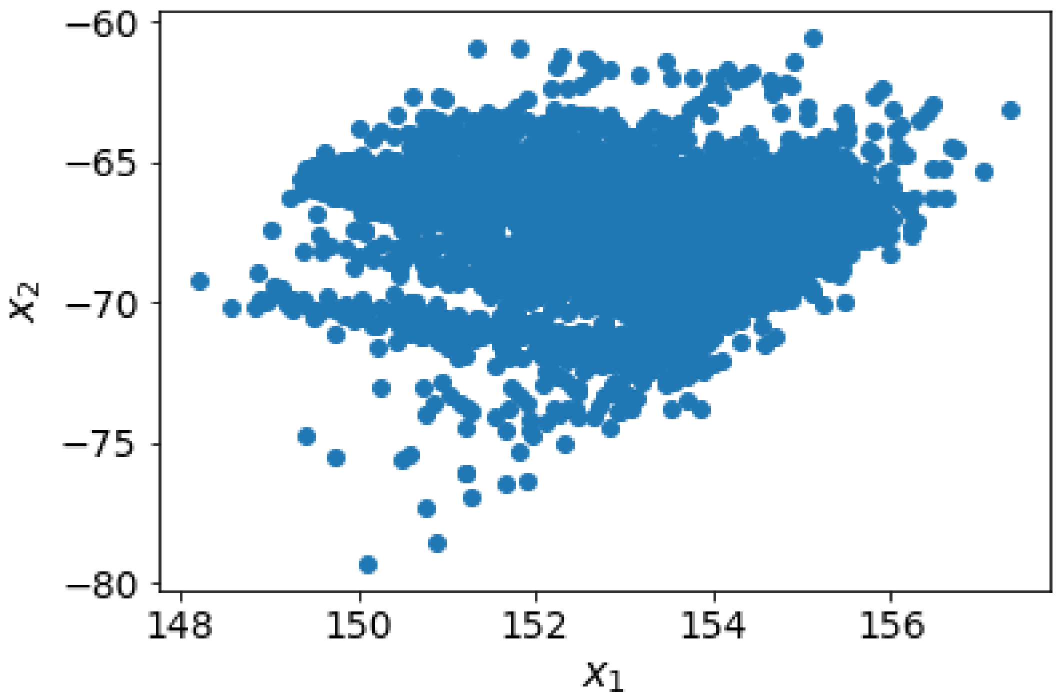
fit_output_summaries

May 20, 2019

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt

        import sys
        sys.path.append('../../../../')
        import set_plot_sizes

        sys.path.append('../../../../cosmosis_wrappers/') # change to correct path
        import ABC_saved_sims_multiparam

        from tqdm import tqdm_notebook as tqdm

        # change to the path where ABCPMC git clone is located
        sys.path.insert(0,'../../../../../abcpmc/')
        import abcpmc # find citation at https://github.com/jakeret/abcpmc

        import corner

In [2]: data_dir = './preloaded_data/ABC_results'

In [3]: def load_ABC_results(modelversion):
            """
            Load the results of 10,000 simulations that have been fed through a trained networ
            and compared with some fiducial simulation
            """
            abc = dict()
            for key in ['summary', 'fisher', 'parameters', 'summaries', 'differences', 'distan
                abc[key] = np.load(f'./preloaded_data/ABC_results/abc{modelversion}{key}.npy')
            return abc

In [4]: abc = load_ABC_results(modelversion=3)

In [5]: plt.scatter(abc['summaries'][:,0],abc['summaries'][:,1]);
        plt.xlabel('$x_1$');
        plt.ylabel('$x_2$');
        plt.savefig('./output.png')
        plt.show()
```

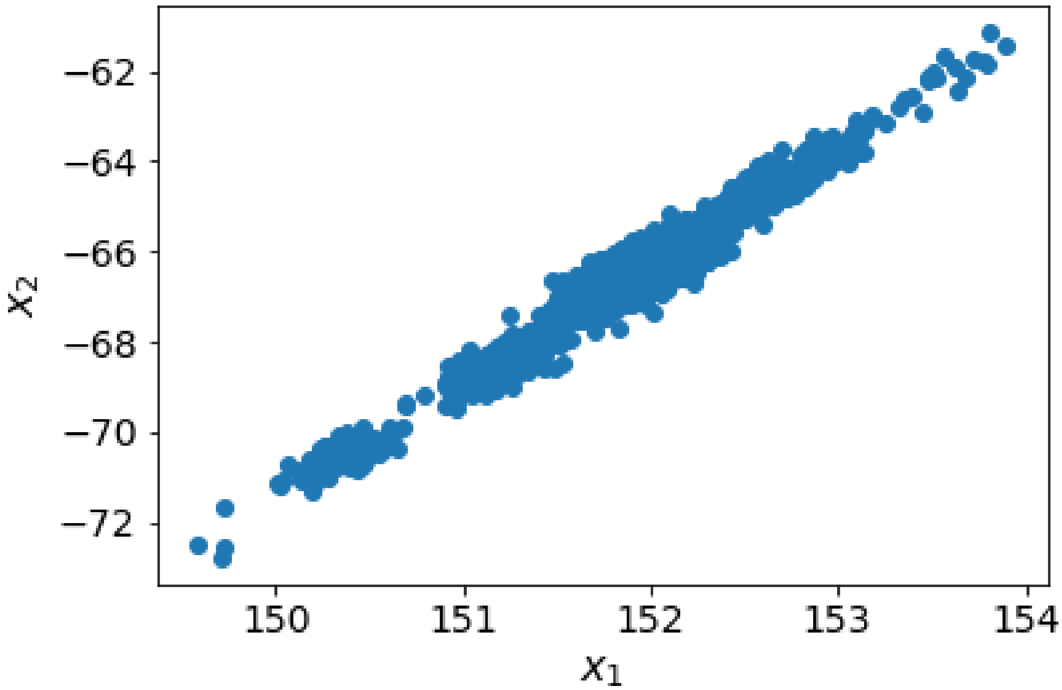



```
In [6]: print (np.corrcoef(abc['summaries'],rowvar=False))

[[ 1.         -0.2900731]
 [-0.2900731  1.        ]]
```

The variables seem negatively correlated quite significantly. Also note the one weird region: (the lower stripe)

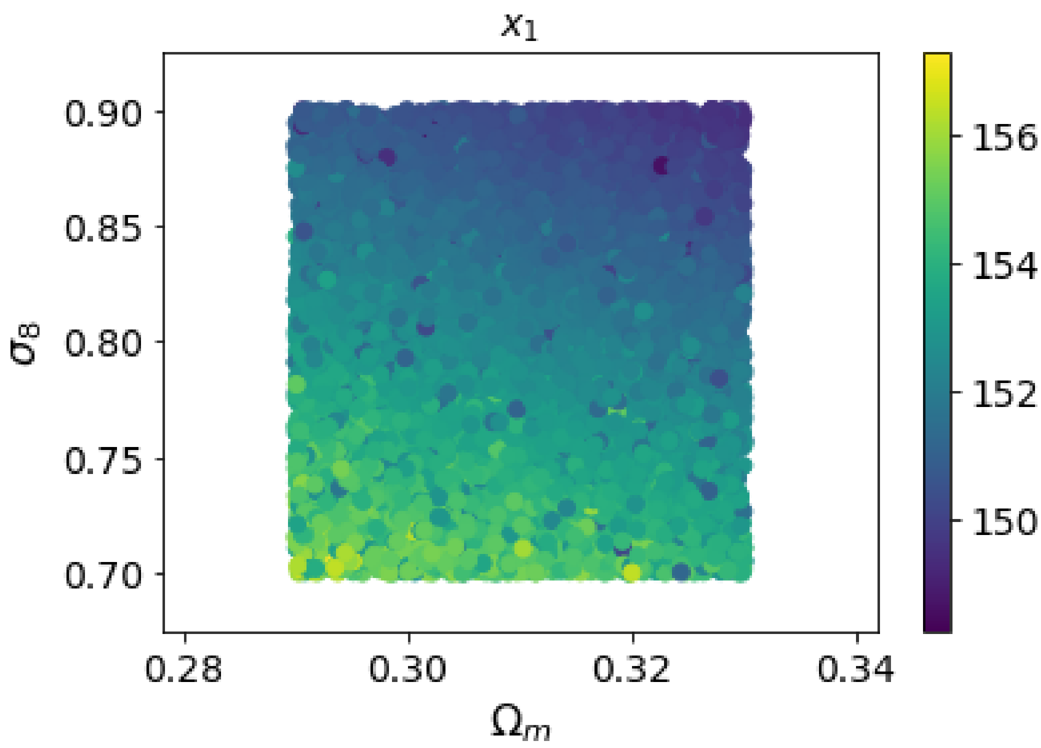
What if we just load noisy versions of the fiducial cosmology?

```
In [7]: # (5000,600). 5000 simulations of flattened noisy fiducial cosmology
        fid_sims = np.load('./fid_sims.npy')
        # (5000,2) summaries of the flattened noisy fiducial cosmology
        fidsummaries = np.load('fid_summaries.npy')

In [8]: plt.scatter(fidsummaries[:,0],fidsummaries[:,1]);
        plt.xlabel('$x_1$');
        plt.ylabel('$x_2$');
        plt.savefig('./output_fid.png')
        plt.show()
        print (np.corrcoef(fidsummaries,rowvar=False))
```

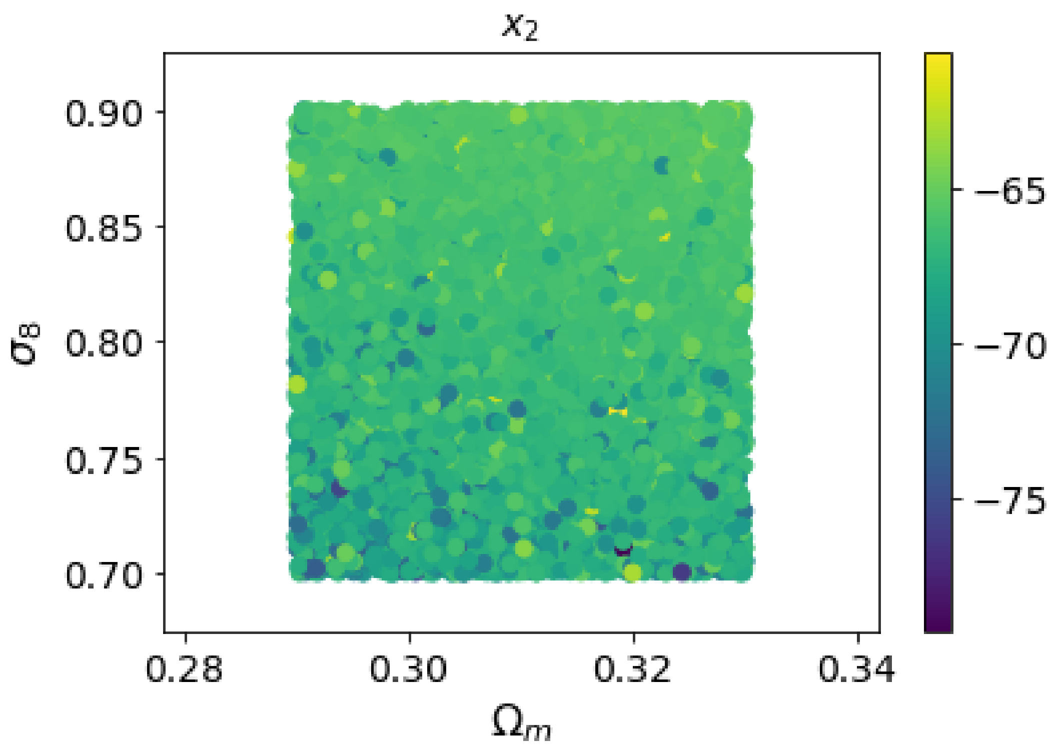



