Robótica grupo2
Clase 27

Facultad de Ingeniería UNAM

M.I. Erik Peña Medina

Derechos reservados

Contenido

Robótica paralela

- Definición de un robot paralelo
- Modelo postura de un robot paralelo
- Modelo cinemático de un robot paralelo
  - Modelo cinemático directo de las velocidades
  - Modelo cinemático inverso de las velocidades
- Modelo dinámico de un robot paralelo

Modelo cinemático de la postura

$$\mathbf{F}_i(X_i, q_i) = {}^0\boldsymbol{\xi}_{P.i} - {}^0\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{F}_i(X_i, q_i) = \begin{pmatrix} {}^0x_P - {}^0x_{0.i} - L_{1.i}\cos(\varphi_i + \theta_{1.i}) - L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^0y_P - {}^0y_{0.i} - L_{1.i}\sin(\varphi_i + \theta_{1.i}) - L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^0\theta_P - \varphi_i - \theta_{1.i} - \theta_{2.i} - \theta_{3.i} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{F}(X, q) = \begin{pmatrix} \mathbf{F}_1(X_1, q_1) \\ \mathbf{F}_2(X_2, q_2) \\ \mathbf{F}_3(X_3, q_3) \end{pmatrix} = \begin{pmatrix} {}^{P.1}\boldsymbol{\xi}_0 - {}^{P.1}\boldsymbol{\xi}_0(q_1) \\ {}^{P.2}\boldsymbol{\xi}_0 - {}^{P.2}\boldsymbol{\xi}_0(q_2) \\ {}^{P.3}\boldsymbol{\xi}_0 - {}^{P.3}\boldsymbol{\xi}_0(q_3) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

Modelo cinemático directo de las velocidades

$$ {}^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i) $$

$$ {}^{P.i}\dot{\boldsymbol{\xi}}_{0} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}} $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}\dot{y}_P \\ {}^{0}\dot{\theta}_P \end{pmatrix} \qquad {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i \qquad \dot{\mathbf{q}}_i = \begin{pmatrix} \dot{\theta}_{1.i} \\ \dot{\theta}_{2.i} \\ \dot{\theta}_{3.i} \end{pmatrix} $$

$$ \mathbf{J}_{\theta i}(q_i) = \begin{pmatrix} -L_{1.i}\sin(\varphi_i+\theta_{1.i}) - L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3,i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{2.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}) - L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & -L_{3.i}\sin(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ L_{1.i}\cos(\varphi_i+\theta_{1.i}) + L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3,i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) & L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i}) \\ 1 & 1 & 1 \end{pmatrix} $$

Modelo cinemático directo de las velocidades

$$ {}^{0}\boldsymbol{\xi}_{P.i} = \boldsymbol{\xi}_{P.i}(q_i) $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \frac{d}{dt}\,{}^{0}\boldsymbol{\xi}_{P.i} = \frac{\partial}{\partial\theta_{1.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{1.\mathrm{i}} + \frac{\partial}{\partial\theta_{2.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{2.\mathrm{i}} + \frac{\partial}{\partial\theta_{3.\mathrm{i}}}\,{}^{0}\boldsymbol{\xi}_{P.i}(\mathbf{q}_i)\dot{\theta}_{3.\mathrm{i}} $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P.i} = \mathbf{J}_{\theta i}(q_i)\dot{\mathbf{q}}_i $$

$$ {}^{0}\dot{\boldsymbol{\xi}}_{P} = \begin{pmatrix} {}^{0}\dot{\boldsymbol{\xi}}_{P.1} \\ {}^{0}\dot{\boldsymbol{\xi}}_{P.2} \\ {}^{0}\dot{\boldsymbol{\xi}}_{P.3} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_{\theta.1}(\mathbf{q}_1) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\theta.2}(\mathbf{q}_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J}_{\theta.3}(\mathbf{q}_3) \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \\ \dot{\mathbf{q}}_3 \end{pmatrix} $$

Modelo cinemático inverso de las velocidades

$$\mathbf{F}_i(X_i, q_i) = {}^0\boldsymbol{\xi}_{P.i} - {}^0\boldsymbol{\xi}_{P.i}(q_i) = \mathbf{0}$$

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(X_i, q_i, \dot{q}_i) = {}^0\dot{\boldsymbol{\xi}}_{P.i} - {}^0\dot{\boldsymbol{\xi}}_{P.i}(q_i, \dot{q}_i) = \mathbf{0}$$

Modelo cinemático inverso de las velocidades

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \dot{\mathbf{F}}_i(X_i, q_i, \dot{q}_i) =$$

$$\begin{pmatrix} {}^0\dot{x}_P + \text{[redacted]} \\ {}^0\dot{y}_P - \dot{\theta}_{1.i}\left(L_{1.i}\cos(\varphi_i+\theta_{1.i}) + L_{2.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}) + L_{3.i}\cos(\varphi_i+\theta_{1.i}+\theta_{2.i}+\theta_{3.i})\right) - \text{[redacted]} \\ {}^0\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \frac{\partial}{\partial {}^0x_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial {}^0y_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial {}^0\theta_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

Modelo cinemático inverso de las velocidades

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) = \begin{pmatrix} \frac{\partial}{\partial {}^0x_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial {}^0y_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial {}^0\theta_P}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{1.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{2.i}}\mathbf{F}_i(X_i, q_i) & \frac{\partial}{\partial \theta_{3.i}}\mathbf{F}_i(X_i, q_i) \end{pmatrix}$$

$$\dot{\Psi}_{T.i} = \begin{pmatrix} \dot{x}_P \\ \dot{y}_P \\ \dot{\theta}_P \\ \dot{\theta}_{1,i} \\ \dot{\theta}_{2,i} \\ \dot{\theta}_{3,i} \end{pmatrix}$$

