A — The formal logical content
B — The intuitive background
C — The applications

Def

If sample space for an experiment is a set S with the property that each physical outcome of the experiment corresponds to exactly one element of S. An element of S is called samplepoint.

Ex

Distribution of two balls in three cells.

1. {ab | - | -}
2. {- | ab | -}
3. {- | - | ab}
4. {a | b | -}
5. {a | - | b}
6. {b | a | -}
7. {- | a | b}
8. {b | - | a}
9. {- | b | a}

Sample Space [S]/[Ω]

1 of 9 sample points

A = {1,2,3} – one cell is mutually occupied.

B – The first cell is not empty. => {1, 4, 5, 6, 8}

Every cell is occupied. <=> impossible event

C – "a" is in the first cell => {1, 4, 5}

Def

Any subset of a sample space is called an event. The empty set $\varnothing$ is called "the impossible set"

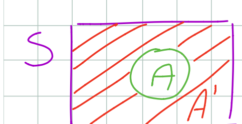
S is called the certain event.

Def

The event consisting of all points not contained in event A will be called the complementary event of A and will be denoted $[A']/[\bar{A}]$. In particular $S' \Leftrightarrow \varnothing$

Ex

B' – The first cell is empty. {2, 3, 7, 9}

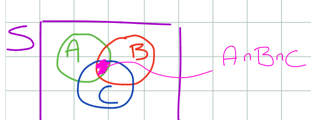


Def

The aggregate of the samplepoints which belong to all the given sets, ABC, will be denoted by $A \cap B \cap C$ and called the intersection or simultanious realization.

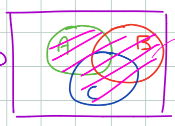
Ex

$A \cap B = \{1\}$



Def
The aggregate of the samplepoints which belong to at least one of the given sets, A B C, will be denoted $A \cup B \cup C$ and called the union.

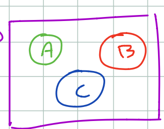
Ex
$A \cup B = \{1,2,3,4,5,6,8\}$

S


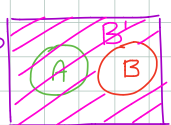
$A \cup B \cup C$

Def
The event A, B, C are mutually exclusive if no two have a point in comon.

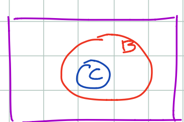
Ex
B and B' are mutually exclusive $B \cap B' = \varnothing$

S


Def
The symbols $A \subset B$ and $B \supset A$ signify that every point of A is contained in B. "A implies B", "B is implied by A".

Ex
S


$A \subset B'$



$C \subset B$
$B \not\subset C$

Ex - Coin toss, 3 random tosses.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

D - Two or more H.

E - Just one T.

Def
$$P(A) = \frac{\text{number of ways A can occur}}{\text{number of ways the experiment can proceed}}$$

$P(D) = \frac{4}{8} = \frac{1}{2}$

$P(E) = \frac{3}{8}$

Def
Relative frequency approximation, $P(A) = \frac{\text{number of times event A occured}}{\text{number of times experiment was run}}$

Axioms of probability

1. Let S denote a sample space. $P(S)=1$
2. $P(A)\geq 0$, for every event A.
3. Let $A_1, A_2, A_3 \dots$ be a finite or infinite collection of mutually exclusive events. Then $P(A_1 \cup A_2 \cup A_3 \dots) = P(\bigcup_{i=1}^{\infty} A_i) = P(A_1)+P(A_2)+\dots$
$= \sum_{i=1}^{\infty} P(A_i)$

Theorems

$P(A') = 1 - P(A)$

Proof

$P(A')+P(A) = \{ax\ 3\} = P(A \cup A') = P(S) = \{ax\ 1\} = 1$

$P(\emptyset) = 0$

Proof

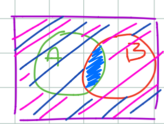
$P(S) = \{S \cup \emptyset = S\} = P(S \cup \emptyset) = \{S \cap \emptyset = 0\} + \{ax\ 3\} = P(S) + P(\emptyset)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof

$P(A) + P(B) = \{ax\ 3\} = P(A \cap B) + P(A \cap B') + P(B \cap A) + P(B \cap A') = P(A \cup B) + P(A \cup B)$

$A = (A \cap B) \cup (A \cap B')$



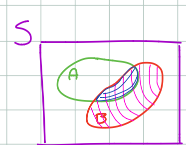
A'
B'
$A \cap B$

Def

Let A and B be events such that $P(B)>0$. The conditional probability of A given B denoted by $P(A|B)$ is defined by: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Ex

$P(A|B) = \frac{P(A \cap B)}{P(B)}$



The outcome will be in B.

$P(A|B)$

$P(B) = \frac{5}{9}$
$P(A \cap B) = \frac{1}{9}$

$\frac{\frac{1}{9}}{\frac{5}{9}} = \frac{9}{9 \cdot 5} = \frac{1}{5}$

$P(A) = \frac{3}{9} = \frac{1}{9}$