```
[[1.         0.95541443]
 [0.95541443 1.        ]]
```

```
In [9]: num = 10000 # number of points to plot
        plt.scatter(abc['parameters'][:num,0],abc['parameters'][:num,1],c=abc['summaries'][:nur
        plt.title("$x_1$")
        plt.colorbar()
        plt.xlabel('$\Omega_m$');
        plt.ylabel('$\sigma_8$')
        # plt.legend(frameon=False);
        plt.show()
```



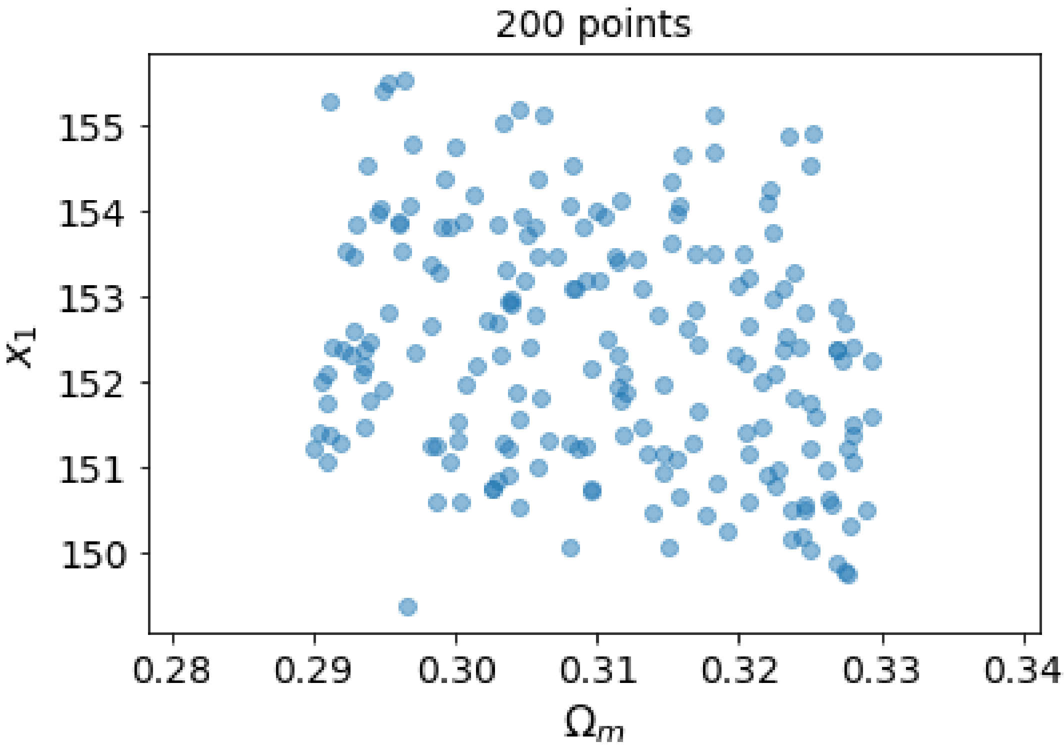

```
In [10]: plt.scatter(abc['parameters'][:num,0],abc['parameters'][:num,1],c=abc['summaries'][:nu
         plt.title("$x_2$")
         plt.colorbar()
         plt.xlabel('$\Omega_m$');
         plt.ylabel('$\sigma_8$')
         # plt.legend(frameon=False);
         plt.show()
```

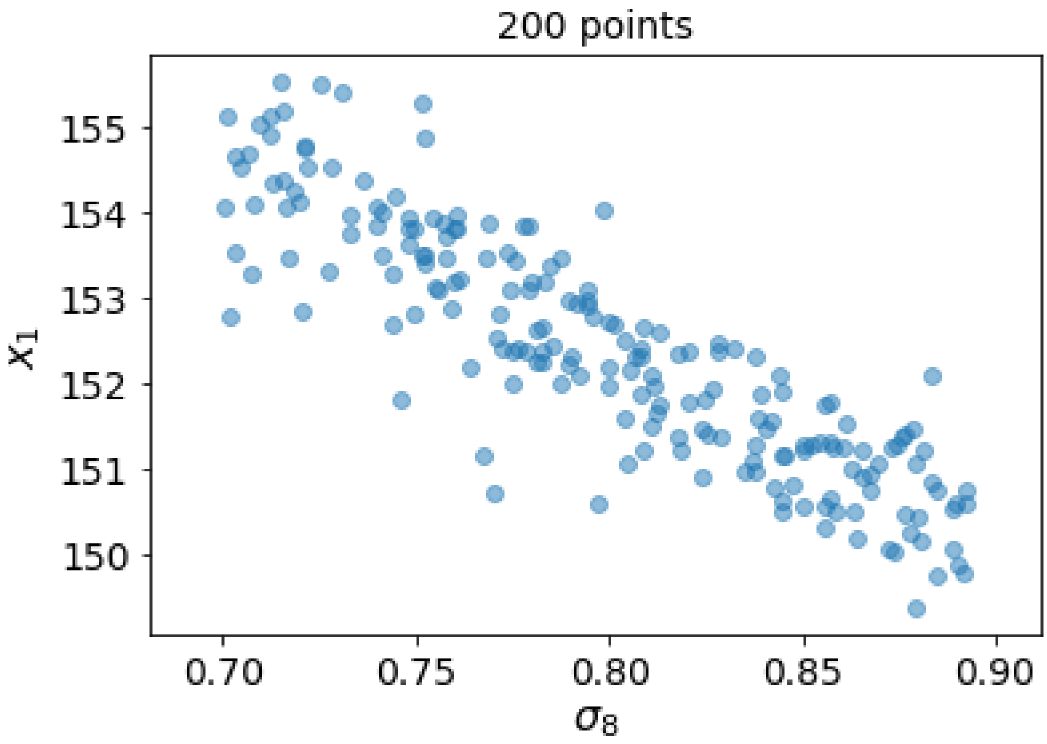


x_1 **seems to be easier to fit with a plane than** x_2**, we shall start with** $x_1 = a\Omega_m + b\sigma_8$**.**

```
In [11]: # num = 1000
         # plt.title(f"{num} points")
         # plt.scatter(abc['parameters'][:num,0], abc['summaries'][:num,0],alpha=0.5)
         # plt.xlabel('$\Omega_m$');
         # plt.ylabel('$x_1$')
         # plt.show()

         # plt.title(f"{num} points")
         # plt.scatter(abc['parameters'][:num,1], abc['summaries'][:num,0],alpha=0.5)
         # plt.xlabel('$\sigma_8$');
         # plt.ylabel('$x_1$')
         # plt.show()

         num = 200
         plt.title(f"{num} points")
         plt.scatter(abc['parameters'][:num,0], abc['summaries'][:num,0],alpha=0.5)
         plt.xlabel('$\Omega_m$');
         plt.ylabel('$x_1$')
         plt.show()

         plt.title(f"{num} points")
```

```
plt.scatter(abc['parameters'][:num,1], abc['summaries'][:num,0],alpha=0.5)
plt.xlabel('$\sigma_8$');
plt.ylabel('$x_1$')
plt.show()
```






```
In [12]: def linleastsquares(X, y):
             """
             Fit linear least squares, given a matrix X
             X = shape (observations,num_params) (1000,2)
             y = observed data values -- shape = (1000,)

             returns beta -- best fit parameters -- shape (2,)
             """
             beta = np.dot((np.dot( np.linalg.inv(np.dot(X.T,X)), X.T)),y)
             return beta

In [13]: def plot_fit_result(model,xindx):
             """
             Plot a few images to show how well the model fits the data

             model -- function: the best fit model, should take two parameters: Omega_m and si
             xindx -- integer:  1 for fitting x1. 2 for fitting x2

             """
             # Calculate values according to the model
             omega_m = np.linspace(0.28,0.34,30)
             sigma_8 = np.linspace(0.70,0.90,30)
             xv, yv = np.meshgrid(omega_m, sigma_8)
```

```
    model_sol = model(xv,yv)

    plt.contourf(omega_m,sigma_8, model_sol);
    plt.title(f"Fit to $x_{xindx}$")
    plt.colorbar()
    plt.xlabel('$\Omega_m$');
    plt.ylabel('$\sigma_8$')
    # plt.legend(frameon=False);
    plt.show()

    # color boundaries hardcoded
    if xindx == 1:
        vmin = 149
        vmax = 157
    if xindx == 2:
        vmin = -79
        vmax = -61

    num = 1000
    model_sol_scatter = model(abc['parameters'][:num,0],abc['parameters'][:num,1])
    plt.scatter(abc['parameters'][:num,0],abc['parameters'][:num,1], c=model_sol_scat
                ,vmin=vmin,vmax=vmax);
    plt.title(f"Fit to $x_{xindx}$")
    plt.colorbar()
    plt.xlabel('$\Omega_m$');
    plt.ylabel('$\sigma_8$')
    # plt.legend(frameon=False);
    plt.show()

    plt.scatter(abc['parameters'][:num,0],abc['parameters'][:num,1],c=abc['summaries']
                ,vmin=vmin,vmax=vmax);
    plt.title(f"Actual $x_{xindx}$")
    plt.colorbar()
    plt.xlabel('$\Omega_m$');
    plt.ylabel('$\sigma_8$')
    # plt.legend(frameon=False);
    plt.show()

def plot_fit_result1D(model,xindx):
    """
    Plot a few images in 1D to show how well the model fits the data

    model -- function: the best fit model, should take two parameters: Omega_m and si
    xindx -- integer:  1 for fitting x1. 2 for fitting x2

    """
    num = 1000
    model_sol_scatter = model(abc['parameters'][:num,0],abc['parameters'][:num,1])
```

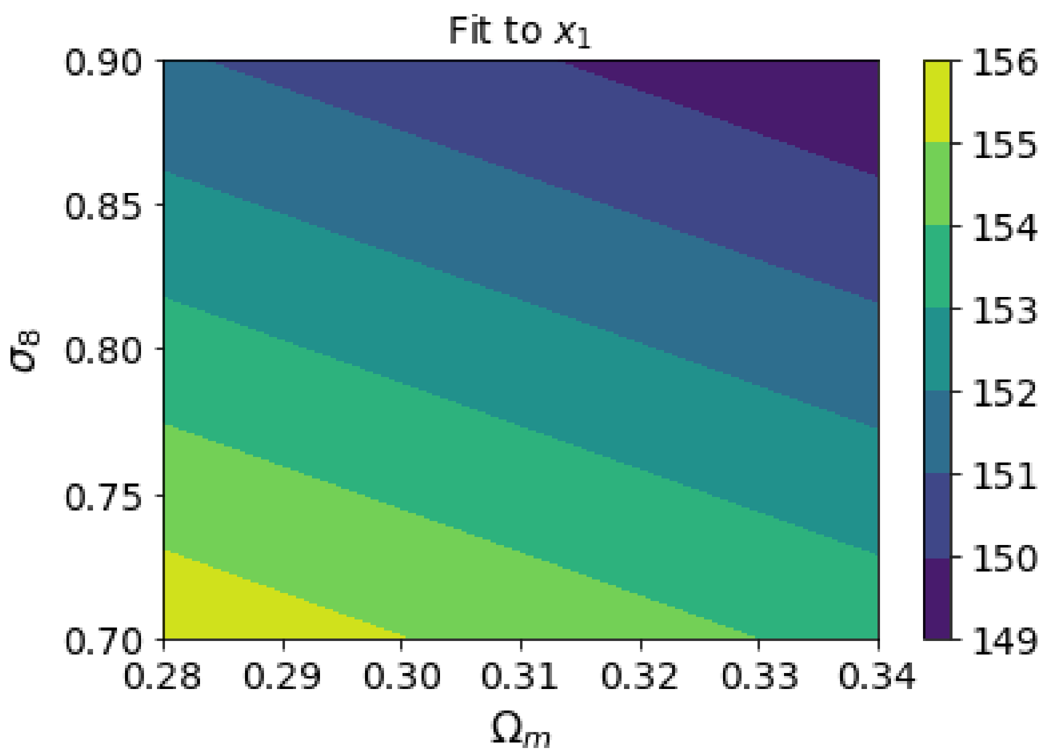
```
    plt.title(f"{num} points")
    plt.scatter(abc['parameters'][:num,0], model_sol_scatter,alpha=0.5)
    plt.xlabel('$\Omega_m$');
    plt.ylabel(f'$x_{xindx}$')
    plt.show()