Modelo cinemático inverso de las velocidades

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0} \qquad \dot{\Psi}_{T.i} = \begin{pmatrix} {}^0\dot{x}_P \\ {}^0y_P \\ {}^0\theta_P \\ {}^0\theta_{1,i} \\ {}^0\theta_{2,i} \\ {}^0\theta_{3,i} \end{pmatrix}$$

$$\mathbf{A}(X_i, q_i, \dot{q}_i) =$$

$$\begin{array}{cccccc}
\dot{x}_P & \dot{y}_P & \dot{\theta}_P & \dot{\theta}_{1.i} & \dot{\theta}_{2.i} & \dot{\theta}_{3.i}
\end{array}$$

$$\begin{pmatrix}
1 & 0 & 0 & L_{1,i}\sin(\varphi_i+\theta_{1,i}) + L_{2,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}) + L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & L_{2,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}) + L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & L_{3,i}\sin(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) \\
0 & 1 & 0 & -L_{1,i}\cos(\varphi_i+\theta_{1,i}) - L_{2,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}) - L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & -L_{2,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}) - L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) & -L_{3,i}\cos(\varphi_i+\theta_{1,i}+\theta_{2,i}+\theta_{3,i}) \\
0 & 0 & 1 & -1 & -1 & -1
\end{pmatrix}$$

$$rak(\mathbf{A}_{q.i}(q_i)) = 3$$

Modelo cinemático inverso de las velocidades

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\begin{aligned}&{}^0\dot{x}_P + \dot{\theta}_{1.i}(L_{1.i}\sin(\varphi_i + \theta_{1.i}) + L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{2.i}((L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \\ &+L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) + \dot{\theta}_{3.i}L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0\end{aligned}$$

$$\begin{aligned}&{}^0\dot{y}_P - \dot{\theta}_{1.i}(L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{2.i}(L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \\ &+L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i})) - \dot{\theta}_{3.i}L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) = 0\end{aligned}$$

$$\dot{\theta}_P - \dot{\theta}_{1.i} - \dot{\theta}_{2.i} - \dot{\theta}_{3.i} = 0$$

Modelo cinemático inverso de las velocidades

$$\mathbf{C}_{q.i}(X_i, q_i, \dot{q}_i) = \mathbf{A}_{q.i}(q_i)\dot{\mathbf{\Psi}}_{T.i} = \mathbf{0}$$

$$\dot{\theta}_{1.i} = \left(\frac{\cos(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i} + \theta_{2.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{3.i})}{L_{1.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{2.\mathrm{i}} = \left(-\frac{L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1,i}\cos(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(-\frac{L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{1.i}\sin(\varphi_i + \theta_{1.i})}{L_{1.i}L_{2,i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P +$$

$$+\left(-\frac{L_{1.i}L_{3.i}\sin(\theta_{2.i} + \theta_{2.i}) + L_{2.i}L_{3.i}\cos(\theta_{3.i})}{L_{1.i}L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

$$\dot{\theta}_{3.\mathrm{i}} = \left(\frac{\cos(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{x}_P + \left(\frac{\sin(\varphi_i + \theta_{1.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{y}_P + \left(\frac{L_{3.i}\sin(\theta_{2.i} + \theta_{3.i}) + L_{2.i}\sin(\theta_{2.i})}{L_{2.i}\sin(\theta_{2.i})}\right){}^0\dot{\theta}_P$$

Modelo cinemático inverso de las velocidades

$$
\dot{\Psi}_{T.i} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}y_P \\ {}^{0}\theta_P \\ {}^{0}\theta_{1,i} \\ {}^{0}\theta_{2,i} \\ {}^{0}\theta_{3,i} \end{pmatrix} = \begin{pmatrix} {}^{0}\dot{x}_P \\ {}^{0}y_P \\ {}^{0}\theta_P \\ \left(\dfrac{\cos(\varphi_i + {}^{0}\theta_{1.i} + {}^{0}\theta_{2.i})}{L_{1.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{x}_P + \left(\dfrac{\sin(\varphi_i + {}^{0}\theta_{1.i} + {}^{0}\theta_{2.i})}{L_{1.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{y}_P + \left(\dfrac{L_{3.i}\sin({}^{0}\theta_{3.i})}{L_{1.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{\theta}_P \\ \left(-\dfrac{L_{2.i}\cos(\varphi_i + {}^{0}\theta_{1.i} + {}^{0}\theta_{2.i}) + L_{1,i}\cos(\varphi_i + {}^{0}\theta_{1.i})}{L_{1.i}L_{2.i}\sin({}^{0}\theta_{2.i})} \right) \text{[redacted]} \\ + \left(-\dfrac{L_{1.i}L_{3.i}\sin({}^{0}\theta_{2.i} + {}^{0}\theta_{3.i}) + L_{2.i}L_{3.i}\cos({}^{0}\theta_{3.i})}{L_{1.i}L_{2.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{\theta}_P \\ \left(\dfrac{\cos(\varphi_i + {}^{0}\theta_{1.i})}{L_{2.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{x}_P + \left(\dfrac{\sin(\varphi_i + {}^{0}\theta_{1.i})}{L_{2.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{y}_P + \left(\dfrac{L_{3.i}\sin({}^{0}\theta_{2.i} + {}^{0}\theta_{3.i}) + L_{2.i}\sin({}^{0}\theta_{2.i})}{L_{2.i}\sin({}^{0}\theta_{2.i})} \right) {}^{0}\dot{\theta}_P \end{pmatrix}
$$

Modelo cinemático inverso de las velocidades

$$
\dot{\Psi}_{T.i} = \begin{pmatrix} {}^0\dot{x}_P \\ {}^0y_P \\ {}^0\theta_P \\ {}^0\theta_{1,i} \\ {}^0\theta_{2,i} \\ {}^0\theta_{3,i} \end{pmatrix} = \begin{pmatrix} {}^0\dot{x}_P \\ {}^0y_P \\ {}^0\theta_P \\ \left(\dfrac{\cos(\varphi_i + {}^0\theta_{1.i} + {}^0\theta_{2.i})}{L_{1.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{x}_P + \left(\dfrac{\sin(\varphi_i + {}^0\theta_{1.i} + {}^0\theta_{2.i})}{L_{1.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{y}_P + \left(\dfrac{L_{3.i}\sin({}^0\theta_{3.i})}{L_{1.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{\theta}_P \\ \left(-\dfrac{L_{2.i}\cos(\varphi_i + {}^0\theta_{1.i} + {}^0\theta_{2.i}) + L_{1,i}\cos(\varphi_i + {}^0\theta_{1.i})}{L_{1.i}L_{2.i}\sin({}^0\theta_{2.i})}\right){}^0 \text{[redacted]} \\ + \left(-\dfrac{L_{1.i}L_{3.i}\sin({}^0\theta_{2.i} + {}^0\theta_{3.i}) + L_{2.i}L_{3.i}\cos({}^0\theta_{3.i})}{L_{1.i}L_{2.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{\theta}_P \\ \left(\dfrac{\cos(\varphi_i + {}^0\theta_{1.i})}{L_{2.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{x}_P + \left(\dfrac{\sin(\varphi_i + {}^0\theta_{1.i})}{L_{2.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{y}_P + \left(\dfrac{L_{3.i}\sin({}^0\theta_{2.i} + {}^0\theta_{3.i}) + L_{2.i}\sin({}^0\theta_{2.i})}{L_{2.i}\sin({}^0\theta_{2.i})}\right){}^0\dot{\theta}_P \end{pmatrix}
$$

