Cal State LAlogo_shield

Experiment 7

Flexure Test: Aluminum I-Beam

Team #3

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Objective

The objective of this experiment was to verify the validity of the Flexure formula, $\sigma = -\frac{My}{I}$ to determine whether stress is dependent on the moment and distance of which the flexure formula implies. The experimental data collected was then to be graphed in a location vs. stress graph and analyzed. Along with this, the collected experimental data was used to show how deflection across the beam at different points of the beam changed. Different gages collected data at different angles on the beam to also show the loads and how they were affected by angles of -45°, 0°, and 45°.

Theory

In previous experiments, the UTM machines were used to perform experimental procedures on different specimens. Using an S-Beam, it is possible to measure the stress value that the beam undergoes given some preliminary information that is given in the experiment. In Figure 1 we can see the given S-Beam from the experiment and in turn note the distance and the other features from the specimen.

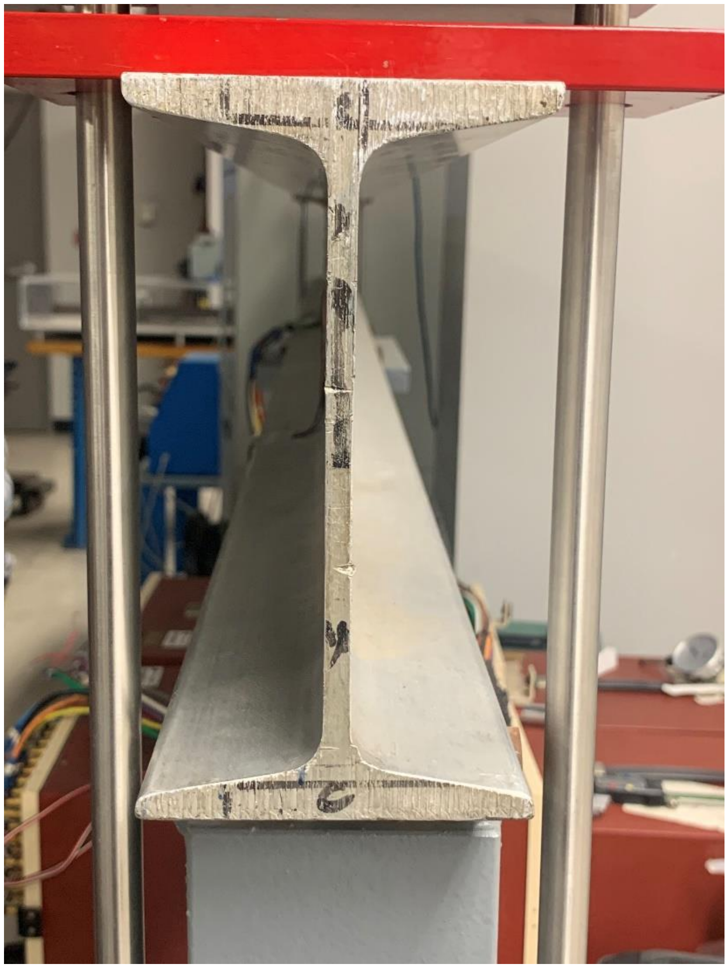


Figure 1. S-Beam Specimen

The beam itself is situated next to a mechanism that will test the flex of the beam and report back with data. Using these fundamental notions, it is possible to find the stresses associated with a given beam. In Figure 2

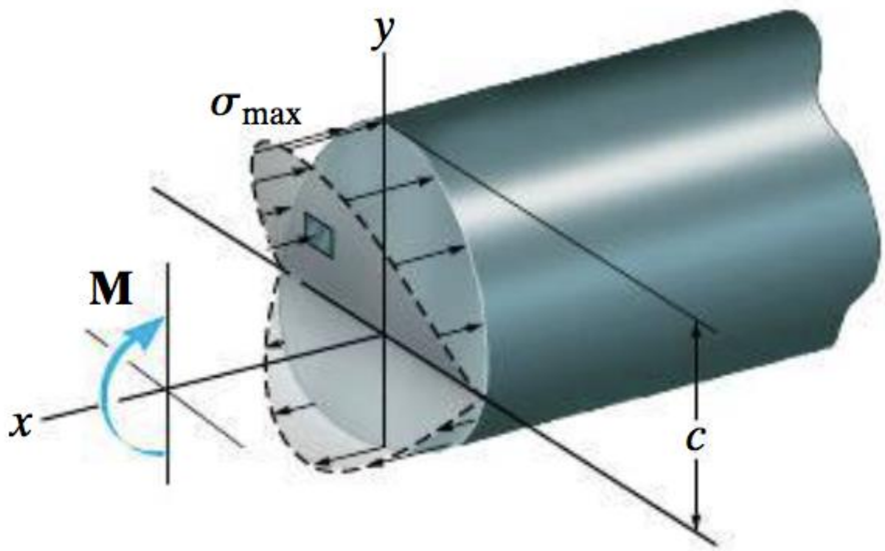


Figure 2. Beam Specimen Showing Forces [1]

By showing all the forces in the given example, it is possible to solve for the given stress on any beam by using the Flexure equation. Equation 1 shows this for a beam that is in tension, where M is the resultant internal moment of inertia, C is the perpendicular distance from the neutral axis to the point of interest and I is the moment of inertia of the cross-sectional area.

$$.\delta_{max} = \frac{M \cdot c}{I}$$

Equation 1. Flexure Formula (In Tension) [1]

If the moment done on the beam is done in the other direction, it will create compression along the beam. This can be described in Equation 2 with the introduction of a negative symbol to define this.

$$\sigma_{\max} = -\frac{M \cdot c}{I}$$

Equation 2. Flexure Formula (In Compression) [1]

Deriving this fundamental value for the moment of Inertia of the interior can be done as:

$$M = \int_A ydF = \int_A y(\delta\, dA) = \int_A y(\frac{y}{c}\delta_{\max})dA$$

$$M = \frac{\delta_{max}}{c}\int_A y^2 dA$$

Equation 3. Derivation of Internal moment of Inertia [1]

Using this information, it can also be used to compare the bending stress by utilizing Hooke's Law for the given system in turn returns with the following equation. Where "E" is the young's modulus of the system, and "e" is the strain placed on the system.

