Robótica grupo2
Clase 26

Facultad de Ingeniería UNAM

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Derechos reservados

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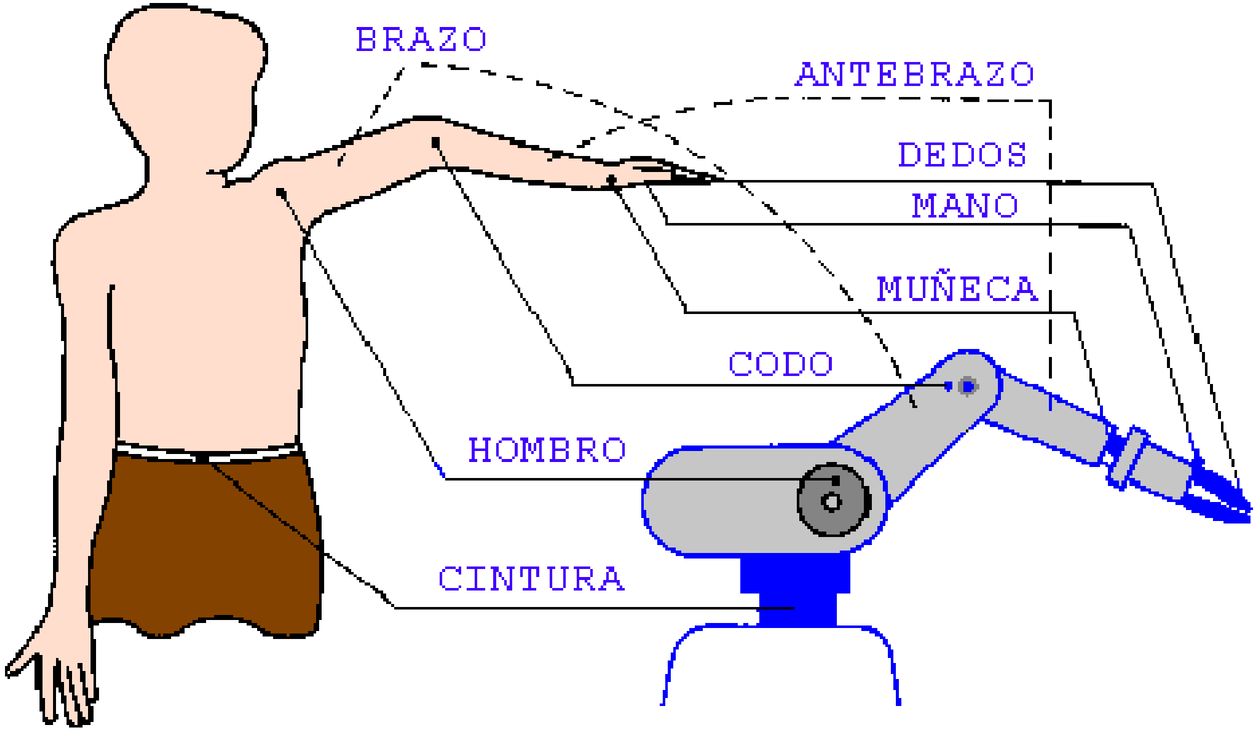
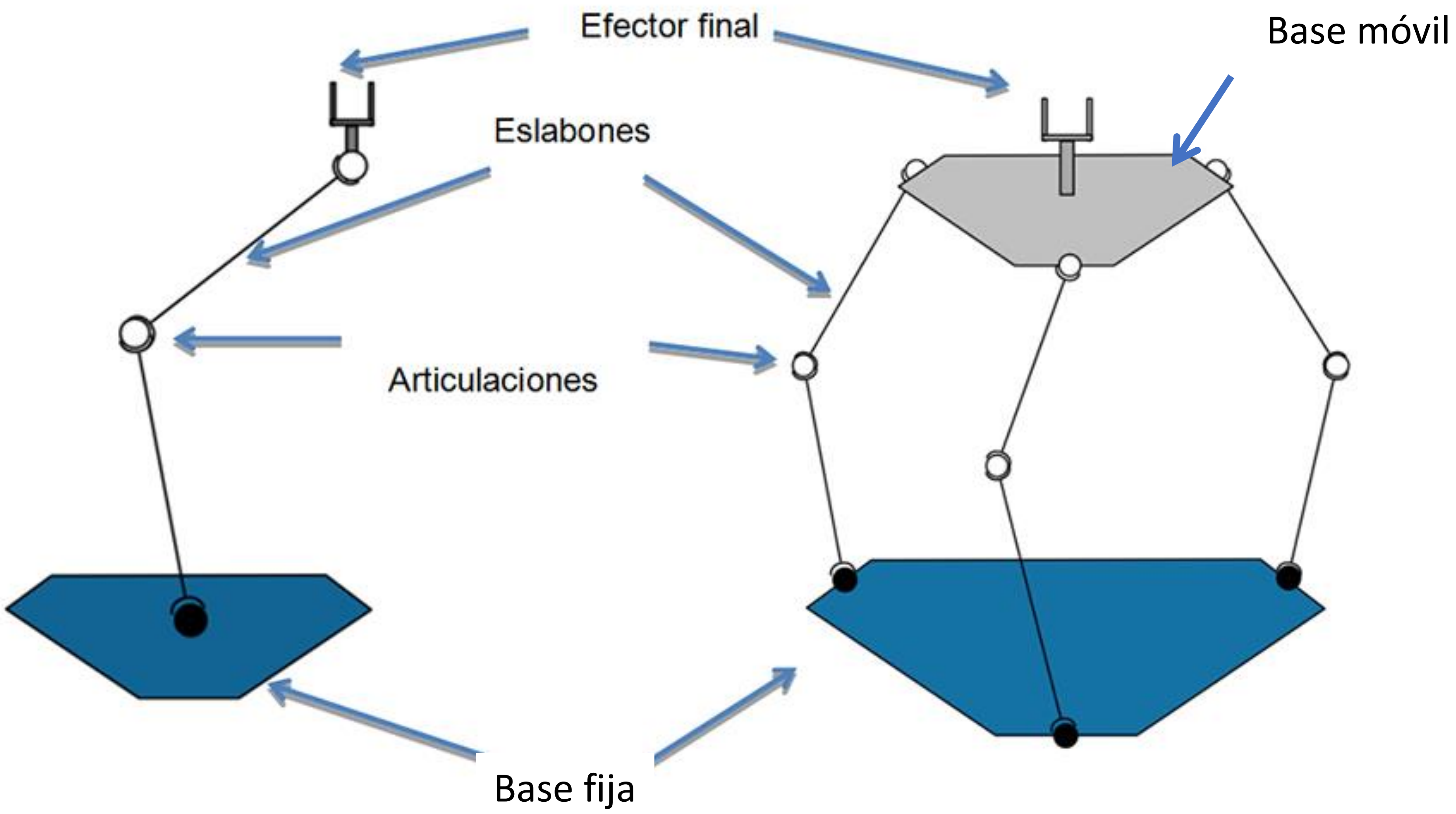
Robótica paralela

- Definición de un robot paralelo
- Modelo postura de un robot paralelo
- Modelo cinemático de un robot paralelo
  - Modelo cinemático directo de las velocidades
  - Modelo cinemático inverso de las velocidades
- Modelo dinámico de un robot paralelo

Definición de un robot paralelo



Definición de un robot paralelo



Definición de un robot paralelo

Robot serial

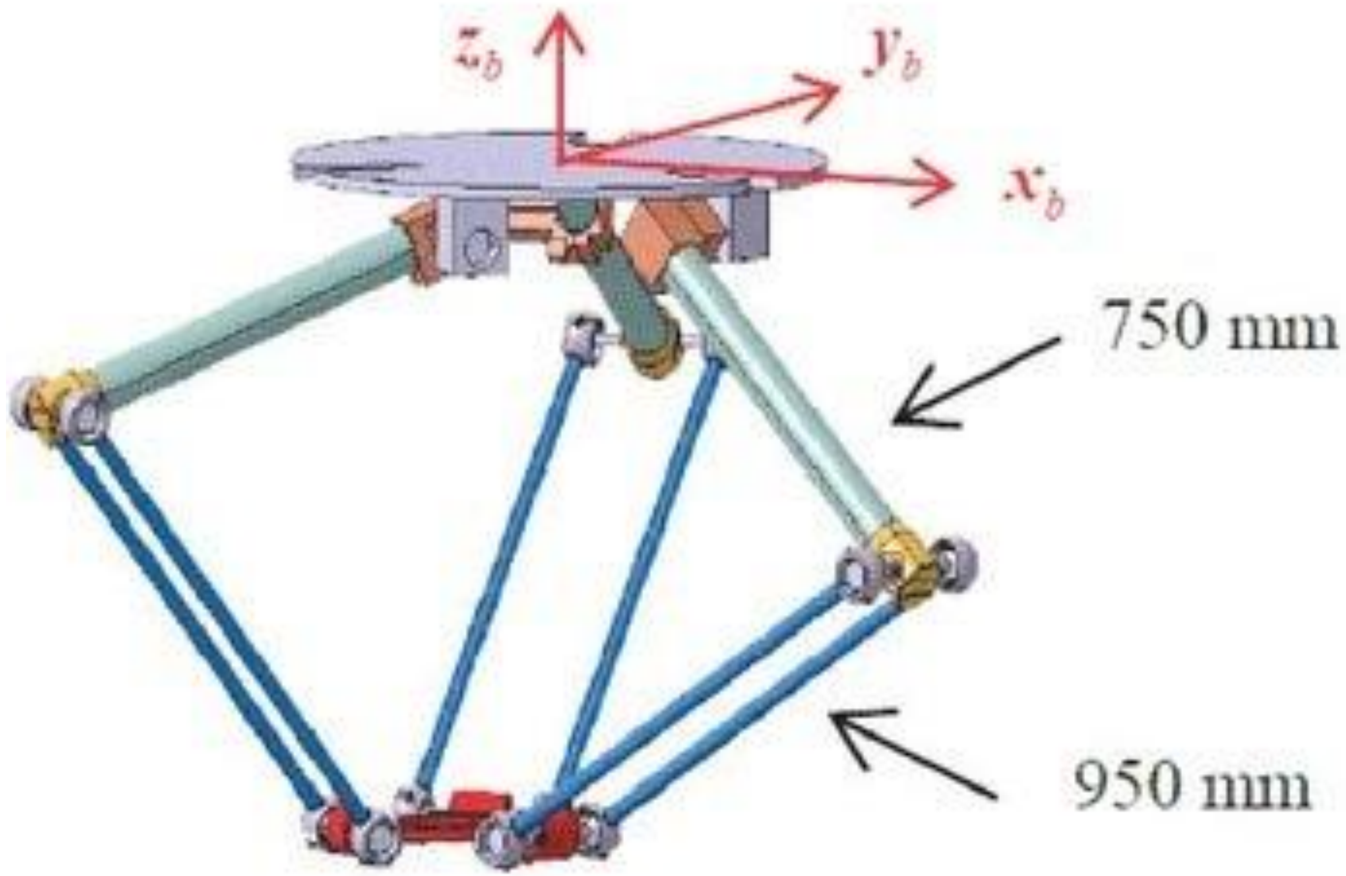
Un robot serial esta compuesto por una cadena cinemática “abierta”, donde cada una de sus juntas es actuada con el fin de mover un efector final con el fin de realizar una tarea.