Modelo cinemático inverso de las velocidades

$$\dot{\mathbf{\Psi}}_{Ti} = \mathbf{S}_{Ti}(q_i)\mathbf{u}_i$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \dfrac{\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\mathrm{L}_{3,i}\sin({}^0\theta_{3,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} \\ -\dfrac{\mathrm{L}_{2,i}\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}}) + \mathrm{L}_{1,i}\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & -\dfrac{\mathrm{L}_{2,i}\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}}) + \mathrm{L}_{1,i}\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & -\dfrac{\mathrm{L}_{1,i}\mathrm{L}_{3,i}\sin({}^0\theta_{2,\mathrm{i}} + \theta_{23\mathrm{i}}) + \mathrm{L}_{2,i}\mathrm{L}_{3,i}\cos({}^0\theta_{3,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} \\ \dfrac{\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\mathrm{L}_{3,i}\sin({}^0\theta_{2,\mathrm{i}} + {}^0\theta_{3,\mathrm{i}}) + \mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} \end{pmatrix} \begin{pmatrix} {}^0\dot{x}_P \\ {}^0\dot{y}_P \\ {}^0\dot{\theta}_P \end{pmatrix}$$

Modelo cinemático inverso de las velocidades

$$\dot{\mathbf{\Psi}}_{Ti} = \mathbf{S}_{Ti}(q_i)\mathbf{u}_i$$

$$\mathbf{S}_{Ti}(q_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \dfrac{\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\mathrm{L}_{3,i}\sin({}^0\theta_{3,\mathrm{i}})}{\mathrm{L}_{1,i}\sin({}^0\theta_{2,\mathrm{i}})} \\ -\dfrac{\mathrm{L}_{2,i}\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}}) + \mathrm{L}_{1,i}\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & -\dfrac{\mathrm{L}_{2,i}\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}} + {}^0\theta_{2,\mathrm{i}}) + \mathrm{L}_{1,i}\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & -\dfrac{\mathrm{L}_{1,i}\mathrm{L}_{3,i}\sin({}^0\theta_{2,\mathrm{i}} + \theta_{23\mathrm{i}}) + \mathrm{L}_{2,i}\mathrm{L}_{3,i}\cos({}^0\theta_{3,\mathrm{i}})}{\mathrm{L}_{1,i}\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} \\ \dfrac{\cos(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\sin(\varphi_i + {}^0\theta_{1,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} & \dfrac{\mathrm{L}_{3,i}\sin({}^0\theta_{2,\mathrm{i}} + {}^0\theta_{3,\mathrm{i}}) + \mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})}{\mathrm{L}_{2,i}\sin({}^0\theta_{2,\mathrm{i}})} \end{pmatrix}$$

$$\mathbf{u}_i = \begin{pmatrix} {}^0\dot{x}_P \\ {}^0\dot{y}_P \\ {}^0\dot{\theta}_P \end{pmatrix}$$

Modelo cinemático inverso de las velocidades

$$\dot{\mathbf{\Psi}}_T = \mathbf{S}_T(q)\mathbf{u} = \begin{pmatrix} \mathbf{S}_{T1}(q_1) \\ \mathbf{S}_{T2}(q_2) \\ \mathbf{S}_{T3}(q_3) \end{pmatrix}\mathbf{u}$$

Modelo dinámico de un robot paralelo

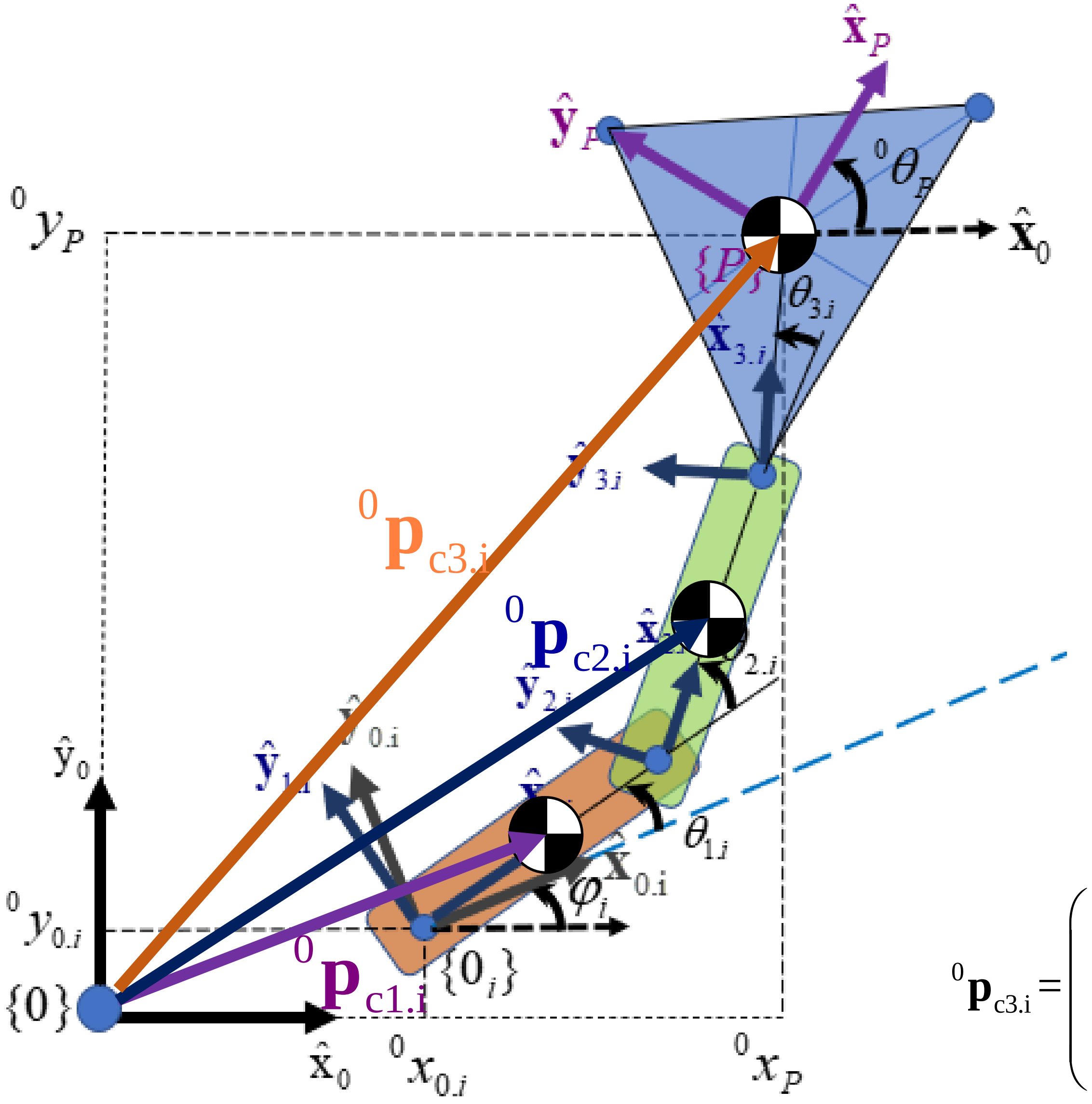
$$\tau_{\theta_i} = \frac{d}{dt}\left(\frac{\partial\Gamma(q,\dot{q})}{\partial\dot{q}_i}\right) - \left(\frac{\partial\Gamma(q,\dot{q})}{\partial q_i}\right)$$

