Robótica grupo2
Clase 25

Facultad de Ingeniería UNAM

M.I. Erik Peña Medina

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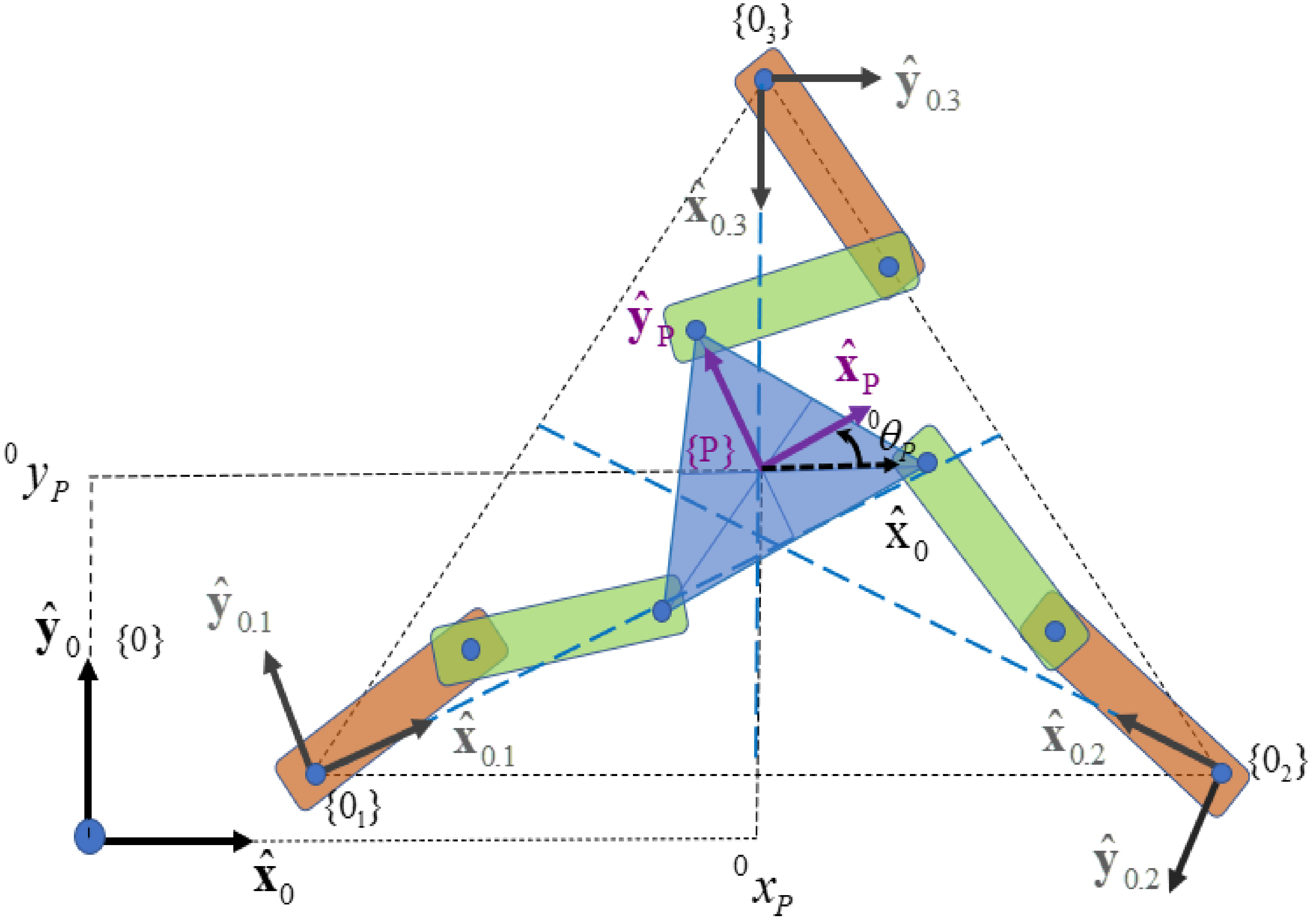
Robótica paralela

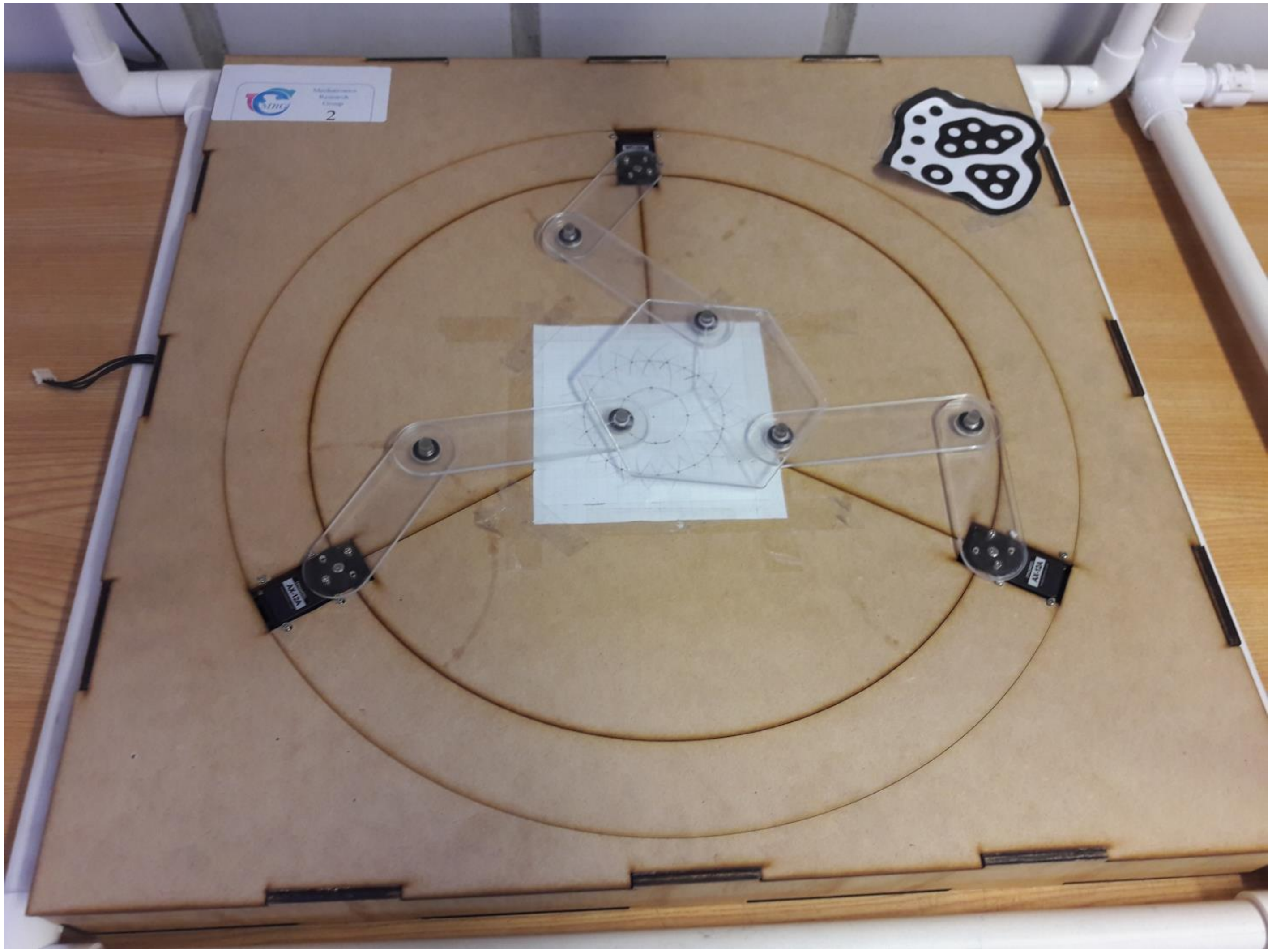
- Definición de un robot paralelo
- Modelo postura de un robot paralelo
- Modelo cinemático de un robot paralelo
  - Modelo cinemático directo de las velocidades
  - Modelo cinemático inverso de las velocidades
- Modelo dinámico de un robot paralelo