Definición de un robot paralelo

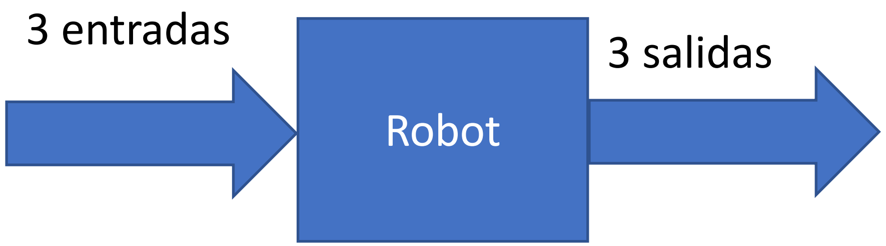
Robot paralelo

Un robot paralelo es un robot serial compuesto por un arreglo de cadenas cinemáticas las cuales comparten una referencia inercial (base fija) en común y una referencia en común relacionada con su efector final (base móvil), para este tipo de configuraciones no todas sus juntas son actuadas.

Definición de un robot paralelo



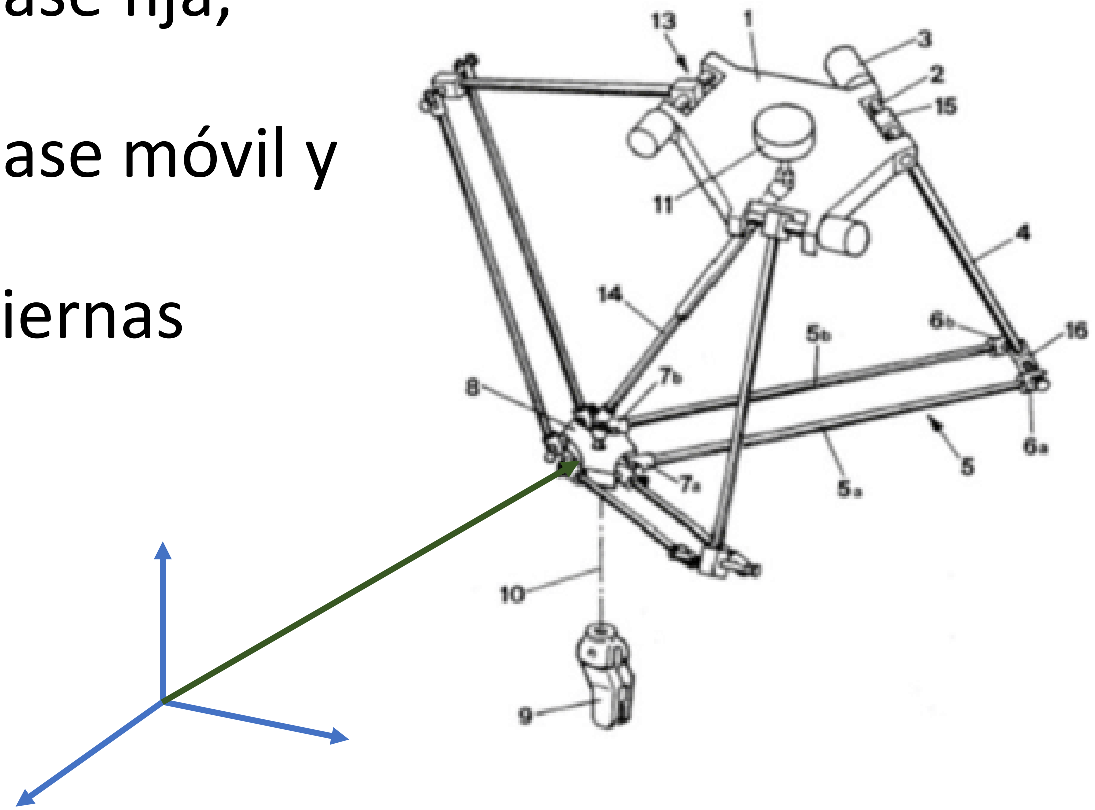
Sistemas MIMO



$$F(x)$$

Robot paralelo

- Base fija,
- Base móvil y
- Piernas



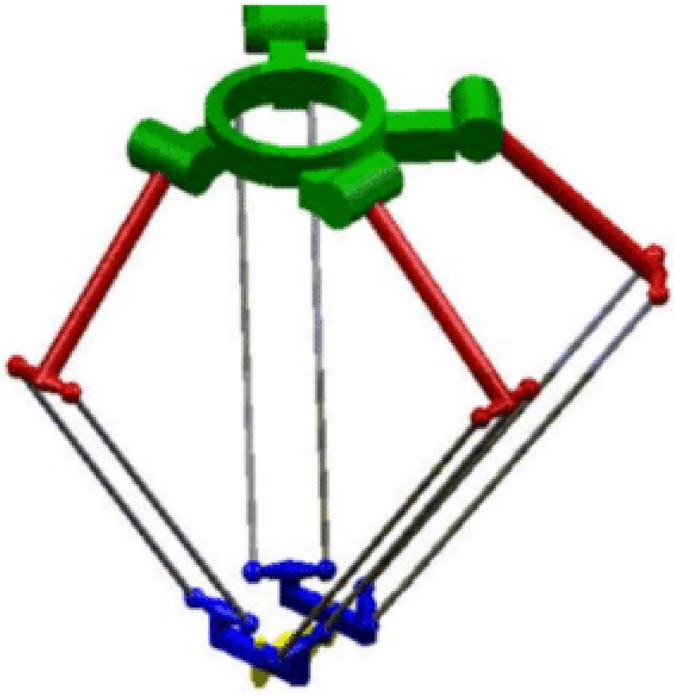
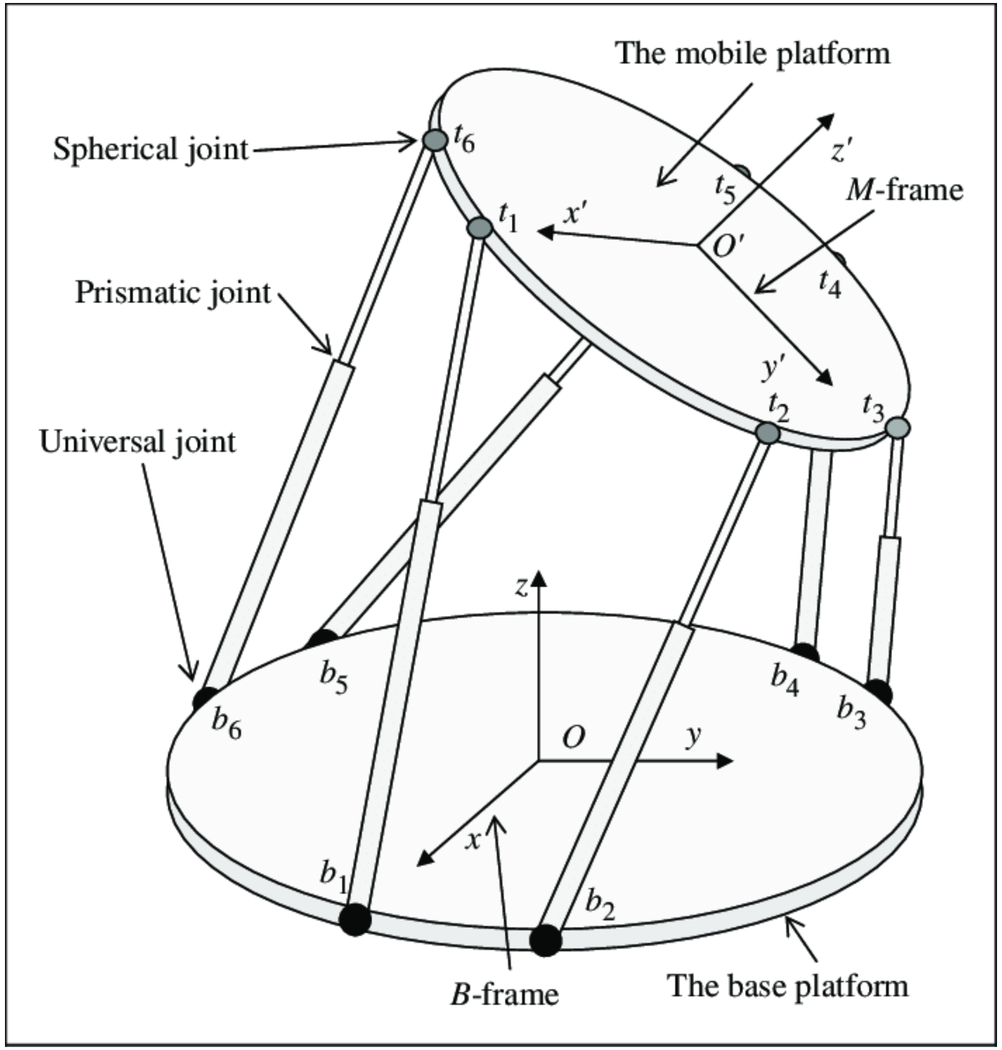


