Problem 4.39 Using Mathematica.

A drug (D) is obtained in a three-stage extraction from the leaves of a tropical plant. About 1000 kg of leaf is required to produce 1 kg of the drug. The extraction solvent (S) is a mixture containing 16.5 wt% ethanol (E) and the balance water (W). The following process is carried out to extract the drug and recover the solvent.

1. 3300 kg of S and 620 kg of leaf are charged to a mixer and stirred for several hours, during which a portion of the drug contained in the leaf goes into solution. The contents of the mixer are then discharged through a filter. The liquid filtrate, which carries over roughly 1% of the leaf fed to the mixer, is pumped to a holding tank, and the solid cake (spent leaf and entrained liquid) is sent to a second mixer. The entrained liquid has the same composition as the filtrate and a mass equal to 15% of the mass of liquid charged to the mixer. The extracted drug has a negligible effect on t he total mass and volume of the spent leaf and the filtrate.

2. The second mixer is charged with the spent leaf from the first mixer and with the filtrate from the previous batch in the third mixer. The leaf is extracted for several more hours, and the contents of the mixer are then discharged to a second filter. The filtrate, which contains 1% of the leaf fed to the second mixer, is pumped to the same holding tank that received the filtrate from the first mixer, and the solid cake—spent leaf and entrained liquid—is sent to the third mixer. The entrained liquid is 15% of the mass of liquid charged to the mixer.

3. The third mixer is charged with the spent leaf from the second mixer and with 2720 kg of solvent S. The mixer contents are filtered; the filtrate, which contains 1% of the leaf fed to the third mixer, is recycled to the second mixer; and the solid cake is discarded. As before, the mass of the entrained liquid in the solid cake is 15% of the mass of the liquid charged to the mixer.

4. The contents of the filtrate holding tank are filtered to remove the carried-over spent leaf, and the wet cake is pressed to recover the entrained liquid. A negligible amount of liquid is lost in this step. The filtrate, which contains D, E, and W, is pumped to an extraction unit (another mixer).

5. In the extraction unit, the alcohol-water-drug solution is contacted with another solvent (F), which is almost but not completely immiscible with ethanol and water. Essentially all of the drug (D) is extracted into the second solvent, from which it is eventually separated by a process of no concern in this problem. Some ethanol but no water is also contained in the extract. The solution from which the drug has been extracted (the raffinate) contains 13.0 wt% E, 1.5% F, 85.5% W. It is fed to a steam stripper for recovery of the ethanol.

6. The feeds to the stripping column are the solution just described and steam. The two streams are fed in a ration such that the overhead product steam from the column contains 20.0 wt% E and 2.6% F, and the bottom product steam contains 1.3 wt% E and the balance W.

Draw and label a flowchart of the process, taking as a basis one batch of leaf processed. Then calculate