    plt.title(f"{num} points")
    plt.scatter(abc['parameters'][:num,1], model_sol_scatter,alpha=0.5)
    plt.xlabel('$\sigma_8$');
    plt.ylabel(f'$x_{xindx}$')
    plt.show()
```

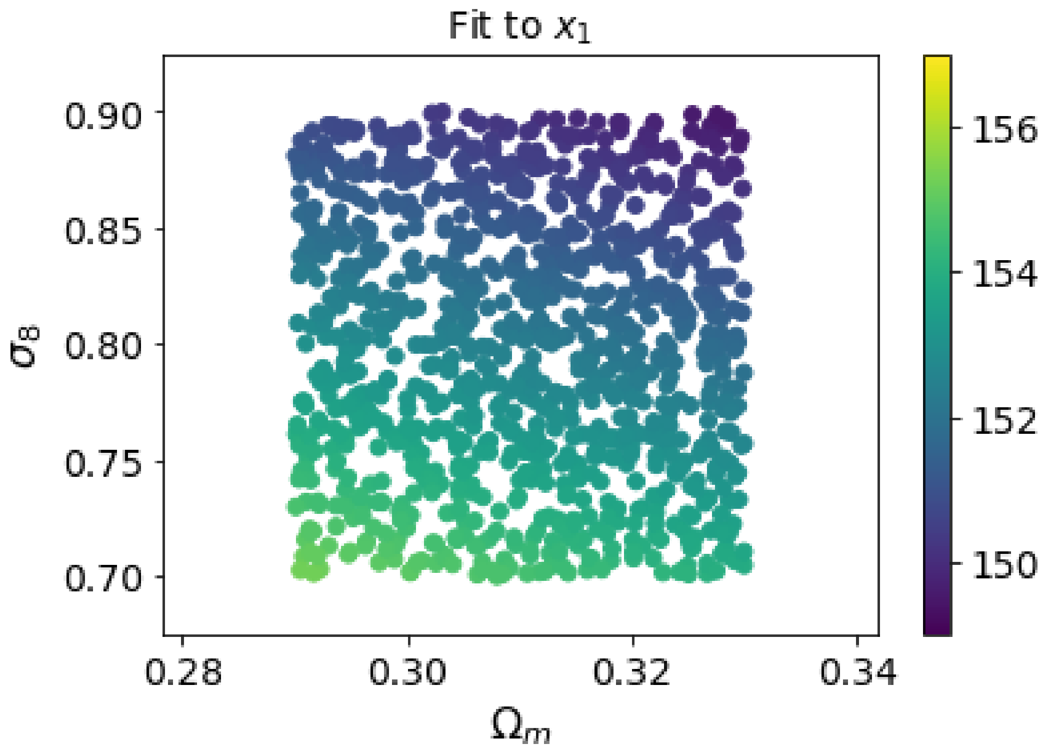
```
In [14]: # Fit to 200 points
         num = 200
         # Fit x1 = a\Omega_m + b\sigma_8 + c
         X = np.array([abc['parameters'][:num,0], abc['parameters'][:num,1], np.ones(num)]).T
         y = abc['summaries'][:num,0] # x1

         ahat, bhat, chat = linleastsquares(X,y)
         print (ahat,bhat, chat)
```

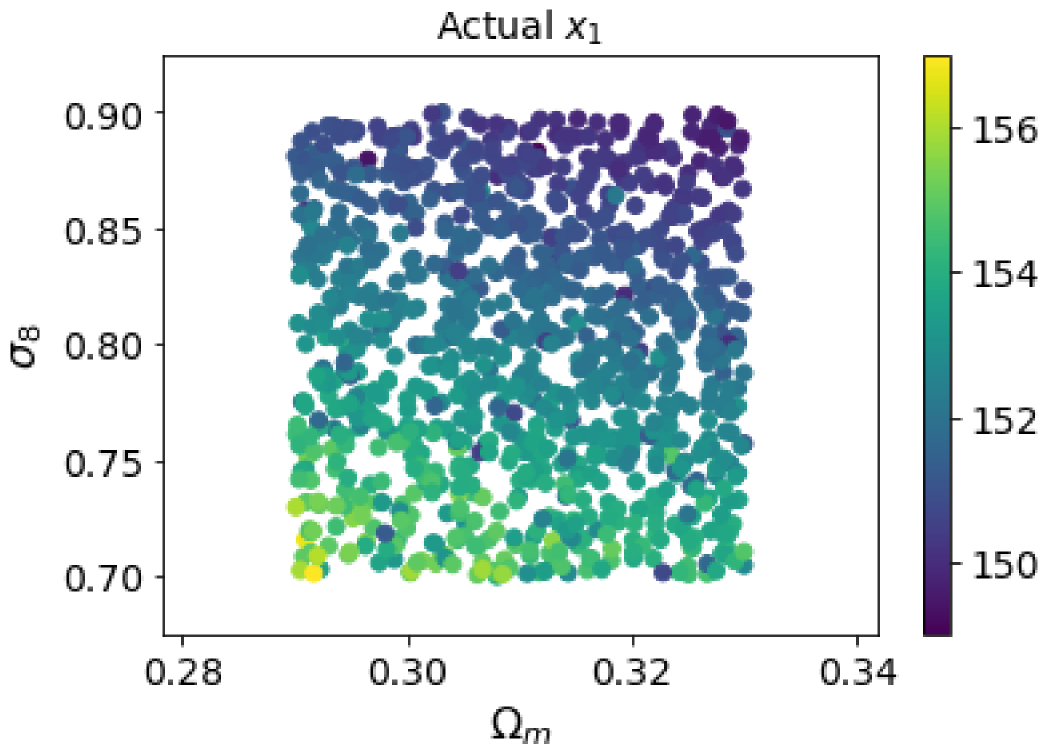
```
-34.111706898950274 -22.986691736333057 181.33919959986176
```

```
In [15]: model = lambda x, y: ahat*x + bhat*y + chat
```

```
In [16]: plot_fit_result(model,xindx=1)
```




Fit to x_1
0.90
0.85
0.80
0.75
0.70
σ_8
0.28
0.30
0.32
0.34
Ω_m
156
154
152
150

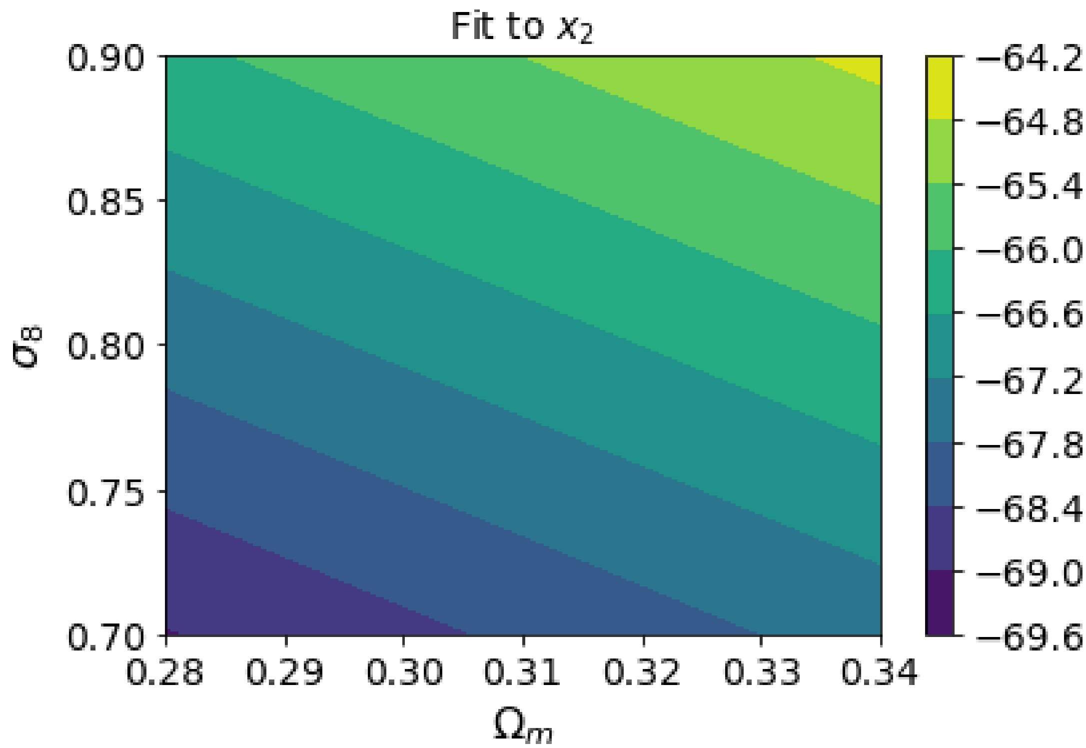


Imo the fit looks quite good.

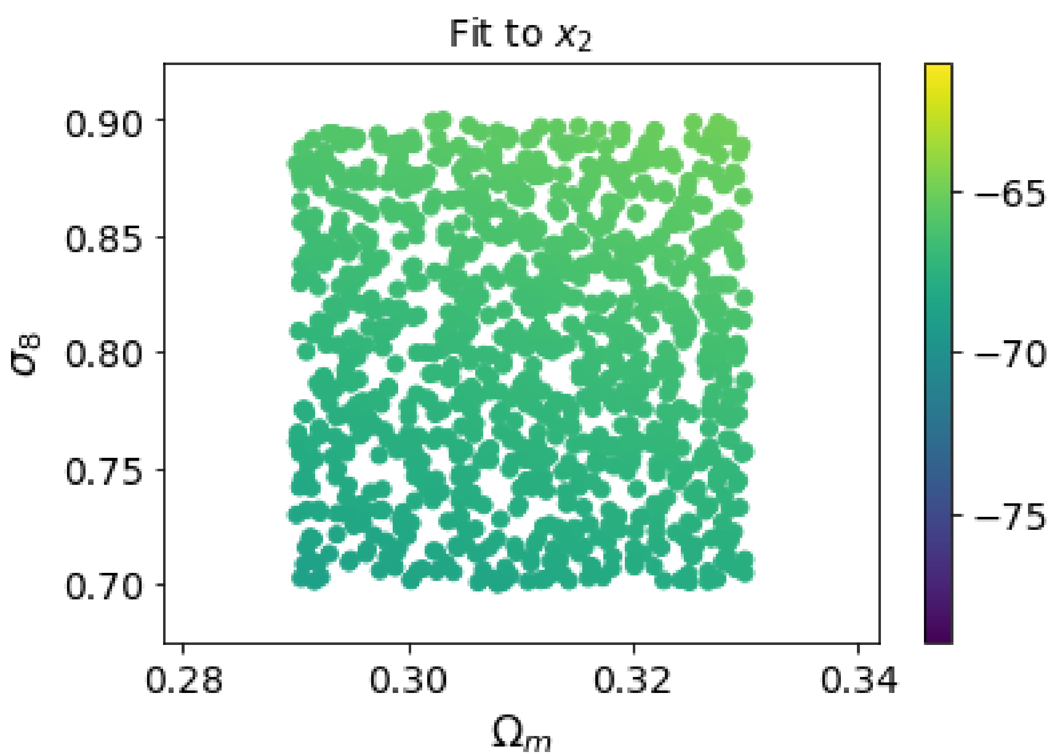
```
In [17]: # Fit to x2, same procedure

         num = 200
         # Fit x1 = a\Omega_m + b\sigma_8 + c
         X = np.array([abc['parameters'][:num,0], abc['parameters'][:num,1], np.ones(num)]).T
         y2 = abc['summaries'][:num,1] # x2

         ahat2, bhat2, chat2 = linleastsquares(X,y2)
         print (ahat2,bhat2, chat2)
```