$$\Gamma(q,\dot{q}) = k(q,\dot{q}) - u(q,\dot{q})$$

$$k(q,\dot{q}) = \sum_{i=1}^{n}\left(\frac{m_i}{2}\mathbf{v}_{ci}{}^T\mathbf{v}_{ci} + \frac{1}{2}\boldsymbol{\omega}_i{}^T\mathbf{I}_{ci}\boldsymbol{\omega}_i\right)$$

$$\mathrm{u_i}(q) = \sum_{i=1}^{n} -m_i\mathbf{g}^T\,{}^0\mathbf{p}_{ci}$$

Modelo dinámico de un robot paralelo

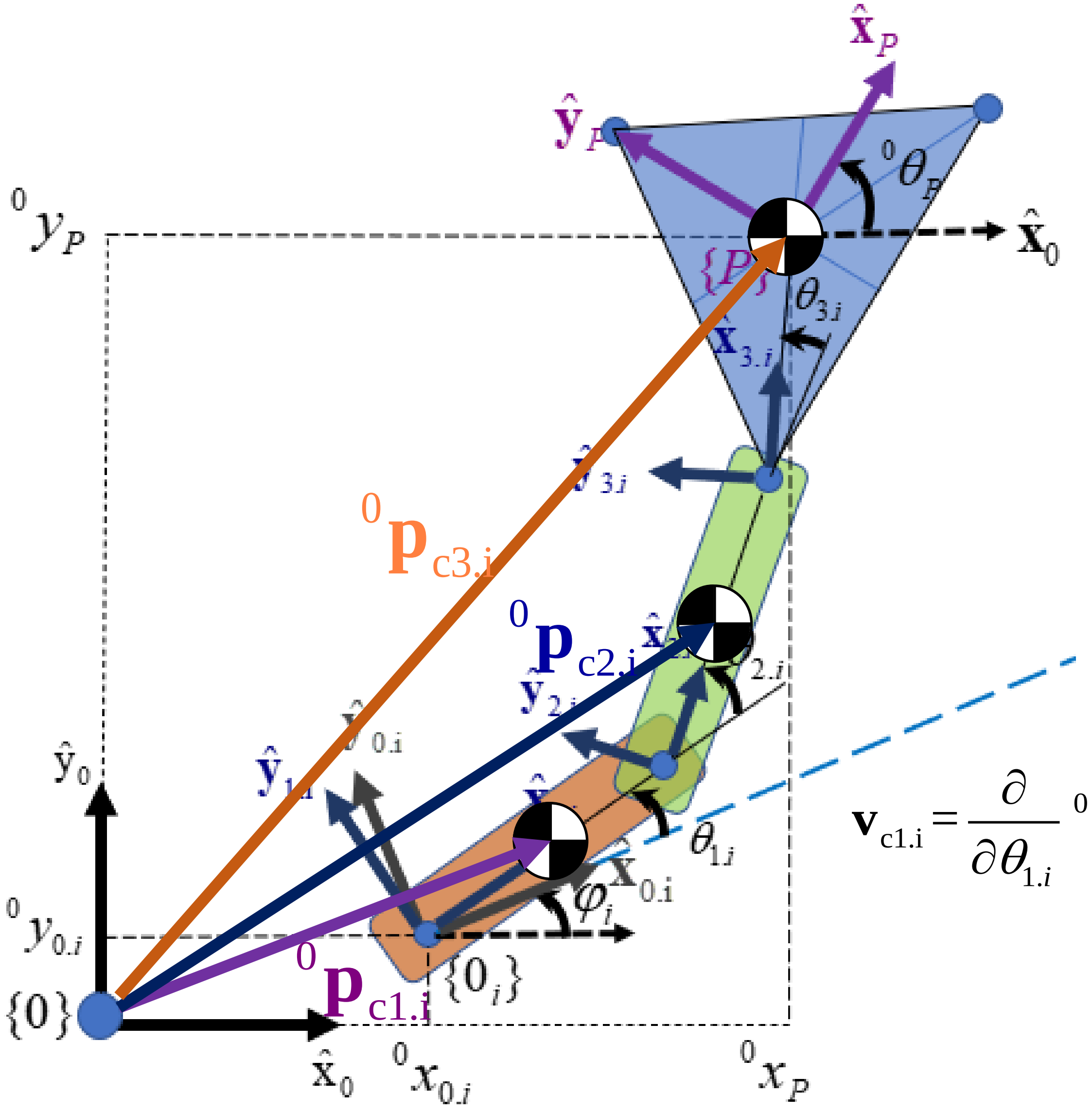


$$
{}^0\mathbf{p}_{c1.i} = \begin{pmatrix} {}^0x_{0.i} + \frac{L_{1.i}}{2}\cos(\varphi_i + \theta_{1.i}) \\ {}^0y_{0.i} + \frac{L_{1.i}}{2}\sin(\varphi_i + \theta_{1.i}) \\ 0 \end{pmatrix}
$$

$$
{}^0\mathbf{p}_{c2.i} = \begin{pmatrix} {}^0x_{0.i} + L_{1.i}\cos(\varphi_i + \theta_{1.i}) + \frac{L_{2.i}}{2}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ {}^0y_{0.i} + L_{1.i}\sin(\varphi_i + \theta_{1.i}) + \frac{L_{2.i}}{2}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ 0 \end{pmatrix}
$$

$$
{}^0\mathbf{p}_{c3.i} = \begin{pmatrix} {}^0x_{0.i} + L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^0y_{0.i} + L_{1.i}\sin(\varphi_i + \theta_{1.i}) + L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ 0 \end{pmatrix}
$$