Figura 2. Robot paralelo de 4 grados de libertad

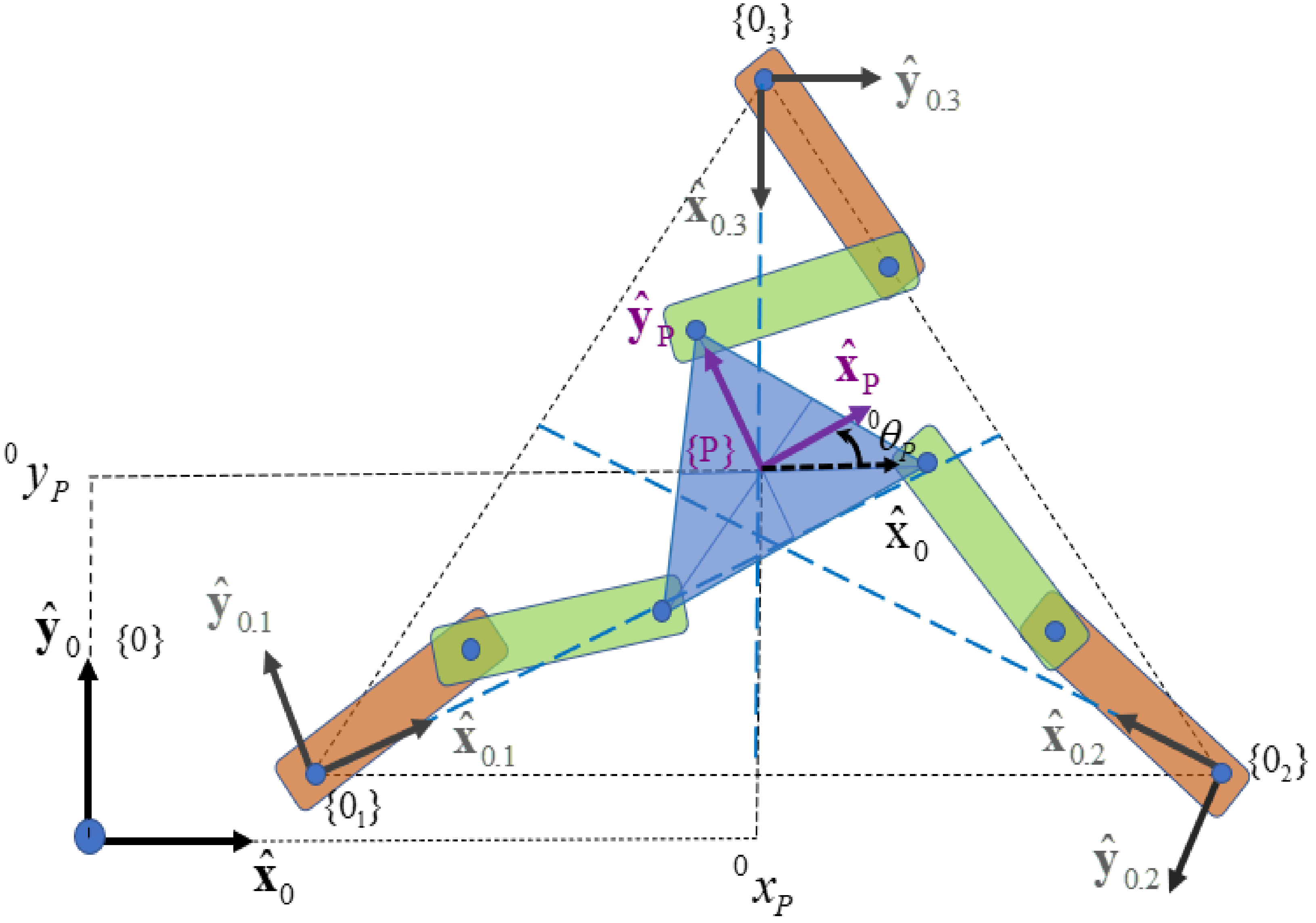
Definición de un robot paralelo



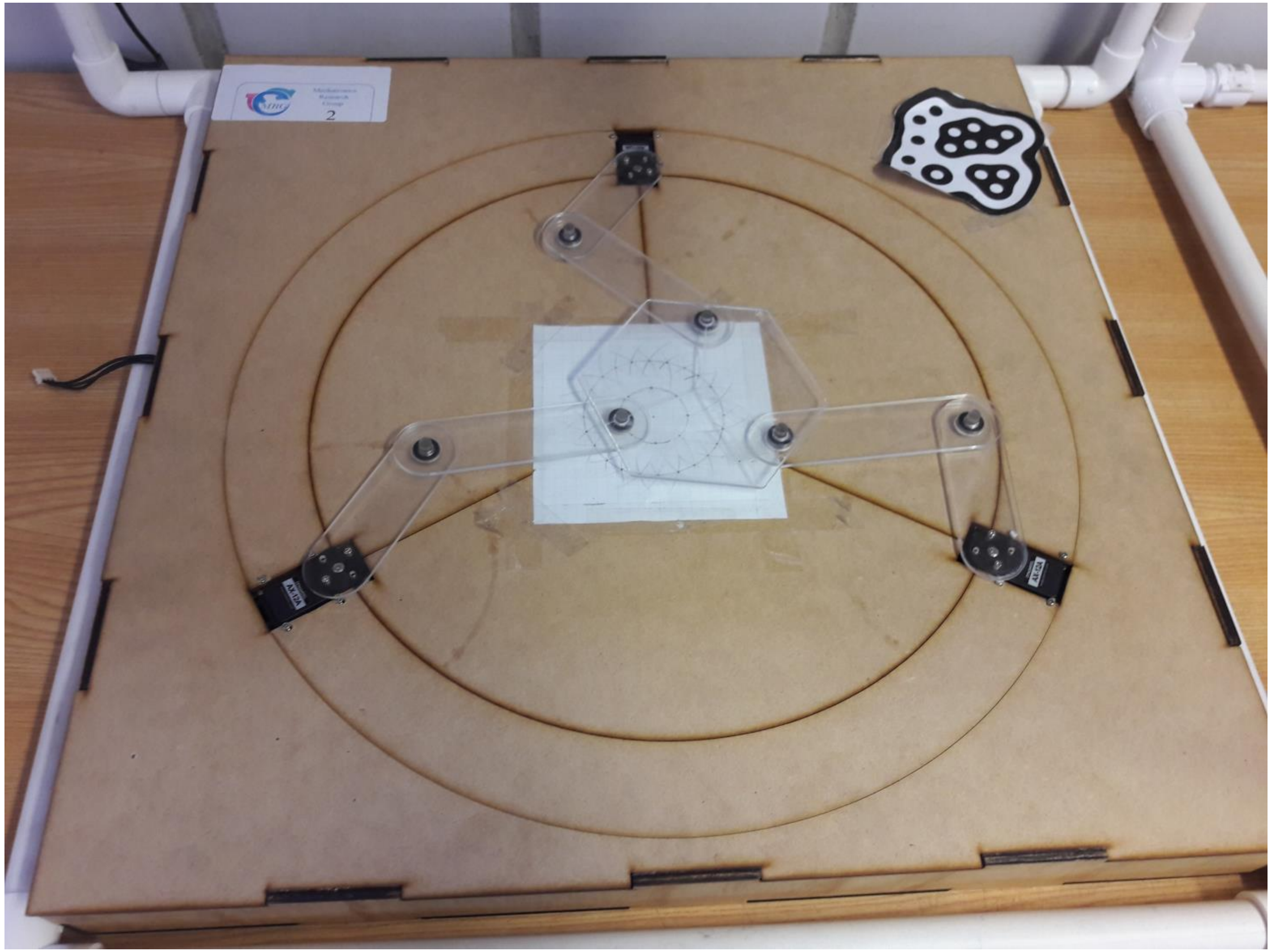
Definición de un robot paralelo

Modelo cinemático de la postura

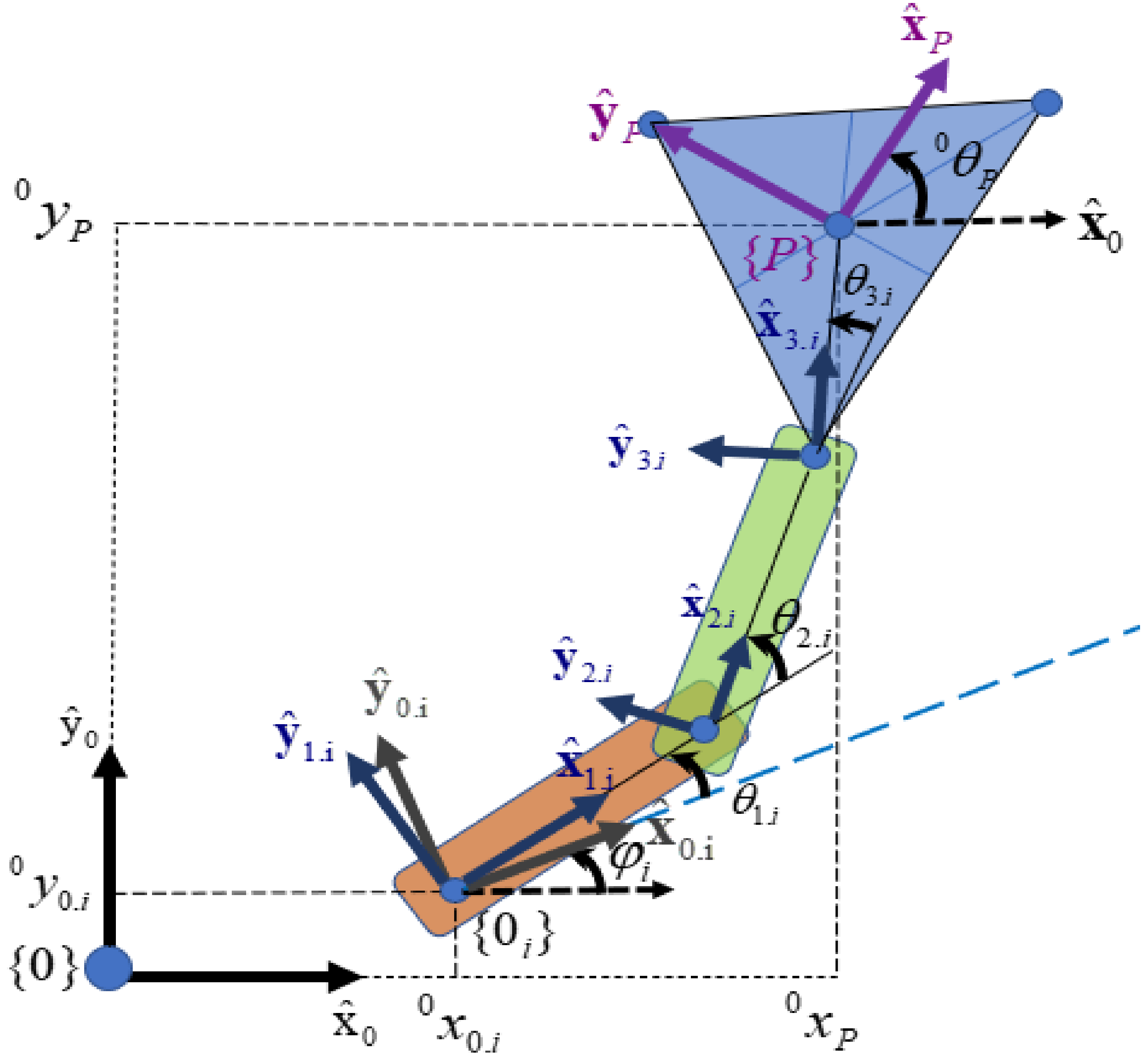
Delta plano

