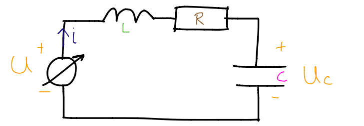
Ex Elektriskt system (forts)



Sökt

U_c

Lösning

- Strukturering
  Inför lämpliga stödvariabler.

- Basekvationer
  Balansekvationer
  Kirchoff
  $U - U_L - U_R - U_c = 0$
  Konstitutiva Samband
  $U_L = L \cdot \frac{di}{dt},\ U_R = R \cdot i,\ i = C \cdot \frac{dU_c}{dt},\quad q = \int_0^t i \, d\tau = \int_0^t C \cdot \frac{dU_c}{dt} d\tau = C \cdot U_c$
  $\frac{dq}{dt} = i,\ \frac{d^2 q}{dt^2} = \frac{di}{dt}$

- Formulera modell

$$U - L\frac{d^2q}{dt^2} - R\frac{dq}{dt} - \frac{1}{C}q = 0$$

$$\Leftrightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = U$$

$$\Leftrightarrow \ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{1}{L}\cdot U$$

$$\mathcal{L} \Rightarrow s^2 Q(s) + \frac{R}{L} s Q(s) + \frac{1}{LC} Q(s) = \frac{1}{L} U(s),\quad Q(s) = C\,U_c(s) \Rightarrow \frac{U_c(s)}{U(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = G(s)$$

Minns det mekaniska systemet: $\frac{Y(s)}{F(s)} = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}$, $\quad R \to b,\ L \to m,\ C \to \frac{1}{k}$

Tillståndsmodeller

System av 1a ordningens differentialekvationer

Ex Elektrisk krets

Mål: $\dot{x} = f(x, u)$ $\quad x$ – tillståndsvariabel, u – insignal

Välj tillståndsvariabler, ex U_c och i $\Rightarrow$

$$\frac{dU_c}{dt} = \frac{1}{C}\underbrace{i}_{\text{tillståndsvariabel}}$$

$$\frac{di}{dt} = \frac{1}{L}U_L = \frac{1}{L}(U - U_c - U_R) = \frac{1}{L}(\underbrace{U}_{\text{insignal}} - \underbrace{U_c - Ri}_{\text{tillståndsvariabel}})$$

Nya variabler: q och i $\Rightarrow$ $\frac{dq}{dt} = i$

$$\frac{di}{dt} = \frac{1}{L}U_L = \frac{1}{L}(U - U_R - U_c) = \frac{1}{L}(U - Ri - \frac{1}{C}\cdot q)$$

$\dot{x} = f(x, u)$
$y = g(x, u)$

Om tv: U_c & $i \Rightarrow y = U_c$
q & $i \Rightarrow y = \frac{1}{C}q$

Intern modell

$u \to x \to y$

$u \to y$

extern modell

"Hur beror de av varandra?"

Linjära system: $\dot{x} = Ax + B\cdot u$
$\quad | = [\ ]\,| + |\,\cdot$

$$\Rightarrow \left(\dot{x} = \right)\begin{bmatrix} \frac{dU_c}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} U_c \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}\cdot u$$

$y = C\,x + D\,u$
$\cdot = \text{—}\,| + \cdots$

$$\Rightarrow y = [1 \ \ 0]\begin{bmatrix} U_c \\ i \end{bmatrix} + [0]\,u$$

Intern modell → Extern

$x = Ax + Bu$
$y = Cx + Du$ $\xleftrightarrow{?}$ $G(s)$

Laplace! ⇒ $sX(s) = AX(s) + BU(s) \Rightarrow (sI - A)X(s) = BU(s) \Rightarrow X(s) = (sI-A)^{-1}BU(s)$
$Y(s) = CX(s) + DU(s)$

$$Y(s) = C(sI-A)^{-1}BU(s) + DU(s) = \left(C(sI-A)^{-1}B + D\right)U(s) = G(s)U(s)$$

Ex Elektriskt system

$$x = \underbrace{\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} u \qquad y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} x + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

$$G(s) = C(sI-A)^{-1}B + D = \begin{bmatrix} 1 & 0 \end{bmatrix}\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}\right]^{-1}\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + 0 = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} s & -\frac{1}{C} \\ \frac{1}{L} & s+\frac{R}{L} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} =$$

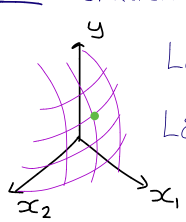
$$\begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s(s+\frac{R}{L}) + \frac{1}{LC}} \begin{bmatrix} s+\frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & s \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} = \frac{\begin{bmatrix} s+\frac{R}{L} & \frac{1}{C} \end{bmatrix}\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}{s(s+\frac{R}{L}) + \frac{1}{LC}} = \frac{\frac{1}{LC}}{s(s+\frac{R}{L}) + \frac{1}{LC}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Notera: Systemets poler bestäms av $\det(sI - A)$ = egenvärdena till A.

Linjärisering

$x = f(x, u)$
$y = g(x, u)$ ⟶ $\begin{cases} x = Ax + Bu \\ y = Cx + Du \end{cases}$

Ex Funktion av flera variabler (2 st)



Lås $x_2 = x_2^\circ$ ⟶ "skiva", $y = f(x_1, x_2^\circ) \approx$ taylorutv $\approx f(x_1^\circ, x_2^\circ) + \left.\frac{\partial f}{\partial x_1}\right|_{x_1^\circ, x_2^\circ} \Delta x_1$

Lås $x_1 = x_1^\circ$ ⟶ "skiva", $y = f(x_1^\circ, x_2) \approx$ taylorutv $\approx f(x_1^\circ, x_2^\circ) + \left.\frac{\partial f}{\partial x_2}\right|_{x_1^\circ, x_2^\circ} \Delta x_2$