Honor Code

Thursday, December 16, 2021 2:46 PM

Final Exam Long

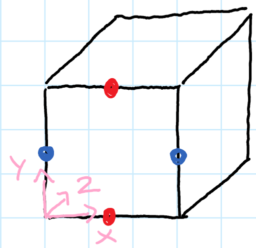
#1) INS in 3D

a) Show where you would store velocity & fluxes.

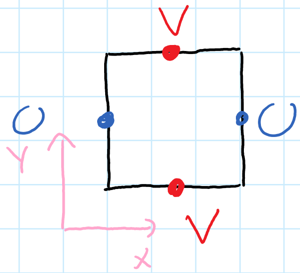
Assume a uniform cube for our mesh cells

Velocity components

cell N



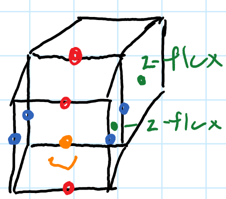
xy-plane



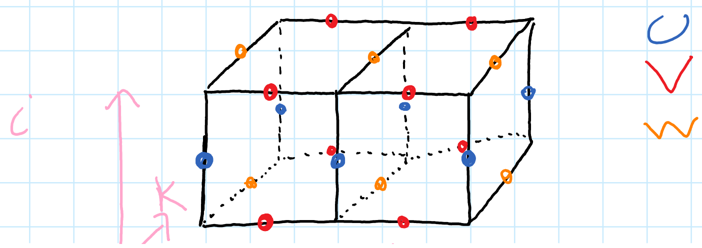
So $\vec{u}$ is stored $\perp$ to x-direction & $\vec{v}$ is stored $\perp$ to y-direction

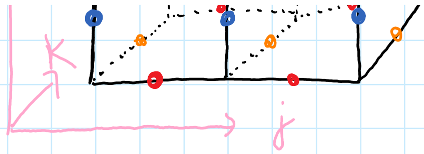
↳ vertical edge ↳ horizontal edge

z-direction is $\perp$ to both, so put it in the center of each cell

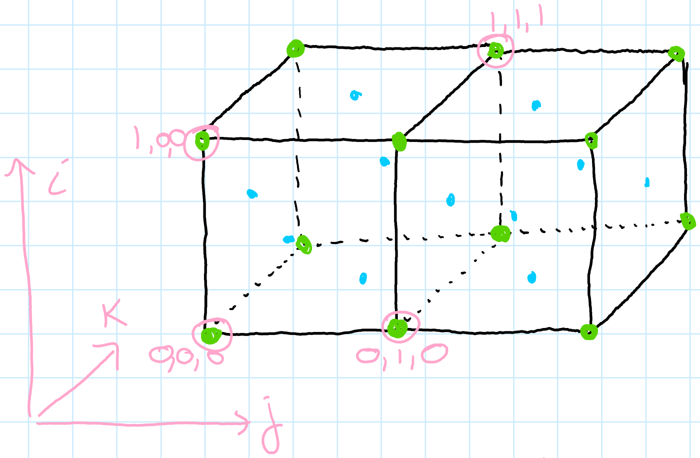


Each cube cell has 4 u & v components, want matching # of w-components for scheme & accuracy, put them on depth edge





The x-y stuff is the same, the z-component is a little different.
All z-fluxes are at vertices



F^{zz}
F^{yz} F^{zy}
F^{xz} F^{zx}
F^{xy} F^{yx} vertex points

P F^{xx} F^{yy} – center of faces

b) Write update for $w^{n+1/2}$

$$w_t + F_x^{zx} + F_y^{zy} + F_z^{zz} + \cancel{P_z} = 0$$ ignore pressure

$$\frac{w^{n+1/2}_{i,j,k+1/2} - w^{n}_{i,j,k+1/2}}{\Delta t} = -F_x^{zx} - F_y^{zy} - F_z^{zz}$$

$$w^{n+1/2}_{i,j,k+1/2} = w^{n}_{i,j,k+1/2} - \Delta t\left(F_x^{zx} - F_y^{zy} + F_z^{zz}\right)$$