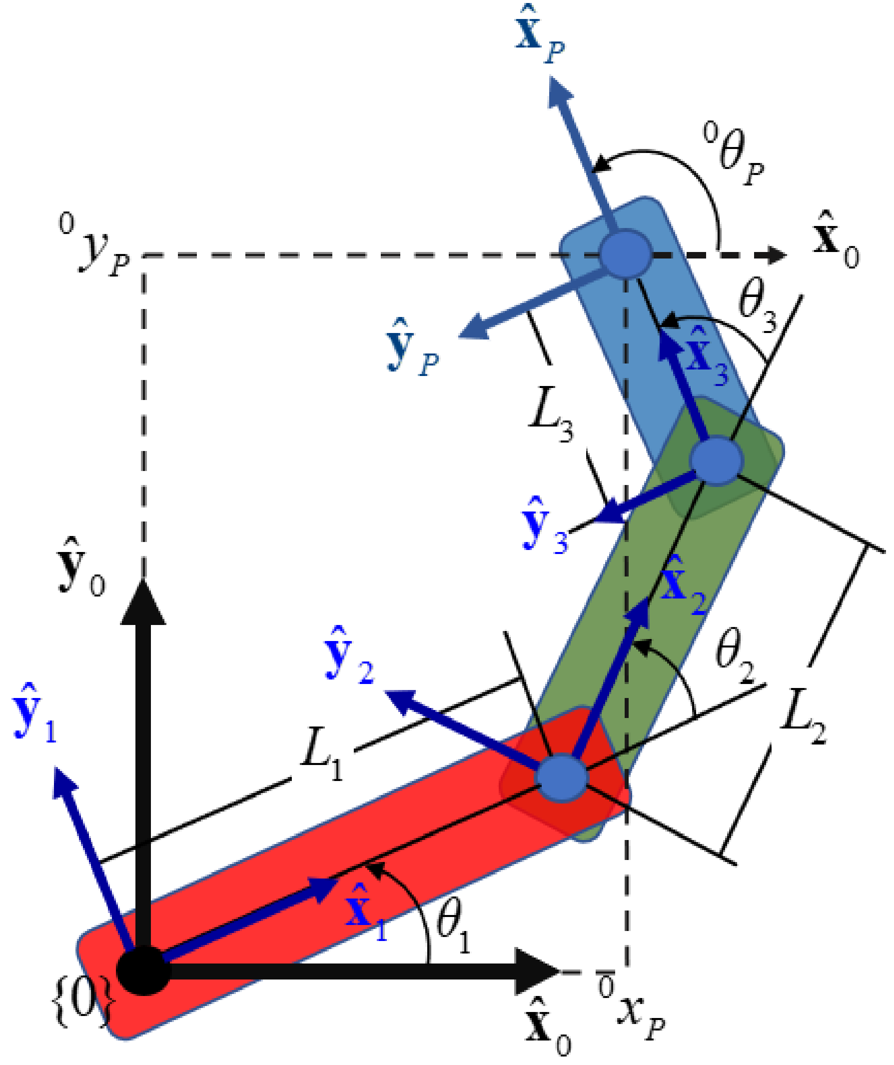


Modelo cinemático de la postura

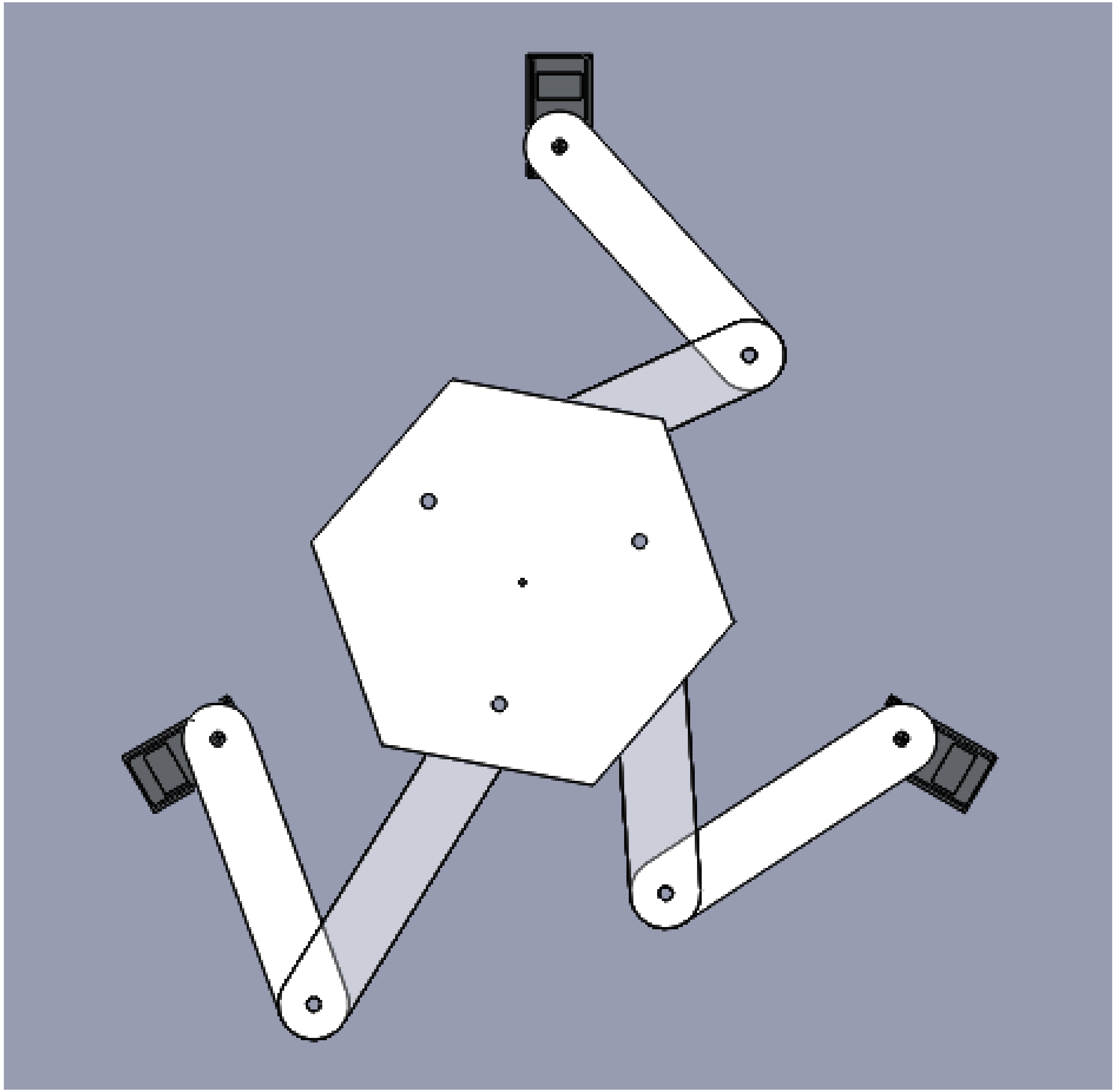


Modelo cinemático de la postura



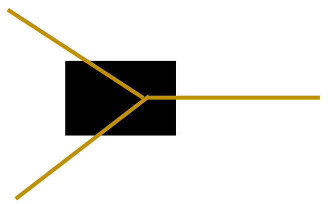


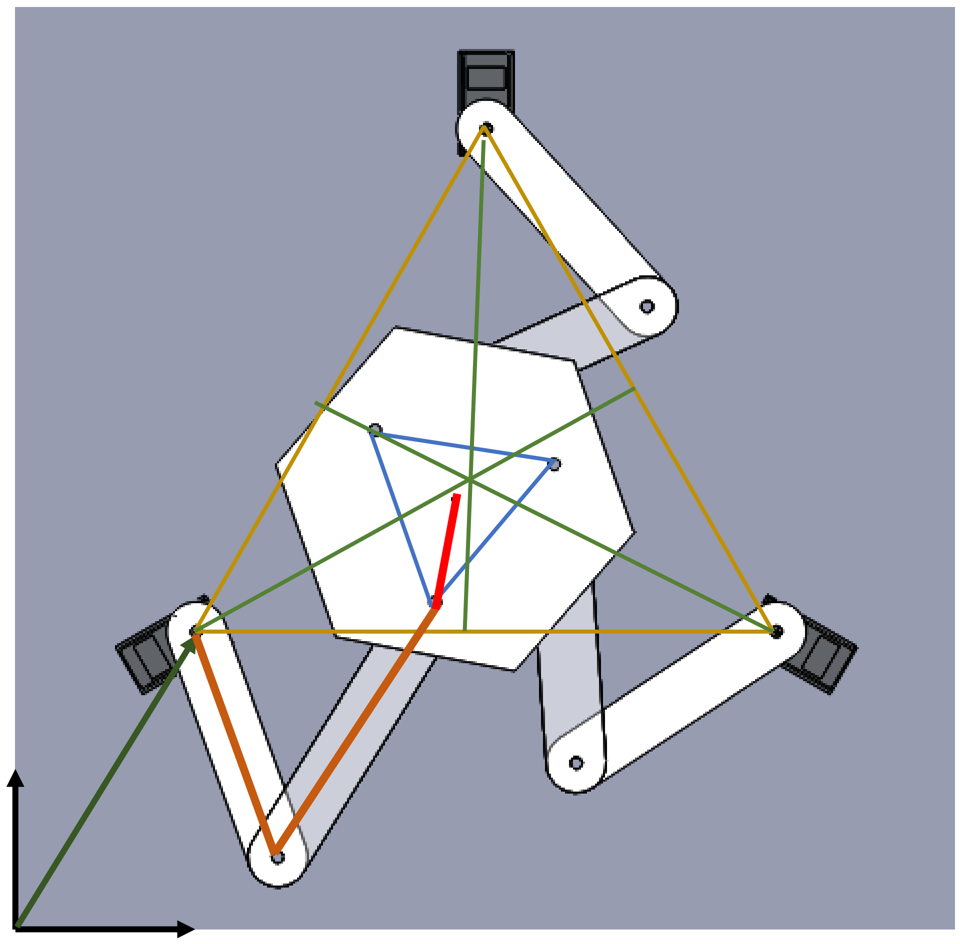
Modelo cinemático de postura



Robot delta plano