```
24.693852408845373 14.539551951223899 -86.13434701648762
```

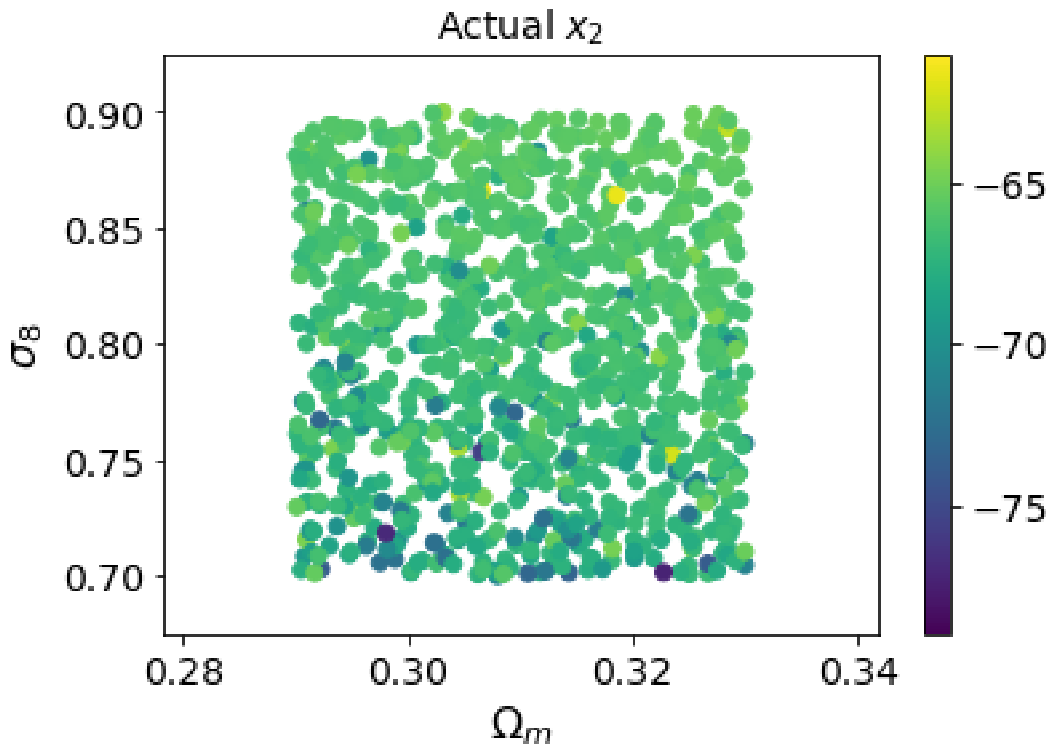
```
In [18]: model2 = lambda x, y: ahat2*x + bhat2*y + chat2
         plot_fit_result(model2,xindx=2)
```



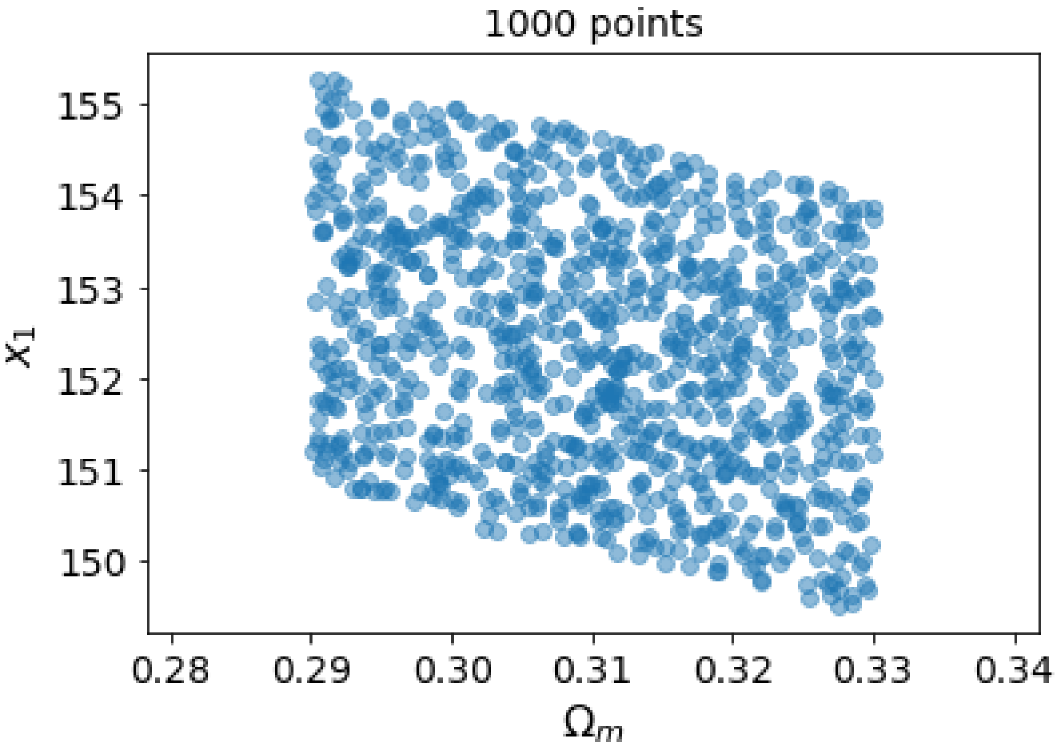
Fit to x_2
0.90
0.85
0.80
0.75
0.70
σ_8
0.28
0.29
0.30
0.31
0.32
0.33
0.34
Ω_m
−64.2
−64.8
−65.4
−66.0
−66.6
−67.2
−67.8
−68.4
−69.0
−69.6

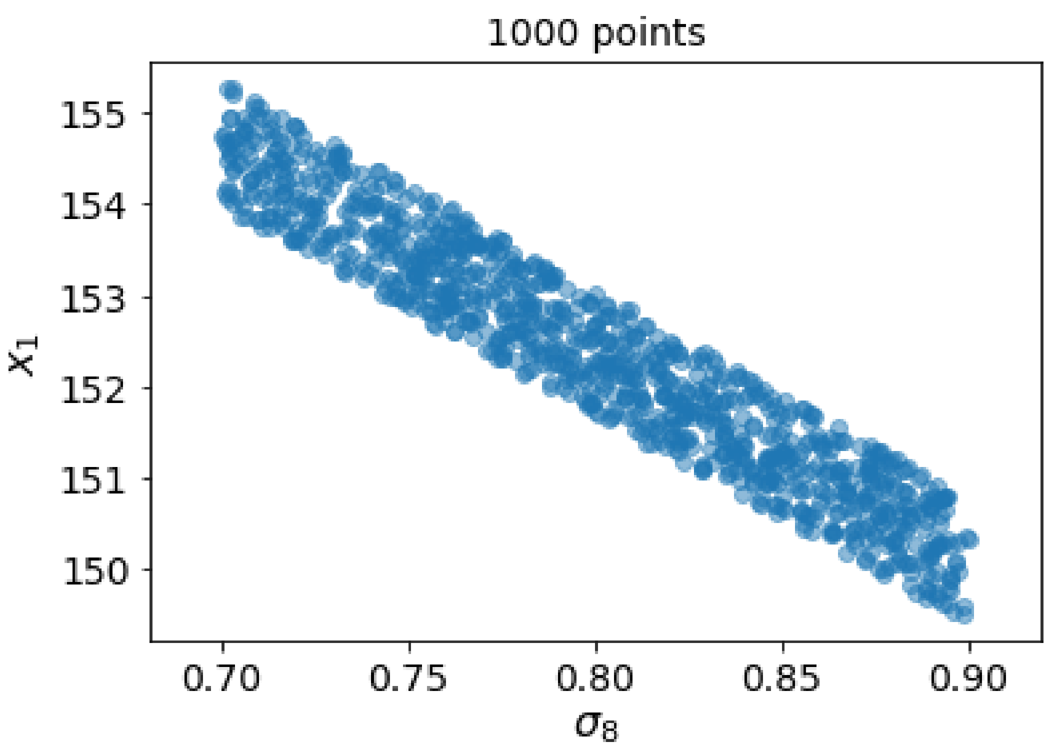


Fit to x_2
0.90
0.85
0.80
0.75
0.70
σ_8
0.28
0.30
0.32
0.34
Ω_m
−65
−70
−75




```
In [19]: # Now in 1D:
         plot_fit_result1D(model,xindx=1)
```


1000 points
155
154
153
152
151
150
x_1
0.28
0.29
0.30
0.31
0.32
0.33
0.34
Ω_m


1000 points
155
154
153
152
151
150
x_1
0.70
0.75
0.80
0.85
0.90
σ_8

Now perform ABC with the function that generates x1 and x2, no cosmosis needed.

In [20]:
```
import tensorflow as tf
# change to the path where the IMNN git clone is located
# new version of IMNN by Tom
sys.path.insert(-1,'../../../../../IMNNv2/IMNN/')
import IMNN.ABC.priors as priors
```

In [21]:
```
def output_summaries(theta,model1,model2):
    """
    Return x1,x2 for a given theta = list of [Omega_m, sigma_8]'s'
    given the two fitted models to x1 x2
    """
    theta = np.array(theta)
    Omega_m = theta[:,0]
    sigma_8 = theta[:,1]
    x1 = model1(Omega_m,sigma_8)
    x2 = model2(Omega_m,sigma_8)

    return np.array([x1,x2]).T # return as array of shape (len(theta),2)
```

In [22]:
```
def ABC_with_model(draws, real_summary, prior, model1, model2, fisher):

    Gaussprior = priors.TruncatedGaussian(prior["mean"],prior["variance"],prior["lowe
                                        ,prior["upper"])
    # Draw params from Gaussian prior
    theta = Gaussprior.draw(draws)
    # Calculate summaries with models
    summaries = output_summaries(theta, model1,model2)
    # Calculate distance
    differences = summaries - real_summary
    distances = np.sqrt(
        np.einsum(
            'ij,ij->i',
            differences,
            np.einsum(
                'jk,ik->ij',
                fisher,
                differences)))
    ABC_dict = dict()
    ABC_dict["summary"] = real_summary
    ABC_dict["fisher"] = fisher
    ABC_dict["parameters"] = theta
    ABC_dict["summaries"] = summaries
    ABC_dict["differences"] = differences
```

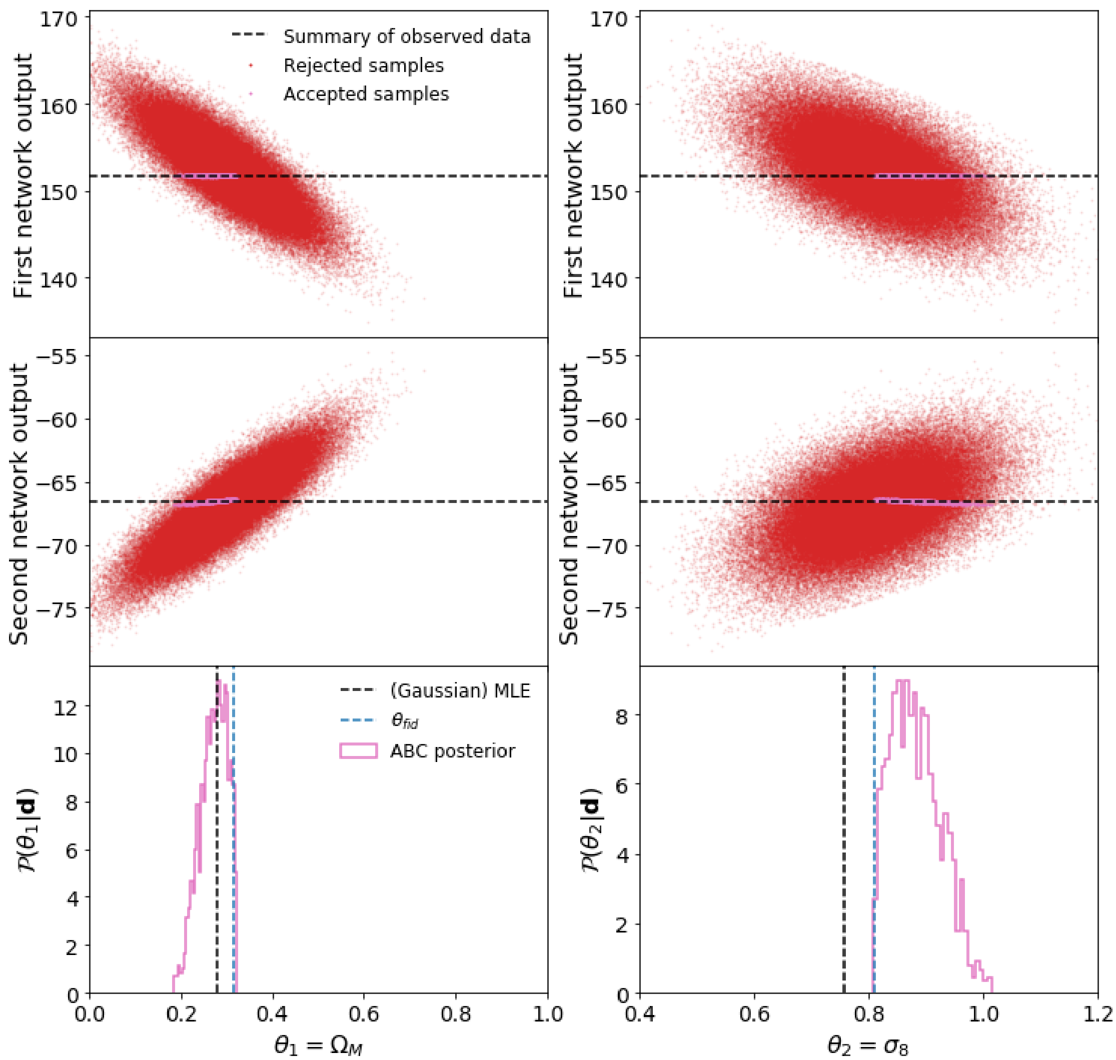
```
        ABC_dict["distances"] = distances

