Robótica grupo2
Clase 13

Facultad de Ingeniería UNAM

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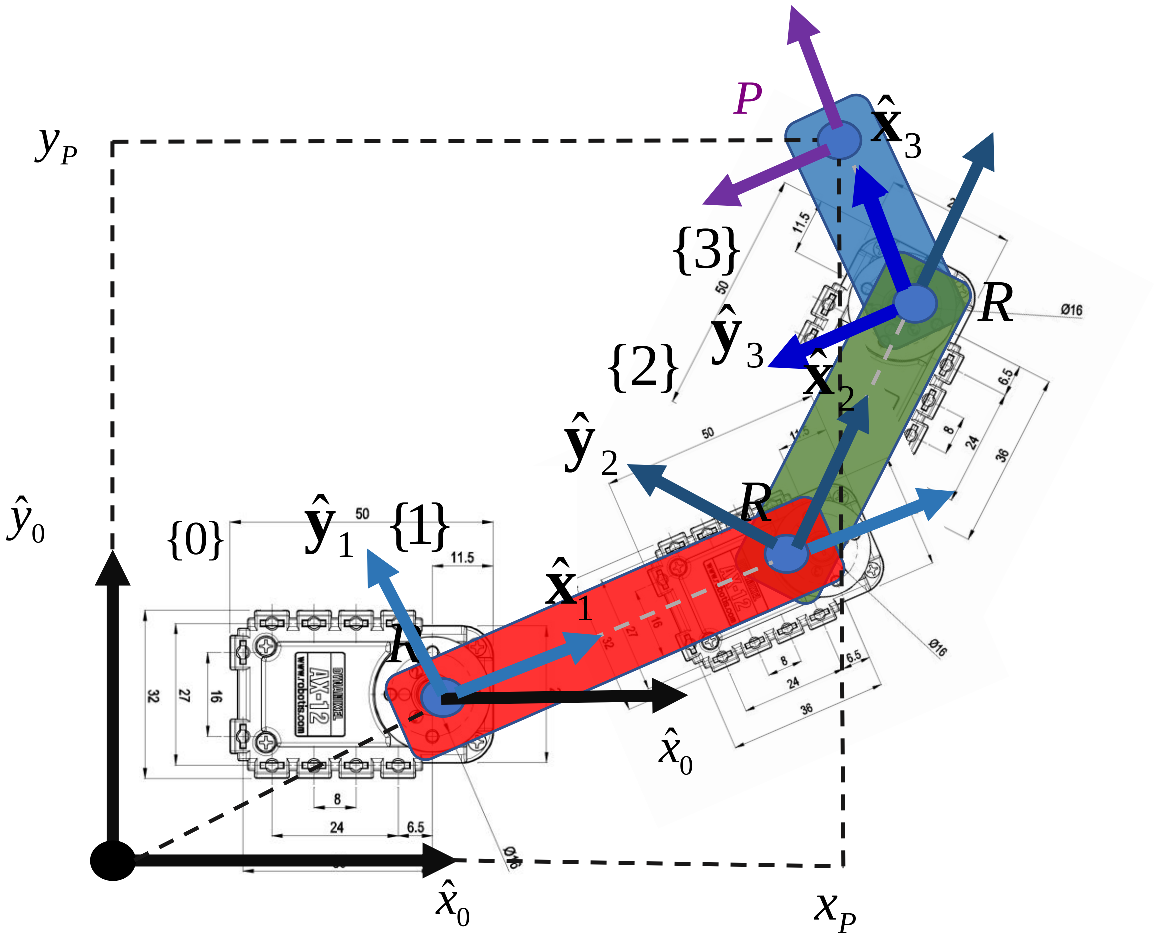
Derechos reservados

Repaso/Elemento base (caso de estudio)

- Repaso de la clases anteriores
  - Planteamiento del modelado del elementos base
    - Planteamiento del modelo de la postura
      - Transformaciones homogéneas
      - Composición de transformaciones
    - Planteamiento del modelo cinemático de las velocidades
    - Planteamiento del modelo dinámico
    - Plantemiento dinámico
- Planteamiento del elemento base en la robótica