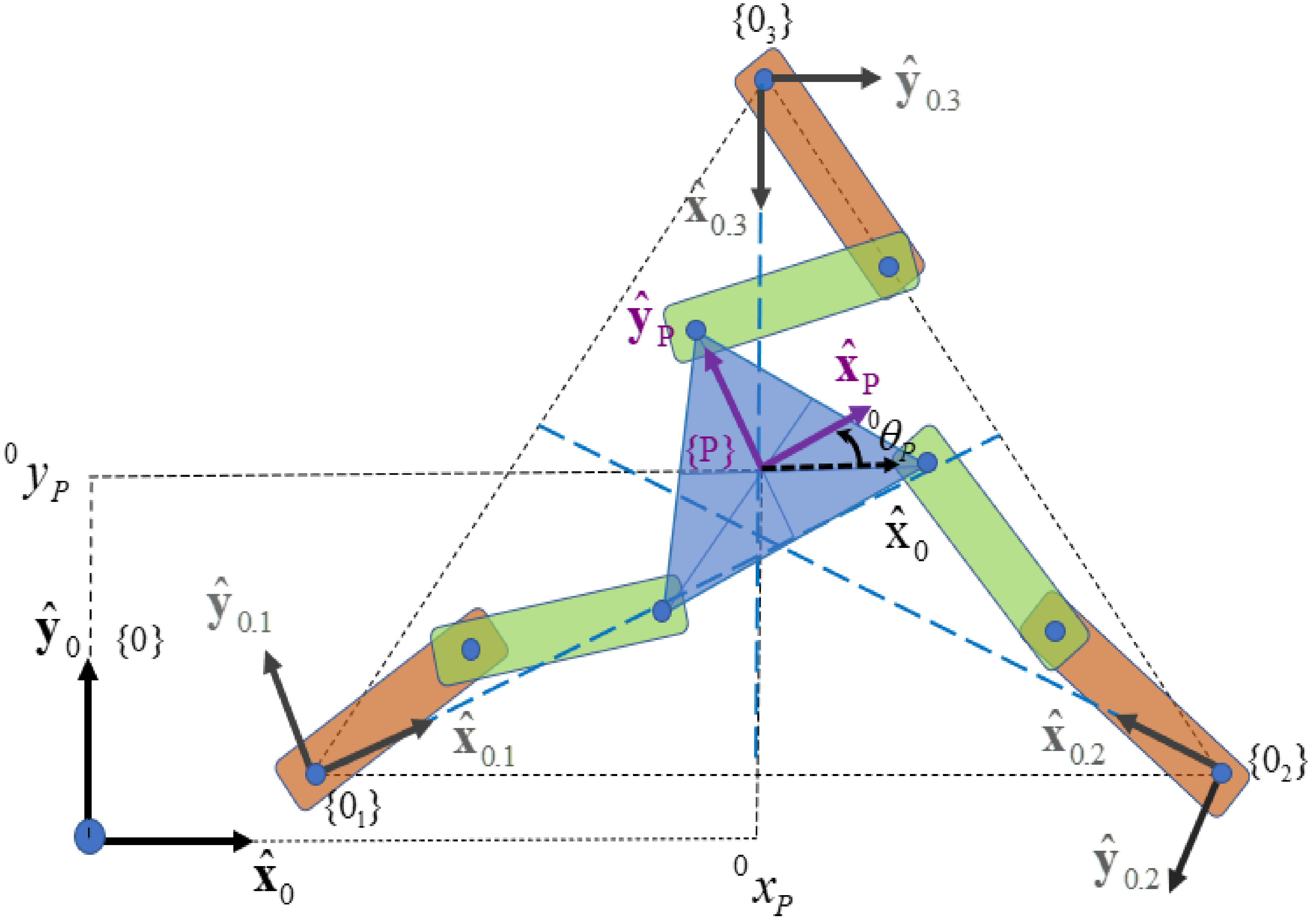
Planteamiento del modelo





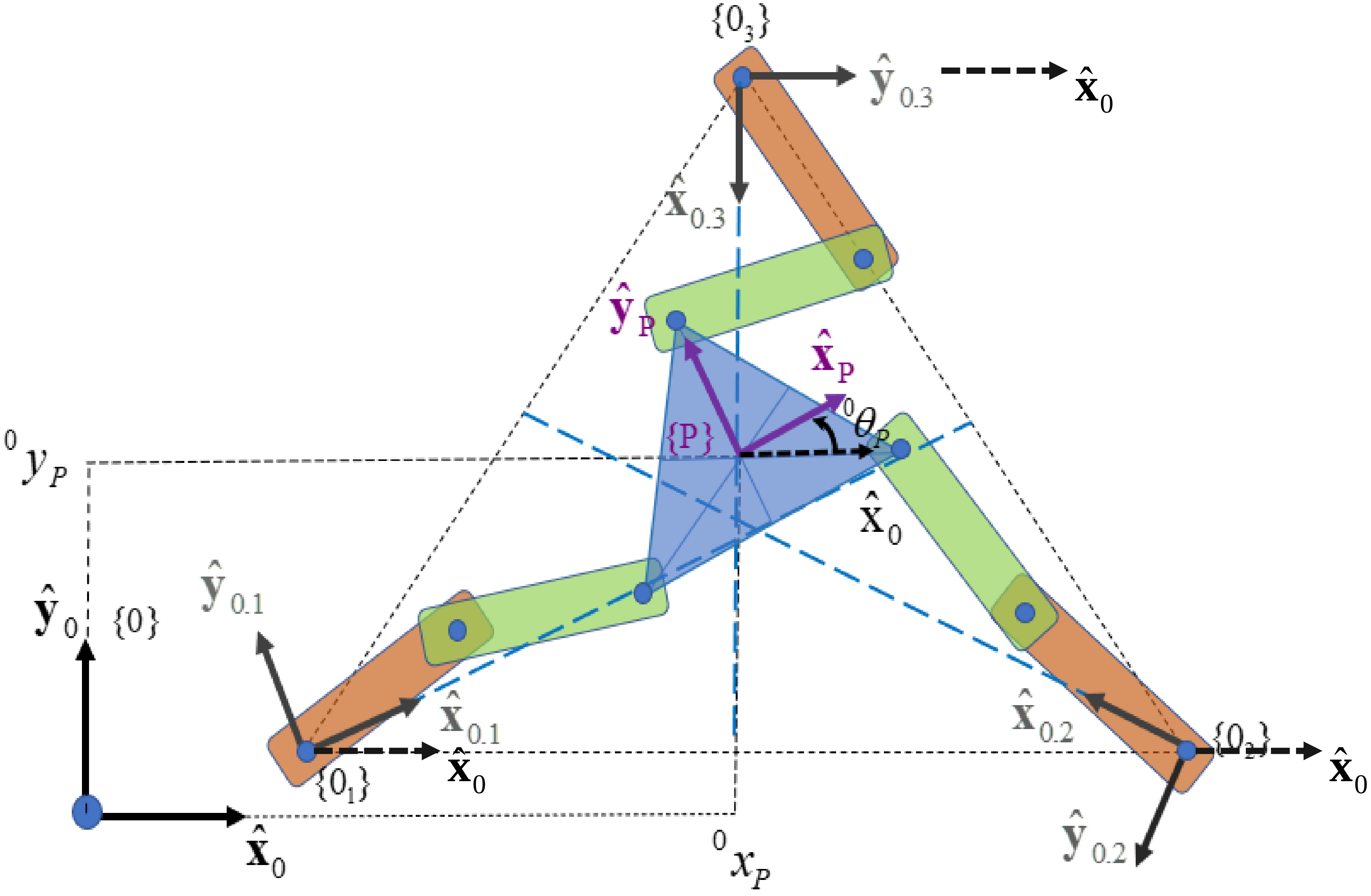
Modelo cinemático de la postura

Delta plano

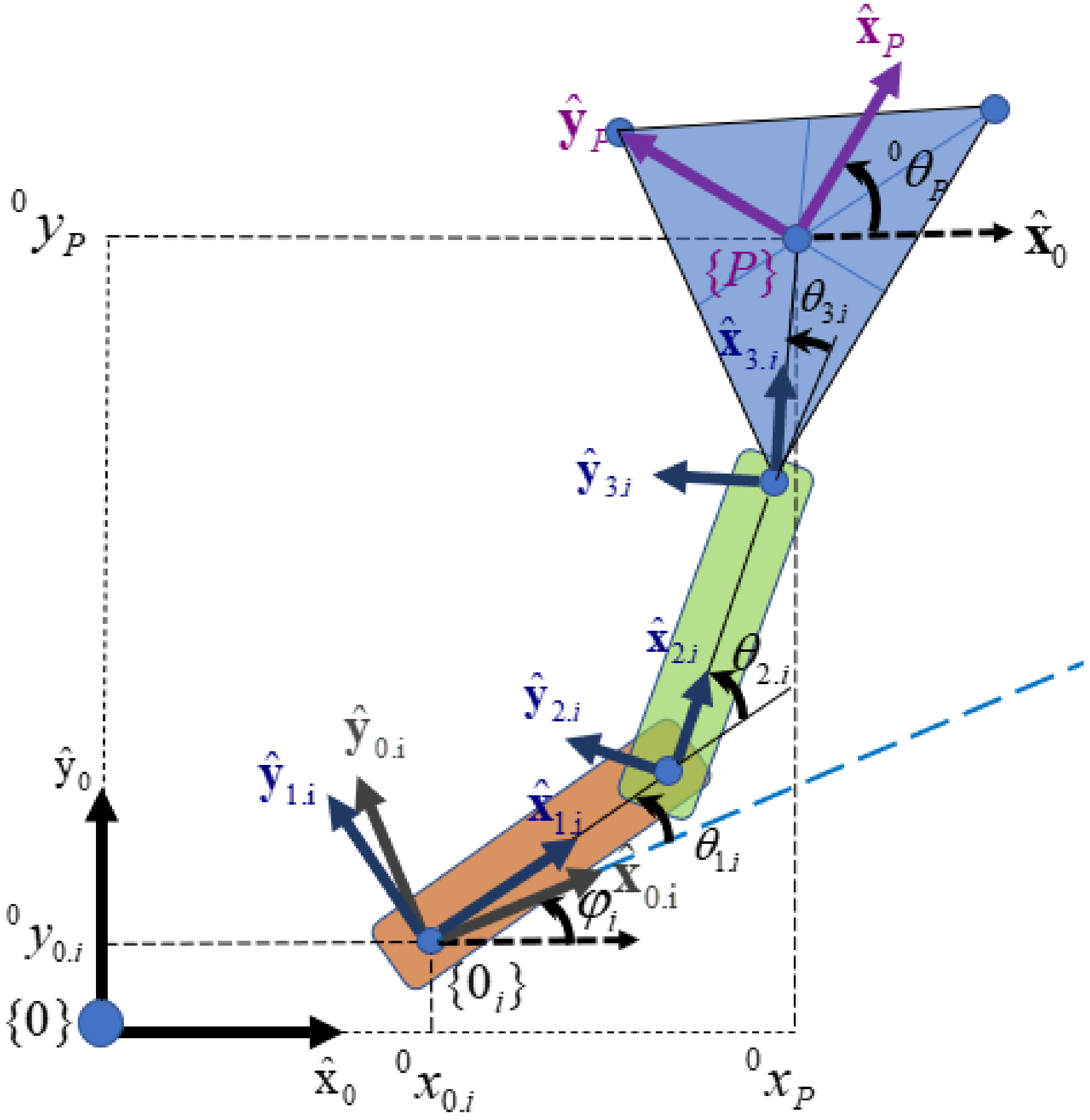


Modelo cinemático de la postura

Delta plano



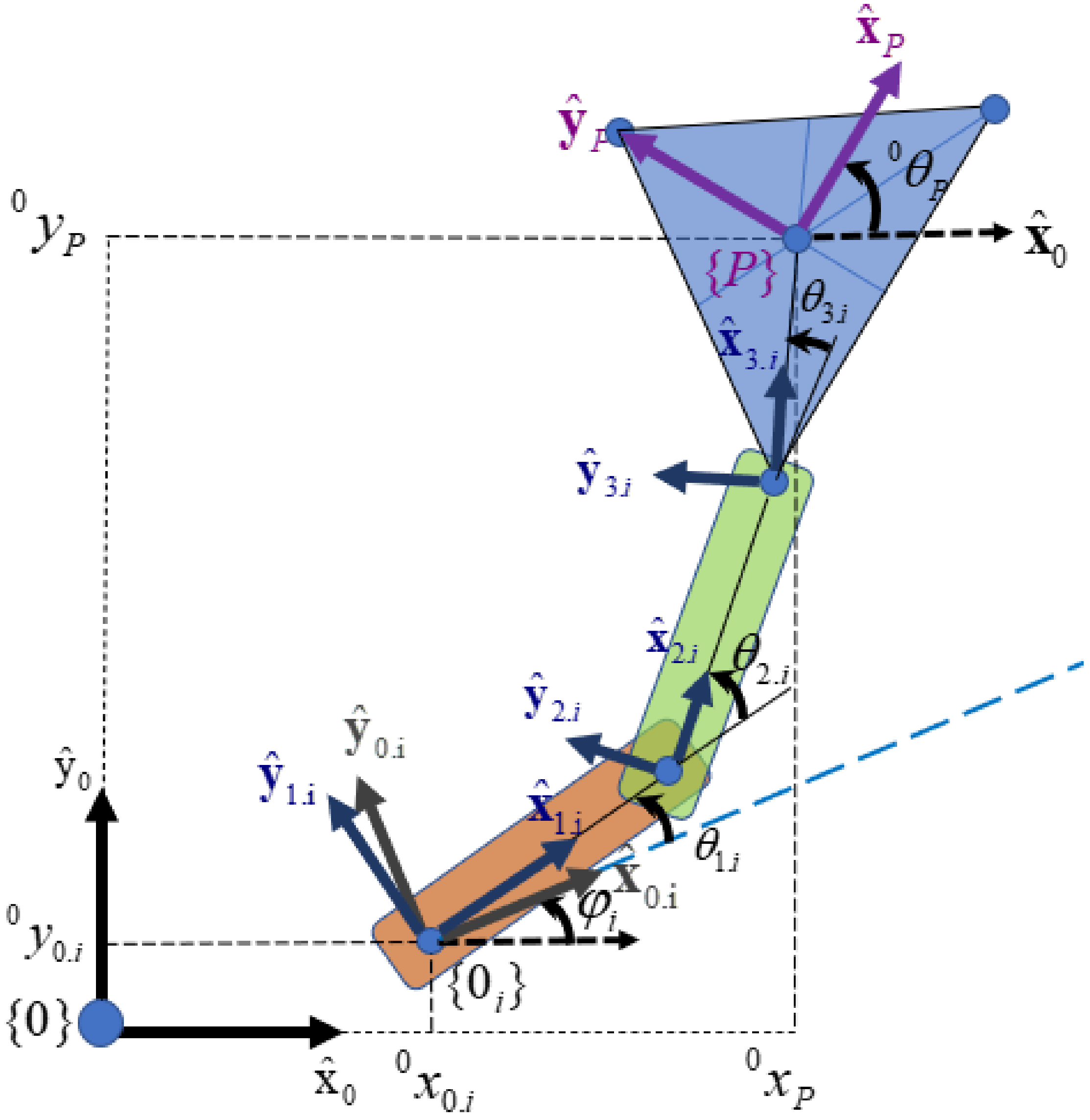
Modelo cinemático de la postura



$$ {}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{i}\mathbf{T}_{j} = \begin{pmatrix} \cos(\theta_j) & -\sin(\theta_j) & 0 & x_i \\ \sin(\theta_j) & \cos(\theta_j) & 0 & y_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