        return ABC_dict
```

```
In [23]: # Variables for ABC
         draws = int(1e5) # amount of draws
         fisher = abc['fisher'] # fisher info
         real_summary = abc['summary'] # summary of 'real' data
         # A Truncated gaussian prior
         prior = {'mean': np.array([0.30,0.805]),
                  'variance': np.array([[0.01,0],[0,0.01]]), # cov matrix
                  'lower': np.array([0.0,0.4]),
                  'upper': np.array([1.0,1.2])
                  }
```

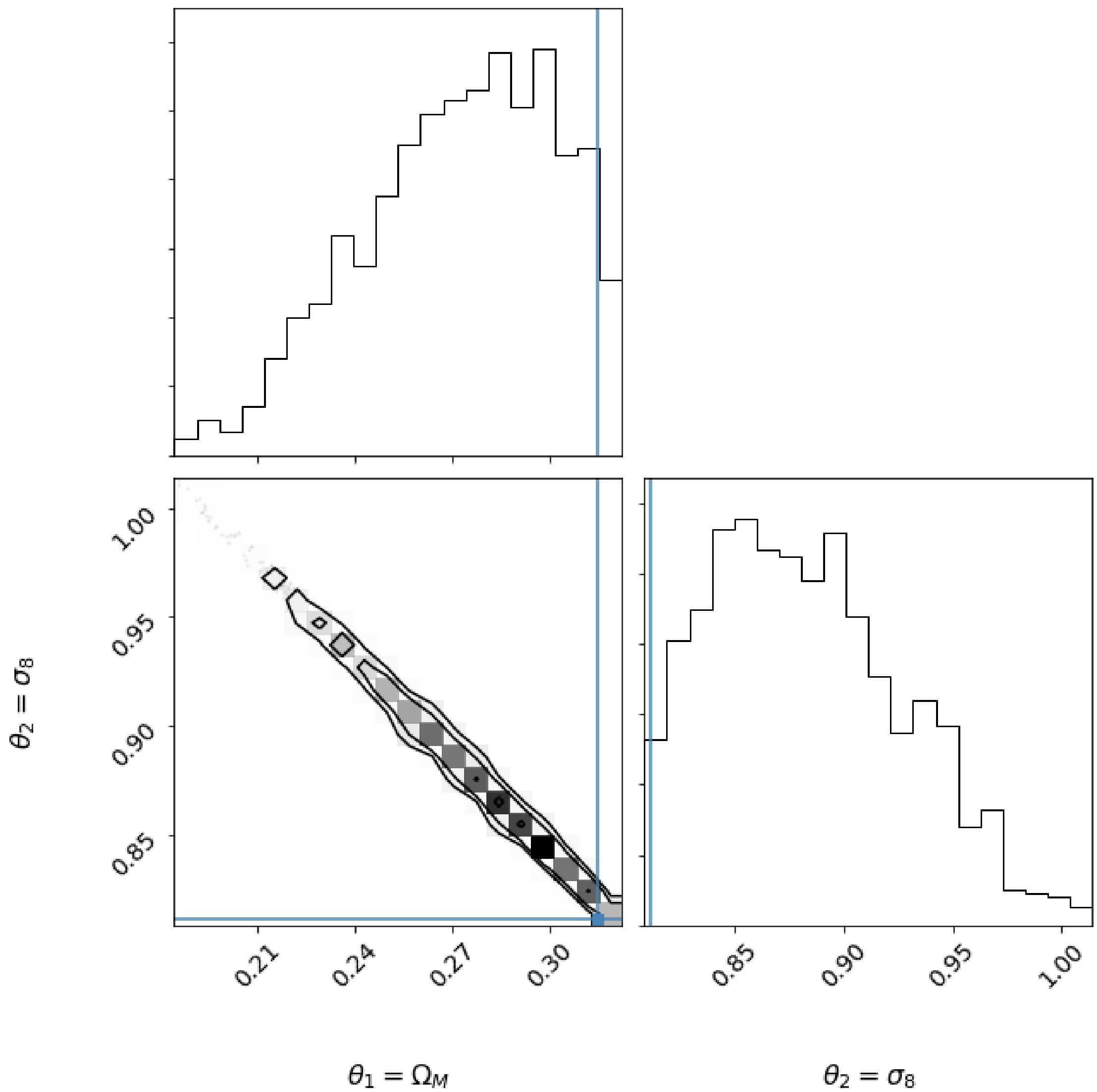
```
In [24]: abc_model = ABC_with_model(draws, real_summary, prior, model, model2, fisher)
```

```
In [25]: class holder(object):
             """Small class because plotting function requires it"""
             def __init__(self,saveversion,figuredir):
                 self.modelversion = saveversion
                 self.figuredir = figuredir


         holder1 = holder(saveversion=3,figuredir='./')
         theta_fid = np.array([0.315, 0.811])
         # plotting function requires this too, does not depend on model fitting though, so st
         abc_model["MLE"] = abc["MLE"]
         ABC_saved_sims_multiparam.plot_ABC_2params(abc_model, holder1, theta_fid, prior, oneD=
                                                  , hbins=30, epsilon=50,show=True)
```

```
Epsilon is chosen to be 50.00
Number of accepted samples =  1303
```



Summary of observed data
Rejected samples
Accepted samples
First network output
Second network output
(Gaussian) MLE
θ_{fid}
ABC posterior
$\mathcal{P}(\theta_1|\mathbf{d})$
$\mathcal{P}(\theta_2|\mathbf{d})$
$\theta_1 = \Omega_M$
$\theta_2 = \sigma_8$




```
In [26]: from importlib import reload
```

```
In [27]: reload(ABC_saved_sims_multiparam)
```

```
Out[27]: <module 'ABC_saved_sims_multiparam' from '../../../../cosmosis_wrappers/ABC_saved_sims
```

Now do ABC-PMC with these models

```
In [28]: # Do ABC-PMC
         def dist_measure(x,y):
             # Calculate distance
             fisher = abc['fisher']
```

```python
    differences = x - y

    distances = np.sqrt(
        np.einsum(
            'ij,ij->i',
            differences,
            np.einsum(
                'jk,ik->ij',
                fisher,
                differences)))

    return distances[0]

# def dist_measure(x,y):
#     # do Euclidian distance to start with


def PMCoutput_summaries(theta,model1,model2):
    """
    Return x1,x2 for a given theta = list of [Omega_m, sigma_8]'s'
    given the two fitted models to x1 x2
    """
    # PMC function only generates one particle at a time
    Omega_m = theta[0]
    sigma_8 = theta[1]
    x1 = model1(Omega_m,sigma_8)
    x2 = model2(Omega_m,sigma_8)

    return np.array([x1,x2]).T # return as array of shape (len(theta),2)

def pmcfunc(theta):
    """wrapper for PMC moduel"""
    return PMCoutput_summaries(theta,model,model2)

pmcprior = abcpmc.GaussianPrior(mu=prior["mean"]
                                ,sigma=prior["variance"])
alpha = 75 #th percentile of the sorted distances = new epsilon
T = 100 # iterations
eps_start = 200.0 # starting threshold, quite arbitrary
eps = abcpmc.ConstEps(T, eps_start)
# creation of new samples

# create instance of sampler,
sampler = abcpmc.Sampler(N=1000, Y=abc['summary']
                         , postfn=pmcfunc
                         , dist=dist_measure, threads=3)

sampler.particle_proposal_cls = abcpmc.OLCMParticleProposal
```

```
        distances = [dist_measure(abc['summary'], pmcfunc(thet))
                     for thet in abc["parameters"][:100]]
```

```
In [29]: # Now we are ready to sample:
         def launch():
             eps = abcpmc.ConstEps(T, eps_start)

             pools = []
             for pool in sampler.sample(pmcprior, eps):
         #                  ,desc='eps: {1:>.4f}, ratio: {2:>.4f}'.format(pool.t, eps(pool
                 print("T: {0}, eps: {1:>.4f}, ratio: {2:>.4f}".format(pool.t, eps(pool.eps), p


         #         for i, (mean, std) in enumerate(zip(*abcpmc.weighted_avg_and_std(pool.theta
         #             print(u"    theta[{0}]: {1:>.4f} \u00B1 {2:>.4f}".format(i, mean,std))

                 eps.eps = np.percentile(pool.dists, alpha) # reduce eps value
                 pools.append(pool)
             sampler.close()
             return pools

         import time
         t0 = time.time()
         pools = launch()
         print ("took", (time.time() - t0))
```

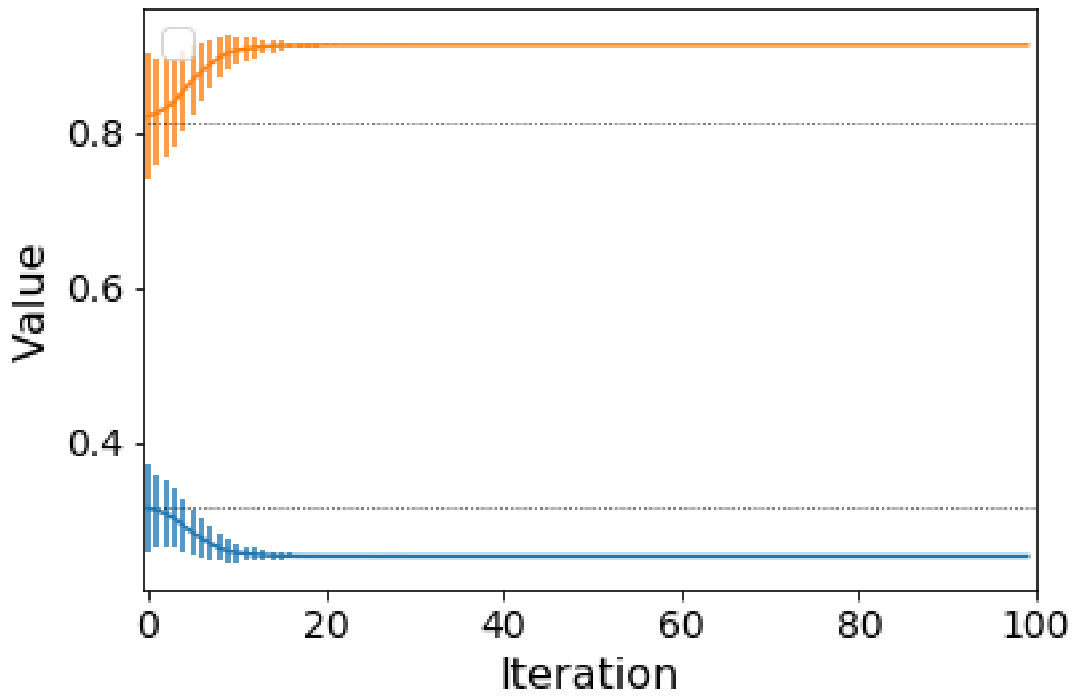
```
T: 0, eps: 200.0000, ratio: 0.1134
T: 1, eps: 156.5586, ratio: 0.6050
T: 2, eps: 121.1994, ratio: 0.5875
T: 3, eps: 95.0597, ratio: 0.5931
T: 4, eps: 75.0622, ratio: 0.5851
T: 5, eps: 60.2092, ratio: 0.5482
T: 6, eps: 47.8182, ratio: 0.5513
T: 7, eps: 37.7878, ratio: 0.5112
T: 8, eps: 30.8664, ratio: 0.5214
T: 9, eps: 24.4409, ratio: 0.5322
T: 10, eps: 19.2433, ratio: 0.5211
T: 11, eps: 15.6736, ratio: 0.5141
T: 12, eps: 12.3733, ratio: 0.5184
T: 13, eps: 9.9903, ratio: 0.5353
T: 14, eps: 7.9758, ratio: 0.5010
T: 15, eps: 6.4559, ratio: 0.5356
T: 16, eps: 5.0993, ratio: 0.5061
T: 17, eps: 3.9922, ratio: 0.5485
T: 18, eps: 3.1824, ratio: 0.5025
T: 19, eps: 2.5118, ratio: 0.5120
T: 20, eps: 1.9902, ratio: 0.4993
```