$$\sigma = Ee$$

Equation 4. Bending Stress Equation [1]

Finally, the moment of the beam can also be expressed using the fundamental equation for a moment where "F" is the force exerted on the beam and "r" is the vector from the point where we will take our moment from.

$$M = F \cdot r$$

Equation 5. Moment Equation [1]

Other properties can also be found with the lab given, using the convention that negative shear stress is on the first quadrant and positive shear stress is on the fourth quadrant we can ascertain a version of Mohr’s Circle which in turn can tell us the average normal stress given on a system. Where Epsilon A and Epsilon C are the strain values found from the system.

$$r = \left(\frac{\varepsilon_a \ + \ \varepsilon_c}{2}\right)$$

Equation 6. Vertical Distance in Mohr’s Circle [1]

Analyzing the beam as it bends, we can also analyze the displacement of the beam system by utilizing Castigliano's Theorem to determine the displacement of the beam as it bends, this can be described in Equation 7. Hence, we can see that the partial derivative of total strain energy is in the numerator with respect to the total force exerted on the beam on the denominator.

$$\sigma = \frac{dM}{dF}\int \frac{M^2}{2El}dx$$

Equation 7. Castigliano's Theorem [1]

Materials Used

- Dial Caliper
- Ruler
- SR-4 Strain Gages
- Switching Box
- Dial Indicators
- Hydraulic Jack on Steel Frame
- 6” Aluminum Alloy I Beam
- Ruler

Procedure [4] [5]

1. Measure, record and verify the dimensions of the I beam used in the experiment to the nearest 0.001”.

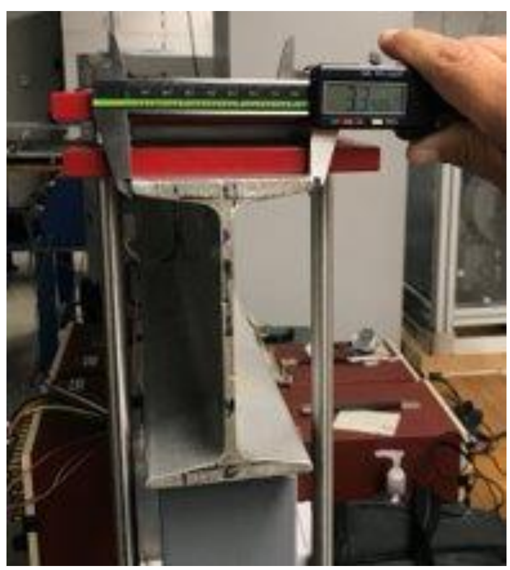


Figure 3: **Measurement of I beam [3]**

2. Take note and record the locations of where each SR-4 electrical strain gages, and dial indicators are located. The recorder will create a diagram/sketch of the entire loading diagram.

Figure 4: Dial indicators and Strain gage location on I beam.

3. Before beginning to put a load on the beam, first balance the testing machine with Zero load on the beam.

4. While there is zero load on the beam, record the readings at all Sr-4 strain gages and dial indicators on the I beam.

   Before proceeding get instructors approval and instruction before proceeding with the experiment

5. At the lowest speed possible, apply a 1000 lbf load on to the beam, via the hydraulic jack. (Note that the loading is obtained by proper used of the loading curve).

Figure 5: Hydraulic Jack on I beam, with loading indicator.

6. Repeat step 4 but apply the load in the following intervals. At 2000, 3000, 4000, 5000, and 6000 lbf. *(Be cautious when loading the beam at 6000 lbf, to not exceed 6000 lbf to avoid plastic deformation) *.
7. Once the last reading has been taken, remove load promptly be releasing pressure off of the hydraulic jack.
8. Record all strain gages and dial indicators at a load of zero, to obtain the true strain at zero load.

Sample Calculations

Maximum Moment on the beam:

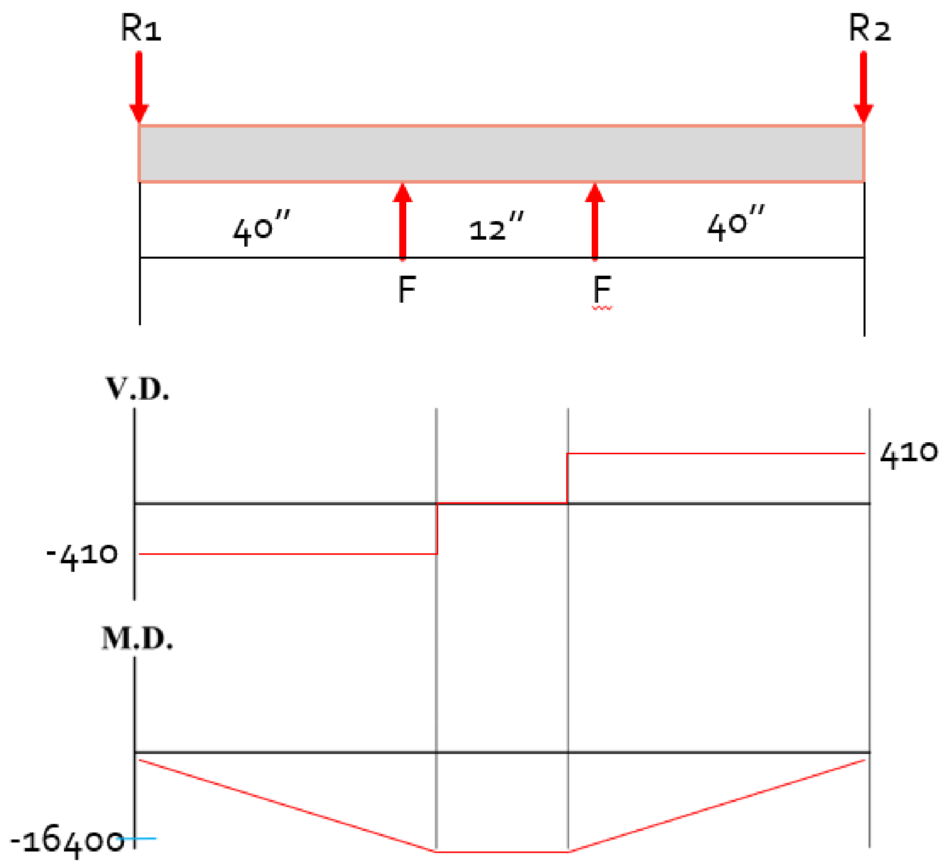


Figure #6 – FBD of Beam/Shear/Moment Diagram

$$M_{max} = R_1(40\ \text{in})$$

$$R_1 = -410\ lbf$$

$$\mathrm{M_{max}} = (-410\ \text{lbf})(40\ \text{in})$$

$$M_{max} = -16,400\ lbf - in$$

Bending Stress from Hooke's Law:

Young Modulus

$$\sigma = Ee$$

$$E = 10.6 * 10^6 \, psi$$

Strain

$$e = 165 * 10^6$$

$$\sigma = (10.6 * 10^6)(165 * 10^{-6})$$

$$\sigma = 1749 \text{ psi}$$

Bending Stress from Flexure:

Maximum Moment

$$\sigma = -\frac{My}{I}$$

$$M = -16,400 \text{ lbf} - \text{in}$$

$$y = 2 \text{ in}$$

$$I = 22.08 \text{ in}^4$$

Moment of Inertia

$$\sigma = -\frac{(-16400 \, lbf - in)(2 \, in)}{(22.08 \, in^4)}$$

$$\sigma = 1,485 \text{ psi}$$

Percent Error:

$$Percent\ Error = \left(\frac{Theoretical\ Stress - Experimental\ Stress}{Theoretical\ Stress}\right) \times 100\%$$

Theoretical Stress Calculated from Hooke's Law

Experimental Stress Calculated from Flexure

$$Theoretical\ Stress = 1485 \text{ psi}$$

$$Experimental\ Stress = 1749 \text{ psi}$$

$$Percent\ Error = \left(\frac{1485 \, psi - 1749 \, psi}{1485 \, psi}\right) \times 100\%$$

$$Percent\ Error = 17.7\ \%$$

Deflection utilizing Singularity Functions:

$$q(x) = -R_1 < x - 0" >^{-1} + F_1 < x - 40" >^{-1} + F_2 < x - 52" >^{-1} - R_2 < x - 92" >^{-1}$$

$$V(x) = -R_1 < x - 0" >^{0} + F_1 < x - 40" >^{0} + F_2 < x - 52" >^{0} - R_2 < x - 92" >^{0} + \cancel{C_1}$$

where $C_1 = 0$

$$M(x) = -R_1 < x - 0" >^{1} + F_1 < x - 40" >^{1} + F_2 < x - 52" >^{1} - R_2 < x - 92" >^{1} + \cancel{C_2}$$

Where $C_2 = 0$

$$EI\theta(x) = -\frac{R_1}{2} < x - 0" >^{2} + \frac{F_1}{2} < x - 40" >^{2} + \frac{F_2}{2} < x - 52" >^{2} - \frac{R_2}{2} < x - 92" >^{2} + C_3$$

$$EIy(x) = -\frac{R_1}{6} < x - 0">^{3} + \frac{F_1}{6} <x-40" >^{3} + \frac{F_2}{6} < x - 52">^{3} - \frac{R_2}{6} <x-92" >^{3} + C_3(x) + C_4$$

$$EIy(x) = -\frac{R_1}{6} < x - 0">^{3} + \frac{F_1}{6} <x-40" >^{3} + \frac{F_2}{6} < x - 52">^{3} - \frac{R_2}{6} <x-92" >^{3} + C_3(x) + C_4$$

$$EIy(35") = -\frac{R_1}{6}(35" - 0")^3 + \cancel{\frac{F_1}{6}(35"-40")^3} + \cancel{\frac{F_2}{6}(35" - 52")^3} - \cancel{\frac{R_2}{6}(35"-92")^3} + C_3(x) + C_4$$

$$C_3 = (1410.7)R_1 - (370.6)F$$

$$C_4 = 0$$

E for Al 6061 = ~ 10.1 to $\sim 10.6\ M\ Psi$ [2]

I for Beam = $22.08\ in^4$

$$R_1 = -410\ lbf$$

$$F = 858\ \text{lbf}$$

Depending on the force applied deflection at 35" will be:

$$EIy(35") = -\frac{(R_1)}{6}(35" - 0")^3 + (1410.7)R_1 - (370.6)F(35")$$

$$EIy(35") = -\frac{(410\ lbf)}{6}(35" - 0")^3 + (1410.7)(410\ lbf) - (370.6)(858\ lbf)(35")$$

$$y = -0.0279\ in$$

Deflection utilizing Castigliano's Theorem:

$$\mu = \mu_{axial} + \mu_{bend} + \mu_{tran-Shear} + \mu_{torison}$$

Only Bending Stress is present and thus, the other factors are negligible

$$\delta = \frac{dM}{dF}\left[\cancel{\int \frac{F^2 L}{2LE}}dx + \int \frac{M^2}{2EI}dx + \cancel{\int \frac{k^2 v^2}{2GA}}dx + \cancel{\int \frac{T^2 L}{2GJ}}dx\right]$$

Derived Equation for calculating maximum deflection using Castigliano's Theorem:

$$\delta = \frac{dM}{dF}\int \frac{M^2}{2EI}dx$$

$$\delta_L = \int_0^L \frac{dM}{dF} * \frac{M^2}{2EI}\, dx$$

$$\delta_L = \int_0^L \frac{dM}{dF} * \frac{2M}{2EI}\, dx$$

$$\delta_L = \frac{1}{EI}\int_0^L M\frac{dM}{dF}dx$$

Moment equation from beam

$$M = -Fx$$

$$\frac{dM}{dF} = -x$$

Partial Derivative of Moment with respect to Force

$$\delta_L = \frac{-F}{EI}\int_0^L x(-x)dx$$

Maximum Deflection of the beam at 35":