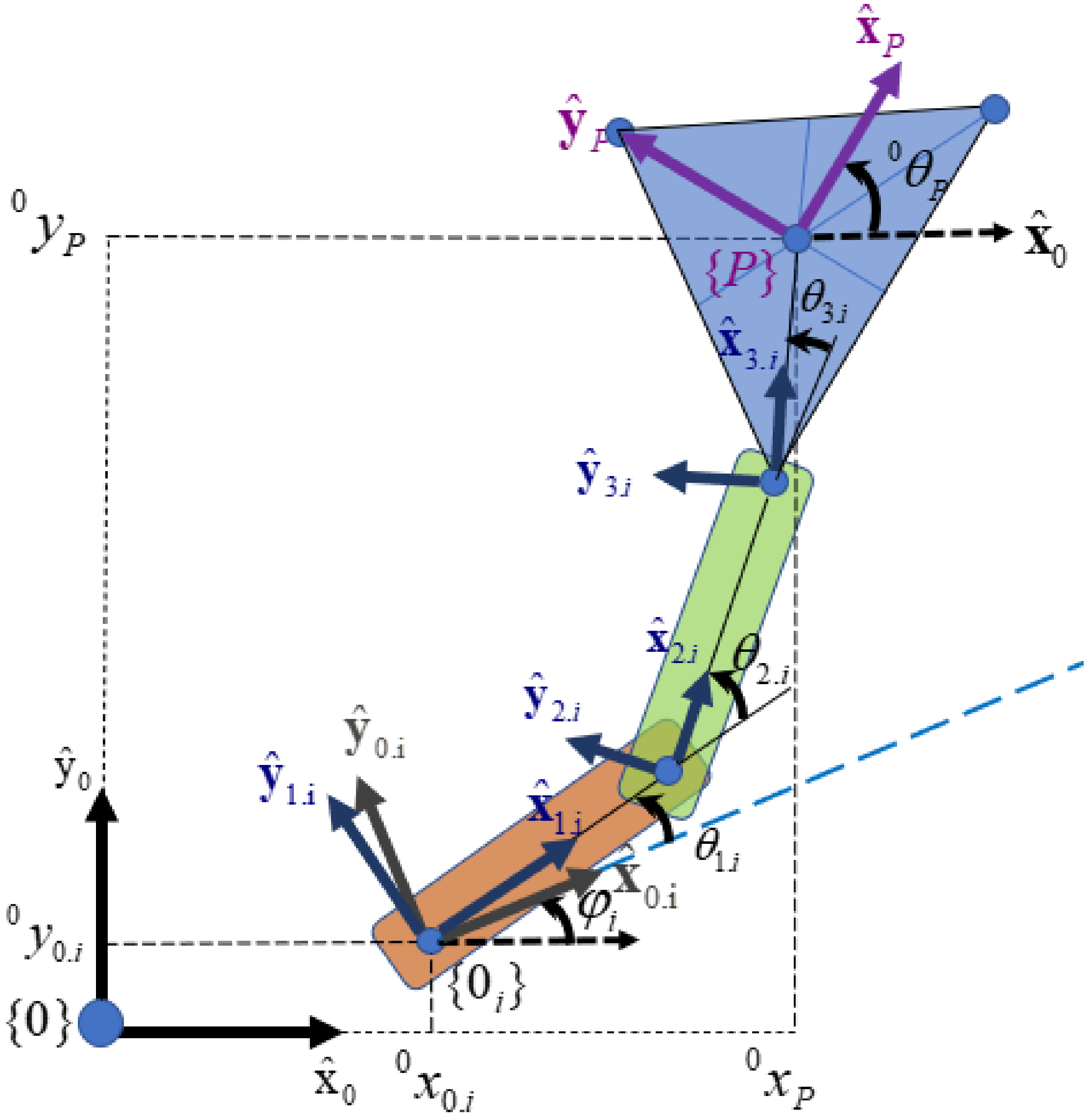


$$^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i}$$

$$^{0}\mathbf{T}_{0.i} = \begin{pmatrix} \cos({}^{0}\varphi_i) & -\sin({}^{0}\varphi_i) & 0 & {}^{0}x_{0.i} \\ \sin({}^{0}\varphi_i) & \cos({}^{0}\varphi_i) & 0 & {}^{0}y_{0.j} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{0.i}\mathbf{T}_{1.i} = \begin{pmatrix} \cos({}^{0.i}\theta_{1.i}) & -\sin({}^{0.i}\theta_{1.i}) & 0 & 0 \\ \sin({}^{0.i}\theta_{1.i}) & \cos({}^{0.i}\theta_{1.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Modelo cinemático de la postura

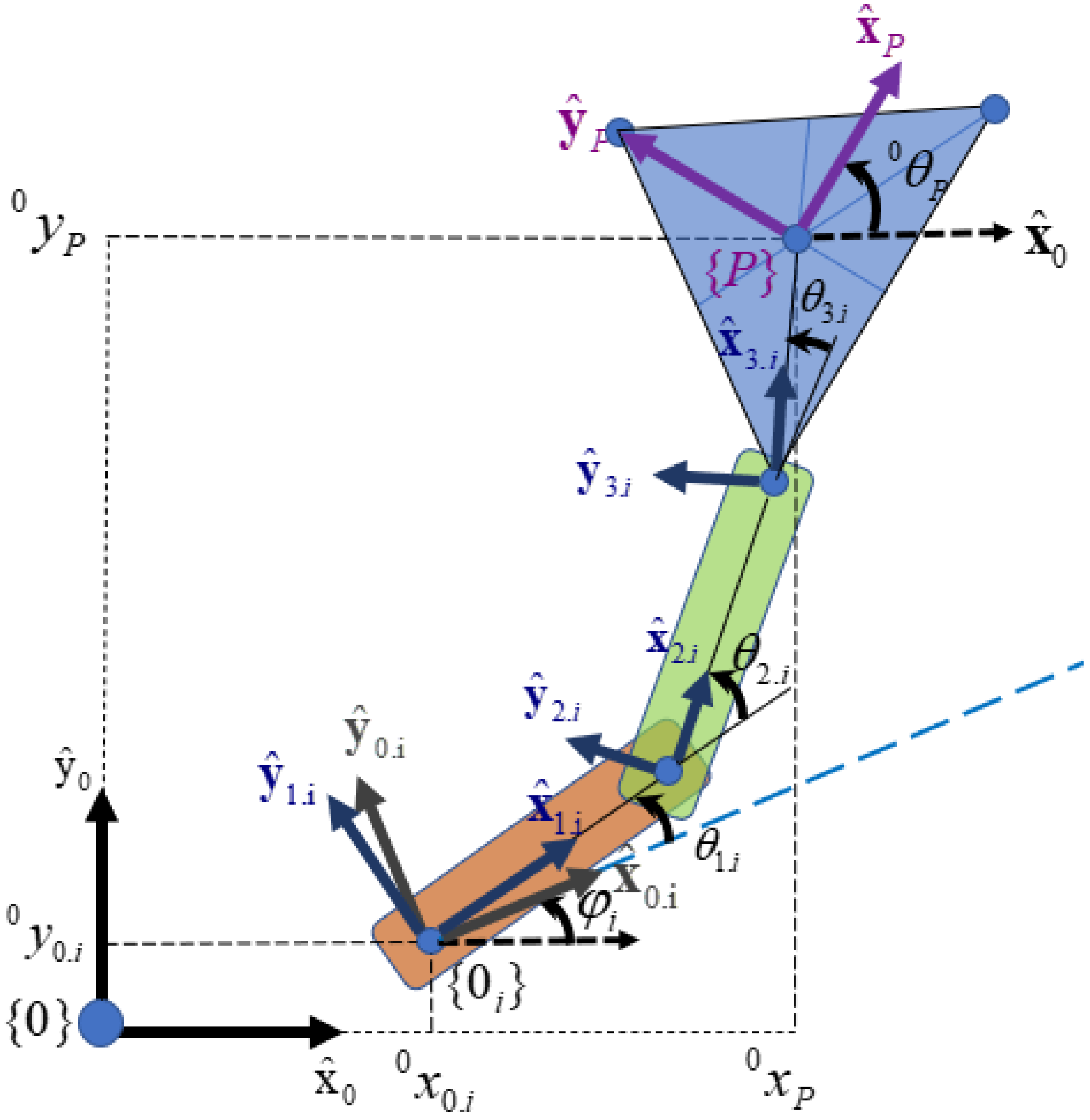


$$ {}^0\mathbf{T}_{P.i} = {}^0\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{1.i}\mathbf{T}_{2.i} = \begin{pmatrix} \cos(\theta_{2.i}) & -\sin(\theta_{2.i}) & 0 & L_{1.i} \\ \sin(\theta_{2.i}) & \cos(\theta_{2.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2.i}\mathbf{T}_{3.i} = \begin{pmatrix} \cos(\theta_{3.i}) & -\sin(\theta_{3.i}) & 0 & L_{2.i} \\ \sin(\theta_{3.i}) & \cos(\theta_{3.i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

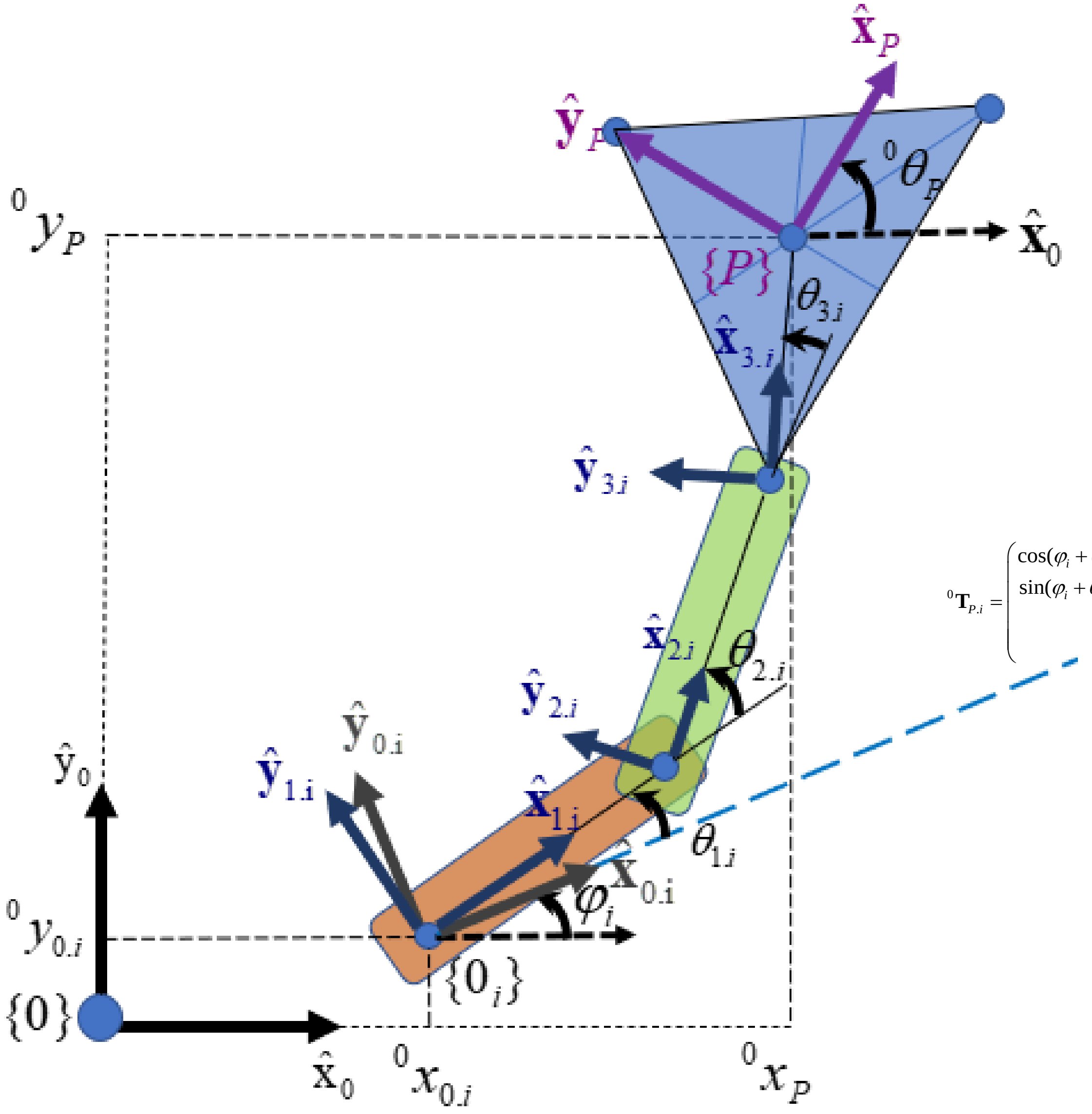
Modelo cinemático de la postura



$$ {}^0\mathbf{T}_{P.i} = {}^0\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{3.i}\mathbf{T}_{P.i} = \begin{pmatrix} 1 & 0 & 0 & L_{3.i} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura