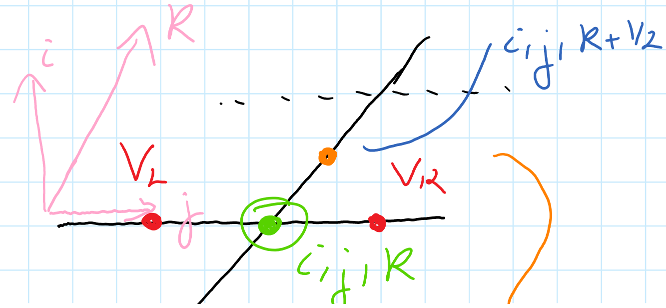
$$F_z^{zz} = \frac{F^{zz}_{i,j,k+1} - F^{zz}_{i,j,k}}{\Delta z} \qquad F^{zx} = uz - \nu w_x \qquad F^{zy} = vz - \nu w_y$$

$$F_x^{zx} = \frac{F^{zx}_{i,j+1} - F^{zx}_{i,j-1,k}}{2\Delta x} \qquad F_y^{zy} = \frac{F^{zy}_{i+1,j,k} - F^{zy}_{i-1,j,k}}{2\Delta y}$$

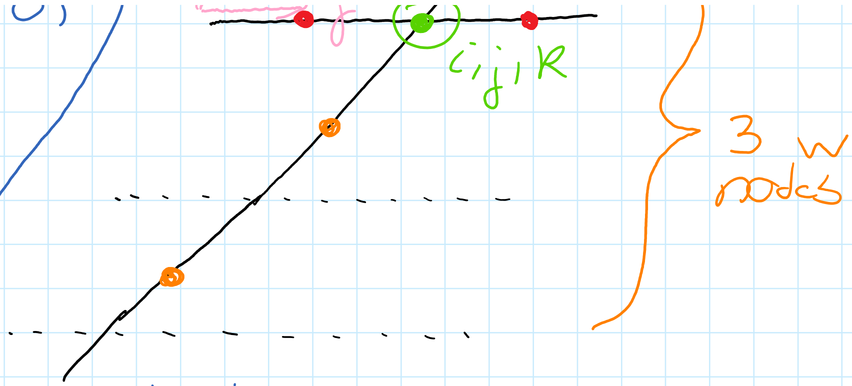
c) Show smart F^{zy} for $q>0$

$F^{zy} = vz - \nu w_y$ – below

$z = \text{SMART}(\phi_{j-1}, \phi_j, \phi_{j+1})$
v>0, index towards 0)



$v>0$, index towards 0)

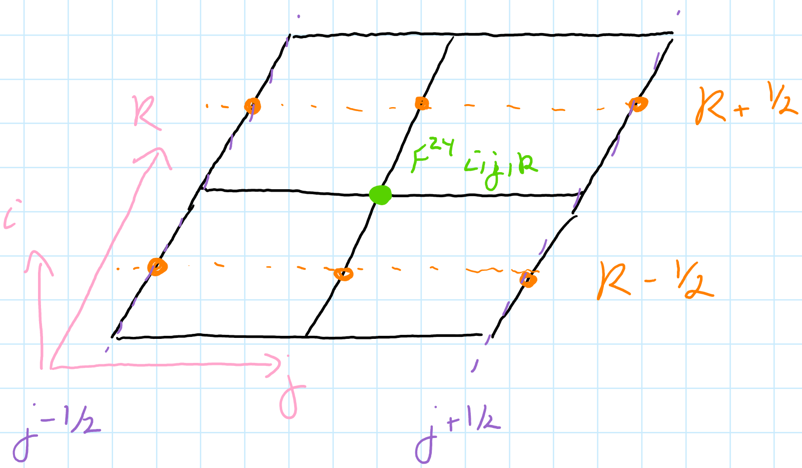


$$q = \frac{V_L + V_R}{2} > 0$$

$$\left.\begin{aligned} \phi_{j-1} &= z_{i,j,k-3/2} \\ \phi_j &= z_{i,j,k-1/2} \\ \phi_{j+1} &= z_{i,j,k+1/2} \end{aligned}\right\}$$

plug into w/ actual values

For w_Y I don't have points to line up, so I can either Taylor series, or average 2 nearby w_Y's