```
T: 21, eps: 1.5825, ratio: 0.5283
T: 22, eps: 1.2538, ratio: 0.5068
T: 23, eps: 0.9883, ratio: 0.5131
T: 24, eps: 0.7937, ratio: 0.5110
T: 25, eps: 0.6190, ratio: 0.5147
T: 26, eps: 0.4854, ratio: 0.5092
T: 27, eps: 0.3836, ratio: 0.5147
T: 28, eps: 0.3007, ratio: 0.5263
T: 29, eps: 0.2295, ratio: 0.5176
T: 30, eps: 0.1805, ratio: 0.5056
T: 31, eps: 0.1437, ratio: 0.5133
T: 32, eps: 0.1126, ratio: 0.5068
T: 33, eps: 0.0906, ratio: 0.5420
T: 34, eps: 0.0698, ratio: 0.4973
T: 35, eps: 0.0559, ratio: 0.5203
T: 36, eps: 0.0439, ratio: 0.5200
T: 37, eps: 0.0352, ratio: 0.5063
T: 38, eps: 0.0281, ratio: 0.5255
T: 39, eps: 0.0219, ratio: 0.5195
T: 40, eps: 0.0176, ratio: 0.5356
T: 41, eps: 0.0141, ratio: 0.5330
T: 42, eps: 0.0113, ratio: 0.5222
T: 43, eps: 0.0089, ratio: 0.5173
T: 44, eps: 0.0071, ratio: 0.5131
T: 45, eps: 0.0055, ratio: 0.5203
T: 46, eps: 0.0044, ratio: 0.5141
T: 47, eps: 0.0035, ratio: 0.5008
T: 48, eps: 0.0028, ratio: 0.4963
T: 49, eps: 0.0022, ratio: 0.5313
T: 50, eps: 0.0018, ratio: 0.5184
T: 51, eps: 0.0014, ratio: 0.5112
T: 52, eps: 0.0011, ratio: 0.5131
T: 53, eps: 0.0009, ratio: 0.5157
T: 54, eps: 0.0007, ratio: 0.5181
T: 55, eps: 0.0005, ratio: 0.5269
T: 56, eps: 0.0004, ratio: 0.5216
T: 57, eps: 0.0003, ratio: 0.5288
T: 58, eps: 0.0003, ratio: 0.4850
T: 59, eps: 0.0002, ratio: 0.5176
T: 60, eps: 0.0002, ratio: 0.5136
T: 61, eps: 0.0001, ratio: 0.5063
T: 62, eps: 0.0001, ratio: 0.5192
T: 63, eps: 0.0001, ratio: 0.5277
T: 64, eps: 0.0001, ratio: 0.5405
T: 65, eps: 0.0001, ratio: 0.5362
T: 66, eps: 0.0000, ratio: 0.5376
T: 67, eps: 0.0000, ratio: 0.5562
T: 68, eps: 0.0000, ratio: 0.5118
```

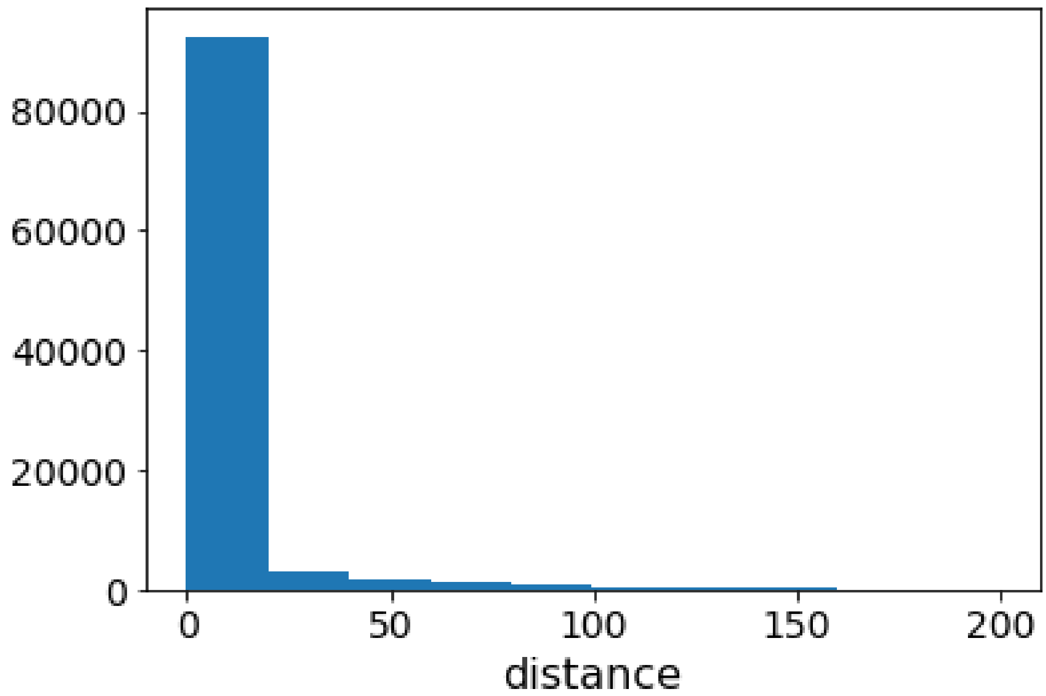
```
T: 69, eps: 0.0000, ratio: 0.5371
T: 70, eps: 0.0000, ratio: 0.5230
T: 71, eps: 0.0000, ratio: 0.5149
T: 72, eps: 0.0000, ratio: 0.4975
T: 73, eps: 0.0000, ratio: 0.5241
T: 74, eps: 0.0000, ratio: 0.5333
T: 75, eps: 0.0000, ratio: 0.5084
T: 76, eps: 0.0000, ratio: 0.5336
T: 77, eps: 0.0000, ratio: 0.4938
T: 78, eps: 0.0000, ratio: 0.5299
T: 79, eps: 0.0000, ratio: 0.5400
T: 80, eps: 0.0000, ratio: 0.5187
T: 81, eps: 0.0000, ratio: 0.4933
T: 82, eps: 0.0000, ratio: 0.5203
T: 83, eps: 0.0000, ratio: 0.5092
T: 84, eps: 0.0000, ratio: 0.5173
T: 85, eps: 0.0000, ratio: 0.5184
T: 86, eps: 0.0000, ratio: 0.5230
T: 87, eps: 0.0000, ratio: 0.5200
T: 88, eps: 0.0000, ratio: 0.5260
T: 89, eps: 0.0000, ratio: 0.5294
T: 90, eps: 0.0000, ratio: 0.5028
T: 91, eps: 0.0000, ratio: 0.5280
T: 92, eps: 0.0000, ratio: 0.5099
T: 93, eps: 0.0000, ratio: 0.5074
T: 94, eps: 0.0000, ratio: 0.5094
T: 95, eps: 0.0000, ratio: 0.5263
T: 96, eps: 0.0000, ratio: 0.5297
T: 97, eps: 0.0000, ratio: 0.5092
T: 98, eps: 0.0000, ratio: 0.5131
T: 99, eps: 0.0000, ratio: 0.5092
took 99.50151085853577
```

In [30]:
```python
# Postprocessing
for i in range(np.shape(abc['summary'])[1]):
    moments = np.array([abcpmc.weighted_avg_and_std(
        pool.thetas[:,i], pool.ws, axis=0) for pool in pools])
    plt.errorbar(range(T), moments[:, 0], moments[:, 1])
plt.hlines(theta_fid, 0, T, linestyle="dotted", linewidth=0.7)
plt.xlim([-.5, T])
plt.xlabel("Iteration")
plt.ylabel("Value")
plt.legend()
plt.show()
```

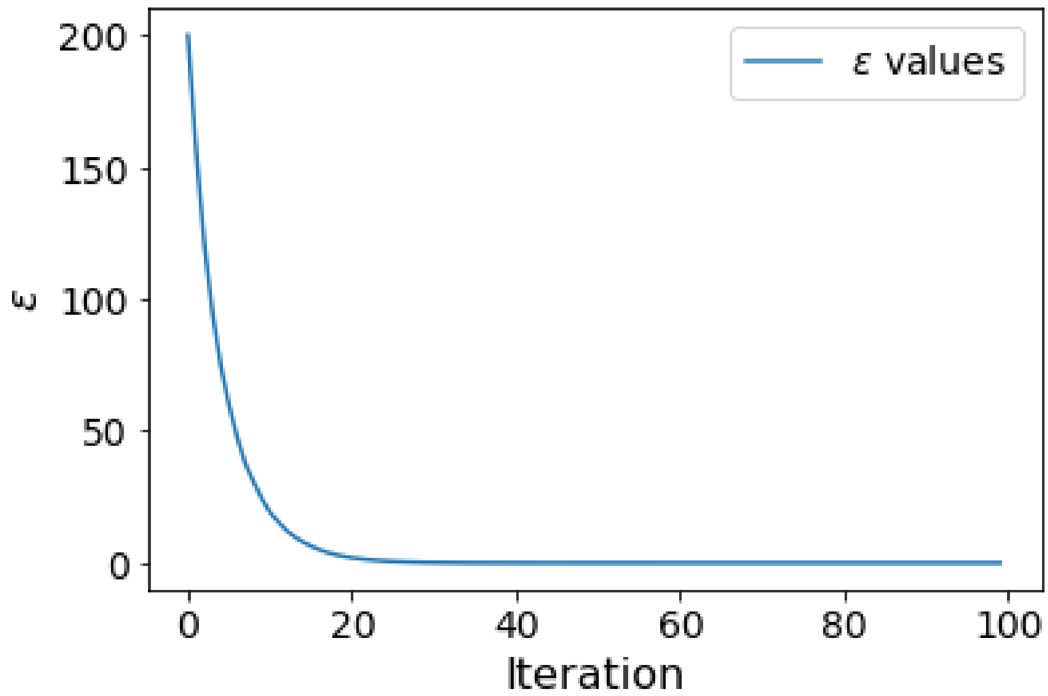
```
No handles with labels found to put in legend.
```



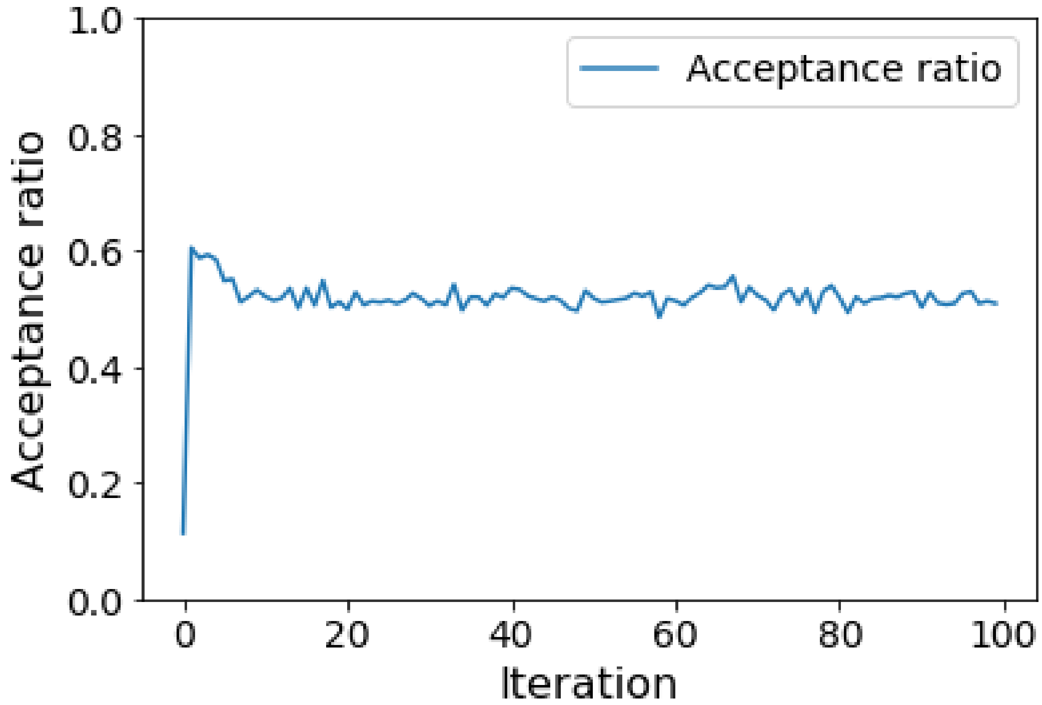

