Laboratory work:≪Numerecial methods, lab4 ≫

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Contents

Purposes

1. Solve the boubdary value problem for a second-order differential equatuon on one-dimention segment using TDMA algorith

Problem statement

Mathematical model

$$\begin{cases} \frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + \sin(\pi x) \\ u(0,t) = 0 \\ u(1,t) = 0 \\ u(x,0) = 0 \end{cases} \tag{1}$$

$x \in [0,1]$
$h = \frac{\pi}{400}$
N=100
Exact solution:

$$U(x,t) = \frac{1}{\pi^2}(1 - \exp(-\pi^2 t))\sin(\pi x) \tag{2}$$

Implict

$$\begin{cases} \frac{u_i^{k+1} - u_i^k}{\tau} = a\frac{u_{i+1}^{k+1} - 2u_i^{k+1} + u_{i-1}^{k+1}}{h^2} + \sin(\pi x_i) \\ u_i^0 = 0 \\ u_0^{k+1} = 0 \\ u_N^{k+1} = 0 \end{cases} \tag{3}$$

$$\begin{cases} a_i = (1 + \frac{2a\tau}{h^2}) \\ b_i = \frac{a\tau}{h^2} \\ c_i = \frac{a\tau}{h^2} \\ f_i = sin(\pi x_i) \end{cases} \tag{4}$$

Explicit

$$\begin{cases} \frac{u_i^{k+1} - u_i^k}{\tau} = a\frac{u_{i+1}^{k} - 2u_i^{k} + u_{i-1}^{k}}{h^2} + \sin(\pi x_i) \\ u_i^0 = 0 \\ u_0^{k+1} = 0 \\ u_N^{k+1} = 0 \end{cases} \tag{5}$$

Krank-Nikolson

$$\begin{cases} \frac{u_i^{k+1}-u_i^k}{\tau} = a\frac{u_{i+1}^{k+1}-2u_i^{k+1}+u_{i-1}^{k+1}}{2h^2} + a\frac{u_{i+1}^k-2u_i^k+u_{i-1}^k}{2h^2} + \sin(\pi x_i) \\ u_i^0 = 0 \\ u_0^{k+1} = 0 \\ u_N^{k+1} = 0 \end{cases} \tag{6}$$

a

$$\begin{cases} a_i = (\frac{1}{\tau} + \frac{2a}{h^2}) \\ b_i = \frac{a}{2h^2} \\ c_i = \frac{a}{2h^2} \\ f_i = u_i^k(\frac{1}{\tau} - \frac{a}{h^2}) + + \frac{a}{2h^2}(u_{i+1}^k + u_{i-1}^k) sin(\pi x_i) \end{cases} \tag{7}$$

$$\begin{cases} a_0 = 1 \\ b_0 = 0 \\ c_0 = 0 \\ f_0 = 0 \end{cases} \tag{8}$$

$$\begin{cases} a_N = 1 \\ b_N = 0 \\ c_N = 0 \\ f_N = 0 \end{cases} \tag{9}$$

$u_i = 0$

TDMA

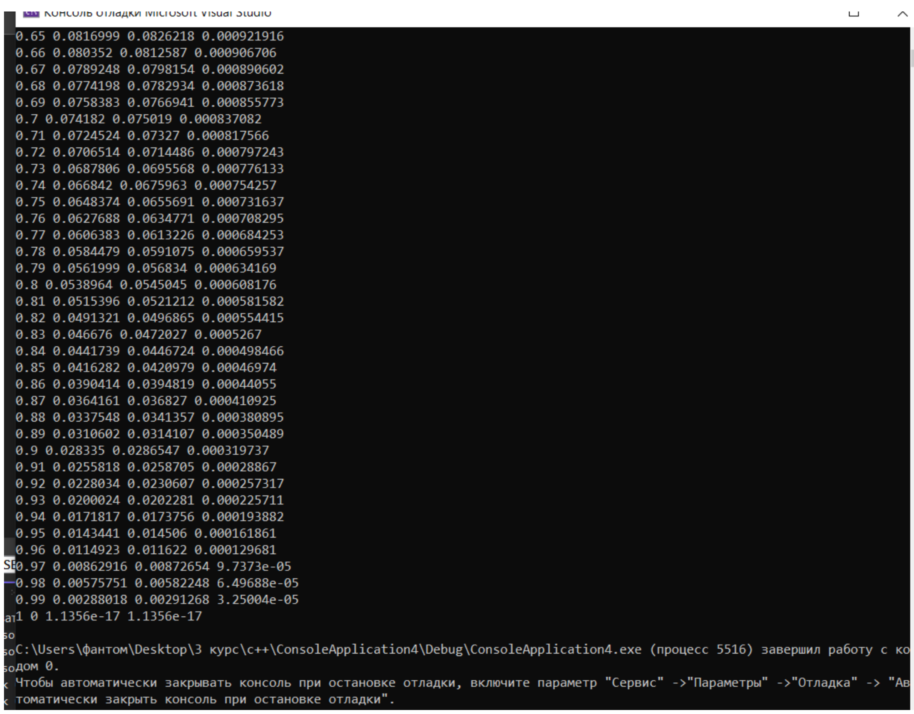
$a_i y_i = b_i y_{i+1} + c_i y_{i-1} + f_i$
$P_0 = \frac{b_0}{a_0}$
$Q_0 = \frac{f_0}{a_0}$
$P_i = \frac{b_i}{a_i - c_i P_{i-1}}$
$Q_i = \frac{f_i + c_i Q_{i-1}}{a_i - c_i P_{i-1}}$
$y_N = Q_N$
$y_i = P_i y_{i+1} + Q_i$