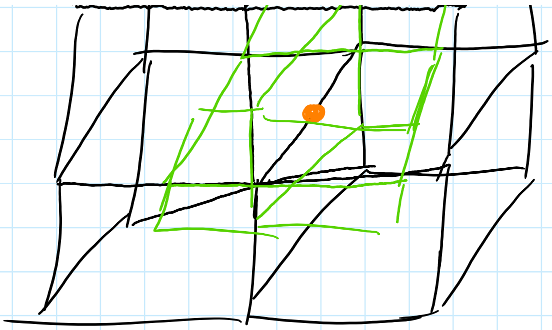


along $k+1/2$ $\quad W_Y = \dfrac{W_{i,j+1/2,k+1/2} - W_{i,j-1/2,k+1/2}}{2\Delta Y}$

along $k-1/2$ $\quad W_Y = \dfrac{W_{i,j+1/2,k-1/2} - W_{i,j-1/2,k-1/2}}{2\Delta Y}$

then average these to get w_Y, $\quad W_Y = \dfrac{W_{Y\,k+1/2} + W_{k-1/2}}{2}$ this is for F^{2Y}

d) Write formula & sketch z-momentum residual



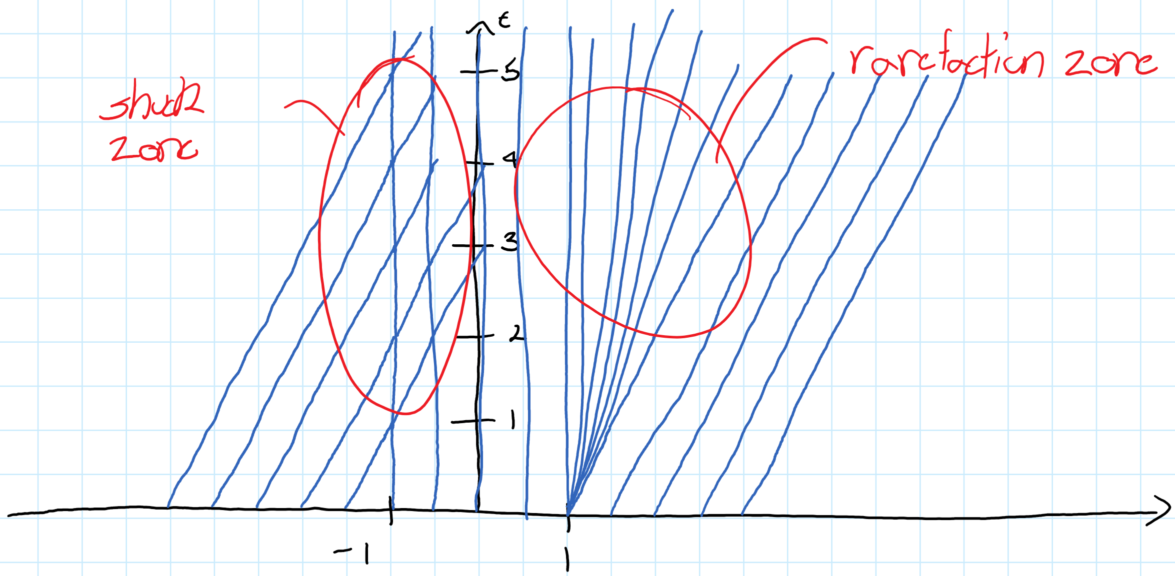
You have this weird pseudo-cell between all other cells for each w.

Need pressure terms, cross momentum & non

$$\boxed{R_i = 1 \oint [\text{Pressure side 1} - \text{Side 2}] + \Delta F^{22} + \Delta \text{cross mom}}$$