$$\delta_{35''} = \frac{F}{EI}\int_0^{35} x^2 dx$$

Young's Modulus of Al 6061

E for Al 6061 = ~ 10.1 to $\sim 10.6\ M\ psi$

I for Beam = $22.08\ in^4$

Inertia

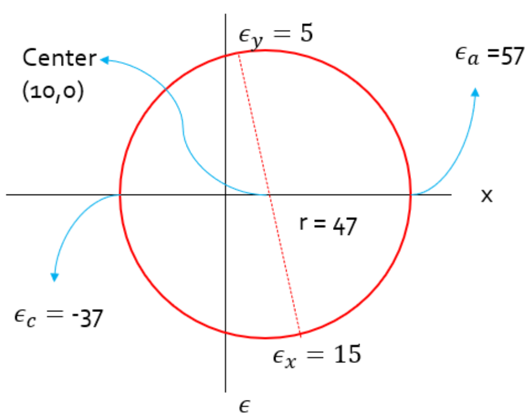
Force = $-410\ lbf$

Force calculated from Newton's 2nd Law

$$\delta_{35''} = \frac{(-410\ lbf)}{(10.6 * 10^6\ psi)(22.08)} * \frac{(35\ in)^3}{3}$$

$$\delta_{35''} = -0.02503\ in$$

Mohr's Circle:



$$\epsilon_a = 57$$
$$\epsilon_b = 15 = \epsilon_x$$
$$\epsilon_c = -37$$

$$r = \frac{\epsilon_a + \epsilon_c}{2}$$

$$r = \frac{57 + (-37)}{2}$$

$$r = \frac{57 + (-37)}{2}$$

$$r = 47$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$

$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$

$$\epsilon_y = 57 + (-37) - 15$$

$$\epsilon_y = 5$$

$$\Upsilon_1 = \Upsilon_2$$

$$\Upsilon_1 = 5 + 15 - 2(57)$$

$$\Upsilon_1 = -97$$

Results

Table #1 – Deflection Data for I-Beam

Dial Indicators (in)	Location from R_1 (in)			Load (lbf)			
		820	1640	2460	3280	4100	4920
1	5	0.012	0.021	0.031	0.04	0.05	0.058
2	10	0.022	0.034	0.05	0.065	0.067	0.119
3	15	0.033	0.061	0.088	0.115	0.142	0.166
4	20	0.042	0.077	0.113	0.147	0.182	0.214
5	25	0.05	0.092	0.136	0.177	0.22	0.258
6	30	0.059	0.107	0.157	0.205	0.252	0.288
7	35	0.064	0.116	0.17	0.235	0.298	0.336

Graph #1 – Plot of Given Deflections for Varying Loads

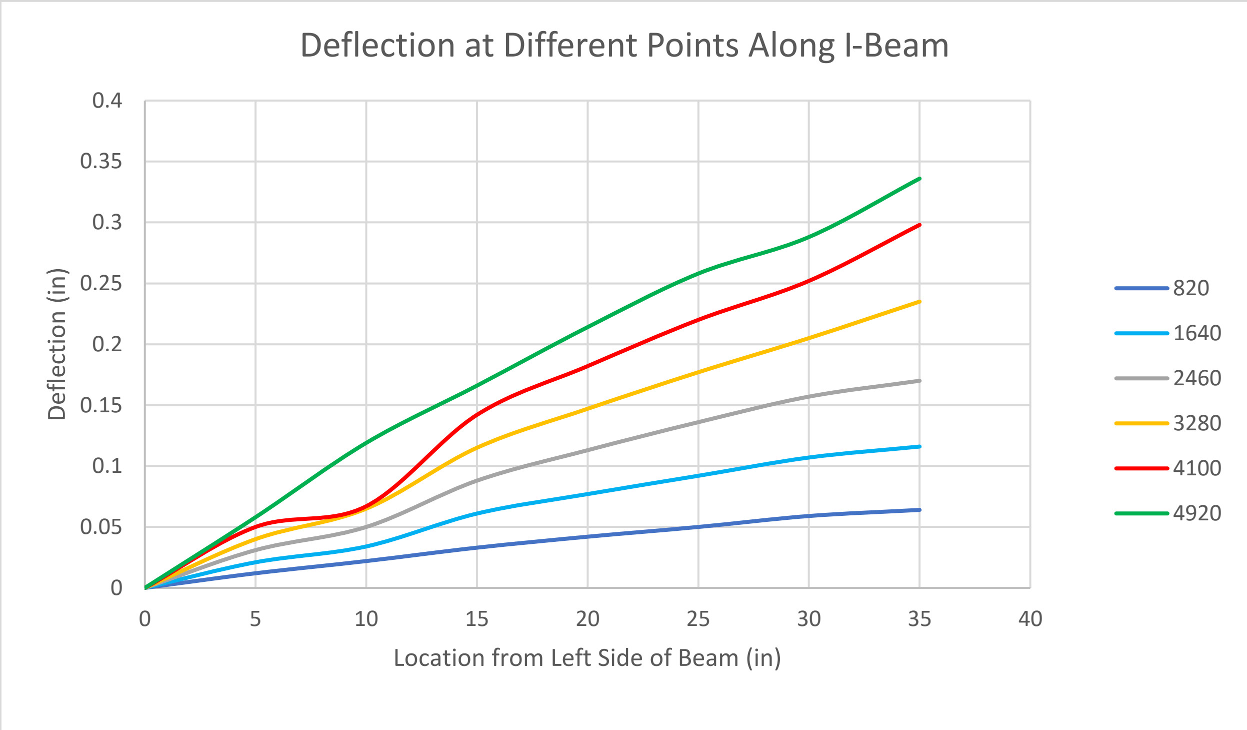


Table #2 – Transverse Gage Readings on the I-Beam

Gage (Transverse) (in/in)*10^-6	Location (in)			Load (lbf)			
		820lbf	1640lbf	2460	3280	4100	4920
1	3	240	447	654	853	1053	1233
2	2	165	309	442	584	717	837
3	1	80	152	224	294	366	424
4	-1	-64	-125	-181	-240	-298	-359
5	-2	-140	-263	-379	-495	-619	-732
6	-3	-223	-408	-586	-761	-947	-1112

Table #3 – Strain Gage Readings Converted to Force

Gage (Transverse) (in/in)*10^-6	Location (in)			Stress (psi)			
		820lbf	1640lbf	2460	3280	4100	4920
1	3	2E+09	5E+09	7E+09	9E+09	1.06E+10	1E+10
2	2	2E+09	3E+09	4E+09	6E+09	7E+09	8E+09
3	1	8E+08	2E+09	2E+09	3E+09	4E+09	4E+09
4	-1	-6E+08	-1E+09	-2E+09	-2E+09	-3E+09	-4E+09
5	-2	-1E+09	-3E+09	-4E+09	-5E+09	-6E+09	-7E+09
6	-3	-2E+09	-4E+09	-6E+09	-8E+09	-1E+10	-1E+10
7	0	0E+00	0E+00	0E+00	0E+00	0E+00	0E+00
8	-45	6E+08	1E+09	1E+09	2E+09	2E+09	3E+09
9	0	2E+08	2E+08	2E+08	4E+08	4E+08	4E+08
10	45	-4E+08	-7E+08	-1E+09	-1E+09	-2E+09	-2E+09

