Algorithmic Price Discrimination

Paper written by Cummings, Devanor, Huang and Wang (2020)

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Agents - Review Bergemann et al. (2015)

SWISS

Buyer ⟺ Monopoly Seller

Buyer

- **Buyers with different valuations**
- Decide on purchase (myopic)

Monopoly Seller

- Observes **segmentation of valuations**
- Sets monopoly price for segments

Extension of Assumptions

- Assumptions - Bergemann et al. (2015)
  1. The exact valuation of buyer is observed.
  2. Segmentation is exogenous.
- Generalized Assumptions - Cummings et al. (2020)
  1. Only a noisy signal of the buyer valuation is observed - the **Buyer Type**.
  2. Endogenous Segmentation by a third agent - the **Intermediary**.

Key Questions

1. Which welfare outcomes can be achieved with noise?
2. How can an intermediary construct an optimal segmentation with different levels of information about buyers valuation?

Noisy Signal

Noisy Signal

Noise in Signal of Buyers Valuation

The buyer valuation is not perfectly known, but indicated by a noisy signal with remaining uncertainty. We call this noisy signal the observable **buyer type**.

Agents - Noisy Buyer Valuation

Buyer

- **Buyer types with different value distr.**
- Decide on purchase (myopic)

$\Longleftrightarrow$

Monopoly Seller

- Observes **segmentation of types**
- Sets monopoly price for segments

Noise Example - Setup

- Value Set $V \in \{1, 2, 3\}$
- Type Set $T \in \{1, 2, 3\}$
- Value Distribution $V \sim \mathit{unif}[1, 3]$
- Noise Function $\mathcal{F}(t)$ with noise $(1 - z) \in [0, 1]$

$$P_{v \sim \mathcal{F}(t)}(v) = \begin{cases} z & \text{if } v = t. \\ \frac{1-z}{2} & \text{otherwise.} \end{cases}$$

Noise Example - Simplex View without Noise

Question: How does noise impact the space of possible segment value distributions?

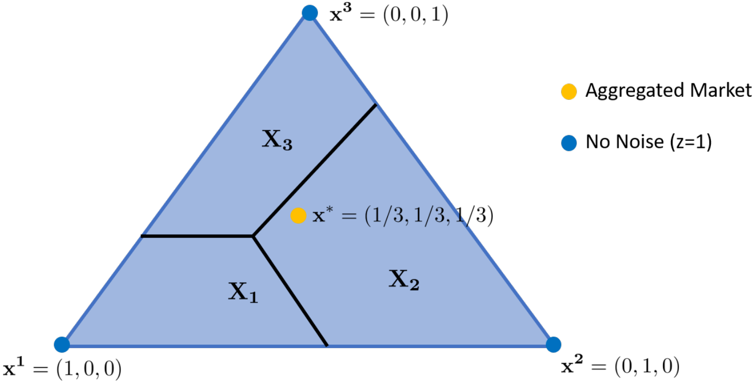


Figure 1: Simplex View without noise. Adapted from Cummings et al. (2020).

Example - Simplex View with Weak Noise

Question: How does noise impact the space of possible segment value distributions?

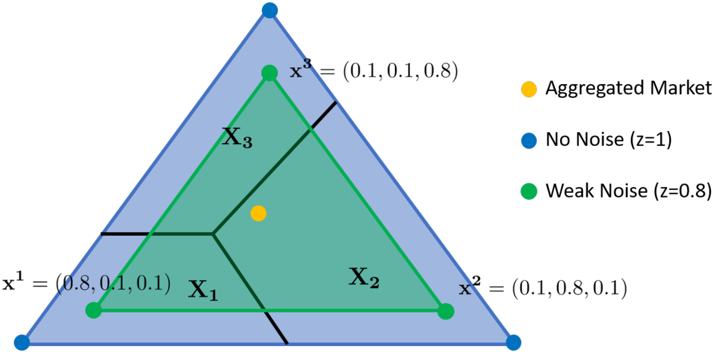


Figure 2: Simplex View with weak noise. Adapted from Cummings et al. (2020).