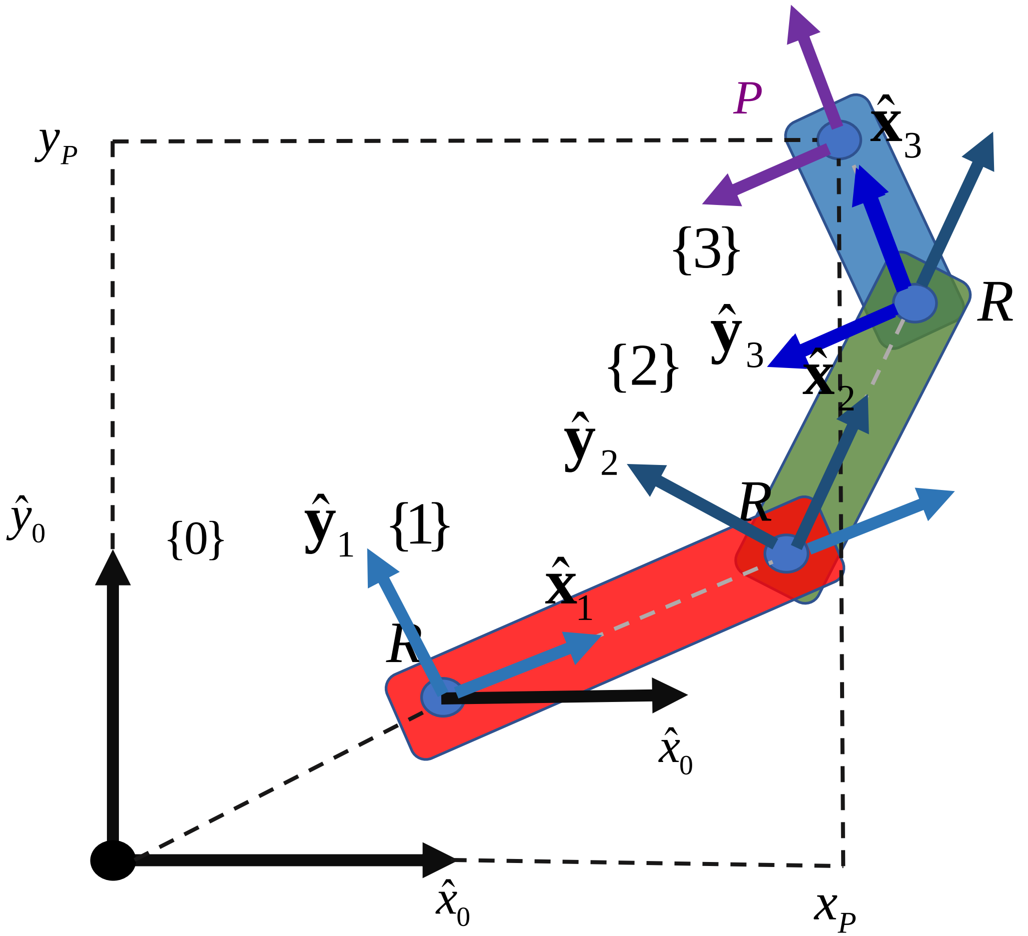
Modelo cinemático de la postura



$$
{}^{i}\mathbf{T}_{j}({}^{i}\alpha_{j}, {}^{i}x_{j}, {}^{i}y_{j}) = \begin{pmatrix} {}^{i}\mathbf{R}_{j} & {}^{i}\mathbf{p}_{j} \\ \mathbf{0}^{T} & 1 \end{pmatrix} =
$$

$$
= \begin{pmatrix} \cos({}^{i}\theta_{j}) & -\sin({}^{i}\theta_{j}) & 0 & {}^{i}x_{j} \\ \sin({}^{i}\theta_{j}) & \cos({}^{i}\theta_{j}) & 0 & {}^{i}y_{j} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

