Lecture 12: Measurement Error

BIO144 Data Analysis in Biology

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A request from your TA's

"Help us help you... (even?) better next year!"

Overview

- Measurement error (ME) in one or more explanatory variable(s) (x)
- Effects of ME on model parameters
- When do you have to worry?
- An example of a method to correct for ME

Course material covered today

The lecture material is partially based on:

- Chapter 6.1 in "Lineare regression" (BC reading)

Sources of measurement error (ME)

- **Measurement imprecision** in the field or in the lab (length, weight, blood pressure, etc.).
- Errors due to **incomplete** or **inaccurate observations** (e.g., self-reported dietary aspects, health history).
- Rounding error, digit preference.
- **Classification error** (e.g., exposure or disease classification).
- ...

"Error" is often used synonymous to "uncertainty".

Yet another assumption…

It is an implicit assumption of most statistical tests that explanatory variables are measured or estimated without error. This is true for:

- correlation
- regression and ANOVA
- Generalized Linear Models (e.g. Poison and binomial GLMs)

Violation of this assumption may lead to:

- biased parameter estimates, standard errors, and thus wrong *p*-values
- incorrect (relative) variable importance, and thus *even more* misguided conclusions

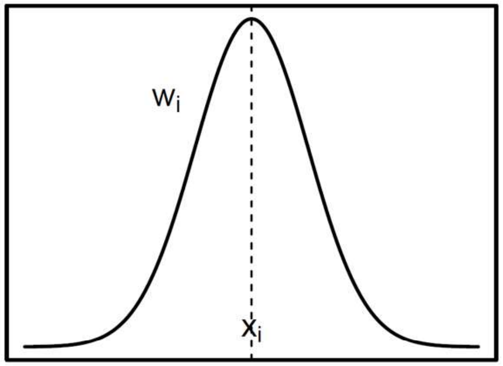
Standard statistics textbooks often do not mention this assumption at all!

Classical measurement error

A very common type of error:

Let x_i be the correct but unobserved variable and w_i the observed variable with error u_i. Then the **classical ME model** is:

$$w_i = x_i + u_i, \qquad u_i \sim N(0, \sigma_u^2)$$



Examples: Inaccurate measurements of a concentration, a mass, a length etc. $\rightarrow$ the observed value w_i varies around the true value x_i.

Illustration of the problem

Find regression parameters β_0 and β_x for the model with explanatory variable $\mathbf{x}$:

$$y_i = 0 + 1 \cdot x_i + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

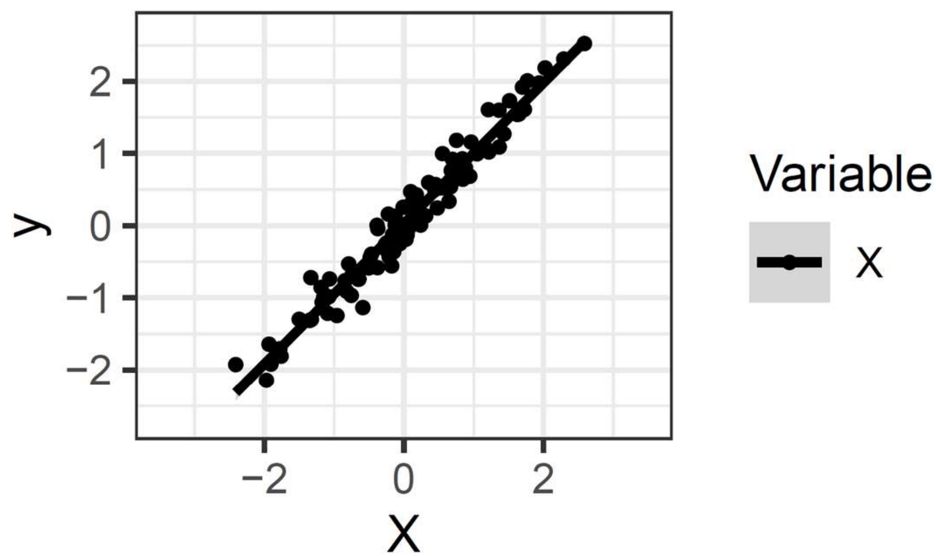
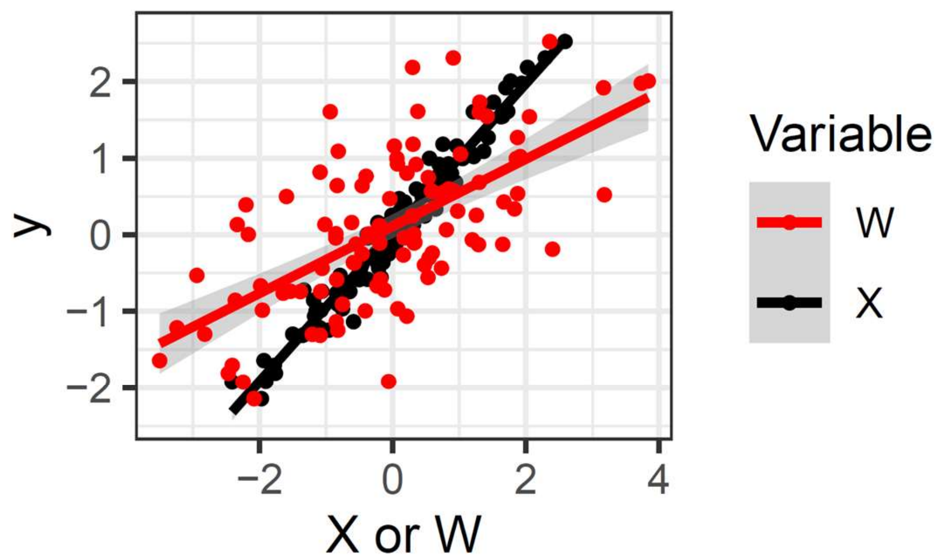