#2) Traffic Flow

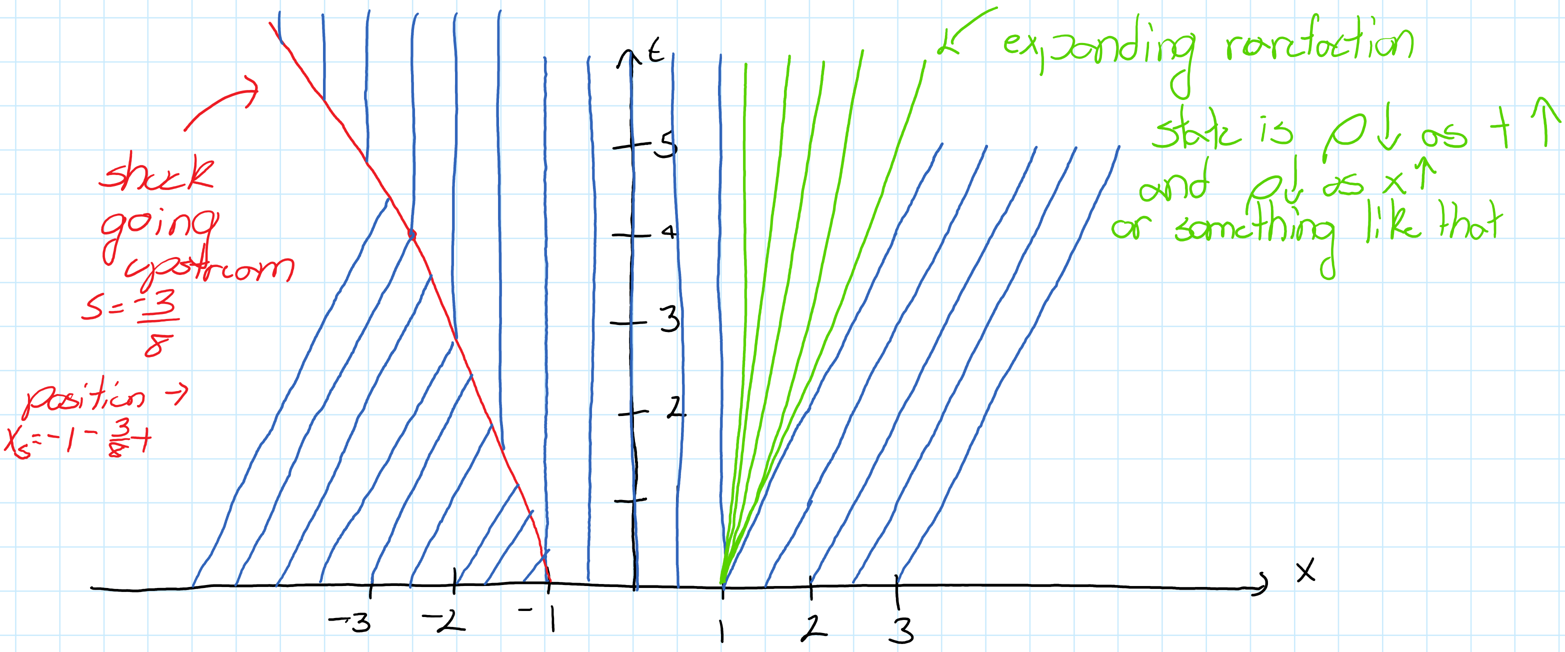
a) Sketch until $t=5$ & include shocks & such



$$s = \frac{f_L - f_R}{\rho_L - \rho_R} \qquad f_L = \frac{1}{2} u_{max}(1 - 1/2) = \frac{u_{max}}{4} = \frac{1}{4}$$