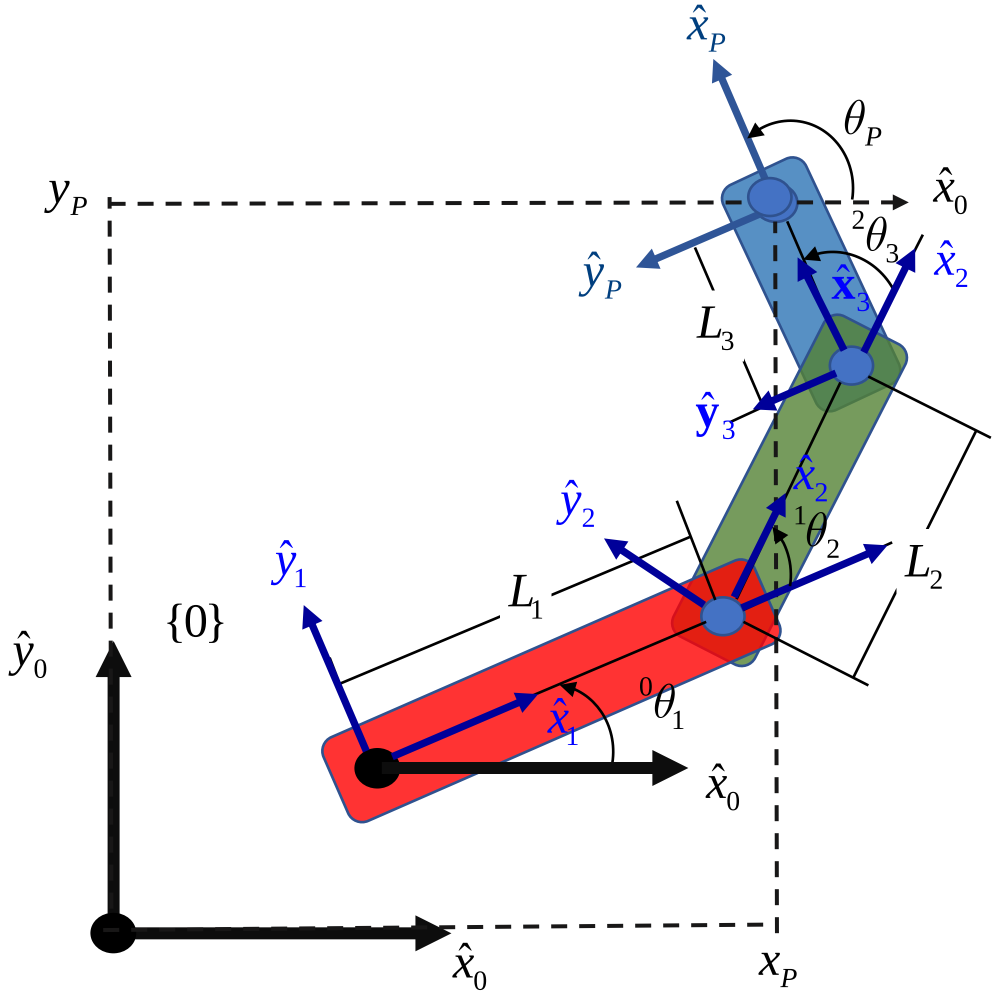
Modelo cinemático de la postura



$$ {}^{i}\mathbf{T}_j({}^{i}\alpha_j, {}^{i}x_j, {}^{i}y_j) = \begin{pmatrix} {}^{i}\mathbf{R}_j & {}^{i}\mathbf{p}_j \\ \mathbf{0}^T & 1 \end{pmatrix} = $$

$$ = \begin{pmatrix} \cos({}^{i}\theta_j) & -\sin({}^{i}\theta_j) & 0 & {}^{i}x_j \\ \sin({}^{i}\theta_j) & \cos({}^{i}\theta_j) & 0 & {}^{i}y_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Elemento base de la robótica (robot RRR)



$$
{}^0\xi_P = \begin{pmatrix} x_P \\ y_P \\ \theta_P \end{pmatrix}
$$

n grados de libertad de un robot

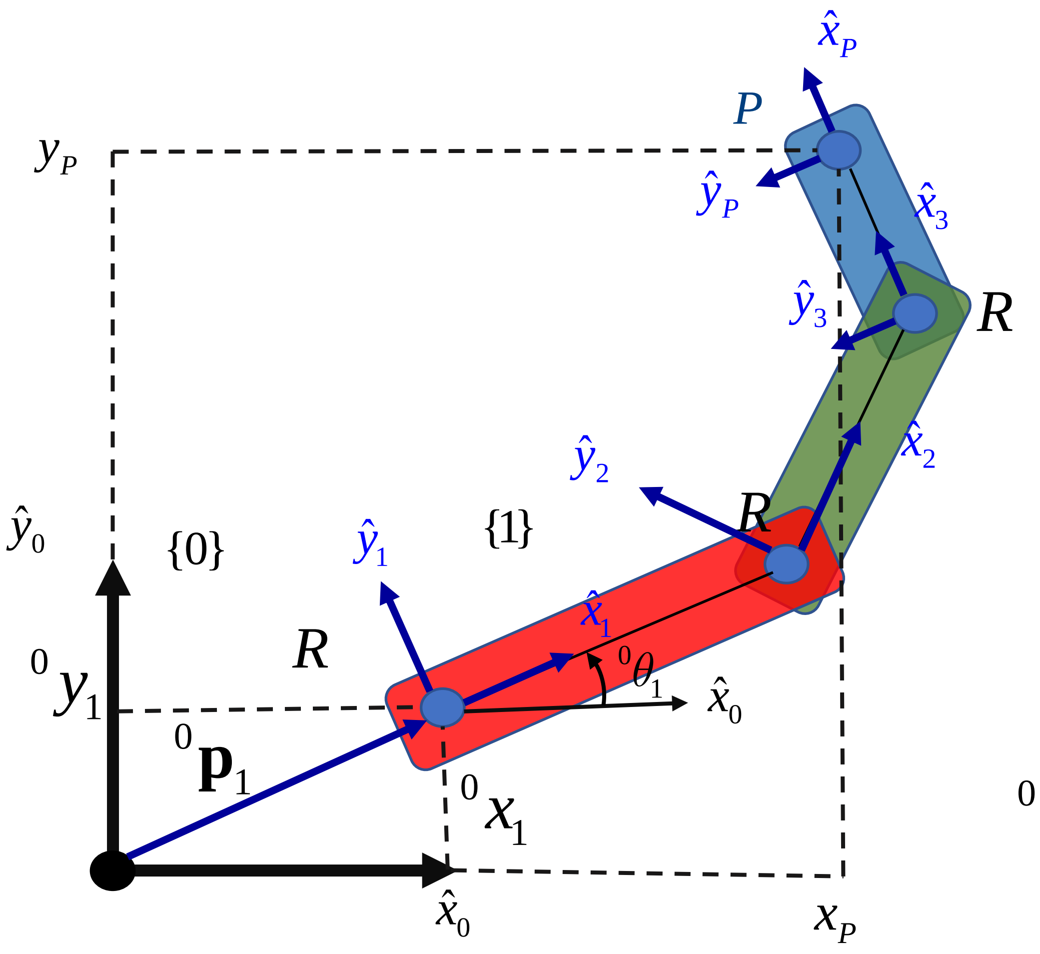
m grados de libertad del espacio de trabajo