Illustration of the problem

However, assume that only an erroneous proxy **w** is observed with classical ME

$$w_i = x_i + u_i, \qquad u_i \sim N(0, \sigma_u^2), \qquad \sigma_u^2 = \sigma_x^2$$



A tool you can play around with...

▸ Illustration in a browser application

Classical measurement error in linear, logit and Poisson regression

Select regression model:

Linear ▾

You can now check the effect of classical measurement error in a covariate of linear, logistic and Poisson regression. The linear predictor of the model is given as

$$\eta = \beta_0 + \beta_1 \cdot x + \epsilon$$

but covariate *x* is not directly observable. Instead, a substitute

$$w = x + u$$

is observed, assuming that

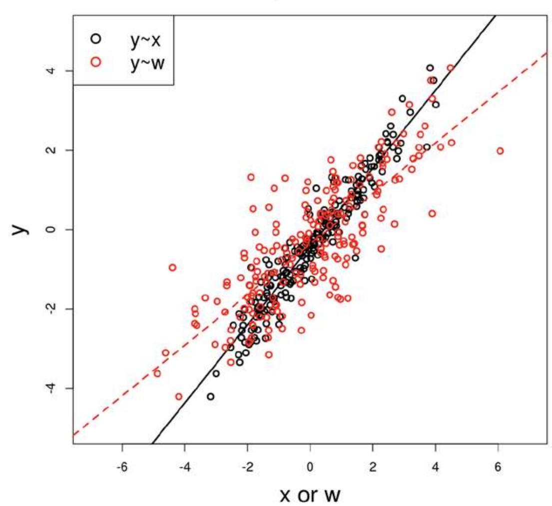
$$u \sim N(0, \sigma_u^2).$$

To check what happens when the error increases, simply move the slider below to increase the error variance in the x covariate.

Select an error variance σ_u^2(while $\sigma_x^2 = 1.$) :

0 1 2 3 4 5 6 7 8

Correct and error-prone data and model fits



The slope parameter of the error prone dataset is estimated as 0.64 (true slope: 1.0).
The residual variance of the error prone model is estimated as 0.89 (true value: 0.25).

The "Triple Whammy of Measurement Error"

(Carroll et al. 2006)

1. Biased parameter estimates
2. Loss of power to detect signals
3. Masks important features of the data, making graphical model inspection difficult

How to correct for ME?

- Generally, to correct for the error you need an **error model** and knowledge of the **error model parameters**.

Example: If classical error $w_i = x_i + u_i$ with $u_i \sim N(0, \sigma_u^2)$ is present, knowledge of the **error variance** σ_u^2 is required.

