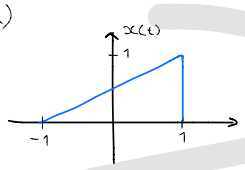
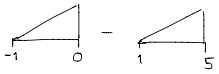
Tentamen 2015-08-26

a)

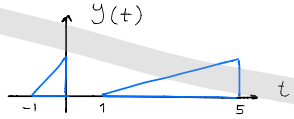


Gör en tydlig skiss av $y(t) = x(0.5(t+1)) - x(2t-3)$

$y(t) = x(0.5t+0.5) - x(2t-3) =$

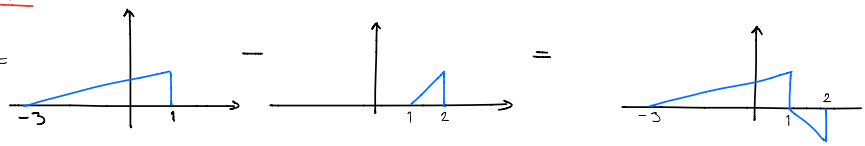


$=$



FACIT

$y(t) =$



(Gör tvärtom)

b)

$x[n] = \underbrace{\cos\left(\frac{9\pi n}{2}\right)}_{x_1[n]} + \underbrace{3\sin\left(\frac{6\pi n}{5} + \frac{\pi}{2}\right)}_{x_2[n]}$

$x_1[n] = x_1[n+N_1]$ $\quad n, N_1, N_2 \in \mathbb{Z}$ $\quad$ Minsta gemensamma period $N = N_1 N_2$

$x_2[n] = x_2[n+N_2]$

$x_1[n+N_1] = \cos\left(\frac{9\pi}{2}(n+N_1)\right) = \cos\left(\frac{9\pi n}{2} + \frac{9\pi N_1}{2}\right) = \left\{\frac{9\pi}{2} N_1 = 2\pi k_1\right\} \Rightarrow N_1 = \frac{4\pi k_1}{9\pi} = \frac{4}{9} k_1 \Rightarrow N_1 = 4$

$x_2[n+N_2] = 3\sin\left(\frac{6\pi n}{5} + \frac{6\pi N_2}{5} + \frac{\pi}{2}\right) = \left\{\frac{6\pi N_2}{5} = 2\pi k_2\right\} \quad N_2 = \frac{10\pi k_2}{6\pi} = \frac{10}{6} k_2 \Rightarrow N_2 = 5$ $\Rightarrow N_1 N_2 = 20$

3

Stegsvaret: $y(t) = 0.1\left(2 - e^{-5t}(2\cos(10t) + \sin(10t))\right)u(t)$

$y(t) = 0.2\,u(t) - 0.2e^{-5t}\cos(10t)u(t) + 0.1e^{-5t}\sin(10t)u(t)$

$Y(s) = \frac{0.2}{s} - 0.2\left(\frac{s+5}{(s+5)^2+10^2}\right) + 0.1\left(\frac{10}{(s+5)^2+10^2}\right) = \frac{1}{10}\left(\frac{2}{s} - \frac{2s+10+10}{(s+5)^2+10^2}\right) = \frac{2}{10}\left(\frac{(s+5)^2+10^2-s^2-10s}{s((s+5)^2+10^2)}\right) = \frac{2}{10}\left(\frac{s^2+10s+25+10^2-s^2-10s}{s((s+5)^2+10^2)}\right) = \frac{2}{10}\left(\frac{125}{s((s+5)^2+10^2)}\right) = \frac{25}{s((s+5)^2+10^2)}$

$x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{25}{s((s+5)^2+10^2)}}{\frac{1}{s}} = \frac{s}{1}\frac{25}{s((s+5)^2+10^2)} = \frac{25}{(s+5)^2+10^2} = 2.5\left(\frac{10}{(s+5)^2+10^2}\right)$

$h(t) = \mathcal{L}^{-1}\{H(s)\} = 2.5\,e^{-5t}\sin(10t)\,u(t)$

4)

$X[k] = \sum_{n=0}^{7} x[n]\,e^{-j\frac{2\pi}{8}kn}$

$T_s = 625\cdot10^{-6} \Rightarrow f_s = 1600$ Hz

$\frac{f}{f_s} = \frac{k}{N} \Rightarrow \frac{400}{1600} = \frac{k}{8} \Rightarrow k = 2$

$X[2] = \sum_{n=0}^{7} x[n]e^{-j\frac{2\pi}{8}2n} = \sum_{n=0}^{7} x[n]e^{-j\frac{\pi}{2}n} = e^{-j\frac{\pi}{2}} + e^{-j\pi} + e^{-j\frac{5\pi}{2}} + e^{-j3\pi} =$

$\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) + \cos(\pi) - j\sin(\pi) + \cos\left(\frac{5\pi}{2}\right) - j\sin\left(\frac{5\pi}{2}\right) + \cos(3\pi) - j\sin(3\pi) =$

$0 - j - 1 + 0 + 0 - j - 1 - 0 = -2 - 2j$