Noise Example - Simplex View with Strong Noise

Question: How does noise impact the space of possible segment value distributions?

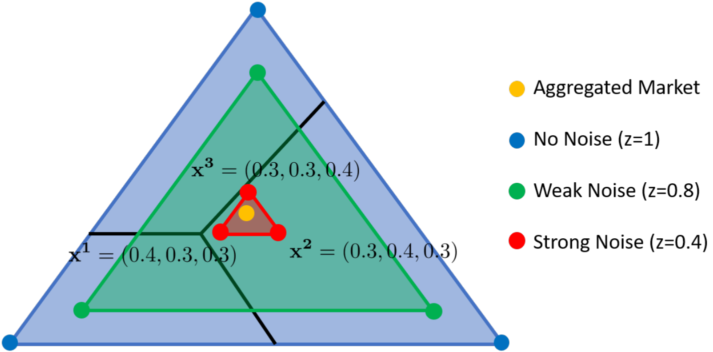
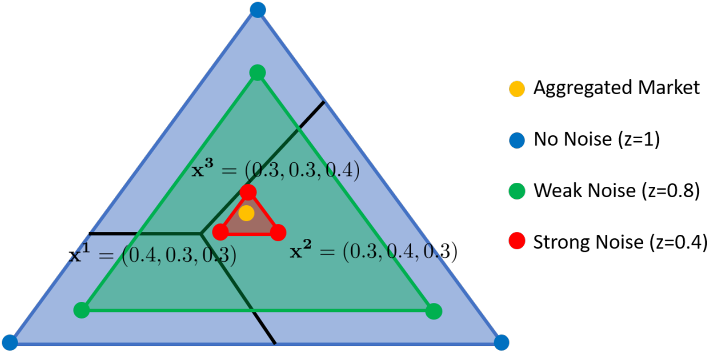


Figure 3: Simplex View with weak noise. Adapted from Cummings et al. (2020).

Noise Example - Simplex View with Strong Noise

Question: How does noise impact the space of possible segment value distributions?



Answer: Noise reduces the space of possible value distributions for segments.

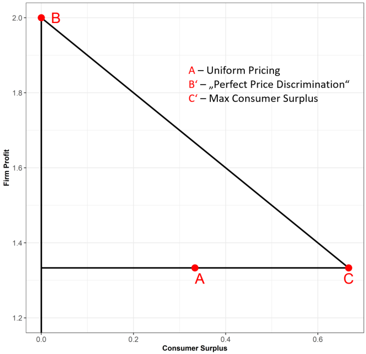
Noise Example - Steps

Question: Which welfare outcomes can be achieved with noise?

1. Generate many possible segmentations.
2. Calculate monopoly prices for segments of all segmentations.
3. Evaluate welfare outcomes of all segmentations.

Noise Example - Results Bergemann

Question: Which welfare outcomes can be achieved with noise?

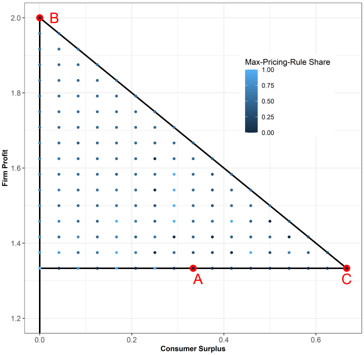


No Noise

Figure 4: Welfare outcome space. Adapted from Bergemann et al. (2015).

Noise Example - Results Simulation I

Question: Which welfare outcomes can be achieved with noise?

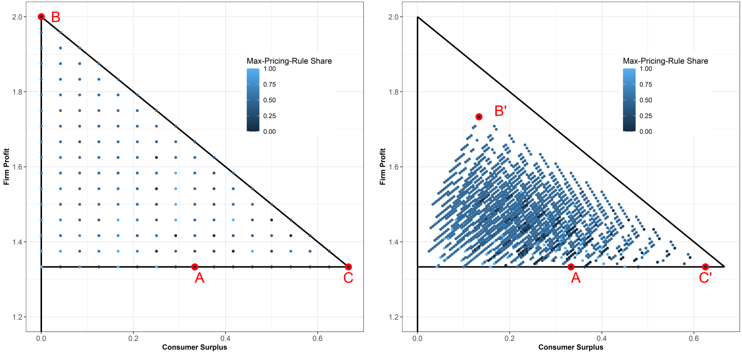


No Noise

Figure 5: Example welfare outcome space. Own representation.