Strategy: Take repeated measurements to estimate the error variance!

- In simple cases, formulas for the bias exist.
- In most cases, such simple relations don't exist, and dedicated error modelling methods are needed.

Attenuation in normal linear regression

Given the simple linear regression equation $y_i = \beta_0 + \beta_x x_i + \epsilon_i$ with $w_i = x_i + u_i$.
Assume that w_i instead of x_i is used in the regression:

$$y_i = \beta_0^\star + \beta_x^\star w_i + \epsilon_i$$

The **naive slope parameter** $\beta_x^\star$ underestimates the true slope β_x by **attenuation factor** λ:

$$\beta_x^\star = \underbrace{\left(\frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}\right)}_{=\lambda} \beta_x$$

$\rightarrow$ knowing σ_u^2 and σ_x^2, the correct slope can be retrieved!

Example: $\sigma_x^2 = 5$, $\sigma_u^2 = 1$, $\rightarrow \lambda = \frac{5}{5+1} = 0.83$

Error modeling

Two common approaches:

- **SIMEX**: SIMulation EXtrapolation, a heuristic and intuitive idea.
- **Bayesian methods**: Information about the error enters the model as a *prior*.

Both, however, require that the error model, and its respective parameters (e.g., σ_u^2) are known!

Thus, information about the error mechanism is essential, and potential sources of error must be identified at the planning stages of a study!

SIMEX: An intuitive idea

Suggested by Cook & Stefanski (1994), SIMEX takes a two-step approach:

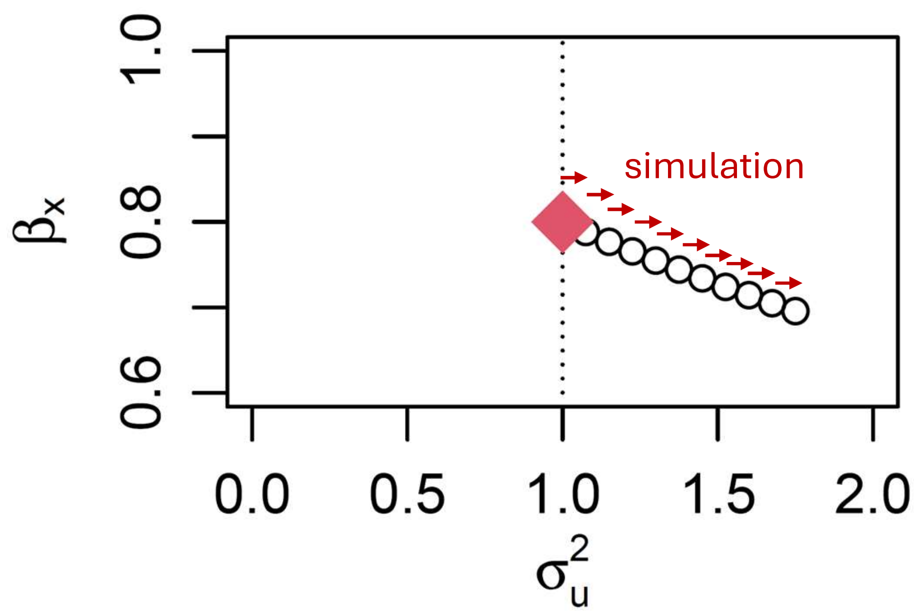
1. **Simulation phase:** The error in the data is progressively aggravated in order to determine how the model parameter of interest is affected.
2. **Extrapolation phase:** The simulated trend is then extrapolated back to a hypothetical error-free value of the model parameter.

