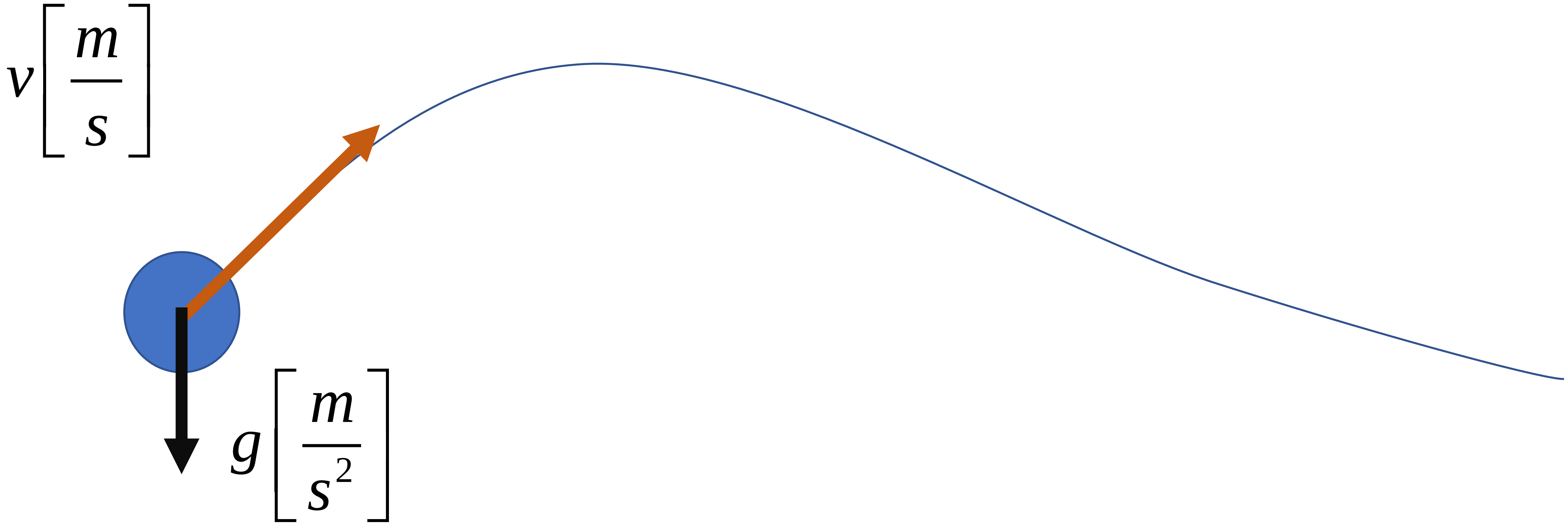


Vector de velocidades del eslabón

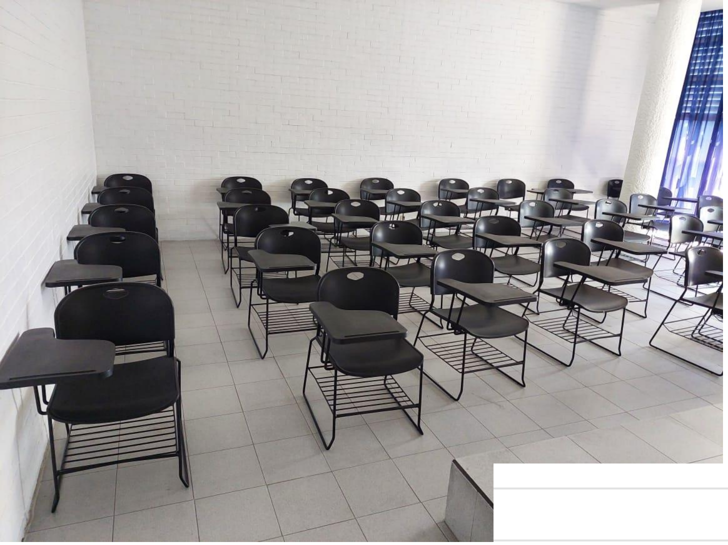
$$ {}^0\dot{\xi}_P = \frac{d}{db_1}\,{}^0\xi_P\,\dot{b}_1 = \begin{pmatrix} \dot{b}_1 \\ 0 \\ 0 \end{pmatrix} $$