Modelo dinámico de un robot paralelo



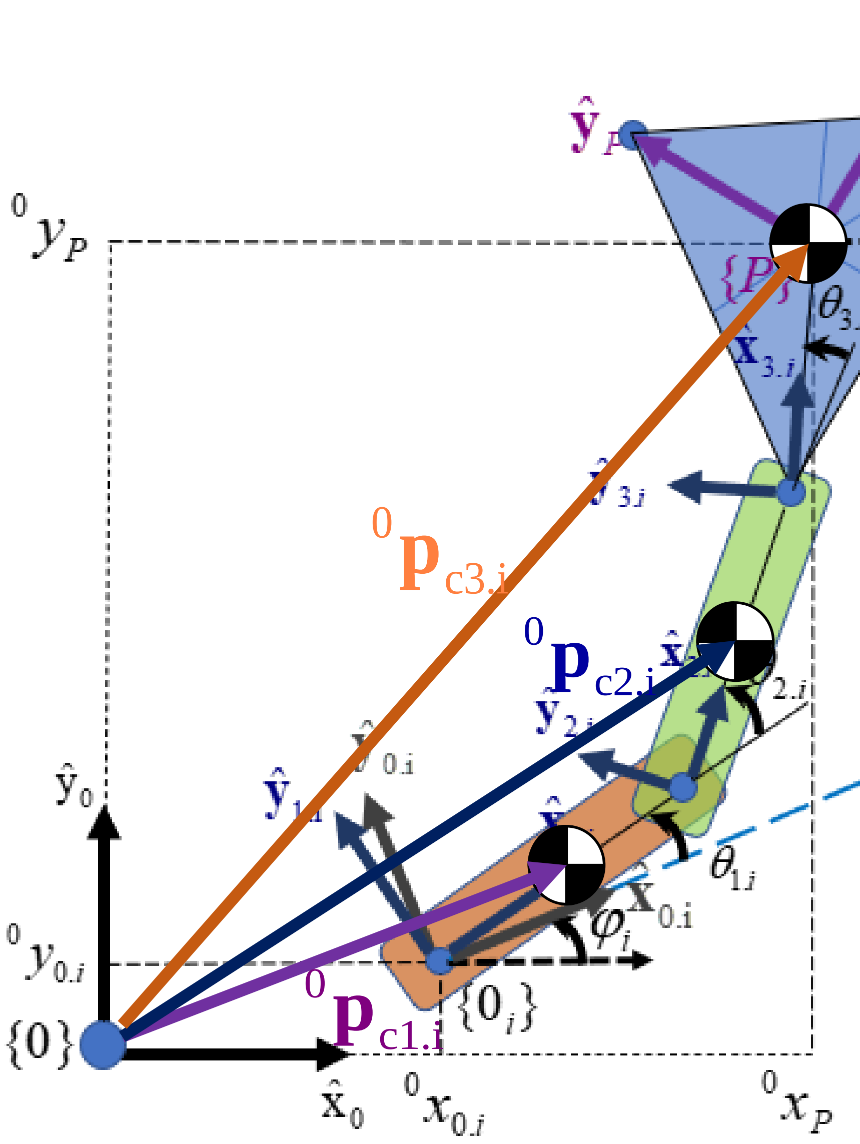
$$
{}^0\mathbf{p}_{c1.i} = \begin{pmatrix} {}^0x_{0.i} + \dfrac{L_{1.i}}{2}\cos(\varphi_i + \theta_{1.i}) \\ {}^0y_{0.i} + \dfrac{L_{1.i}}{2}\sin(\varphi_i + \theta_{1.i}) \\ 0 \end{pmatrix}
$$

$$
\mathbf{v}_{c1.i} = \frac{\partial}{\partial \theta_{1.i}}\, {}^0\mathbf{p}_{c1.i} \cdot
$$
[redacted]

$\partial\theta_{2.i}$ $\partial\theta_{3.i}$

$$
\begin{pmatrix} \dfrac{-L_{1.i}}{2}\sin(\varphi_i + \theta_{1.i}) \\ 2 \\ 0 \end{pmatrix}
$$

