IN3190 - Exam Preparation Questions H23
University of Oslo

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Contents

1 Essential Topics

1.1 Discrete time

1.1.1 Sine, cosine and exponential functions

Mathematical formula that establishes the fundamental relationship between the trigonometric functions and the complex exponential function.
Euler's formula states that for any real number x:

$$e^{ix} = \cos(x) + i\sin(x) \tag{1}$$

When $x = \pi$, Euler's formula yields Euler's identity:

$$e^{i\pi} + 1 = 0 \tag{2}$$

1.1.2 Elementary discrete signals

Unit impulse   Also known as the dirac delta function.

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \quad \text{may also be written as:} \quad \delta[n] = u[n] - u[n-1] \tag{3}$$
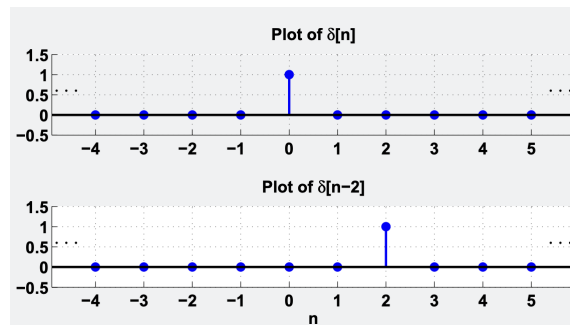

Figure 1: Unit impulse

Step function   Also known as unit step, unit step function or heaviside function. The value which is zero for negative arguments and one for positive arguments.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} \quad \text{may also be written as:} \quad u[n] = \sum_{k=0}^{\infty} \delta[n-k] \tag{4}$$
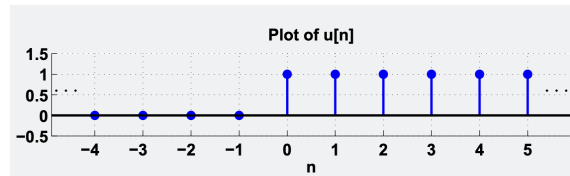
Figure 2: Unit step function

Ramp function   Also known as the unit-ramp or unit ramp function. Graph shaped like a ramp.

$$u_r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases} \tag{5}$$
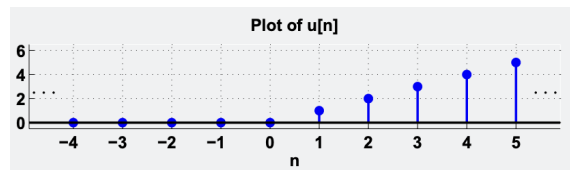

Figure 3: Unit ramp function

Periodic sequences $x[n]$ is periodic if and only if $x[n] = x[n + N]$

- **Fundamental period**:
  Smallest positive integer N which fulfills the relation above

- **Sinusoidal sequences**:
  $x[n] = A\cos(\omega_0 n + \phi)$, where

  - A is amplitude,
  - ω_0 is the angular frequency
  - and ϕ is the phase of $x[n]$.

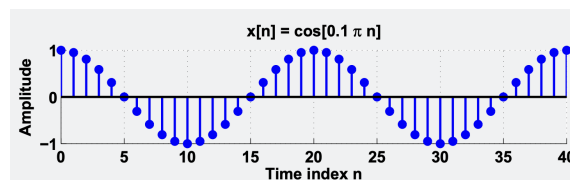 $x[n]$ is periodic if and only if $\omega_0 N = 2\pi k$, for N and k as positive integers.

Figure 4: Sinusoidal sequences

3 of 5

1.1.3 LTI systems, including characteristics via the transformation between input and output

Linearity: The relationship between the input $x(t)$ and the output $y(t)$, both being regarded as functions. If a is a constant then the system output to $ax(t)$ is $ay(t)$. Linear system if and only if it $H\{\cdot\}$ is both additive and homogeneous, in other words: If it fulfills the superposition principle. That is:

$$H\{ax_1(t) + bx_x(t)\} = aH\{x_1(t)\} + bH\{x_2(t)\} \tag{6}$$

Time-invariance: A linear system is time-invariant or shift-invariant means that wether we apply an input to the system now or T seconds from now, the output will be identical except for a time delay of S seconds. In other words, if $y(t)$ is the output of a system with a input $x(t)$, then the output of the system with input $x(t - T)$ is $y(t - T)$. The system is invariant becasue the output does not depend on the particular time at which the input is applied.

The fundamental result in LTI system theory is that any LTI system can be characterized entirely by a single function called the system's impulse response $h(t)$. The output of the system $y(t)$ is simply the convolution of the input to the system $x(t)$ with the systems's impulse response $h(t)$.

LTI system can also be characterized in the frequency domain by the system's transfer function $H(s)$, which is the Laplace transform of the system's impulse response $h(t)$. Or Z-transform in the case of discrete time systems.
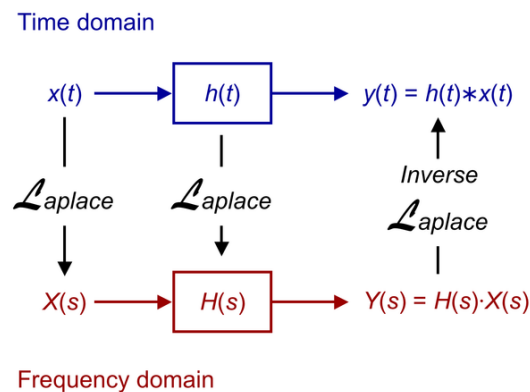

Figure 5: Relationship between the time domain and the frequency domain

1.2   Z-transform

1.3   Frequency analysis and DFT

1.4   Filter design

1.5   Sampling and reconstruction

2   Additional Topics

2.1   Discrete time

2.1.1   Symmetrical signals

2.2   LTI systems and characteristics

2.3   Convolution and correlation

2.4   Z-transform

2.5   Frequency analysis and DFT

2.6   Filter design

2.7   Sampling and reconstruction