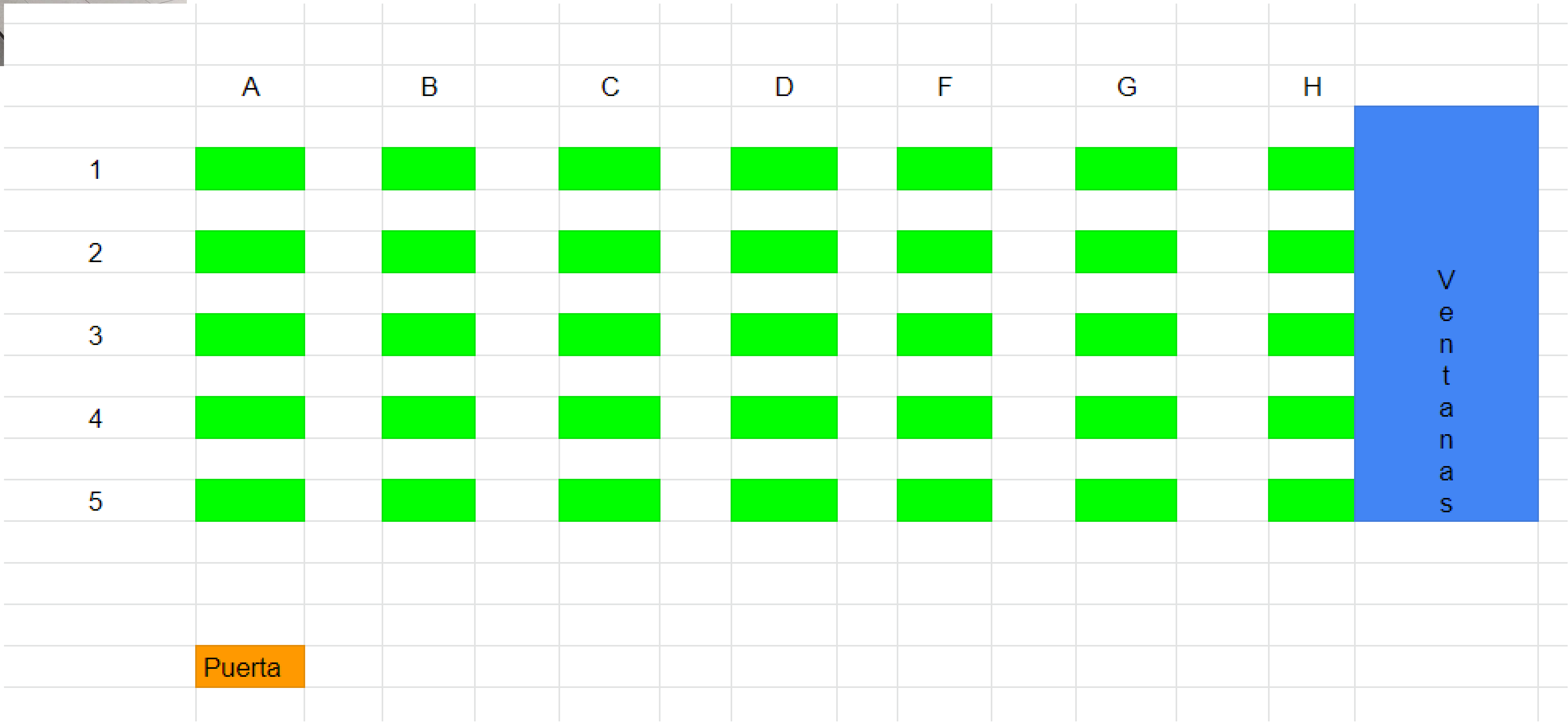
Vector de aceleraciones del eslabón

$$ {}^0\ddot{\xi}_P = \frac{d}{dt}\,{}^0\dot{\xi}_P = \frac{\partial}{\partial \dot{b}_1}\,{}^0\dot{\xi}_P\,\ddot{b}_1 = \begin{pmatrix} \ddot{b}_1 \\ 0 \\ 0 \end{pmatrix} $$

