Abstract

Brief explanation of what this stuff is...

Introduction

This is a fairly detailed collection of all the subjects, concepts, theorems and tools I'we studied during my educational trip. There is no request of completeness as well as this "few" concepts must be intended as a leading guide for the student that is approaching mathematics and/or physics from scratch.

1 Elementary Math

- Sum and Difference.
- Multiplication and Division.
- Fraction.
- Powers.
- Logarithm and exponential.

2 Logic

- Unary operators: *not*: $\neg$; tautology: $\top$
- Binary operators: and, or, nor, nand, ...
- Equivalence Class
- Axiom of Choice

3 Linear Algebra

- CayleyHamilton theorem

4 Single-valued Real Analysis

- Topology, open and closed sets.

5 Mechanics

6 Thermodynamics

7 Algorithms and Computations

- Computational cost.
- List, Stack, Arrays, Queries.
- Bubble sort, Merge Sort, Quick Sort.

8 Multi-valued Analysis

- Partial derivatives, derivatives vector ∇.
- Curl and Divergence.

9 Numerical Analysis

10 Algebra

Even if this (sometimes *very*) abstract subject is considered almost selfcontained and useful only for very narrow fields purposes, I wanted to be very precises in the subsections in order to underline the extreme importance that Algebra and (Lie) Groups in general has assumed in the last few decades. The ones listed belows are nowaday unavoidable topics for future theoreticians willing to completely understand all the Symmetry Properties in Nature, from Special and General Relativity to Classical and Quantum Mechanics as well as Particle Physics.

10.1 Group Theory

- Cayley Diagram
- Generators
- Klein Group
- Cyclic Group
- Abelian Group
- Dehydral Group
- Coset
- Normal Subgroup
- Quotient Group
- Semidirect Product
- Group Representation
- Irreducible Group Class

11 Geometry

12 Probability

13 Dynamical Systems

14 Electromagnetism

15 Fluid Dynamics

16 Wave Mechanics

- Wave Equation $\partial_{tt}u - c^2\Delta u = 0$
- Planar wave
- Poynting Vector

17 Complex Analysis

18 Numerical Analysis for (Partial) Differential Equations

19 Stochastic Processes

- Markov Process
- Poisson Process
- Birth & Death Process

20 Differential Geometry

- Differentiable Atlas
- Orientable Atlas
- Tangent plane
- Normal versor
- First Fundamental Form: lengths and area
- Geodesic curvature and normal curvature
- Normal sections and Meusnier Theorem
- Principal Curvatures, Gaussian curvature, Mean curvature: minimal surfaces
- Theorema Egregium
- Geodetics
- Free vector space
- Tensor product of two vector spaces
- Tensor product of n vector spaces
- Tensor Algebra
- Transformation of the componenents of a tensoriale
- Mixed tensors
- Symmetric tensors
- Antysimmetric (alternating) tensors
- Exterior Algebra
- Determinant
- Area and Volume
- Definition and examples
- Classification of 1-manifolds
- Classification of simply-connected 2-manifolds
- Product and quotient spaces
- Differentiable maps

- Tangent space and tangent bundle
- Vector field on a manifold
- Tensor field
- Exterior Algebra on manifolds
- Riemannian Manifolds
- Metric Tensor
- Orientations
- Volume
- Exterior derivative
- De Rham Cohomology
- Homotopy
- Affine connection
- Parallel transport
- Levi-Civita connection
- Geodetics
- Riemann curvature tensor
- Bianchi identities

21 Functional Analysis

- L^p spaces
- Riesz Lemma
- Fredholm Alternative

22 Mathematical tools for Physics

- Eigenfunctions for the cube and for the cylinder
- Bessel Equation
- Bessel Functions of the first and second kind: $J_\alpha(x)$ and $Y_n(x)$
- Fourier-Bessel Series
- Eigenfunction for the sphere
- Laplacian in Spherical Coordinates
- Legendre Equation
- Legendre Polynomials
- Rodriguez Formula
- Fourier-Legendre Series
- Recurrence Relations

- Associated Legendre Functions
- Spherical harmonics $Y_{lm}(\theta, \varphi)$
- Perturbation Theory
- Fourier Transform in $\mathbb{R}^n$
- Green Function(s)
- Spectral Representation of Green (homogeneous) Functions
- 1^{st} and 2^{nd} Green Formulas

23 Partial Differential Equations

- Characteristics Method

24 Stochastic Differential Equations

- Itô Integral

25 Advanced Numerical Analysis

26 Analytical Mechanics

27 Quantum Mechanics

- Schrödinger Equation
- Probability Density $\partial_t \psi^* \psi$
- Probability Current Density $\vec{\nabla} \cdot \vec{S}$
- Infinity conditions for the wave function
- Stationary States for a quantum mechanical system
- Klein-Gordon Equation
- Schrödinger Solution as a Markov process
- Simple Harmonic Oscillator
- Ladder Operators a and $a^\dagger$
- Hermite Differential Equation
- 1D Square Well Potential
- Forbidden Regions
- Square Potential Barriers
- Tunneling Effect
- Particle in the box
- Concept of classical limit $\hbar \to 0$
- Gauge Transformations and Landau Gauge

- Landau Levels
- Spherical Harmonics
- Pseudo-vectors/Axial Vectors
- Spin-Orbit Coupling
- Shell Model of the Nucleus

28 Statistical Mechanics

29 Solid State Physics

- Boltzman Model
- Einstein Model
- Bebye Model
- Drude Theory

30 Nuclear Physics

31 Nonequilibrium Statistical Mechanics

- Einstein-Smoluchowski relation $D = \mu k_B T$
- Stokes-Einstein equation $D = \frac{k_B T}{6\pi \eta r}$
- Ornstein-Uhlenbeck process

32 Advanced Quantum Theory

- Coupling Basis
- Clebsch-Gordan Coefficients
- Isospin
- Coherent State
- Displacement Operator
- Squeezing Operator
- Cross section amplitude coefficient σ

33 Quantum Field Theory

34 Advanced Quantum Field Theory