Noise Example - Results Simulation II

Question: Which welfare outcomes can be achieved with noise?



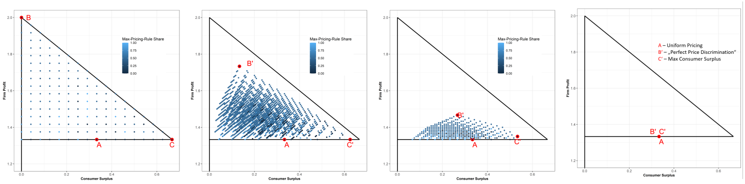
No Noise

Weak Noise

Figure 6: Example Welfare Outcome Space without noise and with weak noise. Own representation.

Noise Example - Results Simulation III

Question: Which welfare outcomes can be achieved with noise?



No Noise Weak Noise Medium Noise Strong Noise

Figure 7: Example welfare outcome space without noise and with weak, medium and strong noise. Own representation.

Noise - General Results

Question: Which welfare outcomes can be achieved with noise?

- **Smaller space of welfare outcomes:** A noisy signal of buyer valuations reduces the space of possible welfare outcomes, especially efficient trade results might no longer be possible.
- **Seller cannot be worse off:** Seller revenue cannot fall below uniform monopoly revenue as in Bergemann et al. (2015). (But: Seller can be worse off compared to no noise case with segmentation.)
- **Noise function:** The space of possible welfare outcomes depends on the noise function.

Bayesian Model

Bayesian Model

Agents - Model with Intermediary

Buyer	⟺	Intermediary	⟺	Monopoly Seller

Buyer

- Buyer types with different value distr.
- Decide on purchase (myopic)

Intermediary

- Observes buyer type
- **Sets segmentation** of types

Monopoly Seller

- Observes segmentation of types
- Sets monopoly price for segments

Bayesian Model

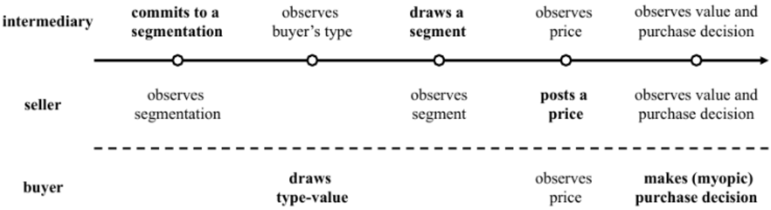


Figure 8: Timeline of a single round of the Bayesian Model. Cummings et al. (2020).

Bayesian Model - Intermediary

Goal of the intermediary is to maximize linear combination of monopoly revenue and consumer surplus:

$$E_{\sigma \sim \mathcal{S}}[\lambda \text{Rev}(\mathcal{F}(\sigma)) + (1-\lambda)\text{CS}(\mathcal{F}(\sigma))]$$

- $\lambda \in [0, 1]$.
- Rev: monopoly revenue.
- CS: consumer surplus.
- $\mathcal{F}(\sigma)$: posterior distribution on all values for each segment σ.

Goal of the Intermediary:

- Wants to create a surplus for the consumer, such that they return in the future.
- Wants to provide the seller with a positive revenue.

Bayesian Model - Main Results

Main result: Lemma 2.2.

There exists an optimal segmentation in such a way that for every area in the simplex, there is at most one segment.

Main result: Theorem 2.1.

We can find such an optimal segmentation in polynomial time by solving a linear program.

- Polynomial time: $O(n^k)$ with n being dependent on the input and $k \in \mathbb{N}$.
- Linear program: Optimization problem with linear objective function and linear constraints.