Table #4 – Force Divided by 2 (Two Upward Forces from Hydraulic Jacks on I-Beam)

Gage (Transverse) (in/in)*10^-6	Location (in)			Force from Stress			
		410	820	1230	1640	2050	2460
1	3	1E+09	2E+09	3E+09	4E+09	5E+09	6E+09
2	2	8E+08	2E+09	2E+09	3E+09	4E+09	4E+09
3	1	4E+08	8E+08	1E+09	1E+09	2E+09	2E+09
4	-1	-3E+08	-6E+08	-9E+08	-1E+09	-2E+09	-2E+09
5	-2	-7E+08	-1E+09	-2E+09	-2E+09	-3E+09	-4E+09
6	-3	-1E+09	-2E+09	-3E+09	-4E+09	-5E+09	-6E+09

Table #5 – Theoretical Stress from Flexure Formula

Gage (Transverse) (in/in)*10^-6	Location (in)			Theoretical Stress (psi)			
		410	820	1230	1640	2050	2460
1	3	2E+03	4E+03	7E+03	9E+03	1E+04	1E+04
2	2	1486	3E+03	4E+03	6E+03	7E+03	9E+03

Gage (Transverse) (in/in)*10^-6	Location (in)			Theoretical Stress (psi)			
3	1	7E+02	1E+03	2E+03	3E+03	4E+03	4E+03
4	-1	-7E+02	-1E+03	-2E+03	-3E+03	-4E+03	-4E+03
5	-2	-1E+03	-3E+03	-4E+03	-6E+03	-7E+03	-9E+03
6	-3	-2E+03	-4E+03	-7E+03	-9E+03	-1E+04	-1E+04

Table #6 – Experimental Stress from Hooke's Law

Gage (Transverse) (in/in)*10^-6	Location (in)			Experimental Stress (psi)			
		410	820	1230	1640	2050	2460
1	3	2E+03	5E+03	7E+03	9E+03	10635	1E+04
2	2	1667	3E+03	4E+03	6E+03	7E+03	8E+03
3	1	8E+02	2E+03	2E+03	3E+03	4E+03	4E+03
4	-1	-6E+02	-1E+03	-2E+03	-2E+03	-3E+03	-4E+03
5	-2	-1E+03	-3E+03	-4E+03	-5E+03	-6E+03	-7E+03
6	-3	-2E+03	-4E+03	-6E+03	-8E+03	-1E+04	-1E+04

Graph #2 - Plot of Theoretical and Experimental Stress

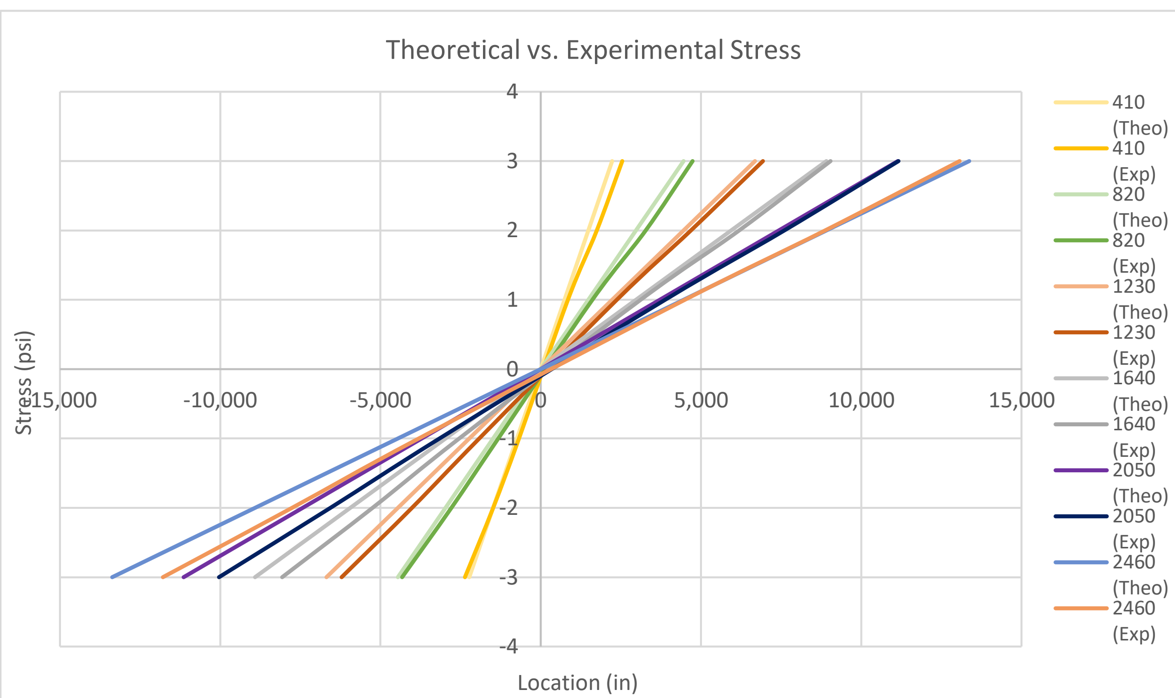


Table #7 – Percent Error Between Theoretical and Experimental Stress

Percent Error							
Gage Indicator	Gage Location (in)	Load (lbf)					
		410	820	1230	1640	2050	2460
1	3	14.2%	6.3%	3.7%	1.4%	0.2%	2.2%
2	2	17.7%	10.2%	5.1%	4.2%	2.3%	0.5%
3	1	14.2%	8.5%	6.6%	4.9%	4.5%	0.8%
4	-1	8.7%	10.8%	13.9%	14.4%	14.9%	14.6%
5	-2	0.1%	6.2%	9.9%	11.7%	11.7%	12.9%
6	-3	6.1%	3.0%	7.1%	9.5%	9.9%	11.8%

