

Lecture 12: Measurement Error

BIO144 Data Analysis in Biology

Owen Petchey & Stephanie Muff

University of Zurich

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- ▶ Measurement error (ME) in covariates (x) and in the response (y) of regression models.
- ▶ Effects of ME on regression parameters.
- ▶ When do I have to worry?
- ▶ Simple methods to correct for ME.

Course material covered today

The lecture material of today is partially based on the following literature:

- ▶ Chapter 6.1 in “Lineare regression” (BC reading)

Sources of measurement error (ME)

- ▶ **Measurement imprecision** in the field or in the lab (length, weight, blood pressure, etc.).
- ▶ Errors due to **incomplete** or **biased observations** (e.g., self-reported dietary aspects, health history).
- ▶ Biased observations due to **preferential sampling or repeated observations**.
- ▶ Rounding error, digit preference.
- ▶ **Misclassification error** (e.g., exposure or disease classification).
- ▶ ...

"Error" is often used synonymous to "uncertainty".

Another fundamental assumption (often neglected!)

- ▶ It is a **fundamental assumption** that explanatory variables are measured or estimated **without error**, for instance for
- ▶ the calculation of correlations.
- ▶ linear regression and ANOVA.
- ▶ Generalized linear and non-linear regressions (e.g. logistic and Poisson).
- ▶ Violation of this assumption may lead to **biased** parameter estimates, altered standard errors and p -values, incorrect covariate importances, and to **misleading conclusions**.
- ▶ Even standard statistics textbooks do often not mention these problems.

Measurement error in the covariates (x) violates an assumption of standard regression analyses!!

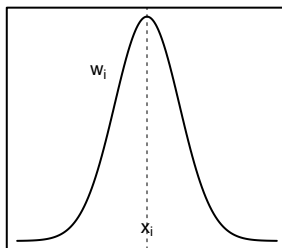
Classical measurement error

A very common error type:

Let x_i be the *correct but unobserved* variable and w_i the observed variable with error u_i .
Then

$$w_i = x_i + u_i, \quad u_i \sim N(0, \sigma_u^2)$$

is the **classical ME model**.



Examples: Imprecise measurements of a concentration, a mass, a length etc. → The observed value w_i varies around the true value x_i .

Illustration of the problem

Find regression parameters β_0 and β_x for the model with covariate \mathbf{x} :

$$y_i = 1 \cdot x_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)$$

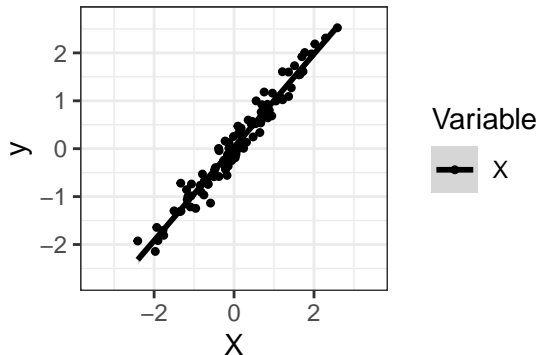
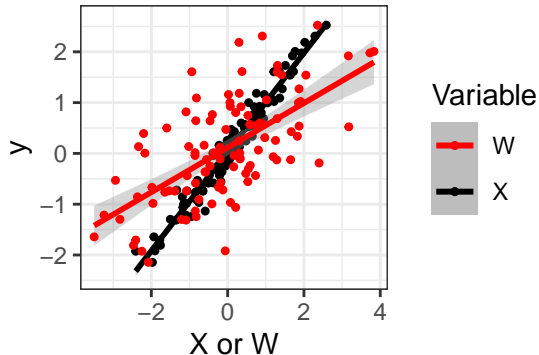


Illustration of the problem II

However, assume that only an erroneous proxy \mathbf{w} is observed with classical ME

$$w_i = x_i + u_i, \quad u_i \sim N(0, \sigma_u^2), \quad \sigma_u^2 = \sigma_x^2$$



A tool you can have a play with...