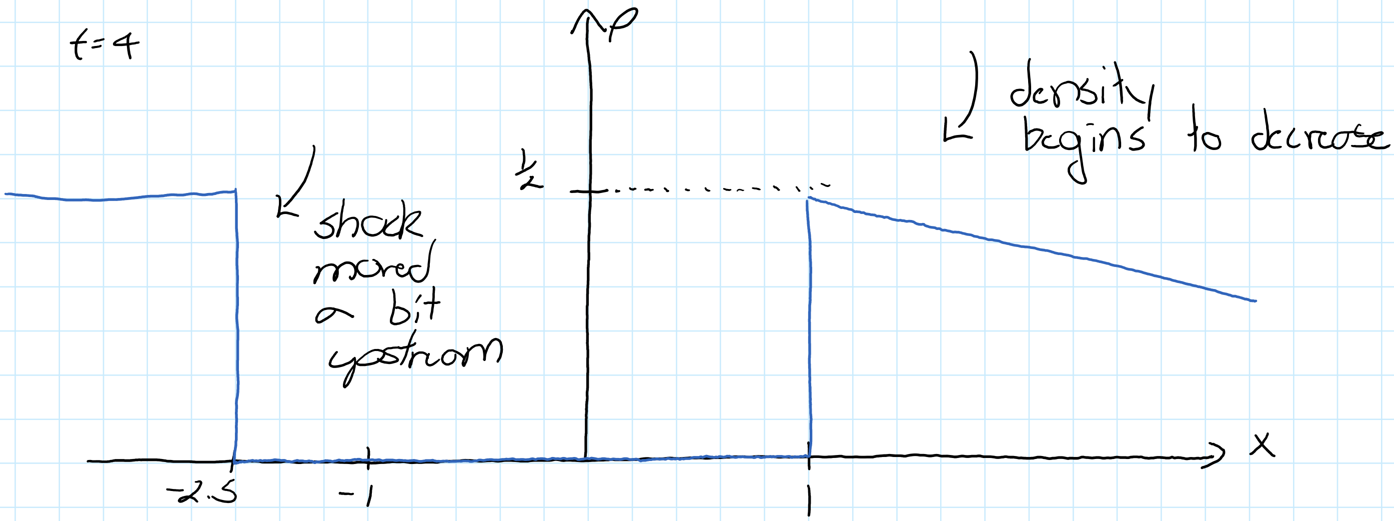
$$f_R = 1 \qquad \rho_L = 1/2 \qquad \rho_R = 0$$

$$s = \frac{1/4 - 1}{1/2} = \frac{-3/4}{1/2} = \frac{-3}{8}$$ shock moves left w/ time

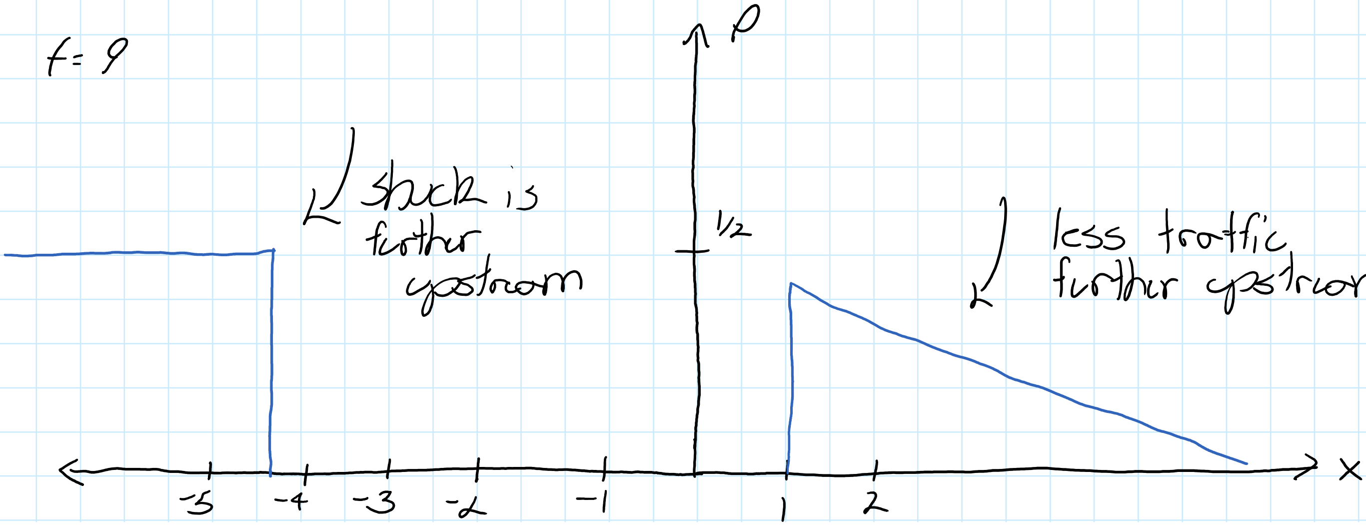


b) Sketch at $t=4$ & $t=9$

$t=4$



$t=9$



Thanks for semester, still don't understand 523 MOC, but sort of did 520 MOC

Sad we didn't do any supersonic flows

Hopefully, you teach 623 in the Fall