Graph #3 – Plot of Applied Stress vs. Force

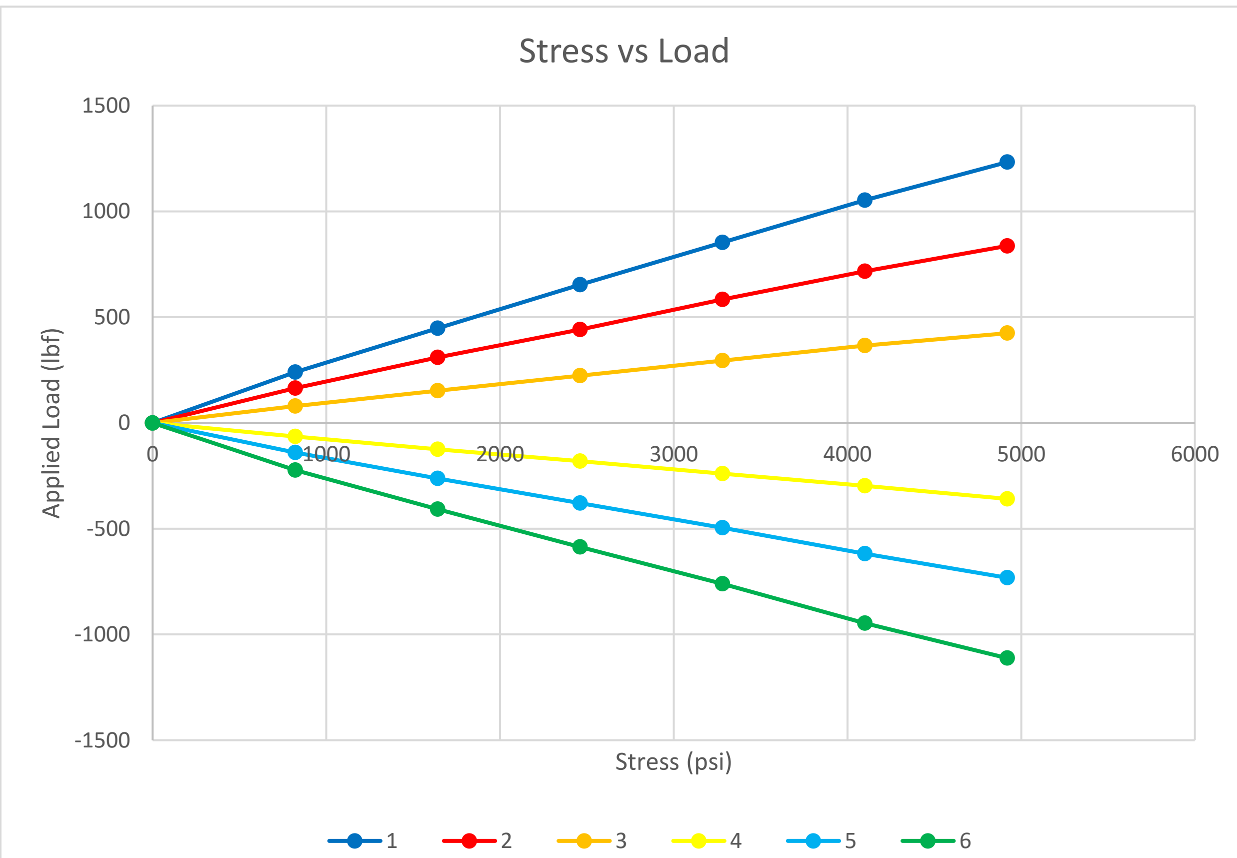


Table #8 – Singularity Function Data

Gage	Location from R_1 (in)			Load (lbf)			
		410	820	1230	1640	2050	2460
1	5	-0.01807	-0.04106	0.006009	-0.06336	-0.09155	-0.11109
2	10	-0.01786	-0.04823	-0.00364	-0.07053	-0.10476	-0.12708
3	15	-0.00309	-0.04144	0.005254	-0.06374	-0.09276	-0.11298
4	20	0.01234	-0.00132	0.063832	-0.02362	-0.01547	-0.01689
5	25	0.0356	0.032878	0.112968	0.010576	0.04810	0.06296
6	30	-0.01115	0.080419	0.180987	0.058118	0.13584	0.17151
7	35	-0.02789	-0.0223	0.033452	-0.0446	-0.05575	-0.0669

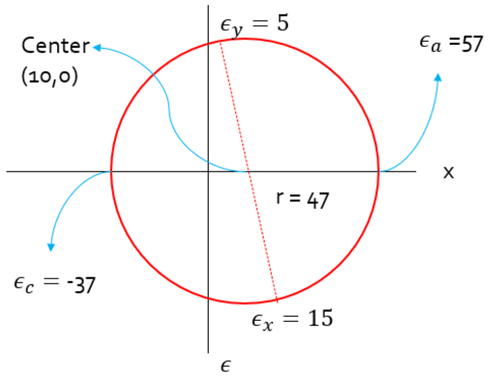
Table #9 – Castigliano's Theorem Data

Gage (in)	Location from R_1 (in)			Load (lbf)			
		410	820	1230	1640	2050	2460
1	5	-7.29E-05	-0.00015	-0.000218	-0.000291	-0.000364	-0.00043
2	10	-0.000583	-0.00117	-0.001751	-0.002335	-0.00291	-0.00350
3	15	-0.00197	-0.00394	-0.005912	-0.007882	-0.00985	-0.01182
4	20	-0.00467	-0.00934	-0.014014	-0.018685	-0.02335	-0.02802
5	25	-0.00912	-0.01825	-0.027371	-0.03649	-0.0456	-0.05474
6	30	-0.0157	-0.03153	-0.04729	-0.06306	-0.078829	-0.09459
7	35	-0.0250	-0.05007	-0.075107	-0.100143	-0.12517	-0.15021

Table #10 – Strain Gage Readings at Different Angles

Gage (Transverse) (in/in)*10^-6				Load (lbf)			
		820lbf	1640lbf	2460	3280	4100	4920
8	-45°	57	105	146	188	233	268
9	0°	15	19	22	35	39	42
10	45°	-37.6	-71	-105	-137	-173	-207