Program code

link to code in github: github.

Results

The graphs almost coincide, the error is small, therefore the methods can be considered accurate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0.65 0.0816999 0.0826218 0.000921916
0.66 0.080352 0.0812587 0.000906706
0.67 0.0789248 0.0798154 0.000890602
0.68 0.0774198 0.0782934 0.000873618
0.69 0.0758383 0.0766941 0.000855773
0.7 0.074182 0.075019 0.000837082
0.71 0.0724524 0.07327 0.000817566
0.72 0.0706514 0.0714486 0.000797243
0.73 0.0687806 0.0695568 0.000776133
0.74 0.066842 0.0675963 0.000754257
0.75 0.0648374 0.0655691 0.000731637
0.76 0.0627688 0.0634771 0.000708295
0.77 0.0606383 0.0613226 0.000684253
0.78 0.0584479 0.0591075 0.000659537
0.79 0.0561999 0.056834 0.000634169
0.8 0.0538964 0.0545045 0.000608176
0.81 0.0515396 0.0521212 0.000581582
0.82 0.0491321 0.0496865 0.000554415
0.83 0.046676 0.0472027 0.0005267
0.84 0.0441739 0.0446724 0.000498466
0.85 0.0416282 0.0420979 0.00046974
0.86 0.0390414 0.0394819 0.00044055
0.87 0.0364161 0.036827 0.000410925
0.88 0.0337548 0.0341357 0.000380895
0.89 0.0310602 0.0314107 0.000350489
0.9 0.028335 0.0286547 0.000319737
0.91 0.0255818 0.0258705 0.00028867
0.92 0.0228034 0.0230607 0.000257317
0.93 0.0200024 0.0202281 0.000225711
0.94 0.0171817 0.0173756 0.000193882
0.95 0.0143441 0.014506 0.000161861
0.96 0.0114923 0.011622 0.000129681
0.97 0.00862916 0.00872654 9.7373e-05
0.98 0.00575751 0.00582248 6.49688e-05
0.99 0.00288018 0.00291268 3.25004e-05
1 0 1.1356e-17 1.1356e-17

C:\Users\фантом\Desktop\3 курс\c++\ConsoleApplication4\Debug\ConsoleApplication4.exe (процесс 5516) завершил работу с ко
дом 0.
Чтобы автоматически закрывать консоль при остановке отладки, включите параметр "Сервис" ->"Параметры" ->"Отладка" -> "Ав
томатически закрыть консоль при остановке отладки".
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Figure 1: result of program

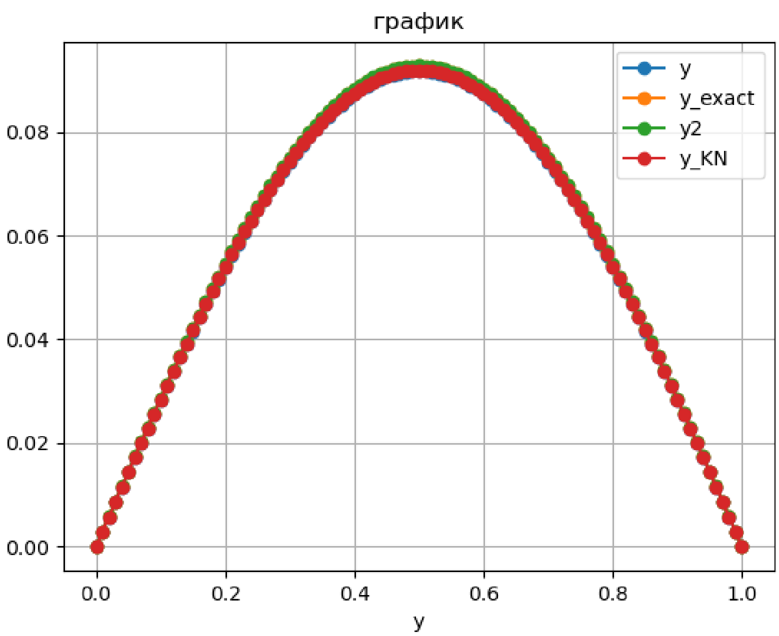

Figure 2: result of program