Modelo cinemático de la postura

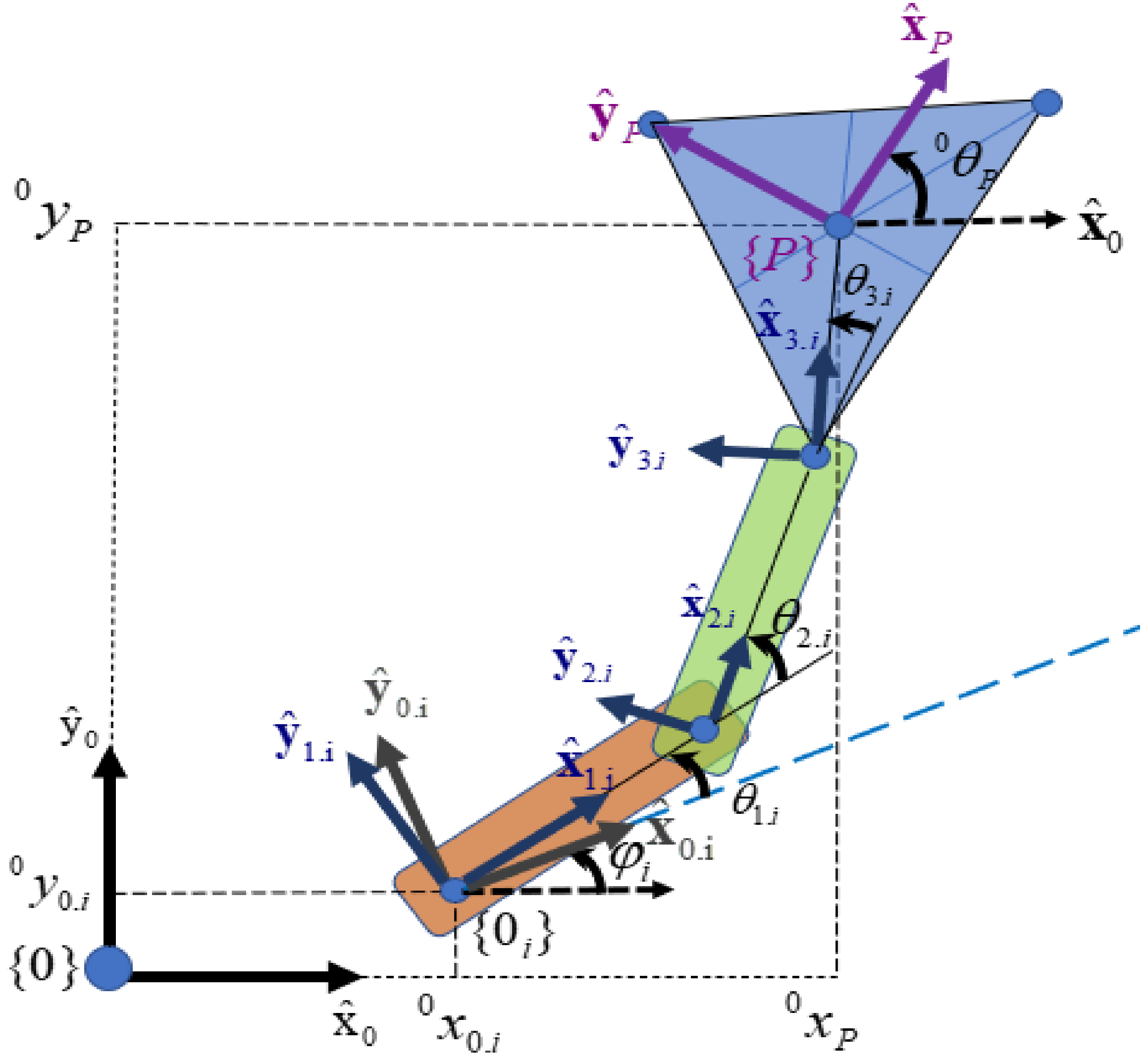
Delta plano

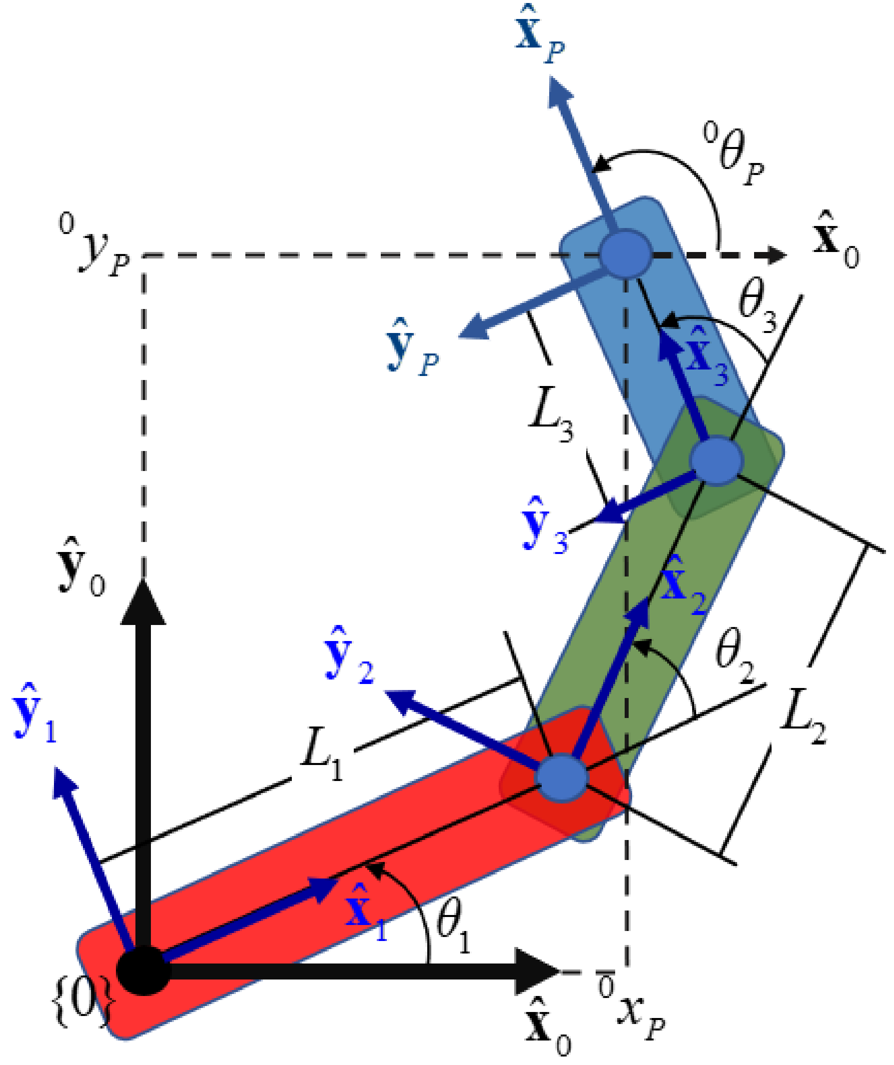


Modelo cinemático de la postura

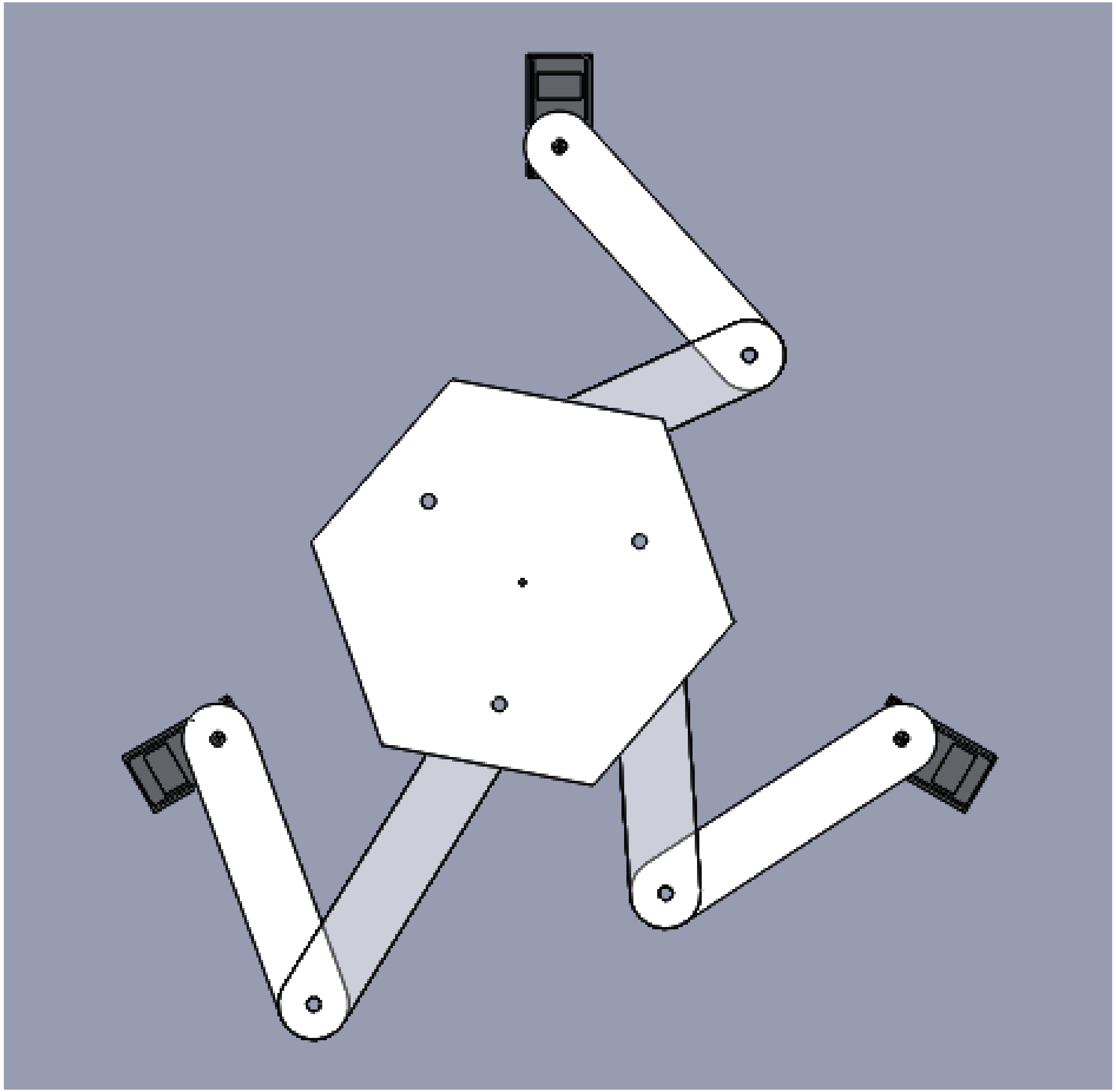


Modelo cinemático de la postura