Modelo dinámico de un robot paralelo

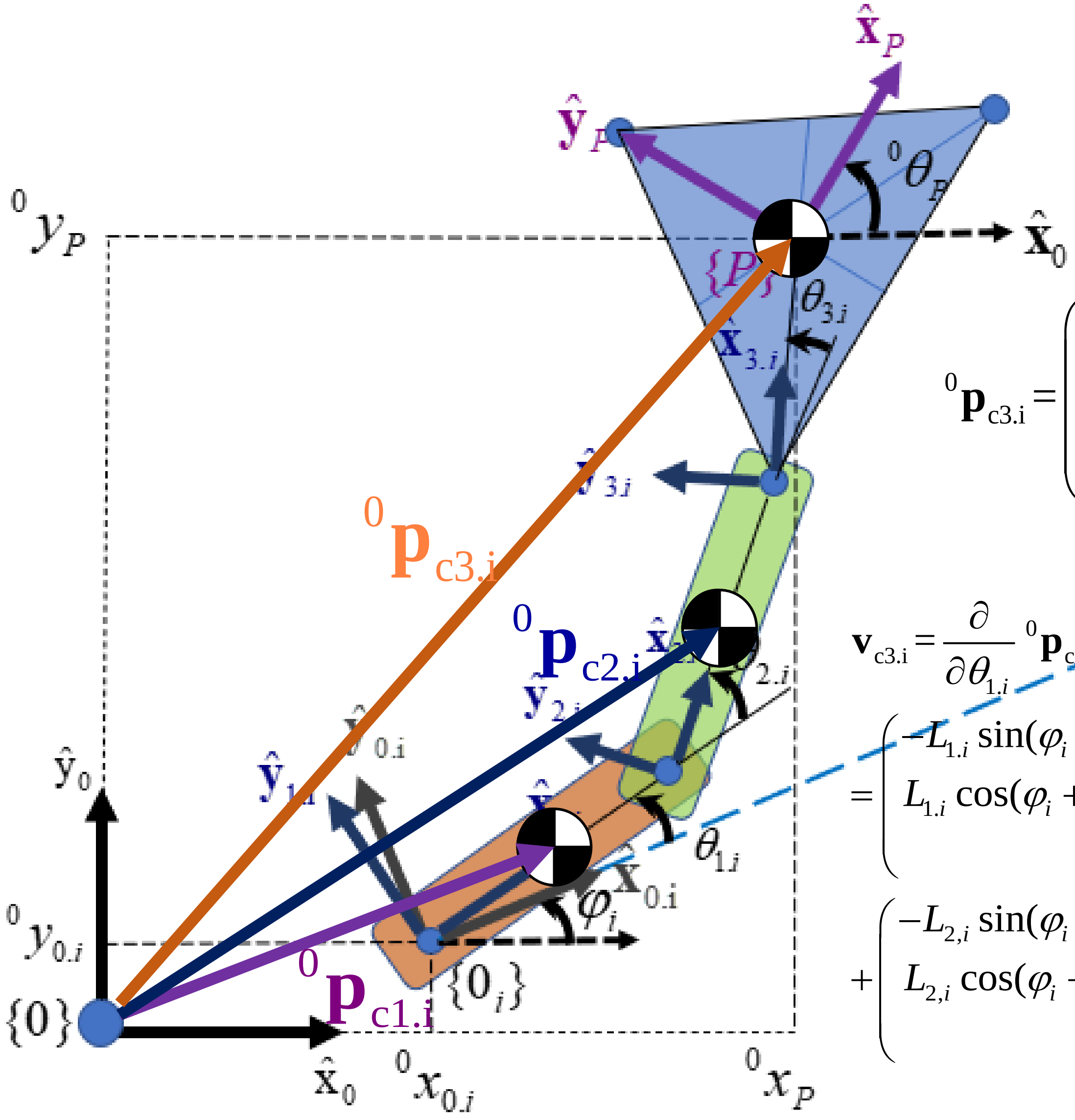


$$
{}^0\mathbf{p}_{c2.i} = \begin{pmatrix} {}^0x_{0.i} + L_{1.i}\cos(\varphi_i + \theta_{1.i}) + \dfrac{L_{2.i}}{2}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ {}^0y_{0.i} + L_{1.i}\sin(\varphi_i + \theta_{1.i}) + \dfrac{L_{2.i}}{2}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ 0 \end{pmatrix}
$$

$$
\mathbf{v}_{c2.i} = \frac{\partial}{\partial \theta_{1.i}}\,{}^0\mathbf{p}_{c2.i}\cdot\dot{\theta}_{1.i} + \frac{\partial}{\partial \theta_{2.i}}\,{}^0\mathbf{p}_{c2.i}\cdot\dot{\theta}_{2.i} + \frac{\partial}{\partial \theta_{3.i}}\,{}^0\mathbf{p}_{c2.i}\cdot\dot{\theta}_{3.i} =
$$

$$
= \begin{pmatrix} -L_{1.i}\sin(\varphi_i + \theta_{1.i}) - \dfrac{L_{2.i}}{2}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ L_{1.i}\cos(\varphi_i + \theta_{1.i}) + \dfrac{L_{2.i}}{2}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ 0 \end{pmatrix}\dot{\theta}_{1.i} + \begin{pmatrix} -\dfrac{L_{2.i}}{2}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ \dfrac{L_{2.i}}{2}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) \\ 0 \end{pmatrix}\dot{\theta}_{2.i}
$$

Modelo dinámico de un robot paralelo



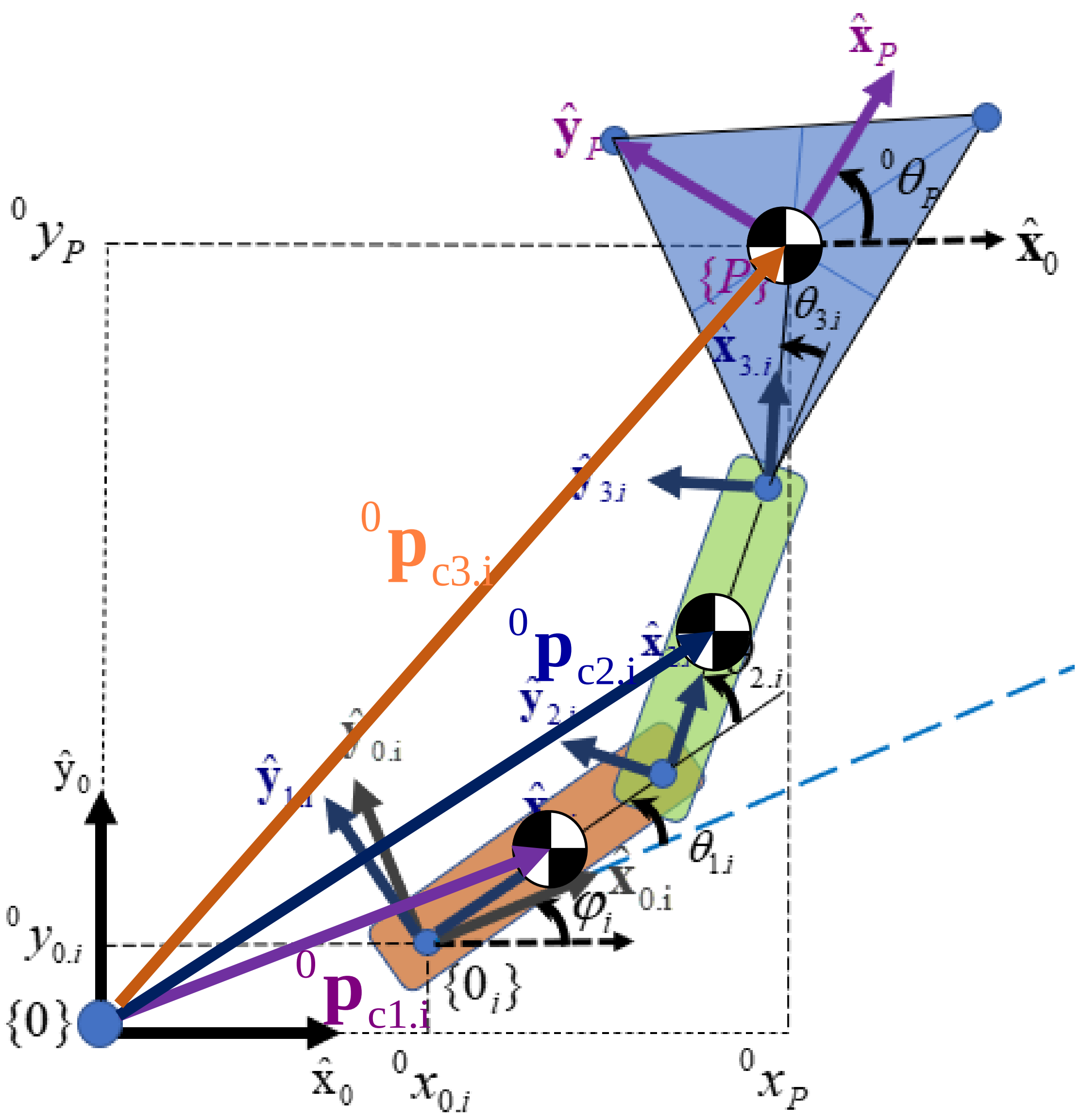
$$
{}^{0}\mathbf{p}_{\mathrm{c3.i}} = \begin{pmatrix} {}^{0}x_{0.i} + \mathrm{L}_{1.i}\cos(\varphi_i + \theta_{1.i}) + \mathrm{L}_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \mathrm{L}_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ {}^{0}y_{0.i} + \mathrm{L}_{1.i}\sin(\varphi_i + \theta_{1.i}) + \mathrm{L}_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) + \mathrm{L}_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ 0 \end{pmatrix}
$$