n < m , robot subactuado

n = m , robot definido

n > m , robot sobreactuado o redundante

Modelo cinemático de la postura

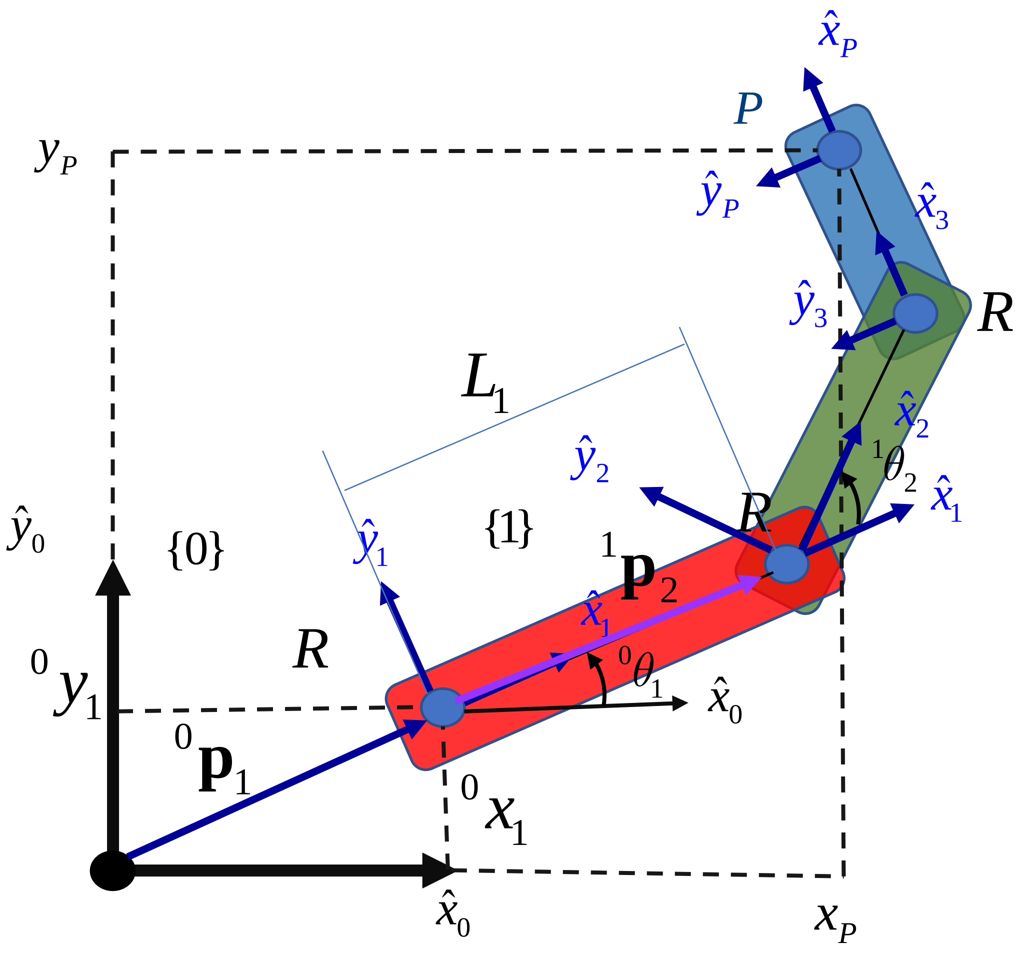


$$
{}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
{}^0\mathbf{p}_1 = \begin{pmatrix} {}^0x_1 \\ {}^0y_1 \\ 0 \end{pmatrix} \quad {}^0R_1 = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