► Illustration in a browser application

Classical measurement error in linear, logit and Poisson regression

Select regression model:

You can now check the effect of classical measurement error in a covariate of linear, logistic and Poisson regression. The linear predictor of the model is given as

$$\eta = \beta_0 + \beta_1 \cdot x + \epsilon$$

but covariate x is not directly observable. Instead, a substitute

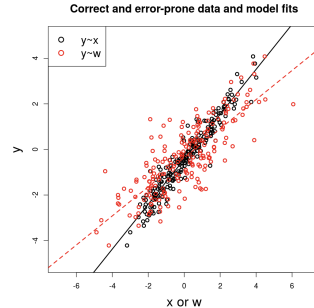
$$w = x + u$$

is observed, assuming that

$$u \sim N(0, \sigma_u^2).$$

To check what happens when the error increases, simply move the slider below to increase the error variance in the x covariate.

Select an error variance σ_u^2 (while $\sigma_\epsilon^2 = 1.$) :

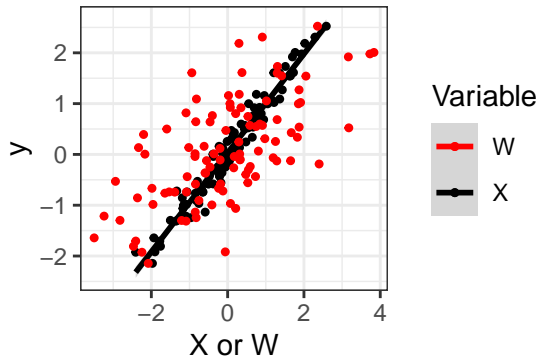


The slope parameter of the error prone dataset is estimated as 0.64 (true slope: 1.0).
 The residual variance of the error prone model is estimated as 0.89 (true value: 0.25).

The “Triple Whammy of Measurement Error”

(Carroll et al. 2006)

- 1 **Bias**: The inclusion of erroneous variables in downstream analyses may lead to biased parameter estimates.
- 2 ME leads to a **loss of power** for detecting signals.
- 3 ME **masks important features** of the data, making graphical model inspection difficult.



How to correct for error?

- ▶ Generally, to correct for the error we need an **error model** and knowledge of the **error model parameters** **Example** If classical error $w_i = x_i + u_i$ with $u_i \sim N(0, \sigma_u^2)$ is present, knowledge of the **error variance** σ_u^2 is needed.

Strategy: Take repeated measurements to estimate the error variance!

- ▶ In **simple cases**, formulas for the bias exist.
- ▶ In most cases, such simple relations don't exist. Specific error modeling methods are then needed!

Attenuation in simple linear regression

Given the simple linear regression equation $y_i = \beta_0 + \beta_x x_i + \epsilon_i$ with $w_i = x_i + u_i$.
Assume that w_i instead of x_i is used in the regression:

$$y_i = \beta_0^* + \beta_x^* w_i + \epsilon_i$$

The **naive slope parameter** β_x^* is then underestimated with respect to the true slope β_x , with **attenuation factor** λ :

$$\beta_x^* = \underbrace{\left(\frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \right)}_{=\lambda} \beta_x$$

→ knowing σ_u^2 and σ_x^2 , the correct slope can be retrieved!

Example: $\sigma_x^2 = 5$, $\sigma_u^2 = 1$, → $\lambda = \frac{5}{6} = 0.83$

Error modeling

The **two most popular approaches**:

- ▶ **SIMEX**: SIMulation EXtrapolation, a heuristic and intuitive idea.
- ▶ **Bayesian methods**: Prior information about the error enters a model. Then use

$$\text{Likelihood} \times \text{prior} = \text{posterior}$$

to calculate the parameter distribution after error correction.

In any case, assessing the biasing effect of the error, as well as error modeling, can be done **only if the error structure (model) and the respective model parameters** (e.g., error variances) **are known!**

Therefore: Information about the error mechanism is essential, and potential errors must be identified already in the planning phase.

SIMEX: A very intuitive idea

Suggested by Cook & Stefanski (1994).

Idea:

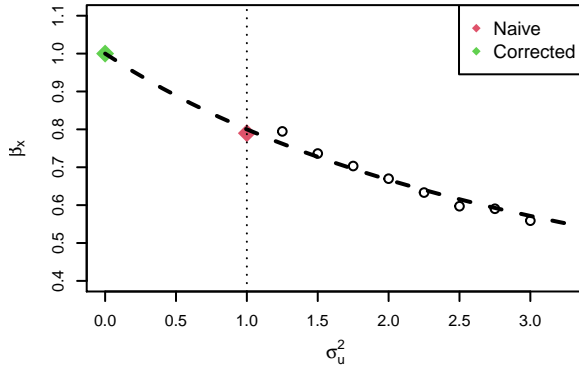
- ▶ **Simulation phase:** The error in the data is progressively aggravated in order to determine how the quantity of interest is affected by the error.
- ▶ **Extrapolation phase:** The observed trend is then extrapolated back to a hypothetical error-free value.