Mohr's Circle (820 lbf):



$$\epsilon_a = 57$$
$$\epsilon_b = 15 = \epsilon_x$$
$$\epsilon_c = -37$$

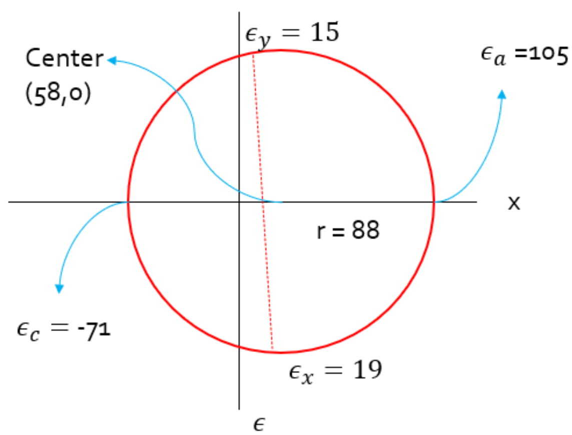
$$r = \frac{\epsilon_a + \epsilon_c}{2}$$
$$r = \frac{57 + (-37)}{2}$$
$$r = \frac{57 + (-37)}{2}$$
$$r = 47$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$
$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$
$$\epsilon_y = 57 + (-37) - 15$$
$$\epsilon_y = 5$$
$$\Upsilon_1 = \Upsilon_2$$

$$\Upsilon_1 = 5 + 15 - 2(57)$$
$$\Upsilon_1 = -97$$

Mohr's Circle (1640 lbf):



$$\epsilon_a = 105$$
$$\epsilon_b = 19 = \epsilon_x$$
$$\epsilon_c = -71$$

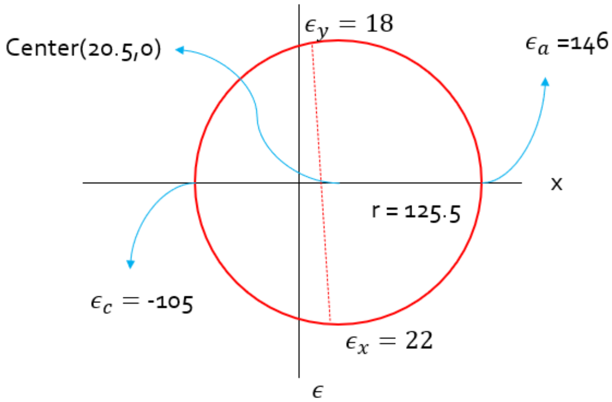
$$r = \frac{\epsilon_a + \epsilon_c}{2}$$
$$r = \frac{105 + (71)}{2}$$
$$r = 88$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$
$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$
$$\epsilon_y = 105 + (-71) - 19$$
$$\epsilon_y = 15$$
$$\Upsilon_1 = \Upsilon_2$$

$$\Upsilon_1 = 15 + 19 - 2(105)$$
$$\Upsilon_1 = -176$$

Mohr's Circle (2460 lbf):



$$r = \frac{\epsilon_a + \epsilon_c}{2}$$

$$r = \frac{146 + (105)}{2}$$

$$r = 125.5$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$

$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$

$$\epsilon_y = 145 + (-105) - 22$$

$$\epsilon_y = 18$$

$$\Upsilon_1 = \Upsilon_2$$

$$\epsilon_a = 146$$

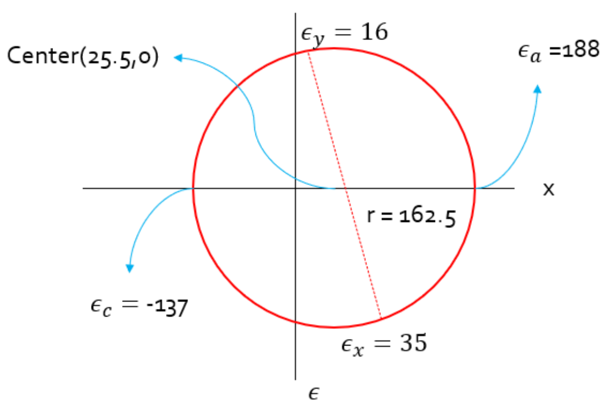
$$\epsilon_b = 22 = \epsilon_x$$

$$\epsilon_c = -105$$

$$\Upsilon_1 = 18 + 22 - 2(146)$$

$$\Upsilon_1 = -252$$

Mohr's Circle (3280 lbf):



$$r = \frac{\epsilon_a + \epsilon_c}{2}$$

$$r = \frac{188 + (137)}{2}$$

$$r = 162.5$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$

$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$

$$\epsilon_y = 188 + (-137) - 35$$

$$\epsilon_y = 16$$

$$\Upsilon_1 = \Upsilon_2$$

$$\epsilon_a = 188$$

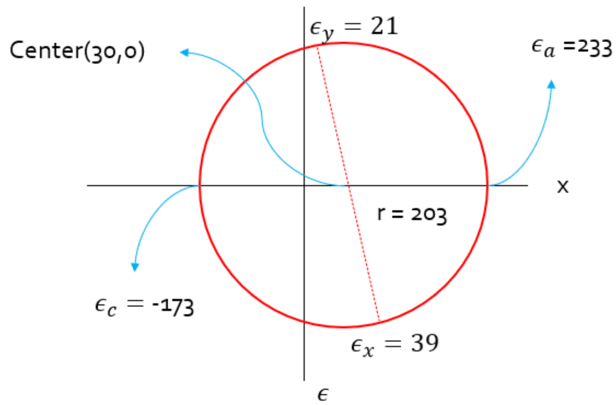
$$\epsilon_b = 35 = \epsilon_x$$

$$\epsilon_c = -137$$

$$\Upsilon_1 = 16 + 35 - 2(188)$$

$$\Upsilon_1 = -325$$

Mohr's Circle (4100 lbf):