```
In [31]: distances = np.array([pool.dists for pool in pools]).flatten()
         plt.hist(distances)
         plt.xlabel("distance")
         plt.show()
```




```
In [32]: eps_values = np.array([pool.eps for pool in pools])
         plt.plot(eps_values, label=r"$\epsilon$ values")
         plt.xlabel("Iteration")
         plt.ylabel(r"$\epsilon$")
         plt.legend(loc="best")

Out[32]: <matplotlib.legend.Legend at 0x7ff7c4e94f28>
```



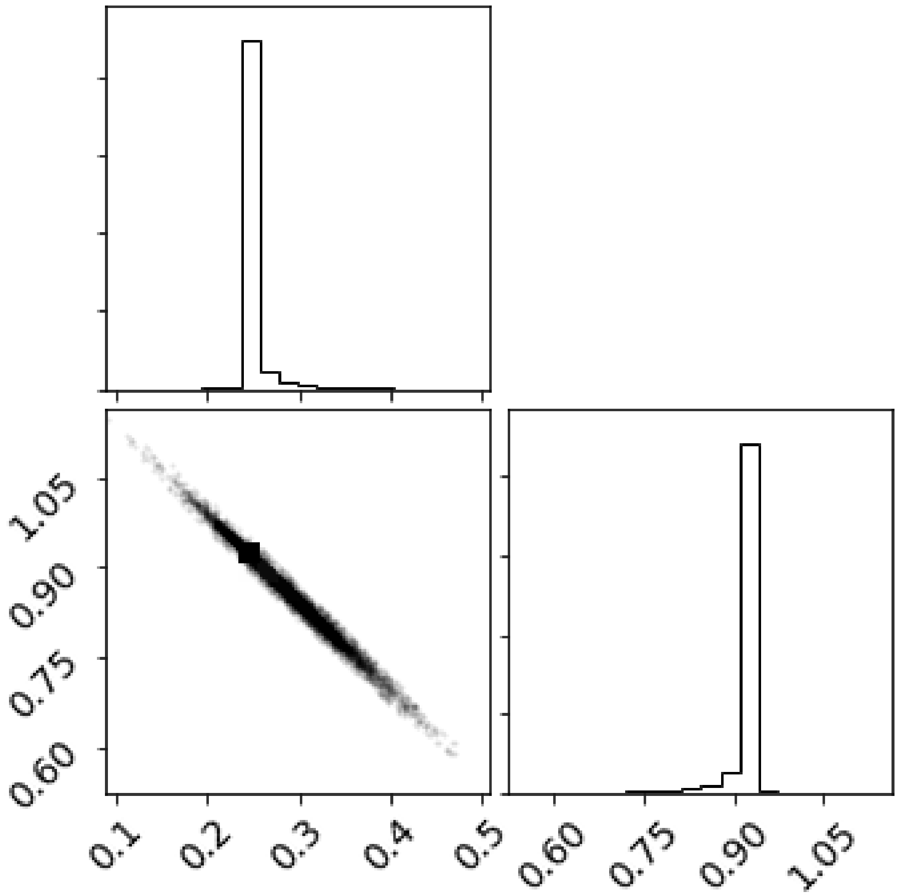

```
In [33]: acc_ratios = np.array([pool.ratio for pool in pools])
         plt.plot(acc_ratios, label="Acceptance ratio")
         plt.ylim([0, 1])
         plt.xlabel("Iteration")
         plt.ylabel("Acceptance ratio")
         plt.legend(loc="best")

Out[33]: <matplotlib.legend.Legend at 0x7ff7c4f3ce80>
```



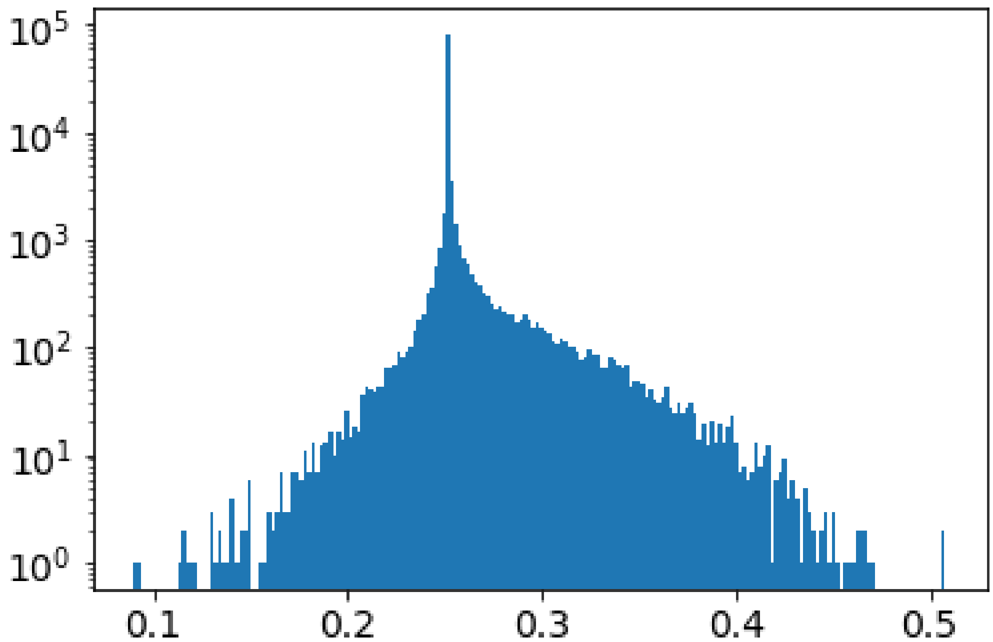

```
In [66]: samples = np.vstack([pool.thetas for pool in pools])
         print (np.shape(samples))
         fig = corner.corner(samples, truths= abc["summary"][0],bins=20)
         plt.show()

         np.sum((samples > 0.25) & (samples < 0.27))
         print (np.where((samples > 0.25) & (samples < 0.27)))
         print (samples[2,0])

         plt.hist(samples[:,0],bins=200)
         plt.yscale('log')
         plt.show()
```

```
WARNING:root:Too few points to create valid contours
```

```
(100000, 2)
```




```
(array([    2,    27,    29, ..., 99997, 99998, 99999]), array([0, 0, 0, ..., 0, 0, 0]))
0.26520722115889783
```



What if we just fit a line to a summmary as a function of Ω_m and as function of σ_8 seperately? Essentially that is saying

$$\vec{x}(\Omega_m, \sigma_8) => \vec{x}(\Omega_m), \vec{x}(\sigma_8) \tag{1}$$

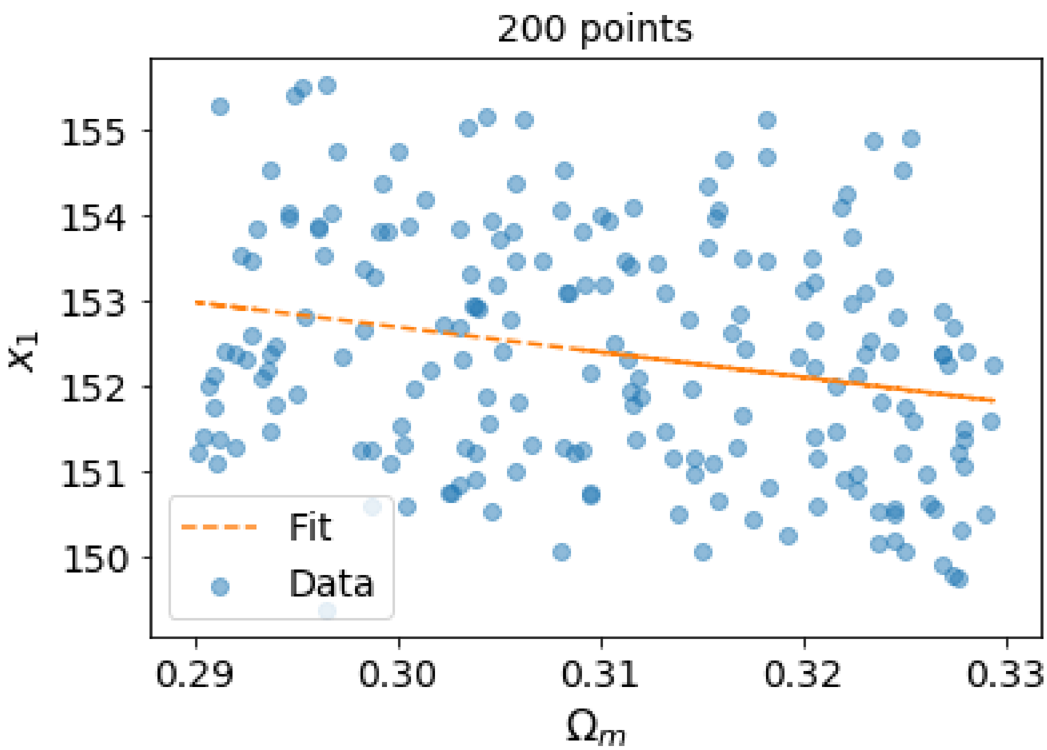
I think the above is already correct, but below is some interesting investigation

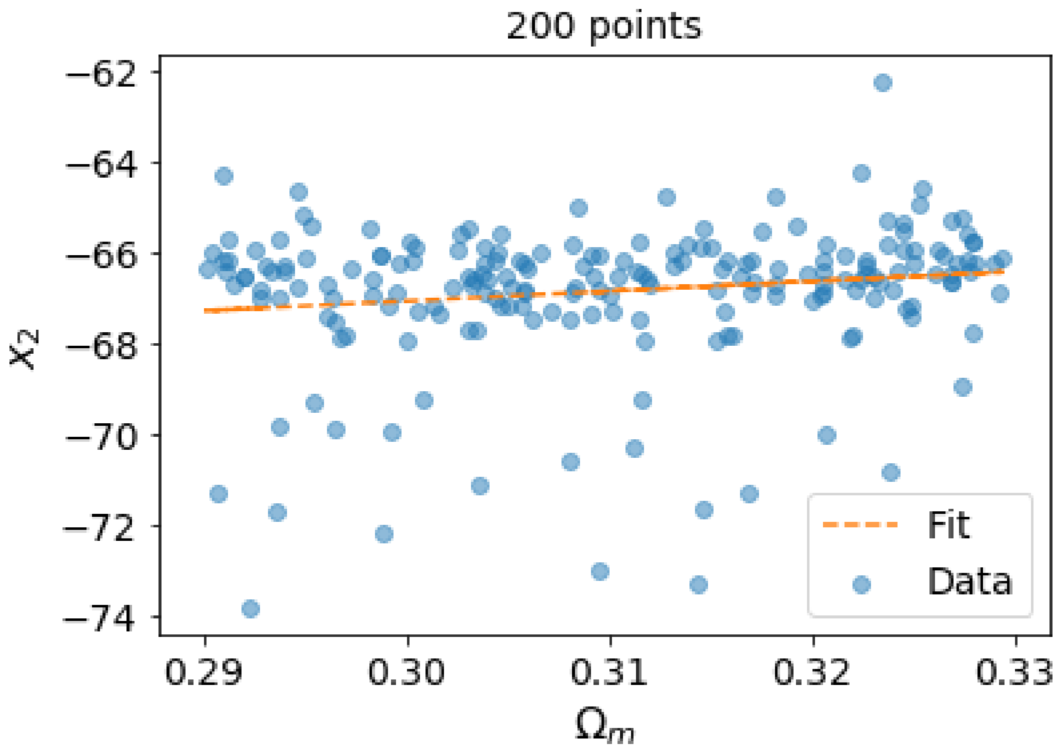
```
In [35]: # Fit to 200 points
         num = 200
         # Fit x1 = a11\Omega_m + b11
         X = np.array([abc['parameters'][:num,0], np.ones(num)]).T # (1000,2)
         y = abc['summaries'][:num,0] # x1

         a11hat, b11hat = linleastsquares(X,y)
         print (a11hat,b11hat)