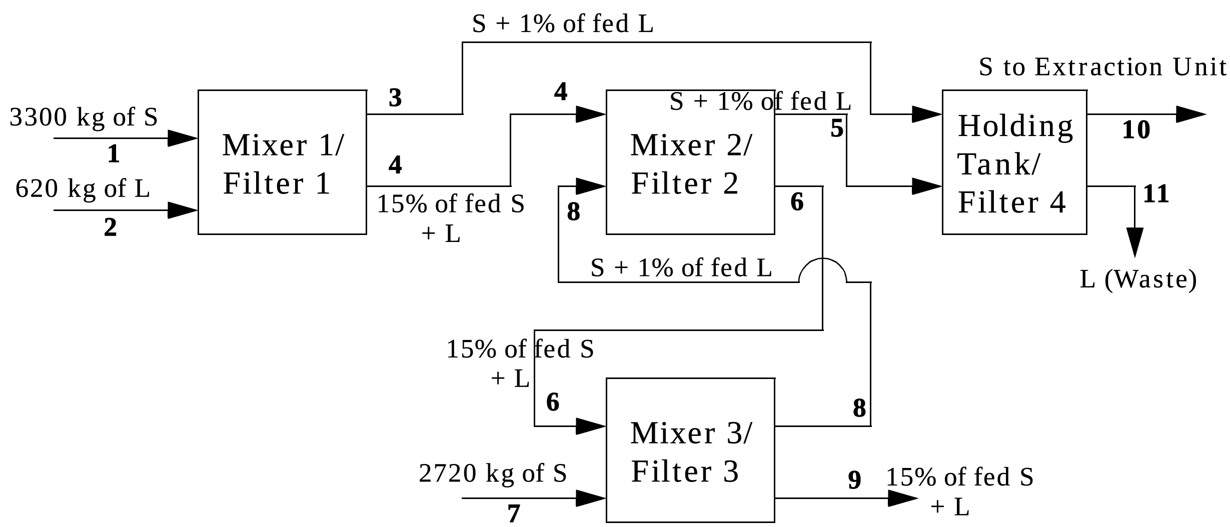
(a) The masses of the components of the filtrate holding tank.
(b) The masses of the components D and E of the extract stream from the extraction unit.
(c) The mass of steam fed to the stripper, and the masses of the stripper overhead and bottoms products

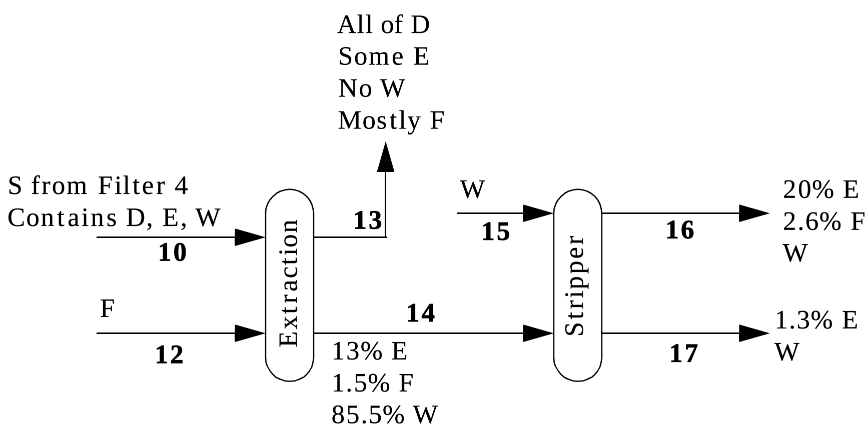
Solution:

Translation of this problem into a flowsheet and mass balances is the toughest part of the problem

First we draw the flowsheet and label the streams.

Flowsheet Drawing





From the flowsheet we can see that there is one recycle loop (Mixer 2 to Mixer 3 and back). The problem can also be separated into two problems: The Mixer/Filters, and the Extraction/Stripper.

Mathematica has two sorts of objects that resemble equations. The first is an assignment statement. For example

In[1]= m3L=5
Out[1]= 5

This statement says to assign the value of *5* to the variable *m3L*.

The second is an equation

In[1]=m3L= =5
Out[1]= m3L= =5

This statement is a conditional or logical relationship between *m3L* and *5*. An equation can be solved for an unknown using either the *Solve* or *FindRoot* command.

For the first problem (the Mixer/Filters) we will use equations exclusively, and them solve them. For the second problem (the Extraction/Stripper) we use assignment statements exclusively. Which is better? When your equations present themselves so that the only unknown in an equation is by itself on one side use assignment statements. In other cases use equations. They can of course be mixed and matched.