Illustration of the SIMEX idea

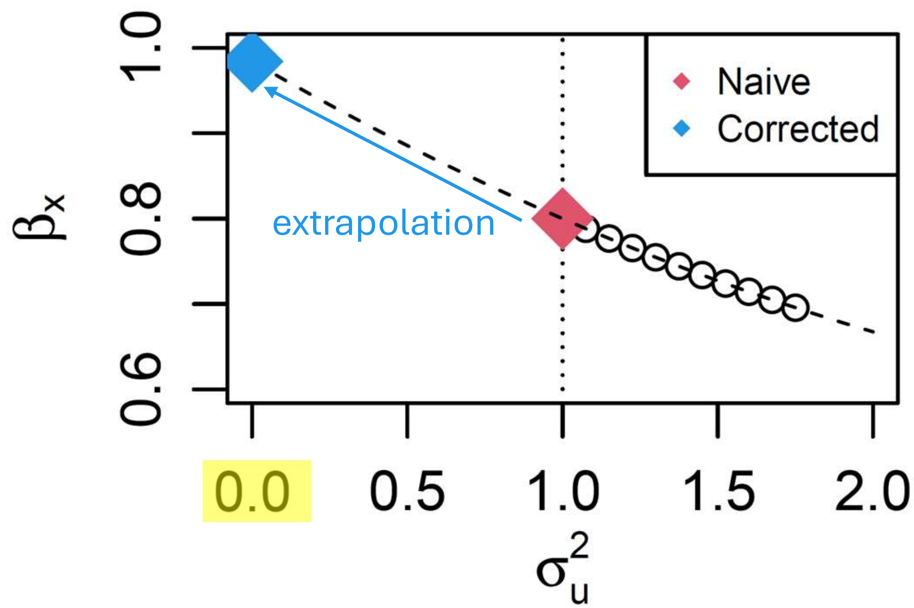
Parameter of interest: β_x (e.g. a regression slope).

Problem: The respective explanatory variable x was estimated with error:

$$w = x + u \ , \quad u \sim N(0, \sigma_u^2)$$



Extrapolate to obtain an estimate of the corrected beta



Example of SIMEX use (part 1)

Let's consider a linear regression model

$$y_i = \beta_0 + \beta_x x_i + \beta_z z_i + \epsilon_i \ , \ \epsilon_i = N(0, \sigma^2)$$

with

- $\mathbf{y} = (y_1, \ldots, y_{100})^\top$: variable with % Bodyfat of 100 individuals.
- $\mathbf{x} = (x_1, \ldots, x_{100})^\top$ the BMI of the individuals.

Problem: The BMI was self-reported and thus suffers from measurement error. Not x_i was observed, but rather

$$w_i = x_i + u_i \ , \quad u_i \sim N(0, \overset{\sigma^2}{4})$$

- $\mathbf{z} = (z_1, \ldots, z_{100})^\top$ a binary explanatory variable that indicates if the i-th person was a male ($z_i = 1$) or female ($z_i = 0$).

$\rightarrow$ Apply the SIMEX procedure!

Simulated example

```
set.seed(3243445)
x<- rnorm(100, 24, 4)
w<- x + rnorm(100, 0, 2)
z<- ifelse(x > 25, rbinom(100, 1, 0.7), rbinom(100, 1, 0.3))

y<- -15 + 1.6*x - 2*z + rnorm(100, 0, 3)

data<- data.frame(cbind(w, z, y))
  names(data)<- c("BMI", "sex", "bodyfat")
```

Any number you want, merely ensures reproducibility

The 'true', but unobserved value

Measurement error in x

z and x are somewhat correlated

Check out the results

Use the error-prone BMI variable to fit a "naive" regression:

```
r.lm <- lm(bodyfat ~ BMI + sex, data, x= TRUE)
summary(r.lm)$coef
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -8.003714 2.07060335 -3.865402 2.005407e-04
## BMI          1.271558 0.08821382 14.414504 7.478782e-26
## sex         -1.951735 0.73625960 -2.650879 9.376840e-03
```