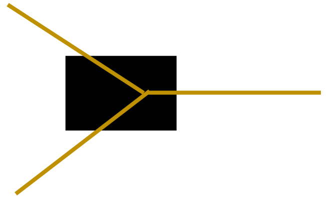


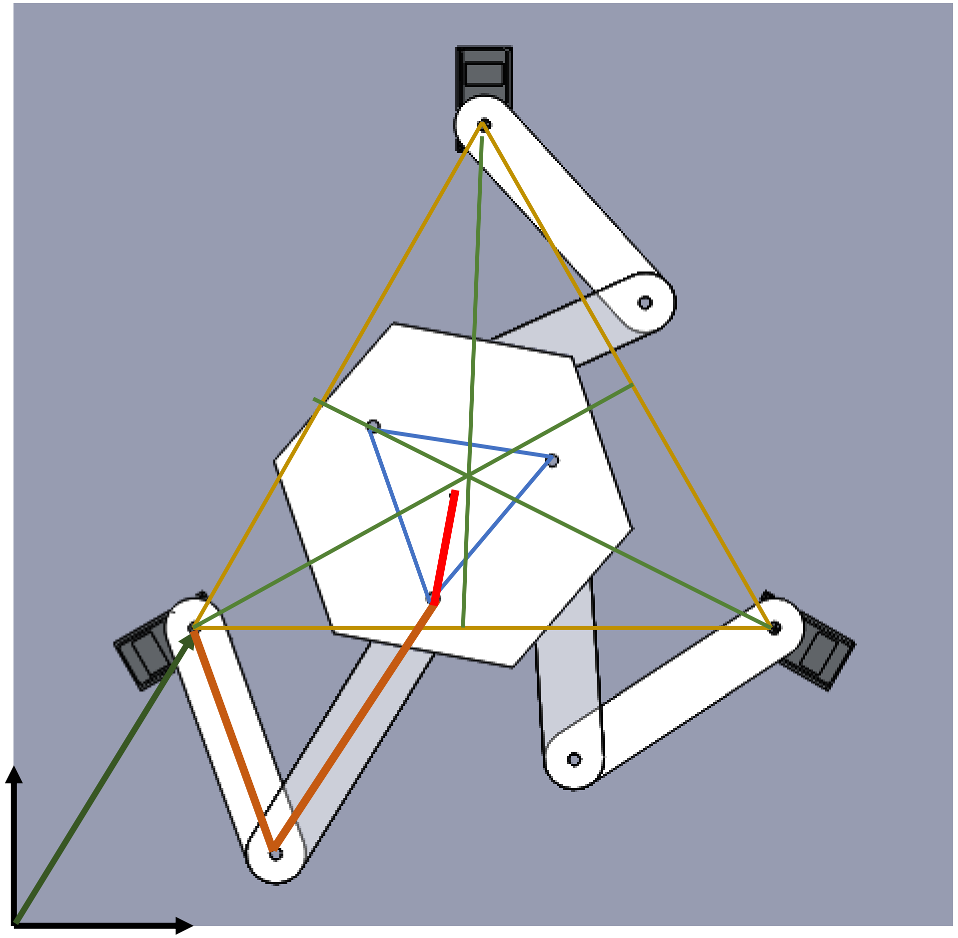
Modelo cinemático de postura



Robot delta plano

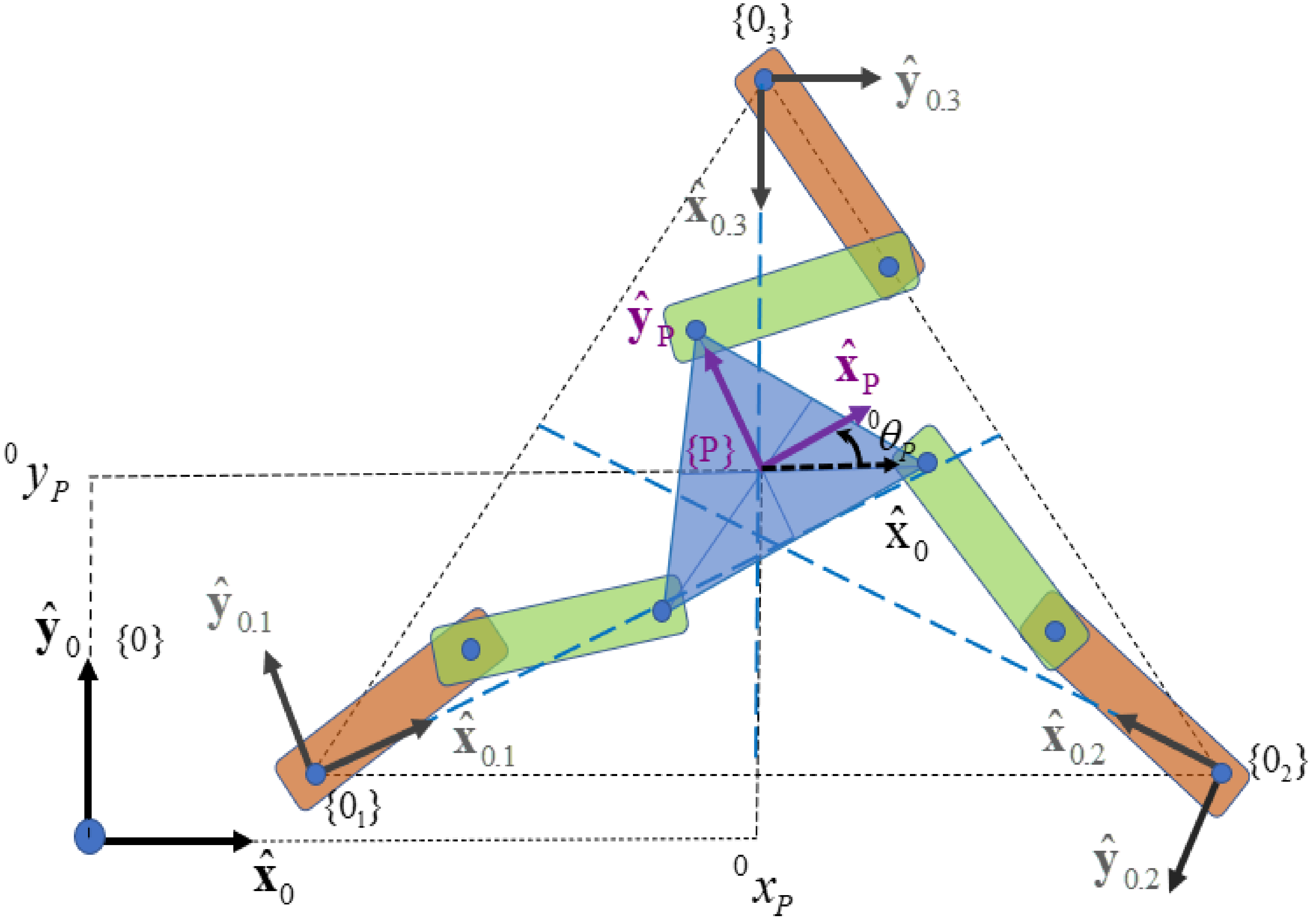
Planteamiento del modelo





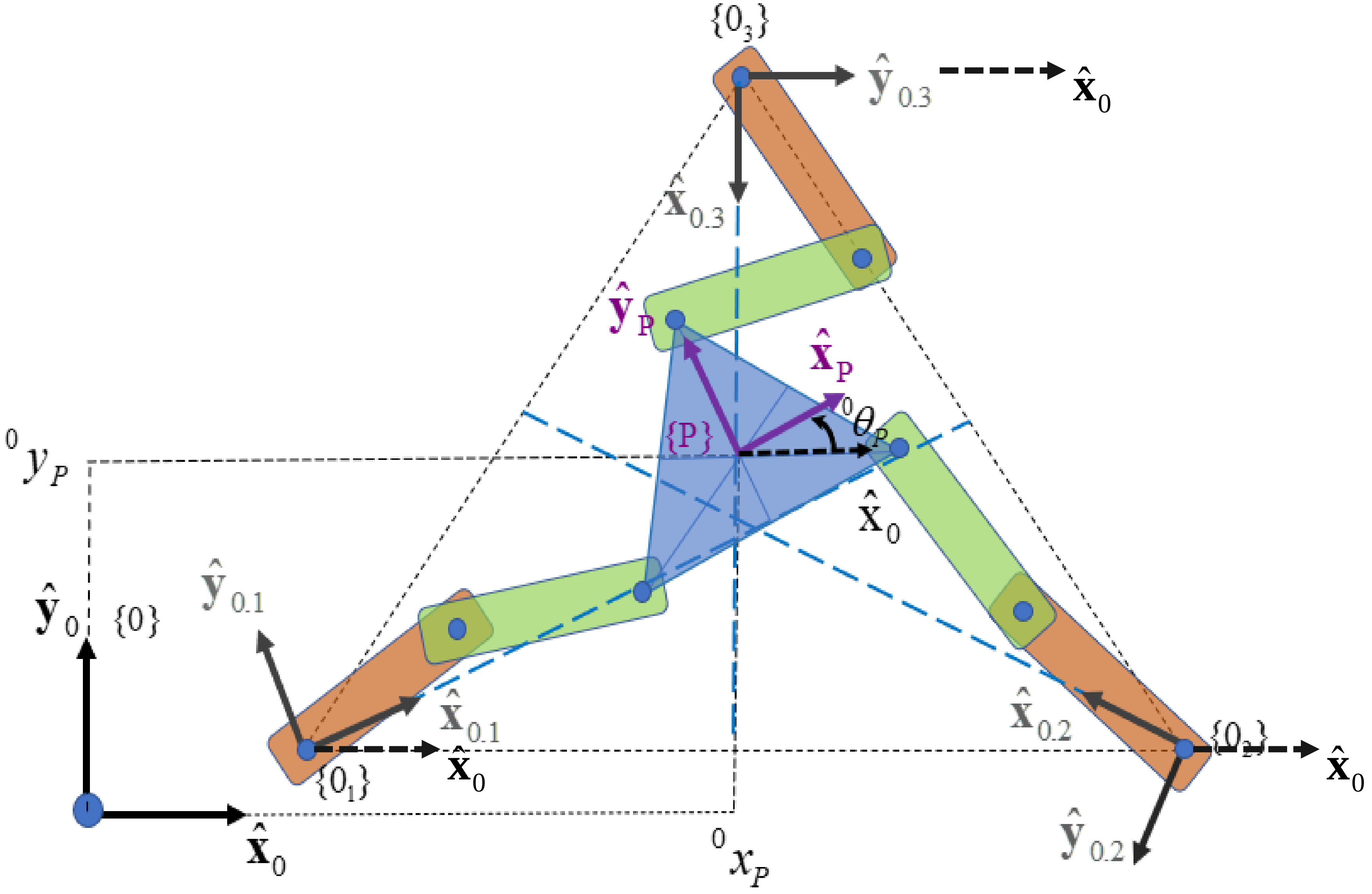