$$
\mathbf{v}_{\mathrm{c3.i}} = \frac{\partial}{\partial \theta_{1.i}}\,{}^{0}\mathbf{p}_{\mathrm{c2.i}} \cdot \dot{\theta}_{1.i} + \frac{\partial}{\partial \theta_{2.i}}\,{}^{0}\mathbf{p}_{\mathrm{c2.i}} \cdot \dot{\theta}_{2.i} + \frac{\partial}{\partial \theta_{3.i}}\,{}^{0}\mathbf{p}_{\mathrm{c3.i}} \cdot \dot{\theta}_{3.i} =
$$

$$
= \begin{pmatrix} -L_{1.i}\sin(\varphi_i + \theta_{1.i}) - L_{2.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ L_{1.i}\cos(\varphi_i + \theta_{1.i}) + L_{2.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ 0 \end{pmatrix} \dot{\theta}_{1.i} +
$$

$$
+ \begin{pmatrix} -L_{2,i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i}) - L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ L_{2,i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i}) + L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ 0 \end{pmatrix} \dot{\theta}_{2.i} + \begin{pmatrix} -L_{3.i}\sin(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ L_{3.i}\cos(\varphi_i + \theta_{1.i} + \theta_{2.i} + \theta_{3.i}) \\ 0 \end{pmatrix} \dot{\theta}_{3.i}
$$

Modelo dinámico de un robot paralelo



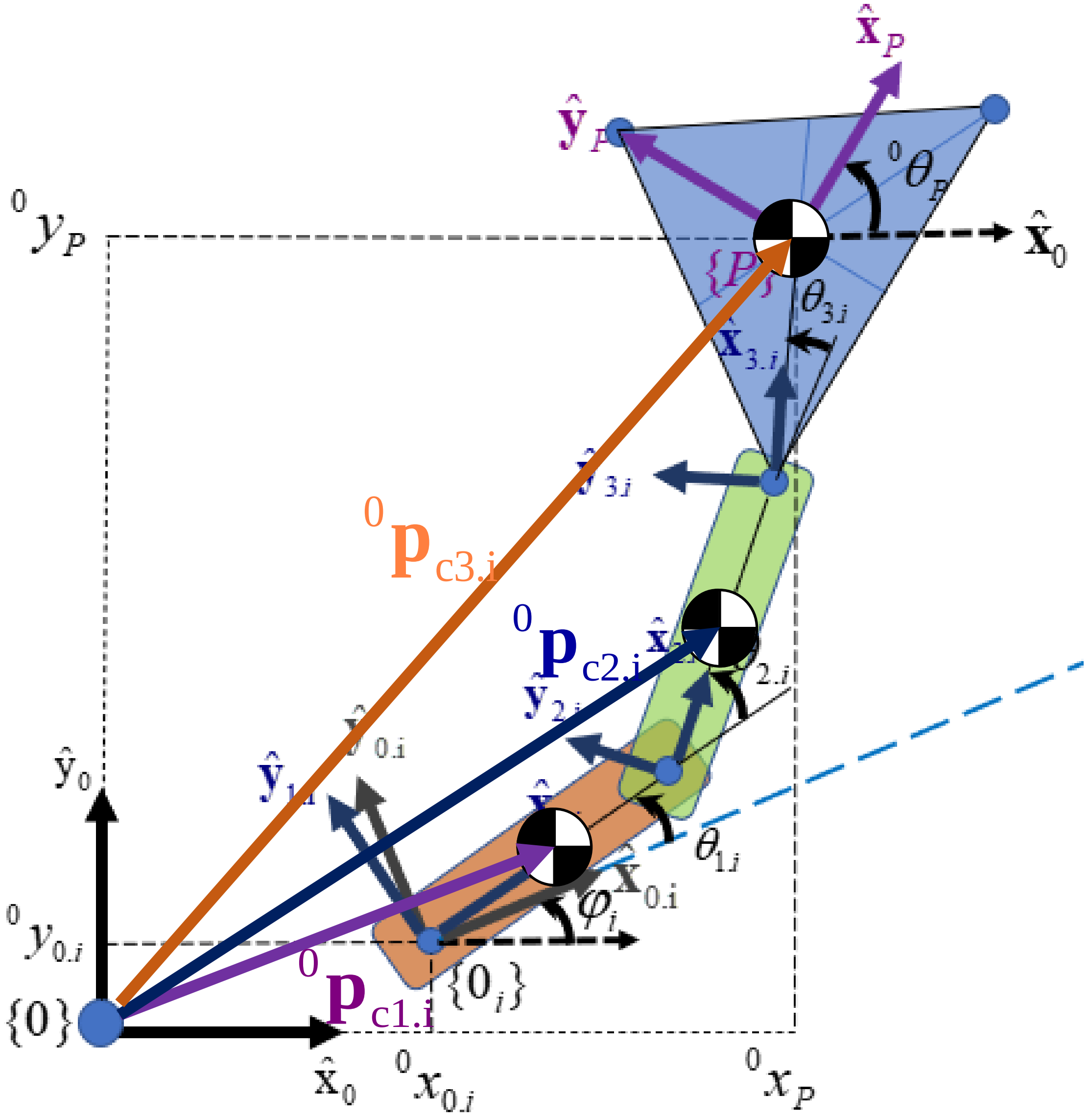
$$ {}^{i}\boldsymbol{\omega}_{\mathrm{i}} = {}^{\mathrm{i}}\mathbf{R}_{\mathrm{i-1}}\,{}^{\mathrm{i-1}}\boldsymbol{\omega}_{\mathrm{i-1}} + \hat{\mathbf{z}}_i\dot{\theta}_{\mathrm{i}} $$

$$ {}^{1.i}\boldsymbol{\omega}_{1.\mathrm{i}} = {}^{1.\mathrm{i}}\mathbf{R}_{0.\mathrm{i}}\,{}^{0.i}\boldsymbol{\omega}_{0.\mathrm{i}} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\dot{\theta}_{1.\mathrm{i}} $$

$$ {}^{2.i}\boldsymbol{\omega}_{2.\mathrm{i}} = {}^{2.\mathrm{i}}\mathbf{R}_{1.\mathrm{i}}\,{}^{1.i}\boldsymbol{\omega}_{1.\mathrm{i}} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\dot{\theta}_{2.\mathrm{i}} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{1.\mathrm{i}} + \dot{\theta}_{2.\mathrm{i}} \end{pmatrix} $$

$$ {}^{3.i}\boldsymbol{\omega}_{3.\mathrm{i}} = {}^{3.\mathrm{i}}\mathbf{R}_{2.\mathrm{i}}\,{}^{2.\mathrm{i}}\boldsymbol{\omega}_{2.\mathrm{i}} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\dot{\theta}_{3.\mathrm{i}} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{1.\mathrm{i}} + \dot{\theta}_{2.\mathrm{i}} + \dot{\theta}_{3.i} \end{pmatrix} $$