Sample Complexity and Bandit Model

Sample Complexity and Bandit Model

Sample Complexity Model

- **Intermediary**: Does not have access to the full $\mathcal{F}(t)$ (distribution over types), but only m samples per type and the test sample.
- **Buyer**: No changes.
- **Seller**: Different levels of information are possible. E.g. knows distribution exactly or knows only m samples per type.

Question:

How many samples are required?

Main result:

Given some properties about the distributions, a polynomial number of samples are sufficient to learn a segmentation that is very close to optimal.

Bandit Model - Setup

The intermediary interacts with the seller and the buyer repeatedly for m rounds for some positive integer m, with the buyer's type-value pair freshly sampled in each round.

- **Intermediary's Information**: Doesn't know value distributions -> must learn through interaction. Only observes purchase decision, but not value.
- **Buyer's Behavior**: Myopic, always buys if $v \geq p$.
- **Seller's Information**: Doesn't know value distribution, must learn such information.
- **Seller's Behavior**: Seller need to pick an optimal price, if there are enough information. Seller also need to explore to discover the value distributions.

Main challenge:

How can the intermediary encourage the seller to explore?

Bandit Model - Reinforcement Learning

- The seller faces a exploration-exploitation dilemma.
- Seller is ϵ_S-canonical learner: **exploits** in at most ϵ_S fraction of the rounds.
- The authors provide an algorithm that arrives at a good results.

Main Result: Theorem 4.1.

The algorithm gets at least Opt - $O(m^{-\frac{1}{19}}\text{poly}(V, T))$ per round on average, provided that the seller is an ϵ_S-canonical learner with $\epsilon_S \leq O(m^{-\frac{6}{19}}\text{poly}(V, T))$

Discussion

Discussion

Summary

Assumptions:

- Myopic buyers
- Monopolistic seller
- Intermediary who can design market segments to maximize any linear combination of consumer surplus and seller revenue

Three different models of information:

- Intermediary can construct a probability distribution of the buyer's value
- Intermediary only sees samples from this distribution
- Intermediary can only observe past purchasing decisions of the buyer, rather than exact value

Main Results

1. Noise reduces the space of possible welfare outcomes by segmentation.
2. For each model with partial information on buyer's value, an algorithm was presented to compute optimal or near optimal market segmentation.

Discussion

Advantages

- **Application:** Can be applied with data, concrete formulas on how to compute segmentation (e.g. in Bergemann: only possible outcomes characterization).
- **Flexible:** Towards models with different forms of partial information.
- **Use of Intermediary:** Can result in a better outcome for buyers without hurting the seller's revenue. Without intermediary intervention, the seller maximizes her own revenue.

Discussion

Limitations

- **In practice:** Setting with monopolist and intermediary is difficult to find in reality.
- **Interest of Intermediary:** Intermediaries might have own interests (max own revenue e.g. ads, fees) in addition to maximizing consumer surplus (except governmental intermediaries).
- **Limited impact of intermediary:** Intermediary draws a segment and shows it to the seller. After doing this, the intermediary does not have any control over the seller and the seller only maximizes her utility.
- **Buyers behavior:** Buyers could potentially be forward-looking and strategize and not myopic.

Future Research

Competitive Market

- Only considering a monopolistic setting.
- Sellers in many marketplaces are in a competitive setting (online marketplaces).
- Sellers can exert some pricing power with **differentiated products** (deadweight loss still incurs).

Question

Can this theory of price discrimination be extended to competitive market with differentiated products?

Future Work

Strategic Buyers
Only considering myopic buyers.

New Setting:

- Seller use past buyer behavior to decide future prices.
- Buyer has repeated interactions with such a seller.
- Buyer may be incentivized to strategize.

Question

What results would we get applying the model to strategic buyers?

References

1. Cummings, Rachel, et al. "Algorithmic Price Discrimination". 31st Annual ACM-SIAM Symposium on Discrete Algorithms (SODA 2020), Salt Lake City, UT, USA, 5-8 January 2020.
2. Bergemann, Dirk, et al. "The Limits of Price Discrimination." The American Economic Review, vol. 105, no. 3, 2015
3. R-Code on our Github (main.R, functions.R), November 2020