Modelo cinemático de la postura

Delta plano

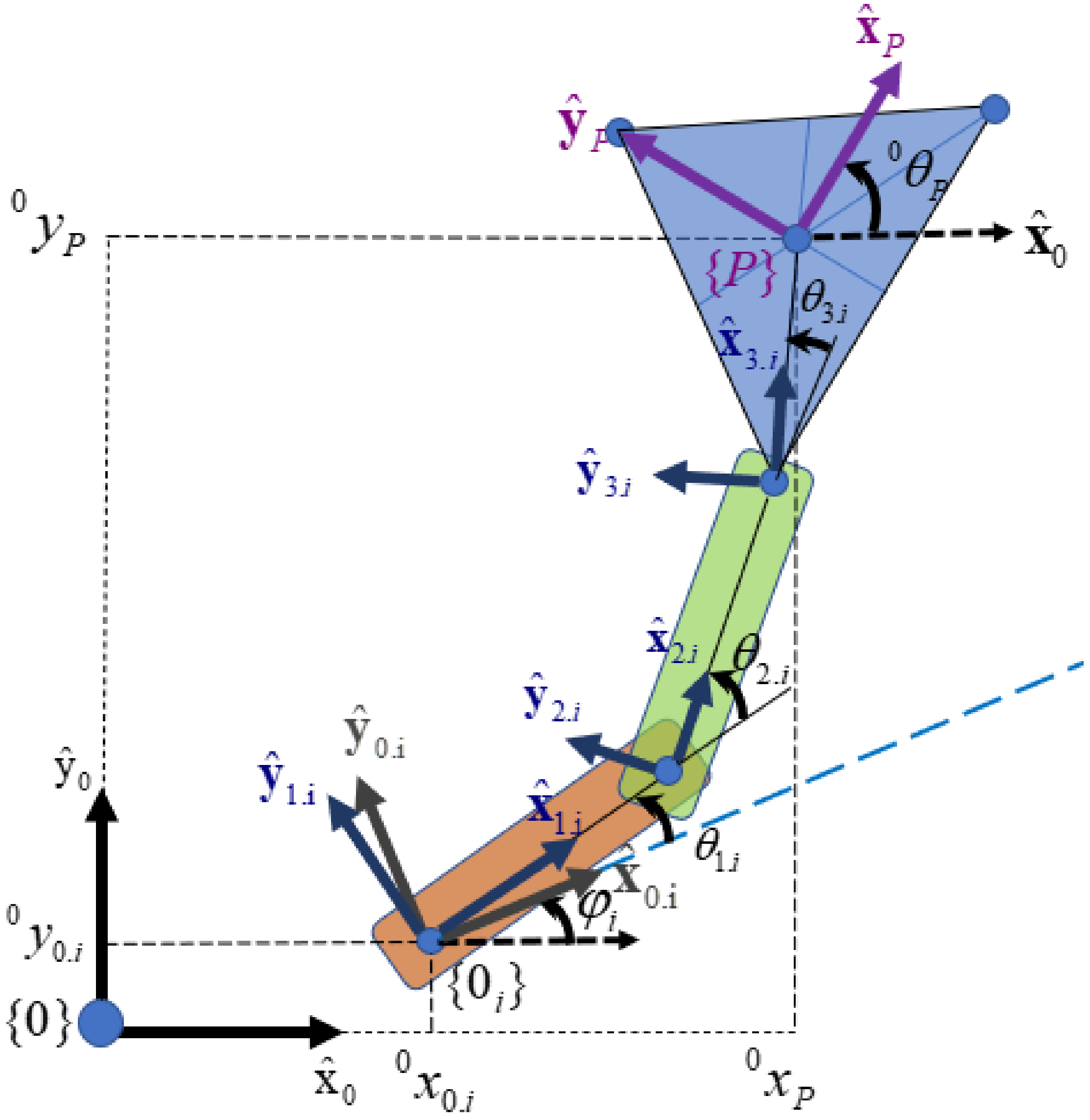


Modelo cinemático de la postura

Delta plano



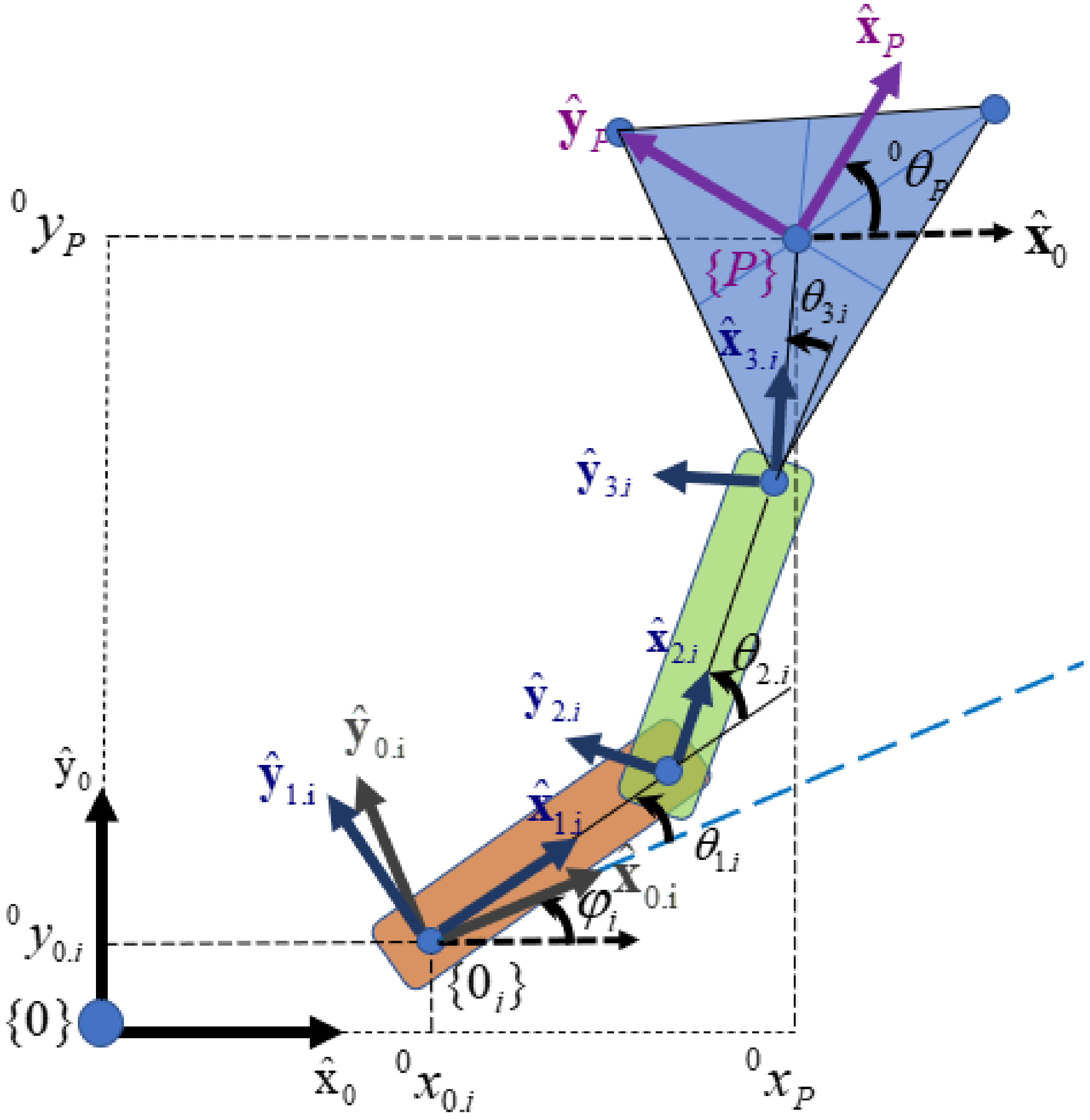
Modelo cinemático de la postura



$$ {}^0\mathbf{T}_{P.i} = {}^0\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{i}\mathbf{T}_{j} = \begin{pmatrix} \cos(\theta_j) & -\sin(\theta_j) & 0 & x_i \\ \sin(\theta_j) & \cos(\theta_j) & 0 & y_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