$$ {}^0\mathbf{T}_{P.i} = {}^0\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^0\mathbf{T}_{P.i} = \begin{pmatrix} \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^0x_{0.i}+L_{1.i}\cos(\varphi_i+\theta_{1.i})+L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^0y_{0.i}+L_{1.i}\sin(\varphi_i+\theta_{1.i})+L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Modelo cinemático de la postura

$$ {}^{0}\mathbf{T}_{P.i} = {}^{0}\mathbf{T}_{0.i}\,{}^{0.i}\mathbf{T}_{1.i}\,{}^{1.i}\mathbf{T}_{2.i}\,{}^{2.i}\mathbf{T}_{3.i}\,{}^{3.i}\mathbf{T}_{P.i} $$

$$ {}^{0}\mathbf{T}_{P.i} = $$

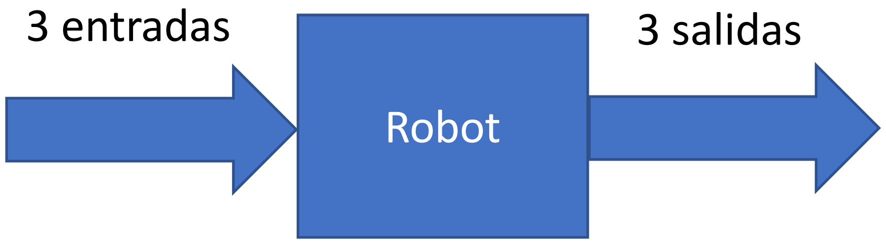
$$ \begin{pmatrix} \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}x_{0.i}+L_{1.i}\cos(\varphi_i+\theta_{1.i})+L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & \cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & 0 & {}^{0}y_{0.i}+L_{1.i}\sin(\varphi_i+\theta_{1.i})+L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{0}\boldsymbol{\xi}_{P.i}(q) = \begin{pmatrix} {}^{0}x_{0.i}+L_{1,i}\cos(\varphi_i+\theta_{1.i})+L_{2,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ {}^{0}y_{0.i}+L_{1,i}\sin(\varphi_i+\theta_{1.i})+L_{2,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i})+L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ \varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i} \end{pmatrix} $$

Modelo cinemático de la postura

$$
{}^{0}\boldsymbol{\xi}_{P.i}(q) = \begin{pmatrix} {}^{0}x_{0.i} + L_{1,i}\cos(\varphi_i + \theta_{1.i}) + L_{2,i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3,i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^{0}y_{0.i} + L_{1,i}\sin(\varphi_i + \theta_{1.i}) + L_{2,i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3,i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ \varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i} \end{pmatrix}
$$

$$
{}^{0}\boldsymbol{\xi}_{P.i} = \begin{pmatrix} {}^{0}x_P \\ {}^{0}y_P \\ {}^{0}\theta_P \end{pmatrix}
$$



$$F(x)$$

$$\mathbf{F}_i(X_{\,i}, q_i) = {}^{0}\boldsymbol{\xi}_{P.i} - {}^{0}\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{F}_i(X_{\,i}, q_i) = {}^{0}\boldsymbol{\xi}_{P.i}(q_i) - {}^{0}\boldsymbol{\xi}_{P.i} = \mathbf{0}$$

Modelo cinemático de la postura

$$\mathbf{F}_i(X_i, q_i) = {}^{0}\boldsymbol{\xi}_{P.i} - {}^{0}\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{F}_i(X_i, q_i) = \begin{pmatrix} {}^{0}x_P - {}^{0}x_{0.i} - L_{1.i}\cos(\varphi_i + \theta_{1.i}) - L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^{0}y_P - {}^{0}y_{0.i} - L_{1.i}\sin(\varphi_i + \theta_{1.i}) - L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^{0}\theta_P - \varphi_i - \theta_{1.i} - \theta_{2.i} - \theta_{3.i} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{F}(X, q) = \begin{pmatrix} \mathbf{F}_1(X_1, q_1) \\ \mathbf{F}_2(X_2, q_2) \\ \mathbf{F}_3(X_3, q_3) \end{pmatrix} = \begin{pmatrix} {}^{P.1}\boldsymbol{\xi}_0 - {}^{P.1}\boldsymbol{\xi}_0(q_1) \\ {}^{P.2}\boldsymbol{\xi}_0 - {}^{P.2}\boldsymbol{\xi}_0(q_2) \\ {}^{P.3}\boldsymbol{\xi}_0 - {}^{P.3}\boldsymbol{\xi}_0(q_3) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

Modelo cinemático directo de las velocidades

$$^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i)$$

$$^{P.i}\dot{\boldsymbol{\xi}}_{0} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}}$$

$$^{0}\dot{\boldsymbol{\xi}}_{P.i} = \begin{pmatrix} ^{0}\dot{x}_P \\ ^{0}\dot{y}_P \\ ^{0}\dot{\theta}_P \end{pmatrix}$$

$$^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i$$

$$\dot{\mathbf{q}}_i = \begin{pmatrix} \dot{\theta}_{1.i} \\ \dot{\theta}_{2.i} \\ \dot{\theta}_{3.i} \end{pmatrix}$$

$$\mathbf{J}_{\theta i}(q_i) = \begin{pmatrix} -L_{1.i}\sin(\varphi_i+\theta_{1.i}) - L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ L_{1.i}\cos(\varphi_i+\theta_{1.i}) + L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 1 & 1 & 1 \end{pmatrix}$$

Modelo cinemático directo de las velocidades

$$ {}^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i) $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}} $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \begin{pmatrix} {}^{0}\dot{\boldsymbol{\xi}}_{P.1} \\ {}^{0}\dot{\boldsymbol{\xi}}_{P.2} \\ {}^{0}\dot{\boldsymbol{\xi}}_{P.3} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_{\theta.1}(\mathbf{q}_1) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\theta.2}(\mathbf{q}_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\theta.3}(\mathbf{q}_3) \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \\ \dot{\mathbf{q}}_3 \end{pmatrix} $$

Modelo cinemático cinemático inverso

$$\mathbf{F}_i(X_i, q_i) = {}^0\boldsymbol{\xi}_{P.i} - {}^0\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(X_i, q_i, \dot{q}_i) = {}^0\dot{\boldsymbol{\xi}}_{P.i} - {}^0\dot{\boldsymbol{\xi}}_{P.i}(q_i, \dot{q}_i) = \mathbf{0}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(X_i, q_i, \dot{q}_i) =$$

$$\begin{pmatrix} {}^0\dot{x}_P + \text{[redacted]} \\ {}^0\dot{y}_P - \dot{\theta}_{1.i}\left(L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})\right) - \text{[redacted]} \\ {}^0\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \dfrac{\partial}{\partial\, {}^0x_P}\mathbf{F}_i(X_i, q_i) & \dfrac{\partial}{\partial\, {}^0y_P}\mathbf{F}_i(X_i, q_i) & \dfrac{\partial}{\partial\, {}^0\theta_P}\mathbf{F}_i(X_i, q_i) & \dfrac{\partial}{\partial \theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \dfrac{\partial}{\partial \theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \dfrac{\partial}{\partial \theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\boldsymbol{\Psi}}_{T.i}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \frac{\partial}{\partial^0 x_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial^0 y_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial^0 \theta_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

$$\dot{\Psi}_{T.i} = \begin{pmatrix} \dot{x}_P \\ y_P \\ \theta_P \\ \theta_{1,i} \\ \theta_{2,i} \\ \theta_{3,i} \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\dot{\Psi}_{T.i} = \begin{pmatrix} \dot{x}_P \\ y_P \\ \theta_P \\ \theta_{1,i} \\ \theta_{2,i} \\ \theta_{3,i} \end{pmatrix}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) =$$

$$\begin{array}{cccccc} \dot{x}_P & \dot{y}_P & \dot{\theta}_P & \dot{\theta}_{1.i} & \dot{\theta}_{2.i} & \dot{\theta}_{3.i} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & L_{1,i}\sin(\varphi_i+\theta_{1,i}) + L_{2,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}) + L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & L_{2,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}) + L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) \\ 0 & 1 & 0 & -L_{1,i}\cos(\varphi_i+\theta_{1,i}) - L_{2,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}) - L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & -L_{2,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}) - L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & -L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) \\ 0 & 0 & 1 & -1 & -1 & -1 \end{pmatrix}$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$^0\dot{x}_P + \dot{\theta}_{1.i}(L_{1.i}\sin(\varphi_i + \theta_{1.i}) + L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{2.i}((L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}$$

$$+L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{3.i}L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0$$

$$^0\dot{y}_P - \dot{\theta}_{1.i}(L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{2.i}(L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) +$$

$$+L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{3.i}L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0$$

$$\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} = 0$$

Modelo cinemático cinemático inverso

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\dot{\theta}_{1.i} = \left(\frac{\cos(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{3.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{2.\mathrm{i}} = \left(-\frac{L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1,i}\cos(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(-\frac{L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1.i}\sin(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2,i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P +$$
$$+\left(-\frac{L_{1.i}L_{3.i}\sin(\theta_{2.i} + \theta_{2.i}) + L_{2.i}L_{3.i}\cos(\theta_{3.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{3.\mathrm{i}} = \left(\frac{\cos(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{2.i} + \theta_{3.i}) + L_{2.i}\sin(\theta_{2.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$