Illustration of the SIMEX idea

Parameter of interest: β_x (e.g. a regression slope).

Problem: The respective covariate x was estimated with error:

$$w = x + u, \quad u \sim N(0, \sigma_u^2)$$



Example of SIMEX use (part 1)

Let's consider a linear regression model

$$y_i = \beta_0 + \beta_x x_i + \beta_z z_i + \epsilon_i, \quad \epsilon_i = N(0, \sigma^2)$$

with

- ▶ $\mathbf{y} = (y_1, \dots, y_{100})^\top$: variable with % Bodyfat of 100 individuals.
- ▶ $\mathbf{x} = (x_1, \dots, x_{100})^\top$ the BMI of the individuals.

****Problem:*** The BMI was self-reported and thus suffers from measurement error! Not x_i are observed, but rather

$$w_i = x_i + u_i, \quad u_i \sim N(0, 4)$$

- ▶ $\mathbf{z} = (z_1, \dots, z_{100})^\top$ a binary covariate that indicates if the i -th person was a male ($z_i = 1$) or female ($z_i = 0$).

→ Apply the SIMEX procedure!

Example of SIMEX use (part 2)

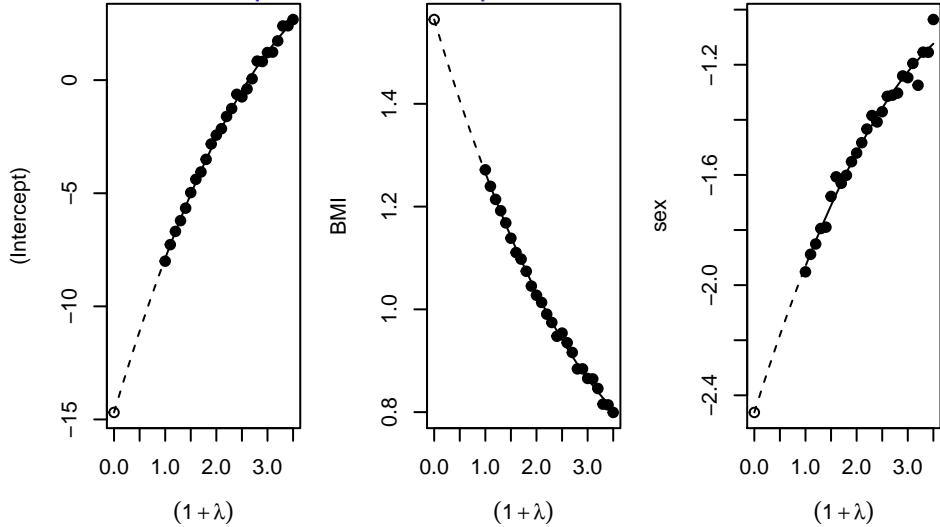
Use the error-prone BMI variable to fit a “naive” regression:

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept) -8.003714  2.07060335  -3.865402  2.005407e-04
## BMI          1.271558  0.08821382  14.414504  7.478782e-26
## sex          -1.951735  0.73625960  -2.650879  9.376840e-03
```

Then run the simex procedure using the `simex()` function from the respective package:

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept) -14.689940  2.6954519  -5.449899  3.825138e-07
## BMI          1.564059  0.1159075  13.494022  5.467540e-24
## sex          -2.462127  0.7906688  -3.113980  2.426632e-03
```

Graphical results with quadratic extrapolation function:



Note: The sex variable has *not* been mismeasured, nevertheless it is affected by the

Practical advice

- ▶ Think about error problems **before** you start collecting your data!
- ▶ Ideally, take **repeated measurements**, maybe of a subset of data points.
- ▶ Figure out if error is a problem and what the bias in your parameters might be. You might need simulations to find out.
- ▶ If needed, model the error. **Seek help from a statistician!**

References

Carroll, R. J., D. Ruppert, L. A. Stefanski, and C. M. Crainiceanu (2006). Measurement Error in Nonlinear Models: A Modern Perspective (2 ed.). Boca Raton: Chapman & Hall.

Cook, J. R. and L. A. Stefanski (1994). Simulation-extrapolation estimation in parametric measurement error models. Journal of the American Statistical Association 89, 1314–1328.