To start we perform a mass balance for S and L around each Mixer/Filter

Mixer 1

$$m_{3_L} = 0.01(m_{1_L} + m_{2_L})$$
$$m_{4_L} = (1 - 0.01)(m_{1_L} + m_{2_L})$$
$$m_{4_S} = 0.15(m_{1_S} + m_{2_S})$$
$$m_{3_S} = (1 - 0.15)(m_{1_S} + m_{2_S})$$

In *Mathematica*-ese:

```
(* A Mathematica solution
   to Problem 4_39 in Felder & Rousseau *)
(* Part 1 - The mixer/filters and
  holding tank. This section uses only equations,
 not assignment statements,
 and solves them all at once. *)
(* Mass Balance around Mixer 1 *)
eq[1] = m3L == 0.01 (m1L + m2L)
```

Actually we have used assignment statements to assign equations to variable names, in this case an array called "*eq*". These assignments will keep us from having to retype the equations later.

Now a mass balance for Mixer 2

$$m_{5_L} = 0.01(m_{4_L} + m_{8_L})$$
$$m_{6_L} = (1 - 0.01)(m_{4_L} + m_{8_L})$$
$$m_{6_S} = 0.15(m_{4_S} + m_{8_S})$$
$$m_{5_S} = (1 - 0.15)(m_{4_S} + m_{8_S})$$

In *Mathematica*-ese:

```
(* Mass Balance around Mixer 2 *)
eq[5] = m5L == 0.01 (m4L + m8L)
m5L == 0.01 (m4L + m8L)
eq[6] = m6L == (1  0.01) (m4L + m8L)
m6L == 0.99 (m4L + m8L)
eq[7] = m6S == 0.15 (m4S + m8S)
m6S == 0.15 (m4S + m8S)
eq[8] = m5S == (1  0.15) (m4S + m8S)
m5S == 0.85 (m4S + m8S)
```

Mixer 3

$$m_{8_L} = 0.01(m_{6_L} + m_{7_L})$$
$$m_{9_L} = (1 - 0.01)(m_{6_L} + m_{7_L})$$
$$m_{9_S} = 0.15(m_{6_S} + m_{7_S})$$
$$m_{8_S} = (1 - 0.15)(m_{6_S} + m_{7_S})$$

In *Mathematica*-ese:

```
(* Mass Balance around Mixer 3 *)
eq[9] = m8L == 0.01 (m6L + m7L)
m8L == 0.01 (m6L + m7L)
eq[10] = m9L == (1 - 0.01) (m6L + m7L)
m9L == 0.99 (m6L + m7L)
eq[11] = m9S == 0.15 (m6S + m7S)
m9S == 0.15 (m6S + m7S)
eq[12] = m8S == (1  .15) (m6S + m7S)
m8S == 0.85 (m6S + m7S)
```

Holding Tank

$$m_{10_L} = 0$$
$$m_{11_L} = m_{3_L} + m_{5_L}$$

$m_{11_S} = 0$

$m_{10_S} = m_{3_S} + m_{5_S}$

In *Mathematica*-ese:

```
(* Mass Balance around Holding Tank *)
eq[13] = m10L == 0
```

```
General::spell1 : Possible spelling error: new symbol
   name "m10L" is similar to existing symbol "m1L".
```

```
m10L == 0
```

```
eq[14] = m11L == m3L + m5L
```

```
General::spell1 : Possible spelling error: new symbol
   name "m11L" is similar to existing symbol "m1L".
```

```
m11L == m3L + m5L
```

```
eq[15] = m11S == 0
```

```
General::spell :
 Possible spelling error: new symbol name "m11S" is
   similar to existing symbols {m11L, m1S}.
```

```
m11S == 0
```

```
eq[16] = m10S == m3S  m5S
```

```
General::spell :
 Possible spelling error: new symbol name "m10S" is
   similar to existing symbols {m10L, m1S}.
```

```
m10S == m3S + m5S
```

In addition, we have three input streams specified.