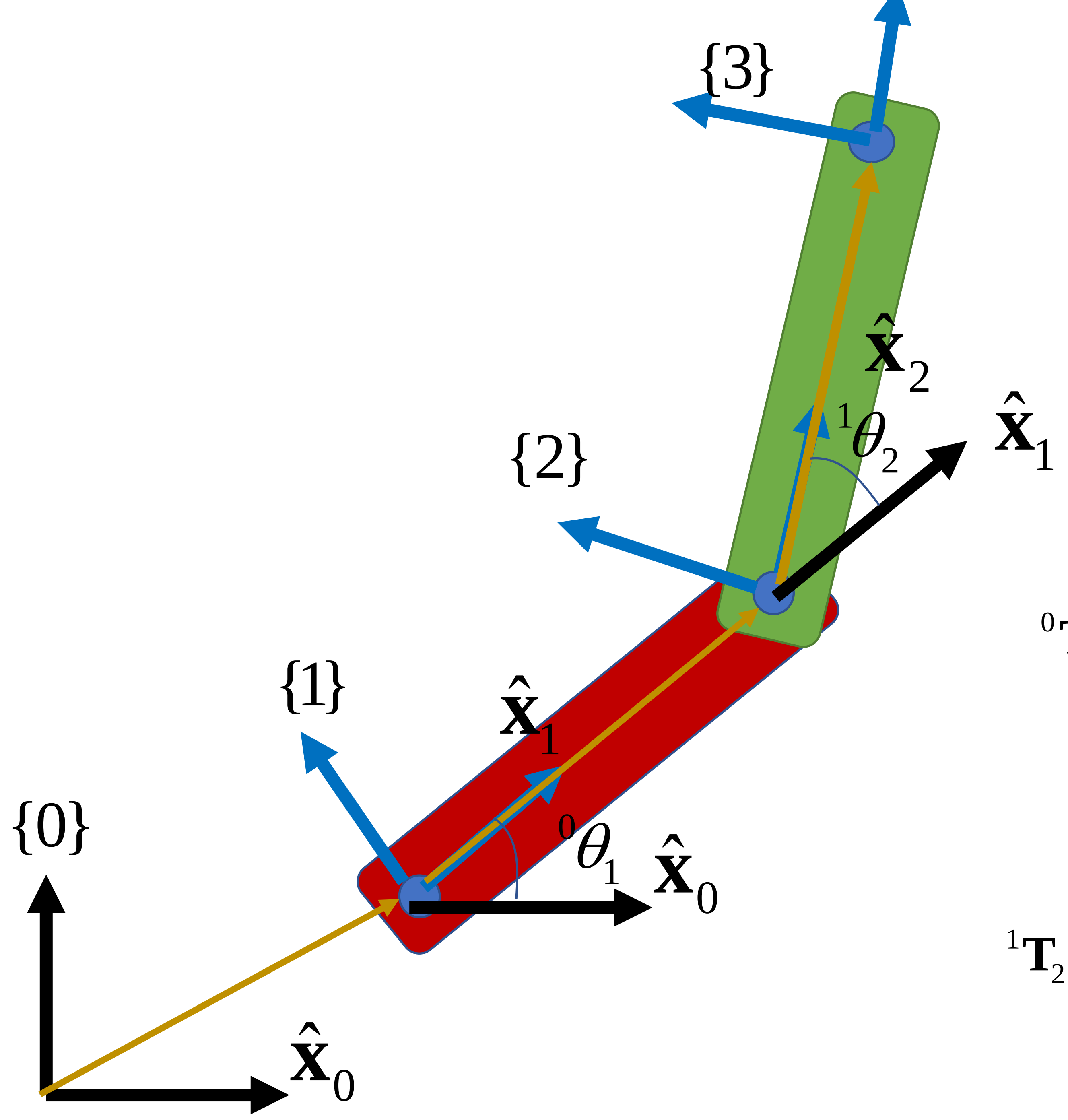

$v\left[\frac{m}{s}\right]$
$g\left[\frac{m}{s^2}\right]$

Planteamiento del ensayo del estado del arte de los robots





A
B
C
D
F
G
H
1
2
3
4
5
Ventanas
Puerta



$$^i\mathbf{T}_j = \begin{pmatrix} \cos(^i\theta_j) & -\sin(^i\theta_j) & 0 & ^ix_j \\ \sin(^i\theta_j) & \cos(^i\theta_j) & 0 & ^iy_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0\mathbf{T}_3 = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_2\,{}^2\mathbf{T}_3$$

$$^0\mathbf{T}_1 = \begin{pmatrix} \cos(^i\theta_j) & -\sin(^i\theta_j) & 0 & ^0x_1 \\ \sin(^i\theta_j) & \cos(^i\theta_j) & 0 & ^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1\mathbf{T}_2 = \begin{pmatrix} \cos(^1\theta_2) & -\sin(^1\theta_2) & 0 & ^1x_2 \\ \sin(^1\theta_2) & \cos(^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$