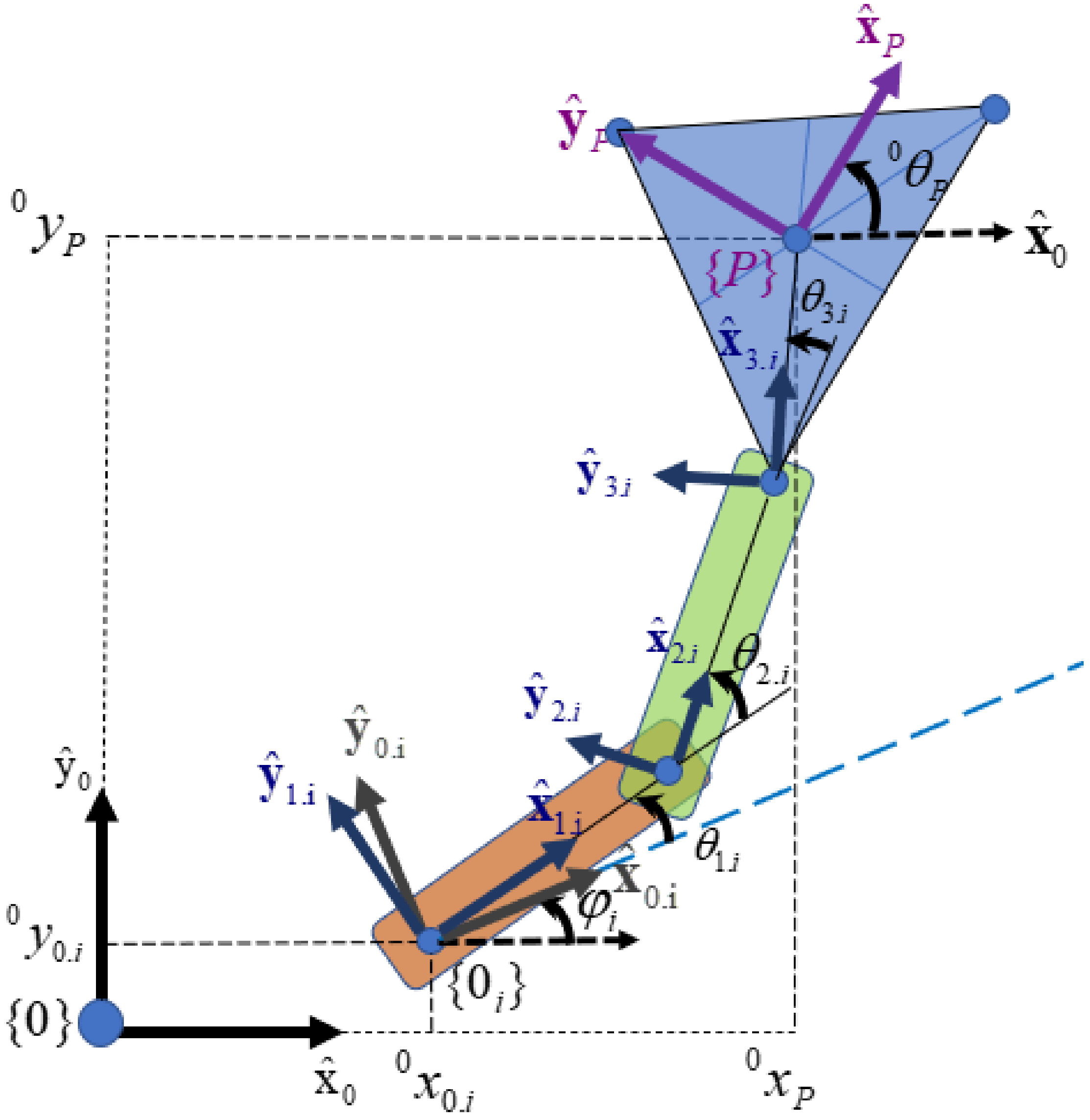


$$ {}^0\mathbf{T}_{P.i} = {}^0\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^0\mathbf{T}_{0.i} = \begin{pmatrix} \cos({}^0\varphi_i) & -\sin({}^0\varphi_i) & 0 & {}^0x_{0.i} \\ \sin({}^0\varphi_i) & \cos({}^0\varphi_i) & 0 & {}^0y_{0.j} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{0.i}\mathbf{T}_{1.i} = \begin{pmatrix} \cos({}^{0.i}\theta_{1.i}) & -\sin({}^{0.i}\theta_{1.i}) & 0 & 0 \\ \sin({}^{0.i}\theta_{1.i}) & \cos({}^{0.i}\theta_{1.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