$m_{1_S} = 3300$

$m_{1_L} = 0$

$m_{2_S} = 0$

$m_{2_L} = 620$

$m_{7_S} = 2720$

$m_{7_L} = 0$

In *Mathematica*-ese:

```
(* Feed Streams *)
eq[17] = m1S == 3300
```

```
m1S == 3300
```

```
eq[18] = m1L == 0
```

```
m1L
```

```
eq[19] = m2S == 0
```

```
m2S == 0
```

```
eq[20] = m2L == 620
```

```
m2L == 620
```

```
eq[21] = m7S == 2720
```

```
m7S == 2720
```

```
eq[22] = m7L == 0
```

```
m7L == 0
```

We now do some bookkeeping to make our lives a little easier. We'll create a list of equations to solve (We only have 22 to this point!). And a list of variables to solve for.

```
(* Set Up Equations to solve *)
EqSet = Table[eq[n], {n, 1, 22}]
```

```
{m3L == 0.01 (m1L + m2L), m4L == 0.99 (m1L + m2L),
 m4S == 0.15 (m1S + m2S), m3S == 0.85 (m1S + m2S),
 m5L == 0.01 (m4L + m8L), m6L == 0.99 (m4L + m8L),
 m6S == 0.15 (m4S + m8S), m5S == 0.85 (m4S + m8S),
 m8L == 0.01 (m6L + m7L), m9L == 0.99 (m6L + m7L),
 m9S == 0.15 (m6S + m7S), m8S == 0.85 (m6S + m7S), m10L == 0,
 m11L == m3L + m5L, m11S == 0, m10S == m3S + m5S, m1S == 3300,
 m1L == 0, m2S == 0, m2L == 620, m7S == 2720, m7L == 0}
```

```
(* Set Up Variables to solve for *)
SolSet = {m1L, m1S, m2L, m2S, m3L, m3S, m4L, m4S,
  m5L, m5S, m6L, m6S, m7L, m7S, m8L, m8S, m9L, m9S,
  m10L, m10S, m11L, m11S}
```

```
{m1L, m1S, m2L, m2S, m3L, m3S, m4L, m4S, m5L, m5S, m6L,
 m6S, m7L, m7S, m8L, m8S, m9L, m9S, m10L, m10S, m11L, m11S}
```

Now if we have enough equations and unknowns, and the equations are consistent (remember linear?) we should be able to solve them all.

```
(* Solution *)
SolnPart1 = Solve[EqSet, SolSet]
```

```
{{m1L → 0., m1S → 3300., m2L → 620., m2S → 0.,
  m3L → 6.2, m3S → 2805., m4L → 613.8, m4S → 495.,
  m5L → 6.19937, m5S → 2734.61, m6L → 613.738,
  m6S → 482.579, m7L → 0., m7S → 2720., m8L → 6.13738,
  m8S → 2722.19, m9L → 607.601, m9S → 480.387,
  m10L → 0., m10S → 5539.61, m11L → 12.3994, m11S → 0.}}
```

We obviously knew the values for some of these, like m_{1_L} and m_{1_S}. However, we now have them all together.

For the second part we'll use assignment statements. We'll have to do more work ourselves. In particular, our equations will have to be arranged so that there is only one unknown at a time.

To solve the extractor and the stripper we first calculate an overall mass balance for D.

$$m_{10_\mathrm{D}} = \frac{1}{1000} m_{2_\mathrm{L}}$$

In *Mathematica*-ese:

```
(* Part 2 - The Extractor/Stripper. This section uses
   only assignment statements. Unlike equations,
 you have to use assignment statements
  in the proper order to avoid recursion *)
m10D = 1 / 1000 m2L /. SolnPart1[[1]]
```

```
General::spell :
 Possible spelling error: new symbol name "m10D" is
   similar to existing symbols {m10L, m10S}.
```

```
0.62
```

We then calculate the the feed (Stream 10) from the solvent composition and the answer to part 1.

$$m_{10_\mathrm{E}} = 0.165 m_{10}$$

$$m_{10_\mathrm{W}} = (1 - 0.165) m_{10}$$

In *Mathematica*-ese:

```
m10 = (m10S + m10L) /. SolnPart1[[1]]
```

```
5539.61
```

```
m10E = 0.165 m10
```

```
General::spell :
 Possible spelling error: new symbol name "m10E" is
   similar to existing symbols {m10, m10D, m10L, m10S}.
```

```
914.036
```

```
m10W =
```

