Project 3: Driven-Cavity Flow

AE 523, Computational Fluid Dynamics, Fall 2021

Due: December 10, 11:59pm, electronically via Canvas

1 Problem Description

In this project you will solve the two-dimensional, lid-driven cavity problem using primitive variables (u, v, p). The geometry for the problem is shown in Figure 1. It consists of a square of side length L, with fixed bottom, left, and right walls, and a top wall that is moving to the right (positive x) at a speed U_{wall}. The flow is governed by the incompressible Navier-Stokes equations. Assume that $\rho = 1$, $L = 1$, and $U_{\text{wall}} = 1$.

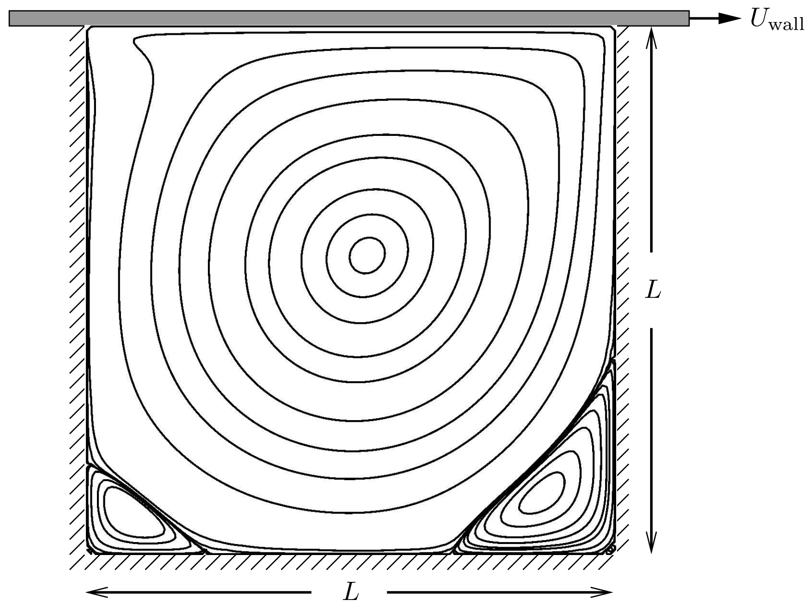

Figure 1: Setup for the driven cavity problem and sample streamline contours.

2 Numerical Method

Use the projection-based, primitive-variable solution algorithm given in the notes (Sections 7.3.3 to 7.3.12), with staggered storage and the SMART limiter. Consider uniform $N \times N$ grids, where N is a power of 2. To solve the pressure Poisson equation, you can build a system and solve it directly (if doing this, make sure you pin the pressure in one cell to a constant value, e.g. zero, otherwise your matrix will be singular), or you can use an iterative technique, such as red-black Gauss-Seidel with successive over-relaxation (SOR). In an iterative method, you may need to remove the average pressure during the iterations to prevent divergence. You will find the best performance if you couple SOR with multigrid.

As a suggestion for data structures, you can use the following:

- The pressure $\mathbf{P}$ is $N \times N$: one value per cell.
- The horizontal velocity $\mathbf{U}$ is $(N+3) \times (N+2)$: one value per vertical edge and one layer of ghost values all around.
- The vertical velocity $\mathbf{V}$ is $(N+2) \times (N+3)$: one value per horizontal edge and one layer of ghost values all around.

- The horizontal x-momentum flux $\mathbf{F}$ is $N \times N$: one value per cell.
- The vertical y-momentum flux $\mathbf{G}$ is $N \times N$: one value per cell.
- The vertical x-momentum flux $\mathbf{H}^x$ is $(N+1) \times (N+1)$: one value per grid node.
- The horizontal y-momentum flux $\mathbf{H}^y$ is $(N+1) \times (N+1)$: one value per grid node.

Set the time step as

$$\Delta t = \beta \min\left(\frac{h^2}{4\nu}, \frac{4\nu}{U_{\text{wall}}^2}\right),$$

where $h = L/N$, and β is a stability factor – you may need to reduce the time step at high Reynolds numbers and/or fine meshes. To enable larger time steps, consider changing the explicit velocity update, Eqns. 7.3.23 and 7.3.24 in the notes, to semi-implicit, by making the viscous fluxes implicit. This will require a Laplace-like system solution, for which you can use a few iterations of successively-over-relaxed red-black Gauss-Seidel.

2.1 Tasks

The minimum required results for this project are converged runs at Reynolds numbers $Re = U_{\text{wall}}L/\nu$ of 100 and 400 on a mesh of size 32×32 (i.e. $N = 32$). A converged run is one for which the sum of the absolute values of the residuals, $|R|_{L_1}$, as defined by Eqn. 7.3.31 in the notes, is below 10^{-5}. For each converged run, present:

1. A semilog-y convergence plot of $|R|_{L_1}$ versus iteration number.
2. Streamfunction contours at the levels given in Table III of Ghia et al[1], and a discussion of the qualitative comparison to those shown in Figure 3 of Ghia et al.
3. Plots of u-velocity along the vertical line through the geometric center, and plots of v-velocity along the horizontal line through the geometric center. Overlay the data from Tables I and II of Ghia et al[2], and discuss the comparison.

The grade breakdown will consist of 40% for the solver code, 15% for post-processing code, 35% for the results and discussion, and 10% for the professionalism of the presentation.

A converged solution at $Re = 1000$ on a 64×64 grid will earn an extra credit of dropping the lowest quiz grade in the class. Additional converged runs at $Re = 3200$ and $Re = 5000$ on a 128×128 grid will earn an additional extra credit of dropping the lowest homework assignment grade. Finally, an additional converged run at $Re = 7500$ on a 512×512 grid will earn an additional extra credit of dropping the lowest exam grade (midterm or final). To be eligible for extra credit, the assignment must be submitted on time, without extensions.

3 Deliverables

You should turn in, electronically via Canvas,

1. A technical report as a `.pdf` file that describes your methods and results and addresses all of the tasks. The report should be professional, complete, and concise. **10%** of your grade will be determined by the professionalism (neatness, labeling, spelling, etc.) of your report.
2. Documented source files of all codes you wrote for this project, as one `.zip` archive.

This is an individual assignment. You can discuss the project at a high level with each other, but you must turn in your own work.

[1] U. Ghia, K.N. Ghia, and C.T. Shin, High-Re Solutions for Incompressible Flow Using the Navier–Stokes Equations and a Multigrid Method, Journal of Computational Physics, Volume 48, pp. 387–411, 1982.

[2] The u-velocity at $Re = 3200$, $y = 0.4531$ is incorrect in Ghia et al. You can omit this value.