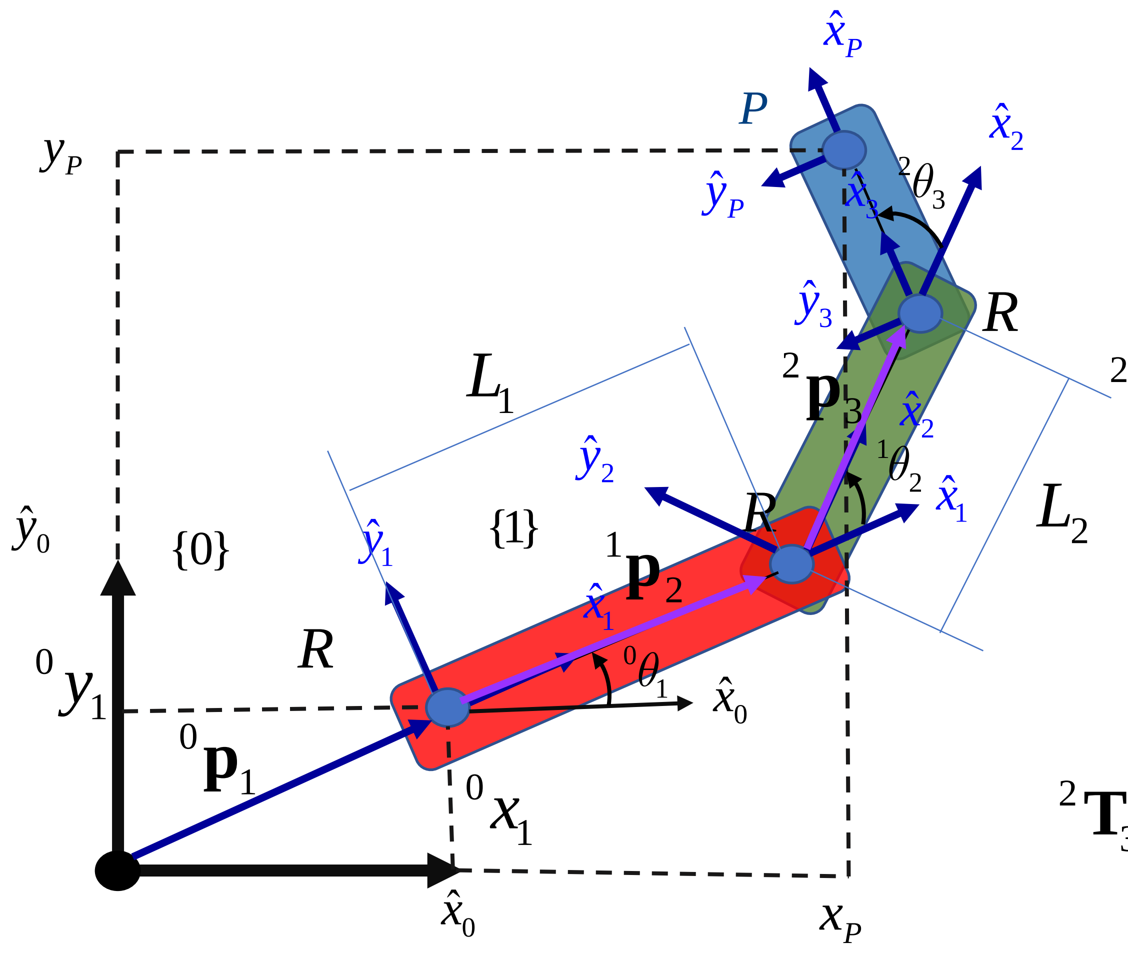


$$ {}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} = $$

$$ {}^1\mathbf{p}_2 = \begin{pmatrix} L_1 \\ 0 \\ 0 \end{pmatrix} \qquad {}^1R_2 = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

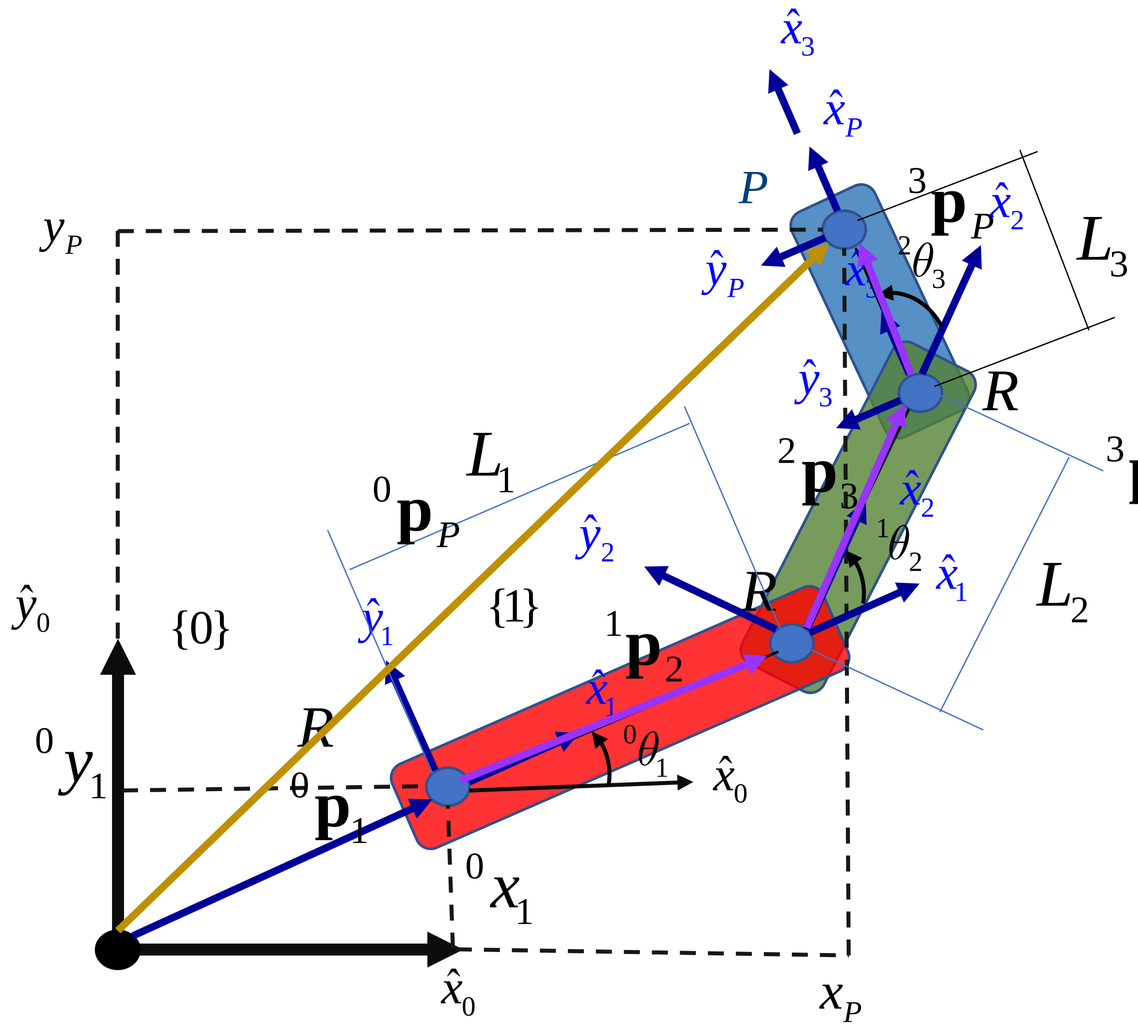


$$
{}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
{}^2\mathbf{p}_3 = \begin{pmatrix} L_2 \\ 0 \\ 0 \end{pmatrix} \quad {}^2R_3 = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