$$r = \frac{\epsilon_a + \epsilon_c}{2}$$

$$r = \frac{233 + (173)}{2}$$

$$r = 203$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$

$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$

$$\epsilon_y = 233 + (-173) - 39$$

$$\epsilon_y = 21$$

$$\Upsilon_1 = \Upsilon_2$$

$$\epsilon_a = 233$$

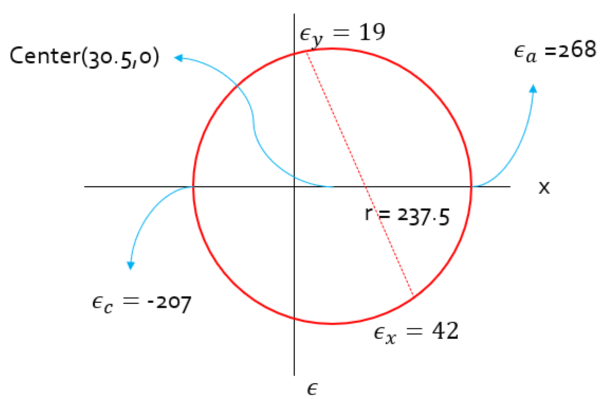
$$\epsilon_b = 39 = \epsilon_x$$

$$\epsilon_c = -173$$

$$\Upsilon_1 = 21 + 39 - 2(233)$$

$$\Upsilon_1 = -406$$

Mohr's Circle (4920 lbf):



$$r = \frac{\epsilon_a + \epsilon_c}{2}$$

$$r = \frac{268 + (207)}{2}$$

$$r = 237.5$$

$$\Upsilon_1 = \epsilon_y + \epsilon_b - 2\epsilon_a$$

$$\Upsilon_2 = 2\epsilon_c - \epsilon_x - \epsilon_y$$

$$\epsilon_y = \epsilon_a + \epsilon_c - \epsilon_b$$

$$\epsilon_y = 268 + (-207) - 42$$

$$\epsilon_y = 19$$

$$\Upsilon_1 = \Upsilon_2$$

$$\epsilon_a = 268$$

$$\epsilon_b = 42 = \epsilon_x$$

$$\epsilon_c = -207$$

$$\Upsilon_1 = 19 + 42 - 2(268)$$

$$\Upsilon_1 = -475$$

Discussion

The I-beam (or more exactly S-beam) made of Aluminum 6061 was put under a force evenly distributed to two points as the Free-Body Diagram (Figure 6) shows. The relationship of the force applied, internal forces and moments, and stress applied gives up ways to solve for the deflection that occurs in the beam internally. The first method used experimentally was to record the deflections using dial indicators along every 5 inches (up to 35 inches) along the beam from the end of the beam toward the center of the beam. This gave us data which we can use to compare to the theoretical values for the deflection that occurs along the beam. This also allows us to see the relationship of the flexure formula of the bending stress at the location of the maximum deflection which happens to be where the maximum internal moment is applied.

The first method used was the flexure formula which uses the maximum internal moment at a given location directly to find the stress. However, there is some assumptions to be made before using this formula. Because we are using the relationship of Young's modulus to get the relationship of the flexure formula, the beam cannot go over the elastic region from the stress-strain curve. If it does, the formula will become void. However, to make sure this is not the case, the lab manual specifically says not to go over 6000lbf which safely keeps the formula in check when doing the experiment. Comparing the flexure formula stress with the stress obtained by Hooke's Law show that the general pattern is the same but has some percent differences.

Then using the dial indicators as an experimental value of the flexure test, two different methods were used to calculate the theoretical values of where the maximum deflection occurs. However, there are some notes to mention. The dial indicators are limited to being set at 35" along the beam. However, the maximum moment occurs a little after 35" based on graphs and calculations. However, to compare to the maximum dial indicator at 35" all calculations using the singularity function and Castigliano's method was adjusted to calculate the deflection that occurs at 35" along the beam. The data calculated shows that the singularity functions show a similar pattern to the experimental data but with higher deflections calculated. Then when compared to the Castigliano's theorem, the theoretical values are relatively close to each other which is relatively close to the experimental value. This confirms that the Dial is calibrated correctly and working. Of course, the experimental values may differ from the theoretical values because the beam might deflect more or less depending on temperature and the condition of the beam. However, the values from the singularity functions and the Castigliano's method do support that the maximum deflection occurs somewhere between 35" to 40". However theoretically the beam should have maximum deflection at the 40" mark where the force is applied.

Then we can use the Mohr's circle to see each principal stresses for each force that was applied. This shows all the possible stresses along the planes in the beam and even shows how the stress would change based on the degree of the deflection. This shows how the flexural testing in a complicated with multiple possibilities of stress and strain that may occur in the beam based on its location unlike the tensile or shear testing. It also shows the relationship of how the top side of our beam was under tension and the bottom of our beam in compression, thus the deflection being a negative number based on our coordinate system of positive y being upwards.

Other things to mention are that the flexure formula shows a linear relationship between the load force and the displacement in the beam when the stress and strain is also in a linear relationship due to being in the elastic region. Therefore, the stress and location relationship should be a linear trend which is shown on the graph 2. However, when it comes to the deflection and displacement, as the dials go from one end to the center (last being at 35"), the defection shows a parabola relationship which makes sense because the beam is also bending parabolically.

Conclusion

The validity of the flexure formula to ascertain whether the stress and moment varies with y is unknown. Looking through the data shown above on the Results Section of the lab report, it is very apparent that there are many discrepancies and uncertainties within the data. The percentage differences for a load of 410-pound force ranges from 0.1% - 14.2% while a load of 820-pound force ranges from 3.0% - 10.8%. In addition, the 1230-pound force ranges from 3.7% - 16.9% with the case of the 1640-pound force ranging from 1.4% - 14.4%. To add on, the applied force of a 2050-pound force ranges 0.2% - 14.9% and the applied force of a 2460-pound force ranges from 0.5% - 14.6%. Now with these set of various ranges of percentage differences between each load, there is a trend of the percentage differences increasing in size depending on the location of the gage from 3 to -3 whilst omitting 0 from the scale due to it being the neutral axis being the centroid of the beam.

The singularity functions, Castigliano's theorem, and deflections points show that there is a similarity in the way deflection occurs throughout the beam, as well. Singularity functions and Castigliano's theorem gave very similar values, as seen in the Results. Mohr's circle was created from the angle readings of the experiment, and the Mohr's circle show different stress values created from the readings. These values were relatively close together, as the Mohr's circles for the 820 and 1640 lbf readings generated a stress of 97 and 176 psi. The deflections for the beam showed a trend of increasing parabolic shape, as well.

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