Modelo dinámico de un robot paralelo

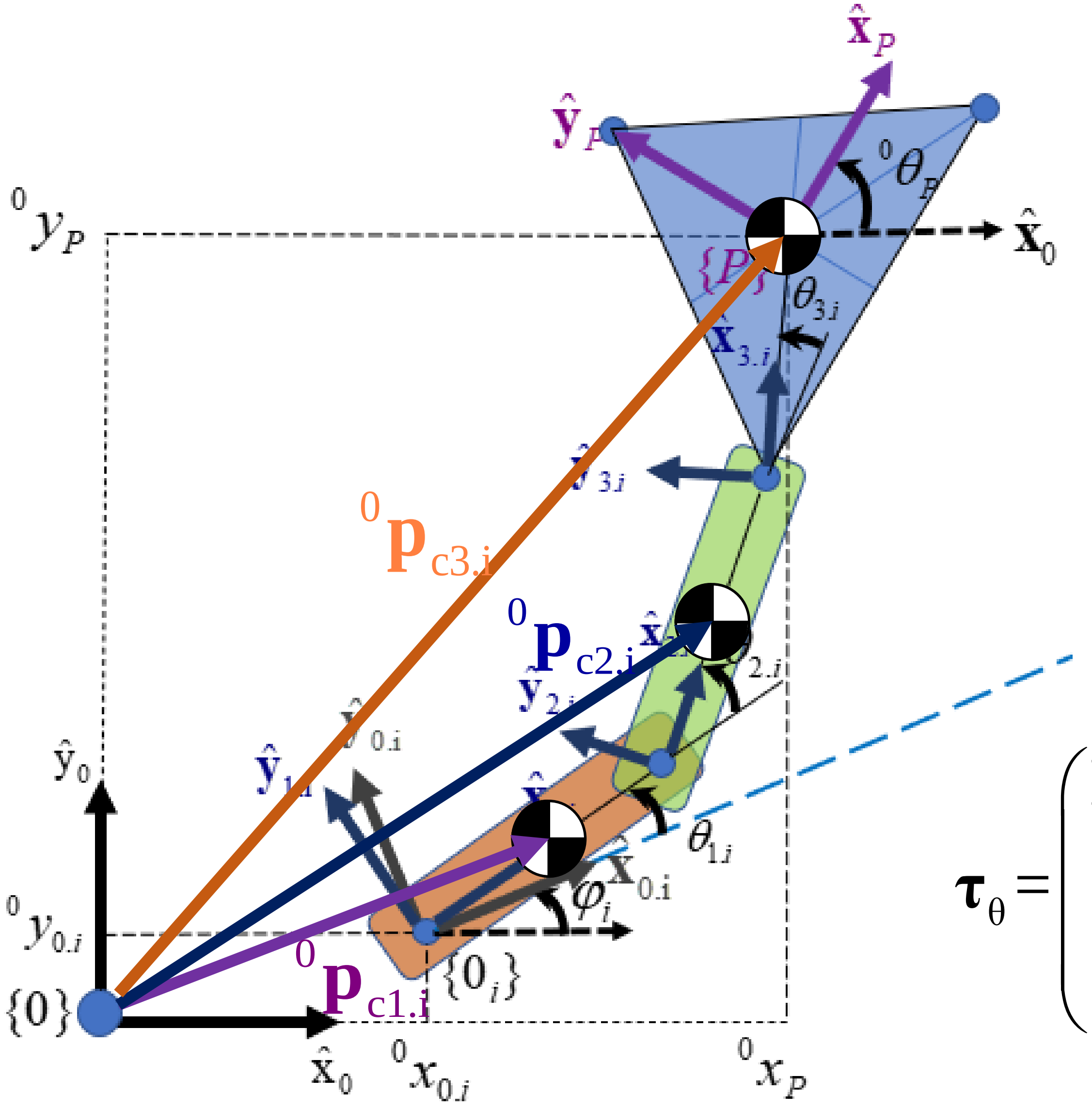


$$\mathbf{I}_{c1.i}=\frac{m_{1.i}}{12}\begin{bmatrix} L_{1.i}^{\,2}+w_{1.i}^{\,2} & 0 & 0 \\ 0 & h_{1.i}^{\,2}+w_{1.i}^{\,2} & 0 \\ 0 & 0 & L_{1.i}^{\,2}+h_{1.i}^{\,2} \end{bmatrix}$$

$$\mathbf{I}_{c2.i}=\frac{m_{2.i}}{12}\begin{bmatrix} L_{2.i}^{\,2}+w_{2.i}^{\,2} & 0 & 0 \\ 0 & h_{2.i}^{\,2}+w_{2.i}^{\,2} & 0 \\ 0 & 0 & L_{2.i}^{\,2}+h_{2.i}^{\,2} \end{bmatrix}$$

$$\mathbf{I}_{c3.i}=\begin{bmatrix} I_{xx.3} & 0 & 0 \\ 0 & I_{yy.3} & 0 \\ 0 & 0 & I_{zz.3} \end{bmatrix}$$

Modelo dinámico de un robot paralelo



$$\boldsymbol{\tau}_{\theta i} = \begin{pmatrix} \tau_{\theta 1.i} \\ \tau_{\theta 2.i} \\ \tau_{\theta 3.i} \end{pmatrix}$$

$$\boldsymbol{\tau}_{\theta.i} = \mathbf{M}_i(q_i)\ddot{\mathbf{q}}_i + \mathbf{V}_i(q_i, \dot{q}_i)$$

$$\boldsymbol{\tau}_{\theta} = \begin{pmatrix} \mathbf{M}_1(q_1) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2(q_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_3(q_3) \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_1 \\ \ddot{\mathbf{q}}_2 \\ \ddot{\mathbf{q}}_3 \end{pmatrix} + \begin{pmatrix} \mathbf{V}_1(q_1, \dot{q}_1) \\ \mathbf{V}_2(q_2, \dot{q}_2) \\ \mathbf{V}_3(q_3, \dot{q}_3) \end{pmatrix}$$