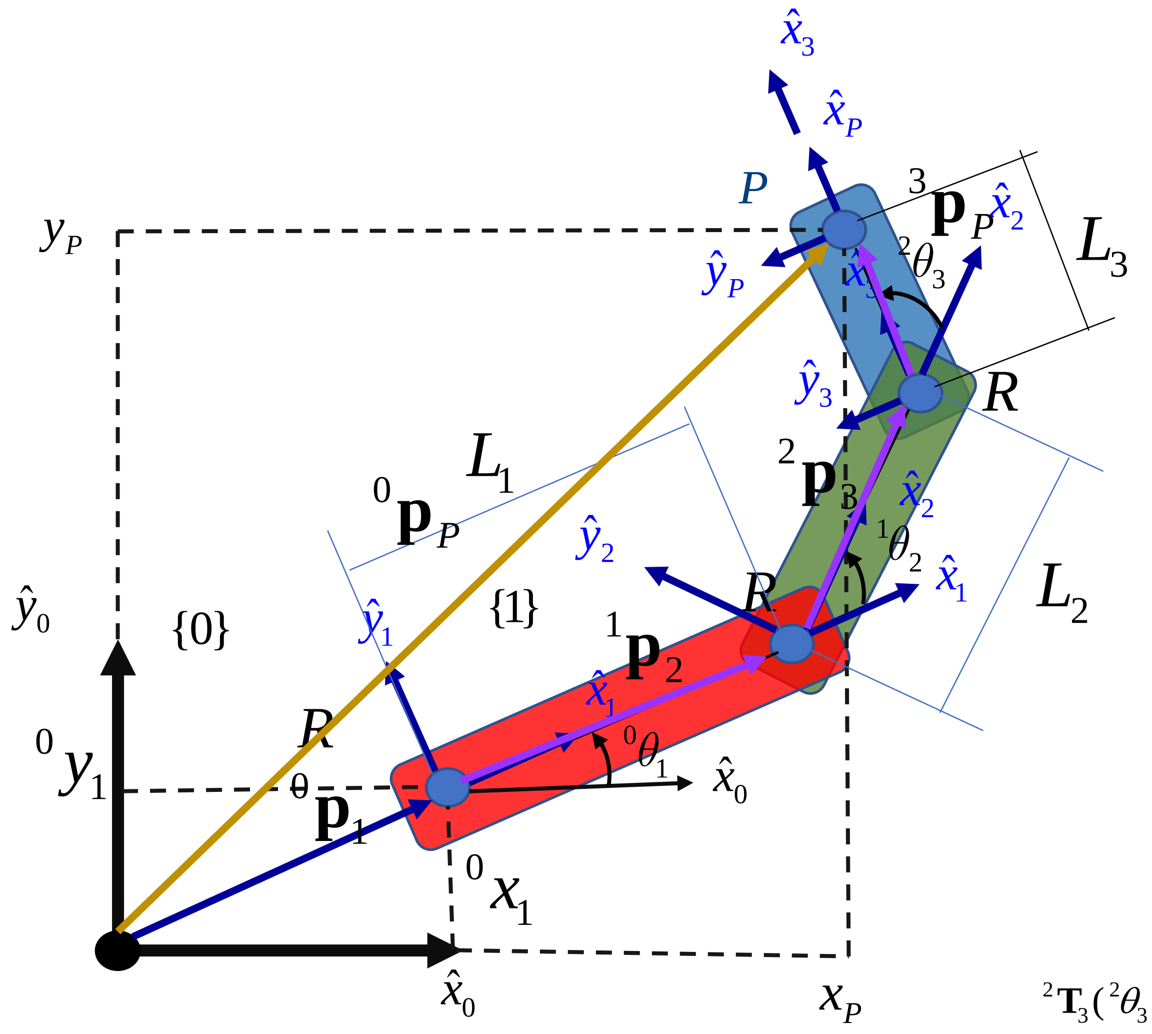


$$
{}^0\mathbf{T}_1({}^i\alpha_j, {}^ix_j, {}^iy_j) = \begin{pmatrix} {}^0\mathbf{R}_1 & {}^0\mathbf{p}_1 \\ \mathbf{0}^T & 1 \end{pmatrix} =
$$

$$
{}^3\mathbf{p}_P = \begin{pmatrix} L_3 \\ 0 \\ 0 \end{pmatrix}
\qquad
{}^2R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura



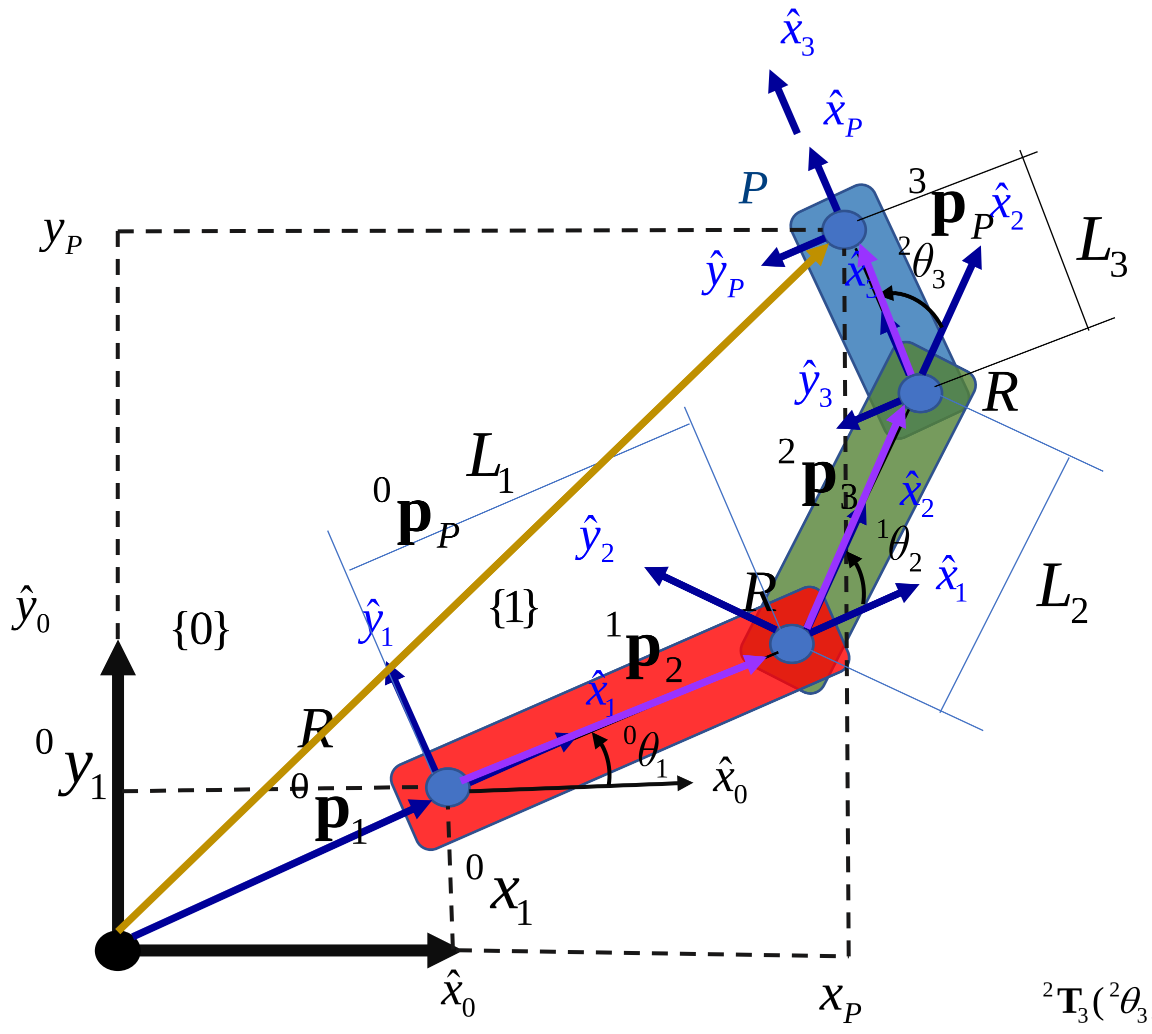
$$
{}^0\mathbf{T}_P = \begin{pmatrix} {}^0\mathbf{R}_P & {}^0\mathbf{p}_P \\ \mathbf{0}^T & 1 \end{pmatrix} = {}^0\mathbf{T}_1 \, {}^1\mathbf{T}_2 \, {}^2\mathbf{T}_3 \, {}^3\mathbf{T}_P
$$

$$
{}^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\qquad
{}^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura



$$
{}^0\mathbf{T}_P = \begin{pmatrix} {}^0\mathbf{R}_P & {}^0\mathbf{p}_P \\ \mathbf{0}^T & 1 \end{pmatrix} = {}^0\mathbf{T}_{\cancel{1}}\,{}^{\cancel{1}}\mathbf{T}_{\cancel{2}}\,{}^{\cancel{2}}\mathbf{T}_{\cancel{3}}\,{}^{\cancel{3}}\mathbf{T}_P
$$

$$
{}^0\mathbf{T}_1({}^0\theta_1, {}^0x_1, {}^0y_1) = \begin{pmatrix} \cos({}^0\theta_1) & -\sin({}^0\theta_1) & 0 & {}^0x_1 \\ \sin({}^0\theta_1) & \cos({}^0\theta_1) & 0 & {}^0y_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^1\mathbf{T}_2({}^1\theta_2, L_1, 0) = \begin{pmatrix} \cos({}^1\theta_2) & -\sin({}^1\theta_2) & 0 & L_1 \\ \sin({}^1\theta_2) & \cos({}^1\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2\mathbf{T}_3({}^2\theta_3, L_2, 0) = \begin{pmatrix} \cos({}^2\theta_3) & -\sin({}^2\theta_3) & 0 & L_2 \\ \sin({}^2\theta_3) & \cos({}^2\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\qquad
{}^3\mathbf{T}_P(0, L_3, 0) = \begin{pmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Modelo cinemático de la postura

$$ {}^{0}\mathbf{T}_P = {}^{0}\mathbf{T}_1\,{}^{1}\mathbf{T}_2\,{}^{2}\mathbf{T}_3\,{}^{3}\mathbf{T}_P = \begin{pmatrix} {}^{0}\mathbf{R}_P & {}^{0}\mathbf{p}_P \\ \mathbf{0}^T & 1 \end{pmatrix} = $$

$$ = \begin{pmatrix} \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & -\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ \sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & \cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) & 0 & {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ q = \{{}^{0}\theta_1, {}^{1}\theta_2, {}^{2}\theta_3\} $$

$$ \mathbf{q}^T = \begin{pmatrix} {}^{0}\theta_1 & {}^{1}\theta_2 & {}^{2}\theta_3 \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P(q) = \begin{pmatrix} {}^{0}\mathbf{p}_P \\ {}^{0}\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3 \end{pmatrix} \qquad {}^{0}\boldsymbol{\xi}_P = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix} $$