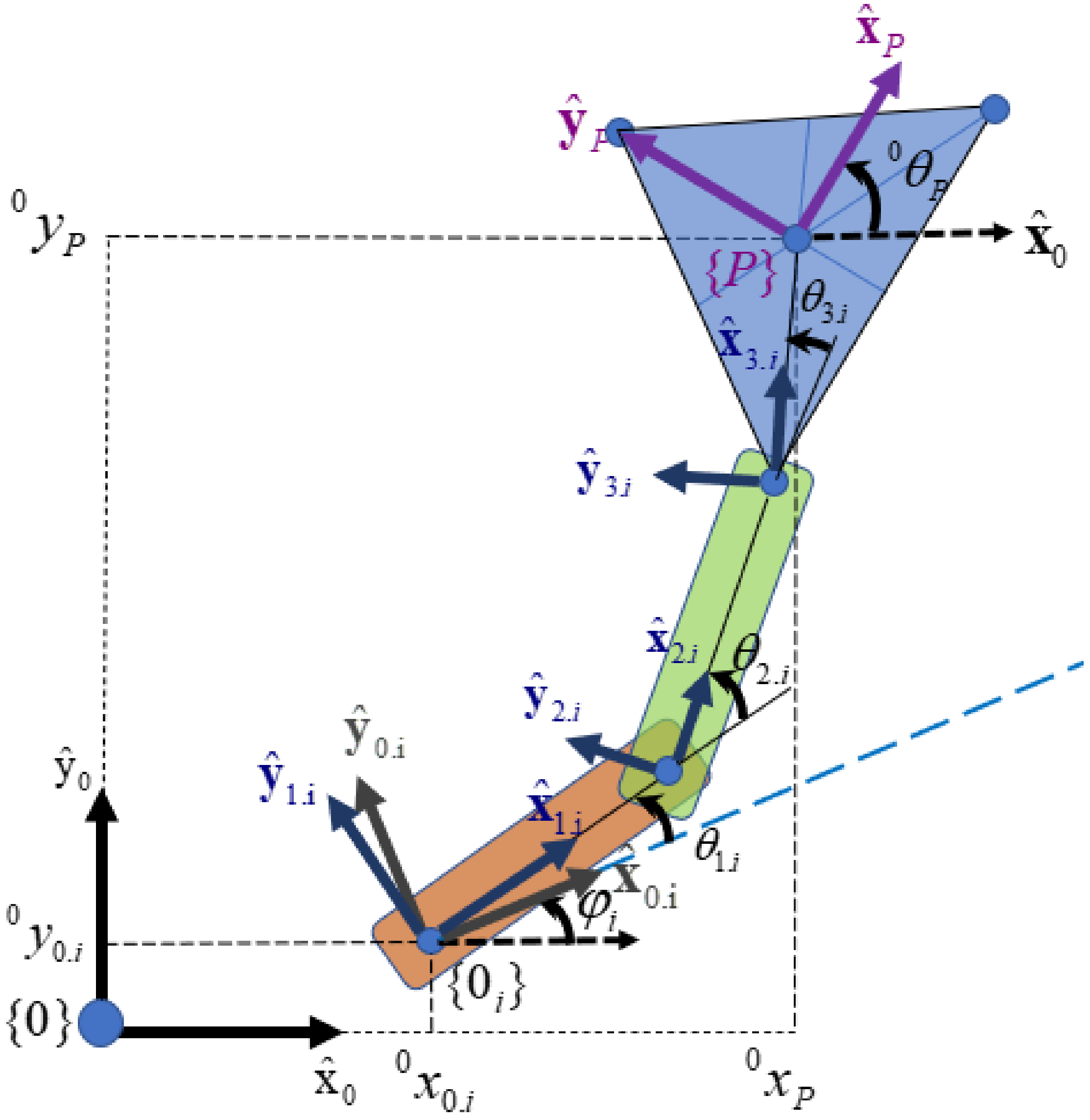


$${}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i}$$

$${}^{1.i}\mathbf{T}_{2.i} = \begin{pmatrix} \cos(\theta_{2.i}) & -\sin(\theta_{2.i}) & 0 & L_{1.i} \\ \sin(\theta_{2.i}) & \cos(\theta_{2.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{2.i}\mathbf{T}_{3.i} = \begin{pmatrix} \cos(\theta_{3.i}) & -\sin(\theta_{3.i}) & 0 & L_{2.i} \\ \sin(\theta_{3.i}) & \cos(\theta_{3.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

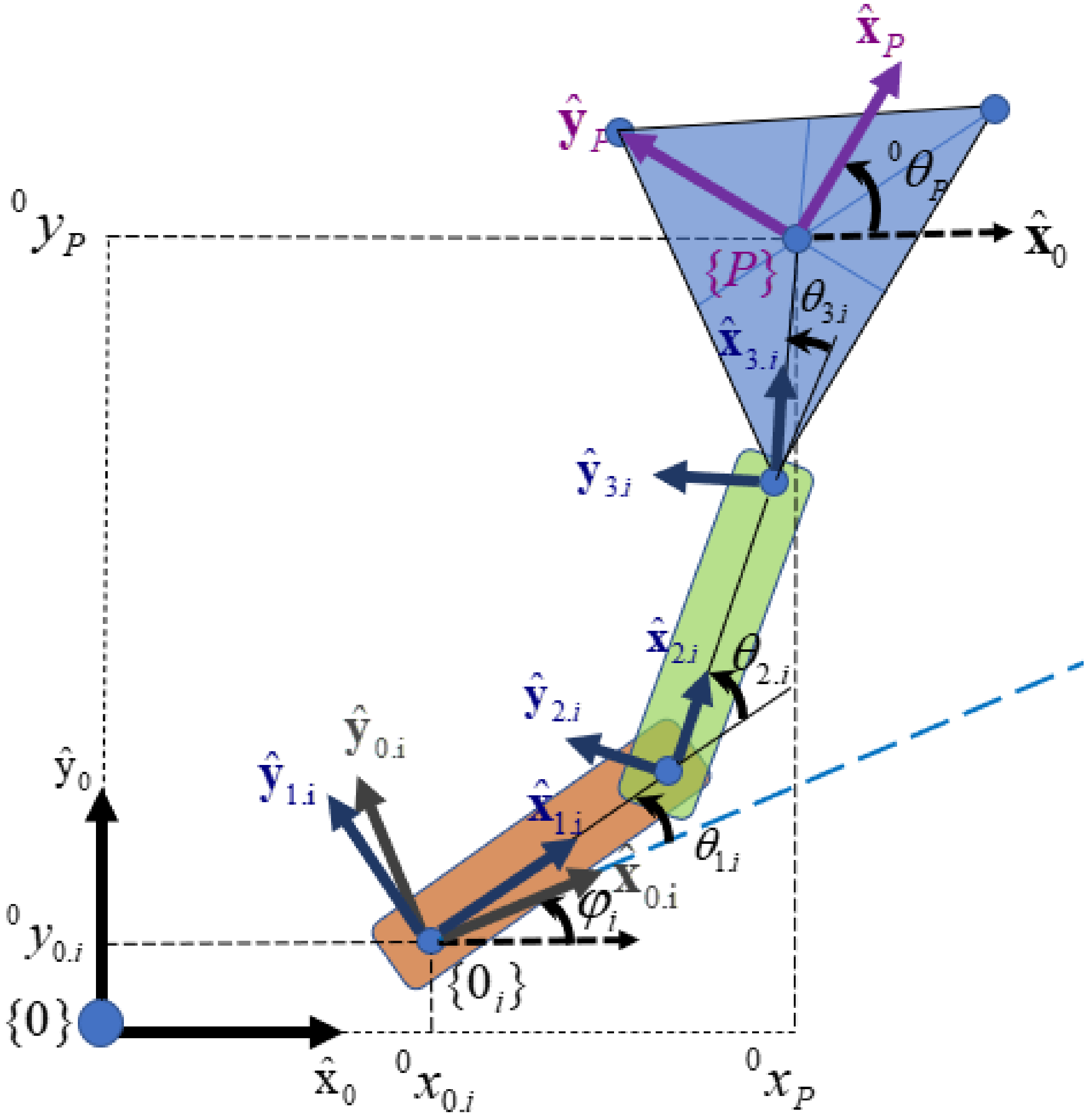
Modelo cinemático de la postura



$${}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\;{}^{0.i}\mathbf{T}_{1.i}\;{}^{1.i}\mathbf{T}_{2.i}\;{}^{2.i}\mathbf{T}_{3.i}\;{}^{3.i}\mathbf{T}_{P.i}$$

$${}^{3.i}\mathbf{T}_{P.i} = \begin{pmatrix} 1 & 0 & 0 & L_{3.i} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la postura



$$ {}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{0}\mathbf{T}_{P.i} = \begin{pmatrix} \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}x_{0.i}+L_{1.i}\cos(\varphi_i+\theta_{1.i})+L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}y_{0.i}+L_{1.i}\sin(\varphi_i+\theta_{1.i})+L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

$$ {}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\, {}^{0.i}\mathbf{T}_{1.i}\, {}^{1.i}\mathbf{T}_{2.i}\, {}^{2.i}\mathbf{T}_{3.i}\, {}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{0}\mathbf{T}_{P.i} = $$

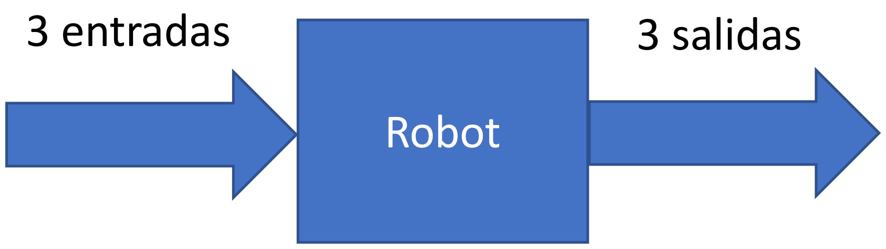
$$ \begin{pmatrix} \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}x_{0.i}+L_{1.i}\cos(\varphi_i+\theta_{1.i})+L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}y_{0.i}+L_{1.i}\sin(\varphi_i+\theta_{1.i})+L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_{P.i}(q) = \begin{pmatrix} {}^{0}x_{0.i}+L_{1,i}\cos(\varphi_i+\theta_{1.i})+L_{2,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ {}^{0}y_{0.i}+L_{1,i}\sin(\varphi_i+\theta_{1.i})+L_{2,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i} \end{pmatrix} $$