         # and x2 = a21\Omega_m + b21
         X = np.array([abc['parameters'][:num,0], np.ones(num)]).T # (1000,2)
         y = abc['summaries'][:num,1] # x2
         a21hat, b21hat = linleastsquares(X,y)
         print (a21hat,b21hat)
```

```
-29.23570211919457 161.46084249888997
21.609679809200884 -73.56088019969843
```

In [36]:
```
# x1 as function of Omega_M
model11 = lambda x: a11hat*x+b11hat
# x2 as function of Omega_M
model21 = lambda x: a21hat*x+b21hat
```

In [37]:
```
num = 200
plt.plot(abc['parameters'][:num,0], model11(abc['parameters'][:num,0]),label='Fit',ls=
plt.title(f"{num} points")
plt.scatter(abc['parameters'][:num,0], abc['summaries'][:num,0],alpha=0.5,label='Data
plt.xlabel('$\Omega_m$');
plt.ylabel('$x_1$')
plt.legend()
plt.show()

plt.plot(abc['parameters'][:num,0], model21(abc['parameters'][:num,0]),label='Fit',ls=
plt.title(f"{num} points")
plt.scatter(abc['parameters'][:num,0], abc['summaries'][:num,1],alpha=0.5,label='Data
plt.xlabel('$\Omega_m$');
plt.ylabel('$x_2$')
plt.legend()
plt.show()
```





In [38]:
```
def output_summaries1D(theta,model1,model2):
    """
    Return x1,x2 for a given theta = list of [Omega_m]'s'
    given the two fitted models to x1 x2
    """
    theta = np.array(theta)
    Omega_m = theta[:,0]
    x1 = model1(Omega_m)
    x2 = model2(Omega_m)

    return np.array([x1,x2]).T # return as array of shape (len(theta),2)

def ABC_with_model1D(draws, real_summary, prior, model1, model2, fisher):
    """
    Only as a function of Omega_m
    """

    Gaussprior = priors.TruncatedGaussian(prior["mean"],prior["variance"],prior["lowe
                                          ,prior["upper"])
    # Draw params from Gaussian prior
    theta = Gaussprior.draw(draws)
    # Calculate summaries with models
    summaries = output_summaries1D(theta, model1,model2)
```

```
        # Calculate distance
        differences = summaries - real_summary
        distances = np.sqrt(
            np.einsum(
                'ij,ij->i',
                differences,
                np.einsum(
                    'jk,ik->ij',
                    fisher,
                    differences)))

        ABC_dict = dict()
        ABC_dict["summary"] = real_summary
        ABC_dict["fisher"] = fisher
        ABC_dict["parameters"] = theta
        ABC_dict["summaries"] = summaries
        ABC_dict["differences"] = differences
        ABC_dict["distances"] = distances

        return ABC_dict
```

In [39]:
```
# Variables for ABC
draws = int(1e5) # amount of draws
fisher = abc['fisher'] # fisher info
real_summary = abc['summary'] # summary of 'real' data
# A Truncated gaussian prior
prior = {'mean': np.array([0.30]),
         'variance': np.array([[0.01]]), # 1x1 covariance matrix
         'lower': np.array([0.27]),
         'upper': np.array([0.34])
         }
```

In [ ]:

In [40]:
```
# Plot results
def plot_1D(ABC_dict, prior, epsilon=None,analytic_posterior = None, param_array = Non
    """
    As function of omega_m only
    """

    if epsilon is None: epsilon = np.linalg.norm(abc["summary"])/2. # chosen quite ar
    accept_indices = np.argwhere(ABC_dict["distances"] < epsilon)[:, 0]
    reject_indices = np.argwhere(ABC_dict["distances"] >= epsilon)[:, 0]

    print ('Epsilon is chosen to be %.2f'%epsilon)
    print("Number of accepted samples = ", accept_indices.shape[0])

    truths = theta_fid
```

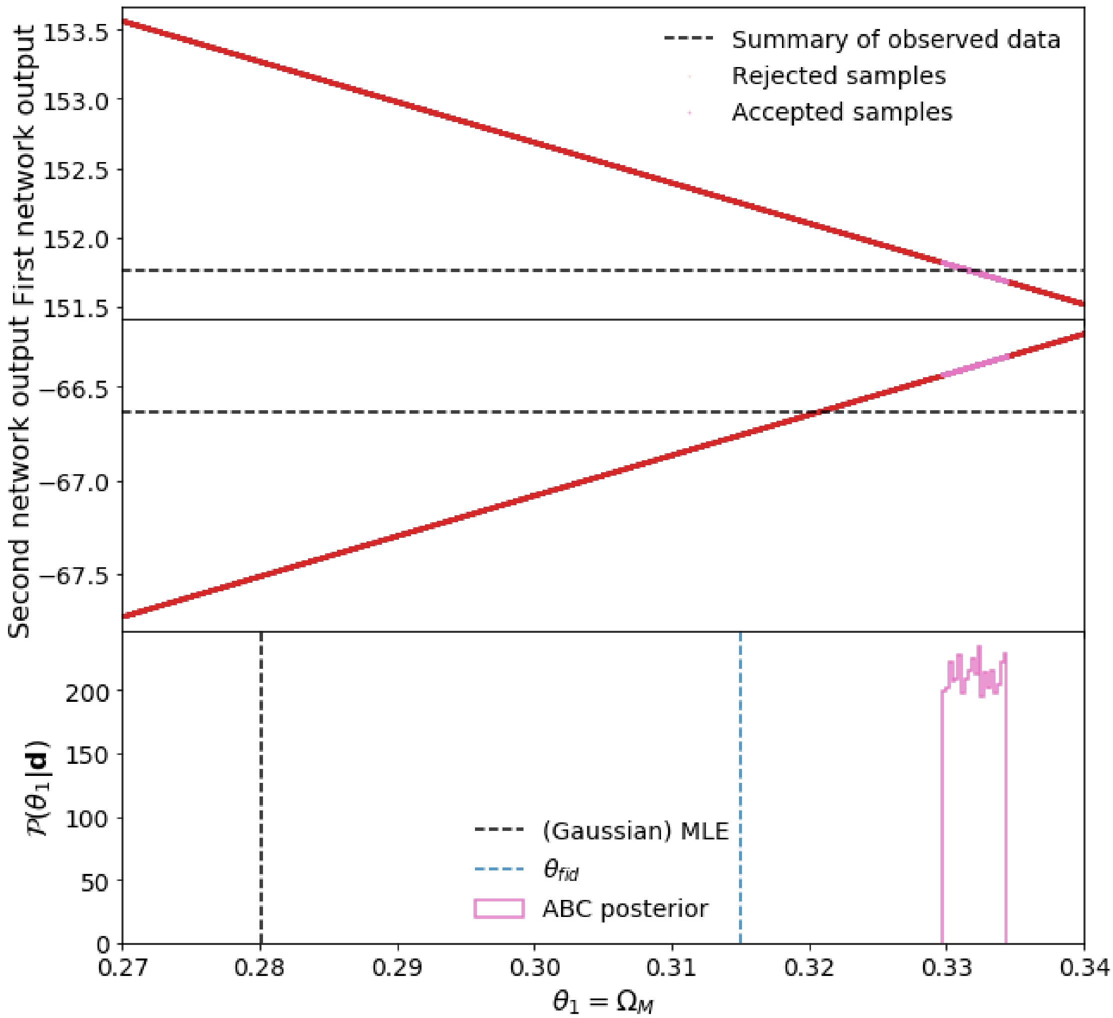
```
fig, ax = plt.subplots(3, 1, sharex = 'col', figsize = (10, 10))
plt.subplots_adjust(hspace = 0)

# Plot the accepted/rejected samples
ax[0].scatter(ABC_dict["parameters"][reject_indices] # Omega_m
    , ABC_dict["summaries"][reject_indices,0] # x1
    , s = 1, alpha = 0.1, label = "Rejected samples", color = "C3")

ax[0].scatter(ABC_dict["parameters"][accept_indices]
 , ABC_dict["summaries"][accept_indices,0]
 , s = 1, label = "Accepted samples", color = "C6", alpha = 0.5)

ax[0].axhline(abc["summary"][0,0]
    , color = 'black', linestyle = 'dashed', label = "Summary of observed data")

ax[0].legend(frameon=False)
ax[0].set_ylabel('First network output', labelpad = 0)
ax[0].set_xlim([prior["lower"][0], prior["upper"][0]])
# ax[0].set_xticks([])

# Plot the accepted/rejected samples
ax[1].scatter(ABC_dict["parameters"][reject_indices] # Omega_m
    , ABC_dict["summaries"][reject_indices,1] # x2
    , s = 1, alpha = 0.1, label = "Rejected samples", color = "C3")

ax[1].scatter(ABC_dict["parameters"][accept_indices]
 , ABC_dict["summaries"][accept_indices,1] # x2
 , s = 1, label = "Accepted samples", color = "C6", alpha = 0.5)

ax[1].axhline(abc["summary"][0,1]
    , color = 'black', linestyle = 'dashed', label = "Summary of observed data")

ax[1].set_ylabel('Second network output', labelpad = 0)

# plot the posterior
ax[2].hist(ABC_dict["parameters"][accept_indices], bins=20, histtype = u'step', de
ax[2].axvline(abc["MLE"][0,0], linestyle = "dashed", color = "black", label = "(Ga
ax[2].set_xlim([prior["lower"][0], prior["upper"][0]])
ax[2].set_ylabel('$\\mathcal{P}(\\theta_1|{\\bf d})$')
# ax[2].set_yticks([])
# ax[2].set_xticks([])|
ax[2].set_xlabel(r"$\theta_1 = \Omega_M$")

# Theta-fid
ax[2].axvline(theta_fid[0], linestyle = "dashed", label = "$\\theta_{fid}$")

if analytic_posterior is not None:
```

```
            ax[2].plot(param_array, analytic_posterior, linewidth = 1.5, color = 'C2'
                , label = "Analytic posterior")


        ax[2].legend(frameon = False)


        fig.suptitle(f"Epsilon = {epsilon}")

        plt.savefig(f'./TEST.png')
        plt.show()
```

```
In [41]: abc_model1D = ABC_with_model1D(draws, real_summary, prior, model11, model21, fisher)
         abc_model1D["MLE"] = abc["MLE"][:,0] # only 1st param
         plot_1D(abc_model1D, prior, epsilon=60,analytic_posterior = None, param_array = None)
```

```
Epsilon is chosen to be 60.00
Number of accepted samples =  6565
```

Epsilon = 60




```
In [ ]:
```

```
In [ ]:
```