Modelo cinemático de la postura

$$ {}^{0}\boldsymbol{\xi}_P = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P(q) = \begin{pmatrix} {}^{0}\mathbf{p}_P \\ {}^{0}\boldsymbol{\theta}_P \end{pmatrix} = \begin{pmatrix} {}^{0}x_1 + L_1\cos({}^{0}\theta_1) + L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_1 + L_1\sin({}^{0}\theta_1) + L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) + L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3 \end{pmatrix} $$

$$ \mathbf{F} = {}^{0}\boldsymbol{\xi}_P - {}^{0}\boldsymbol{\xi}_P(q) = \mathbf{0} = \begin{pmatrix} {}^{0}x_P - {}^{0}x_1 - L_1\cos({}^{0}\theta_1) - L_2\cos({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\cos({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}y_P - {}^{0}y_1 - L_1\sin({}^{0}\theta_1) - L_2\sin({}^{0}\theta_1 + {}^{1}\theta_2) - L_3\sin({}^{0}\theta_1 + {}^{1}\theta_2 + {}^{2}\theta_3) \\ {}^{0}\theta_P - {}^{0}\theta_1 - {}^{1}\theta_2 - {}^{2}\theta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_P = {}^{0}\boldsymbol{\xi}_P(q) $$

Modelo cinemático directo de las velocidades

Modelo de la postura

$$ {}^{0}\boldsymbol{\xi}_{P} = {}^{0}\boldsymbol{\xi}_{P}(q) $$

$$ \frac{d}{dt}\,{}^{0}\xi_{P} = \frac{d}{dt}\begin{pmatrix} {}^{0}x_{P} \\ {}^{0}y_{P} \\ {}^{0}\theta_{P} \end{pmatrix} = \frac{\partial}{\partial\,{}^{0}x_{P}}\begin{pmatrix} {}^{0}x_{P} \\ {}^{0}y_{P} \\ {}^{0}\theta_{P} \end{pmatrix}{}^{0}\dot{x}_{P} + \frac{\partial}{\partial\,{}^{0}y_{P}}\begin{pmatrix} {}^{0}x_{P} \\ {}^{0}y_{P} \\ {}^{0}\theta_{P} \end{pmatrix}{}^{0}\dot{y}_{P} + \frac{\partial}{\partial\,{}^{0}\theta_{P}}\begin{pmatrix} {}^{0}x_{P} \\ {}^{0}y_{P} \\ {}^{0}\theta_{P} \end{pmatrix}{}^{0}\dot{\theta}_{P} = $$

$$ = \begin{pmatrix} {}^{0}\dot{x}_{P} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ {}^{0}\dot{y}_{P} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ {}^{0}\dot{\theta}_{P} \end{pmatrix} = \begin{pmatrix} {}^{0}\dot{x}_{P} \\ {}^{0}\dot{y}_{P} \\ {}^{0}\dot{\theta}_{P} \end{pmatrix} $$

Derivada del modelo

$$ \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P}(q) $$

$$ {}^{0}\dot{\xi}_{P} = \begin{pmatrix} {}^{0}\dot{x}_{P} \\ {}^{0}\dot{y}_{P} \\ {}^{0}\dot{\theta}_{P} \end{pmatrix} $$

Modelo cinemático directo de las velocidades

$$ \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P}(q) = \frac{\partial}{\partial\,{}^{0}\theta_{1}}\,{}^{0}\boldsymbol{\xi}_{P}(q)\,{}^{0}\dot{\theta}_{1} + \frac{\partial}{\partial\,{}^{1}\theta_{2}}\,{}^{0}\boldsymbol{\xi}_{P}(q)\,{}^{1}\dot{\theta}_{2} + \frac{\partial}{\partial\,{}^{2}\theta_{3}}\,{}^{0}\boldsymbol{\xi}_{P}(q)\,{}^{2}\dot{\theta}_{3} = $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}} $$

$$ = \begin{pmatrix} \frac{\partial}{\partial\,{}^{0}\theta_{1}}\,{}^{0}\boldsymbol{\xi}_{P}(q) & \frac{\partial}{\partial\,{}^{1}\theta_{2}}\,{}^{0}\boldsymbol{\xi}_{P}(q) & \frac{\partial}{\partial\,{}^{2}\theta_{3}}\,{}^{0}\boldsymbol{\xi}_{P}(q) \end{pmatrix}\begin{pmatrix} {}^{0}\dot{\theta}_{1} \\ {}^{1}\dot{\theta}_{2} \\ {}^{2}\dot{\theta}_{3} \end{pmatrix} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}} $$

Modelo cinemático directo de las velocidades

Modelo cinemático directo de las velocidades

$$^{0}\dot{\boldsymbol{\xi}}_{P} = \mathbf{J}_{\theta}(q)\dot{\mathbf{q}}$$

Modelo cinemático inverso de las velocidades

$$\dot{\mathbf{q}} = \mathbf{J}_{\theta}{}^{-1}(q)\,^{0}\dot{\boldsymbol{\xi}}_{P}$$

Propagación de velocidades