Simulation of the solar system

In this assignment you will model the motion of a large number of celestial bodies (planets, moons, asteroids) in the solar system by numerically solving Newton's equations of motion for these bodies. The pair force between two bodies i and j with masses m_i and m_j is computed using Newton's law of universal gravitation,

$$\mathbf{F}_{ij} = -G \frac{m_i\, m_j}{r_{ij}^2}\, \hat{\mathbf{r}}_{ij}, \text{where } G = 6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}. \tag{1}$$

This is the force on body i due to the gravitational pull of body j. $\hat{\mathbf{r}}_{ij}$ denotes the unit vector: $\hat{\mathbf{r}}_{ij} = \mathbf{r}_{ij}/r_{ij}$, with $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ and $r_{ij} = |\mathbf{r}_{ij}|$.

First you will create a code for N bodies interacting by means of this force-law and evolving in time (and three spatial dimensions) according to Newton's equations of motion.

Questions: modeling

1. Write down the algorithm to compute the gravitational force on body i due to interaction with all other bodies $j \neq i$.
2. Note that for a large number of lighter bodies the mass is unknown. This mass is put to zero in the supplied data file. How will you handle this issue? (Hint: Note that for particle i its mass appears both in the mass-times-acceleration and gravitational force terms).
3. You are asked to solve the equations of motion numerically by means of the velocity-Verlet scheme. Provide the algorithm you will use to integrate the equations of motion. (You may describe it, include (pseudo)code, use a flow diagram, etc.)
4. Write down an expressions for the total kinetic energy and for the total potential energy of the system.

Questions: C-implementation

5. Use dynamically allocated arrays for the masses, positions, velocities of the bodies and for the forces (on the particles). Only store information related to the current time step. That is, do not store previous time steps. (Remark: You can argue that allocated arrays are not needed, and that it is fine to store all time-steps. However, the purpose of this question is also to show C programming skills. So, to get full points, do as asked. Next to that, for very large particle-based simulations this is the way to do it.)

Two files are provided namely `solar_system_13sept2021.dat` and `solar_system_13sept2022.dat`. These files contain the positions and velocities of the planets and the sun on 13 September 2021 and 2022, respectively. The source is https://ssd.jpl.nasa.gov/horizons.cgi. For the simulations use the 2021 data to initialize positions and velocities. The 2022 data file can be used for validation purposes.

6. Write a function that can read a data file with the provided format and use it to set masses and initialize positions and particles.

7. Write a function that computes the forces on (or accelerations of) the bodies from their positions and also computes the total potential energy.

8. Implement the update of positions and velocities according to the velocity-Verlet algorithm.

9. Write a function to output the particle positions in time (once so many time steps) to a file.

10. Write a function that outputs total kinetic and total potential energy to a file every time step.

Questions: analysis

11. Check the correctness of your implementation by simulating the 2-body system of the earth moving around the sun, and the 3-body systems sun-earth-moon. Are the periods of the earth and moon (approximately) correct? Remarks: For the analysis you can use any software you like, such as, Python, Matlab, Excel, Origin, etc.

12. Provide a visualization of the trajectories in case of the sun-earth-moon system.

13. Initialize the system using `solar_system_13sept2021.dat`. Simulate the system for a year and check energy conservation. How well is energy conserved? Check its dependence on the time-step size.

14. Simulate the motion of the planets for 1 year, and compare the final result with `solar_system_13sept2022.dat`. Describe your observations.

15. Speculate about possible causes of observed deviations between your results and those in `solar_system_13sept2022.dat`.

Report

- Write a concise report where the above questions all addressed.

- Upload the report as a pdf onto Canvas. Provide, separately, the source (i.e. `.h` and `.c`) files as one `.zip` file. (Remark: It is convenient for me when you do it exactly like this. So, do not put the pdf in the zip. Use `.zip` not `.rar` or another format. Do not include `.exe` files in the zip.)