Modelo cinemático de la postura

$$
{}^{0}\boldsymbol{\xi}_{P.i}(q)=\begin{pmatrix} {}^{0}x_{0.i}+L_{1,i}\cos(\varphi_i+\theta_{1.i})+L_{2,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ {}^{0}y_{0.i}+L_{1,i}\sin(\varphi_i+\theta_{1.i})+L_{2,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i} \end{pmatrix}
$$

$$
{}^{0}\boldsymbol{\xi}_{P.i}=\begin{pmatrix} {}^{0}x_{P} \\ {}^{0}y_{P} \\ {}^{0}\theta_{P} \end{pmatrix}
$$



$$F(x)$$

$$\mathbf{F}_i(X_i, q_i)={}^{0}\boldsymbol{\xi}_{P.i}-{}^{0}\boldsymbol{\xi}_{P.i}(q_i)=\mathbf{0}$$

$$\mathbf{F}_i(X_i, q_i)={}^{0}\boldsymbol{\xi}_{P.i}(q_i)-{}^{0}\boldsymbol{\xi}_{P.i}=\mathbf{0}$$

Modelo cinemático de la postura

$$\mathbf{F}_i(X_i, q_i) = {}^0\boldsymbol{\xi}_{P.i} - {}^0\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{F}_i(X_i, q_i) = \begin{pmatrix} {}^0x_P - {}^0x_{0.i} - L_{1.i}\cos(\varphi_i + \theta_{1.i}) - L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^0y_P - {}^0y_{0.i} - L_{1.i}\sin(\varphi_i + \theta_{1.i}) - L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^0\theta_P - \varphi_i - \theta_{1.i} - \theta_{2.i} - \theta_{3.i} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{F}(X, q) = \begin{pmatrix} \mathbf{F}_1(X_1, q_1) \\ \mathbf{F}_2(X_2, q_2) \\ \mathbf{F}_3(X_3, q_3) \end{pmatrix} = \begin{pmatrix} {}^{P.1}\boldsymbol{\xi}_0 - {}^{P.1}\boldsymbol{\xi}_0(q_1) \\ {}^{P.2}\boldsymbol{\xi}_0 - {}^{P.2}\boldsymbol{\xi}_0(q_2) \\ {}^{P.3}\boldsymbol{\xi}_0 - {}^{P.3}\boldsymbol{\xi}_0(q_3) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

Modelo cinemático directo de las velocidades

$$ {}^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i) $$

$$ {}^{P.i}\dot{\boldsymbol{\xi}}_{0} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}} $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}\dot{y}_P \\ {}^{0}\dot{\theta}_P \end{pmatrix} \qquad {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i \qquad \dot{\mathbf{q}}_i = \begin{pmatrix} \dot{\theta}_{1.i} \\ \dot{\theta}_{2.i} \\ \dot{\theta}_{3.i} \end{pmatrix} $$

$$ \mathbf{J}_{\theta i}(q_i) = \begin{pmatrix} -L_{1.i}\sin(\varphi_i+\theta_{1.i}) - L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ L_{1.i}\cos(\varphi_i+\theta_{1.i}) + L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 1 & 1 & 1 \end{pmatrix} $$

Modelo cinemático directo de las velocidades

$$^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i)$$

$$^{0}\dot{\boldsymbol{\xi}}_{P.i} = \frac{d}{dt}\,^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}}$$

$$^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i$$

$$^{0}\dot{\boldsymbol{\xi}}_{P} = \begin{pmatrix} ^{0}\dot{\boldsymbol{\xi}}_{P.1} \\ ^{0}\dot{\boldsymbol{\xi}}_{P.2} \\ ^{0}\dot{\boldsymbol{\xi}}_{P.3} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_{\theta.1}(\mathbf{q}_1) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\theta.2}(\mathbf{q}_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\theta.3}(\mathbf{q}_3) \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \\ \dot{\mathbf{q}}_3 \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{F}_i({}^0\mathbf{p}_{P,i}, {}^0\boldsymbol{\theta}_{P,i}, q_i) = {}^0\boldsymbol{\xi}_{P.i} - {}^0\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$${}^0\boldsymbol{\xi}_{P.i} = \begin{pmatrix} {}^0\mathbf{p}_{P,i} \\ {}^0\boldsymbol{\theta}_{P,i} \end{pmatrix}$$

$$\mathbf{C}_{q.i}(\dot{X}_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(\dot{X}_i, q_i, \dot{q}_i) = {}^0\dot{\boldsymbol{\xi}}_{P.i} - {}^0\dot{\boldsymbol{\xi}}_{P,i}(q_i, \dot{q}_i) = \mathbf{0}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(X_i, q_i, \dot{q}_i) =$$

$$\begin{pmatrix} {}^0\dot{x}_P + \text{[redacted]} \\ {}^0\dot{y}_P - \dot{\theta}_{1.i}\left(L_{1.i}\cos(\varphi_i+\theta_{1.i}) + L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i})\right) - \dot{\theta}_{2.i}\left(L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i})\right) - \dot{\theta}_{3.i}L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ {}^0\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \frac{\partial}{\partial\, {}^0x_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\, {}^0y_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\, {}^0\theta_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \frac{\partial}{\partial\, {}^0x_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\, {}^0y_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial\, {}^0\theta_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

$$\dot{\Psi}_{T.i} = \begin{pmatrix} \dot{x}_P \\ y_P \\ \theta_P \\ \theta_{1,i} \\ \theta_{2,i} \\ \theta_{3,i} \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\dot{\Psi}_{T.i} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}y_P \\ {}^{0}\theta_P \\ {}^{0}\theta_{1,i} \\ {}^{0}\theta_{2,i} \\ {}^{0}\theta_{3,i} \end{pmatrix}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) =$$

$$\begin{array}{cccccc} \dot{x}_P & \dot{y}_P & \dot{\theta}_P & \dot{\theta}_{1.i} & \dot{\theta}_{2.i} & \dot{\theta}_{3.i} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & L_{1,i}\sin(\varphi_i + \theta_{1,i}) + L_{2,i}\sin(\varphi_i + \theta_{1,i} + \theta_{2,i}) + L_{3,i}\sin(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) & L_{2,i}\sin(\varphi_i + \theta_{1,i} + \theta_{2,i}) + L_{3,i}\sin(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) & L_{3,i}\sin(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) \\ 0 & 1 & 0 & -L_{1,i}\cos(\varphi_i + \theta_{1,i}) - L_{2,i}\cos(\varphi_i + \theta_{1,i} + \theta_{2,i}) - L_{3,i}\cos(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) & -L_{2,i}\cos(\varphi_i + \theta_{1,i} + \theta_{2,i}) - L_{3,i}\cos(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) & -L_{3,i}\cos(\varphi_i + \theta_{1,i} + \theta_{2,i} + \theta_{3,i}) \\ 0 & 0 & 1 & -1 & -1 & -1 \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\begin{aligned}&{}^{0}\dot{x}_P + \dot{\theta}_{1.i}(L_{1.i}\sin(\varphi_i + \theta_{1.i}) + L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{2.i}((L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \\ &+L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{3.i}L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0\end{aligned}$$

$$\begin{aligned}&{}^{0}\dot{y}_P - \dot{\theta}_{1.i}(L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{2.i}(L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \\ &+L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{3.i}L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0\end{aligned}$$

$$\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} = 0$$

Modelo cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\dot{\theta}_{1.i} = \left(\frac{\cos(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{3.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{2.\mathrm{i}} = \left(-\frac{L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1,i}\cos(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(-\frac{L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1.i}\sin(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2,i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P +$$
$$+\left(-\frac{L_{1.i}L_{3.i}\sin(\theta_{2.i} + \theta_{2.i}) + L_{2.i}L_{3.i}\cos(\theta_{3.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{3.\mathrm{i}} = \left(\frac{\cos(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{2.i} + \theta_{3.i}) + L_{2.i}\sin(\theta_{2.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$