Now from the specified flows and split fractions in Streams 13 and 14

$$m_{14_D} = 0$$
$$m_{13_D} = m_{10_D}$$
$$m_{13_F} = 0$$
$$m_{13_W} = 0$$
$$m_{14_W} = m_{10_W}$$

In *Mathematica*-ese:

```
m14D = 0
```

```
0
```

```
m13D = m10D
```

```
0.62
```

```
m13F = 0
```

```
General::spell1 : Possible spelling error: new symbol
   name "m13F" is similar to existing symbol "m13D".
```

```
0
```

```
m13W = 0
```

```
General::spell :
 Possible spelling error: new symbol name "m13W" is
   similar to existing symbols {m13D, m13F}.
```

```
0
```

```
m14W = m10W
```

```
General::spell1 : Possible spelling error: new symbol
   name "m14W" is similar to existing symbol "m14D".
```

```
4625.58
```

Now from the specified mole fractions in Stream 14 we can calculate

$$m_{14_F} = \frac{1.5}{85.5} m_{14_W}$$
$$m_{14_E} = \frac{13}{85.5} m_{14_W}$$

In *Mathematica*-ese:

```
m14F = 0.015 / 0.855 m14W
```

```
General::spell :
 Possible spelling error: new symbol name "m14F" is
   similar to existing symbols {m14D, m14W}.
```

```
81.1505
```

```
m14E = 13 / 85.5 m14W
```

```
General::spell :
 Possible spelling error: new symbol name "m14E" is
   similar to existing symbols {m14D, m14F, m14W}.
703.304
```

Finally a mass balance on E around the extractor yields

$$m_{13_E} = m_{10_E} - m_{14_E}$$

In *Mathematica*-ese:

```
m13E = m10E - m14E
General::spell :
 Possible spelling error: new symbol name "m13E" is
   similar to existing symbols {m13D, m13F, m13W}.
210.732
```

On the Stripper we first perform a mass balance on F.

$$m_{16_F} = m_{14_F}$$

In *Mathematica*-ese:

```
m16F = m14F
81.1505
```

From the given compositions of Stream 16 we have

$$m_{16} = \frac{m_{16_F}}{0.026}$$

$$m_{16_W} = (1 - 0.2 - 0.026) m_{16}$$

$$m_{16_E} = 0.2 m_{16}$$

In *Mathematica*-ese:

```
m16 = m16F / 0.026
3121.17
m16W = (1 - .2 - .026) m16
General::spell :
 Possible spelling error: new symbol name "m16W" is
   similar to existing symbols {m16, m16F}.
2415.79
m16E = 0.2 m16
```

```
General::spell :
 Possible spelling error: new symbol name "m16E" is
   similar to existing symbols {m16, m16F, m16W}.
624.234
```

Next a mass balance on E about the Stripper yields

$$m_{17_E} = m_{14_E} - m_{16_E} = m_{14_E} - 0.2 m_{16}$$

In *Mathematica*-ese:

```
m17E = m14E  m16E
79.0697
```

Then from the definition of composition in Stream 17

$$m_{17} = \frac{m_{17_E}}{0.013}$$

$$m_{17_W} = (1 - 0.013) m_{17}$$

In *Mathematica*-ese:

```
m17 = m17E / 0.013
6082.28
m17W = (1 - .013) m17
General::spell :
 Possible spelling error: new symbol name "m17W" is
   similar to existing symbols {m17, m17E}.
6003.21
```

And finally a W balance around the Stripper yields

$$m_{15_W} = m_{16_W} + m_{17_W} - m_{14_W}$$

$$m_{15} = m_{15_W}$$

In *Mathematica*-ese:

```
m15W = m16W + m17W - m14W
3793.42
m15 = m15W    (* Answer for Part c *)
3793.42
m16    (* Answer for Part c *)
3121.17
m17    (* Answer for Part c *)
6082.28
```