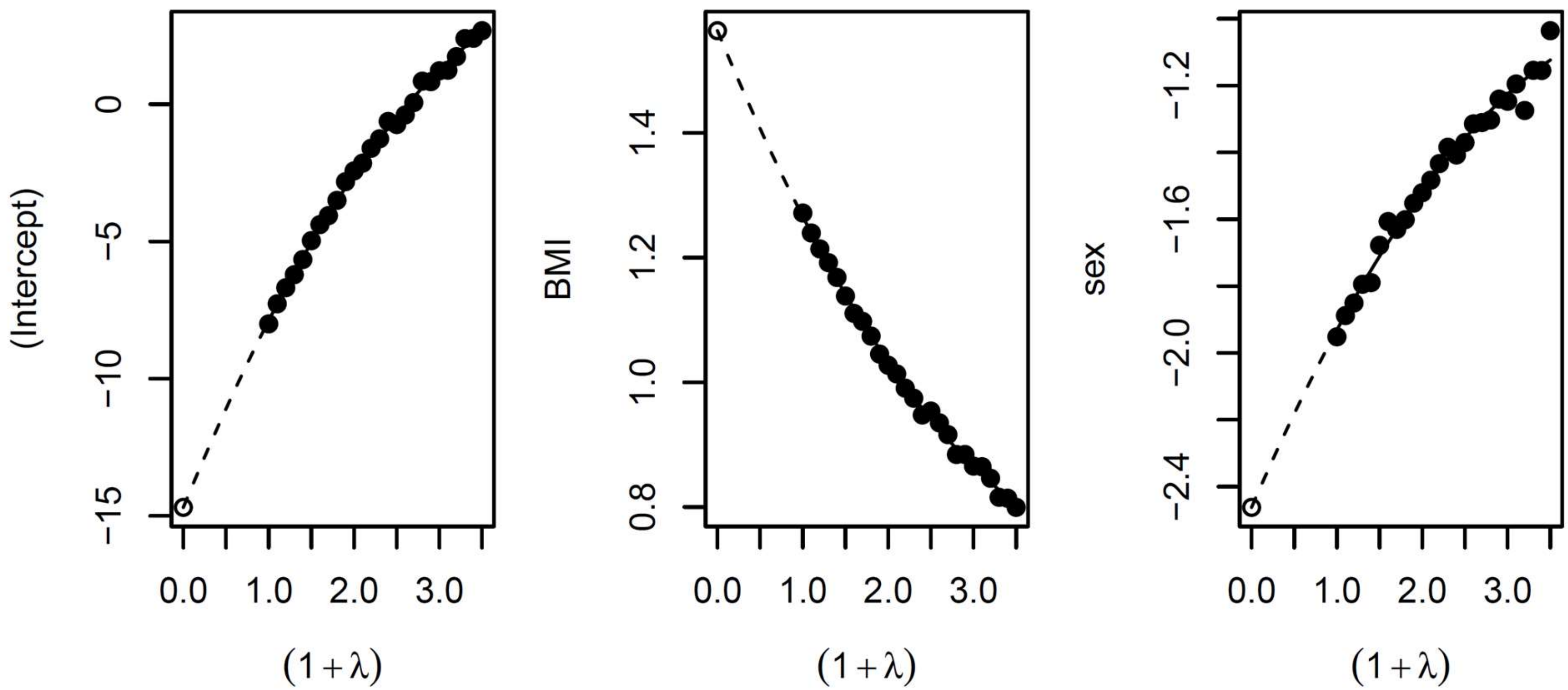
Now run simex procedure

Then run the SIMEX procedure using the `simex()` function:

```
library(simex)
r.simex <- simex(r.lm,
                 SIMEXvariable = "BMI",
                 measurement.error= sqrt(4),
                 lambda= seq(0.1, 2.5, 0.1),
                 B = 100,
                 fitting.method= "quadratic")
summary(r.simex)$coef$asymptotic
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -14.689940  2.6954519 -5.449899 3.825138e-07
## BMI           1.564059  0.1159075 13.494022 5.467540e-24
## sex          -2.462127  0.7906688 -3.113980 2.426632e-03
```

Graphical results with quadratic extrapolation function:



Note: The `sex` variable has *not* been mismeasured, nevertheless it is affected by the error in BMI! **Reason:** `sex` and BMI are correlated.

Practical advice

- Think about measurement error **before** you start collecting your data.
- Ideally, take **repeated measurements**, maybe of a subset of data points
- Figure out if error is a problem and what the bias in your parameters might be. You might need simulations to find out.
- If needed, model the error. **Seek help from a statistician!**

But always use your own common sense
and domain knowledge as well!

References

Carroll, R. J., D. Ruppert, L. A. Stefanski, and C. M. Crainiceanu (2006). Measurement Error in Nonlinear Models: A Modern Perspective (2 ed.). Boca Raton: Chapman & Hall.

Cook, J. R. and L. A. Stefanski (1994). Simulation-extrapolation estimation in parametric measurement error models. Journal of the American Statistical Association 89, 1314–1328.