## 5.3 13

Riemannsumma: $\sum_{k=0}^{n-1} f(c_k)(x_{k+1}-x_k) \qquad x_k \leqslant c_k \leqslant x_{k+1}$

$c_k = \frac{\pi k}{n} = (x_{k+1})$

$x_{k+1} - x_k = \frac{\pi}{n}$

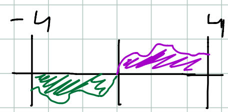
$\sum_{i=1}^{n} \frac{\pi}{n} \cdot \sin \frac{\pi i}{n} \longrightarrow \int \sin(x)\,dx$

$i=1, \quad c_1 = \frac{\pi}{n} \to 0$ om $n \to \infty$

$i=n, \quad c_n = \frac{\pi n}{n} = \pi$

## 5.4 13

$\int_{-4}^{4} e^x - e^{-x}\,dx$, $f(x) = e^x - e^{-x}$ är udda. $f(-x) = e^{-x} - e^{x} = -(e^x - e^{-x}) = f(-x)$




Areorna tar ut varandra.

## 5.5 41

$\frac{d}{dx} \int_{x^2}^{0} \frac{\sin(t)}{t}\,dt$

Låt $F(x)$ vara s.a. $F'(x) = \frac{\sin(x)}{x}$, t.ex. $\int_0^x \frac{\sin(t)}{t}\,dt$

$\frac{d}{dx} \int_{x^2}^{0} \frac{\sin(t)}{t}\,dt = \frac{d}{dx}\left(F(0) - F(x^2)\right) = 0 - F'(x^2)\cdot 2x = \frac{-\sin(x^2)}{x^2}\cdot 2x = \frac{-2\sin(x^2)}{x}$

### 49

$\int_{-1}^{1} \frac{1}{x^2}\,dx = \left[\frac{-1}{x}\right]_{-1}^{1} = \frac{-1}{1} - \left(\frac{-1}{-1}\right) = -1 - 1 = -2$, uppenbarligen fel ty $\frac{1}{x^2} > 0$. Felet är att inter len är generaliserad (typ 2)

$\int_{-1}^{1} \frac{1}{x^2}\,dx = \lim_{a \to 0^-} \int_{-1}^{a} \frac{1}{x^2}\,dx + \lim_{b \to 0^+} \int_{b}^{1} \frac{1}{x^2}\,dx = \lim_{a \to 0^-} \left[-\frac{1}{x}\right]_{-1}^{a} + \lim_{b \to 0^+} \left[-\frac{1}{x}\right]_{b}^{1} = \lim_{a \to 0^-} -\frac{1}{a} - 1 - 1 - \lim_{0^+} -\frac{1}{b} = +\infty - 1 - 1 + \infty = \infty - 2$

## 5.6 19

$\int \tan x \cdot \ln(\cos(x))\,dx$

$\int \tan(x) \cdot \ln(\cos(x))\,dx = \int \frac{\sin(x)}{\cos(x)}\, n(\ os\ x)\,dx = \left[\begin{array}{l} \cos x = u \\ \frac{du}{dx} = -\sin x \Rightarrow du\ -\sin x dx \end{array}\right] = -\int \frac{1}{u} \ln(u)\,du = \left[\begin{array}{l} \ln(u) = t \\ \frac{dt}{du} = \frac{1}{u} \Rightarrow dt = \frac{du}{u} \end{array}\right] = -\int t\,dt$

$= \frac{-t^2}{2} = \frac{-\ln^2(\cos x)}{2}$

### 23

$\int \sin^3 x \cdot \cos^5 x\,dx = \sin^2 x \cdot \sin x \cdot \cos^5 x\,dx = \int \sin x\,(1-\cos^2 x) \cdot \cos^5 x\,dx = \left[\begin{array}{l} \cos x = u \\ \frac{du}{dx} = -\sin x \Rightarrow du = -\sin x\,dx \end{array}\right] = -\int (1-u^2)\cdot u^5\,du = -\int u^5 - u^7\,du =$

$-\frac{u^6}{6} + \frac{u^8}{8} = \frac{-\cos^6 x}{6} + \frac{\cos^8 x}{8}$

eller:

$\int \sin^3 x \cdot \cos^4 x \cdot \cos x\,dx = \left[\begin{array}{l} \sin x = u \\ \frac{du}{dx} = \cos x \end{array}\right] = \int u^3 (1-u^2)^2\,du \ldots$

## 6.1 35

Pre ex: $\int \sin x \cdot \cos x\,dx = \int \frac{\sin 2x}{2}\,dx = \frac{-\cos 2x}{4}$

$\left.\begin{array}{l} \text{eller: } \left[\begin{array}{l} \cos x = u \\ \frac{du}{dx} = -\sin x \Rightarrow du = -\sin x\,dx \end{array}\right] = -\int u\,du = \frac{-u^2}{2} = \frac{-\cos^2 x}{2} \\ \text{eller: } \left[\begin{array}{l} \sin x = u \\ \frac{du}{dx} = \cos x \Rightarrow du = \cos x\,dx \end{array}\right] = \int u\,du = \frac{u^2}{2} = \frac{\sin^2 x}{2} \end{array}\right\} \quad \frac{\sin^2 x}{2} + \frac{\cos^2 x}{2} = \frac{1}{2}$ (varför C spelar roll)

eller, Part int: $I = \sin x \cdot \sin x - \int \cos x \cdot \sin x\,dx = \sin^2 x - I$, $\quad I = \sin^2 x - I$, $\ 2I = \sin^2 x$, $\ I = \frac{\sin^2 x}{2}$

## 35

$I_n = \int \frac{1}{(x^2+a^2)^n}\,dx$

Rationella funktioner => Partialbråk.

$\frac{1}{x-a} \Rightarrow \ln|x-a|$

$\frac{1}{(x-a)^n} \Rightarrow \frac{1}{(x-a)^{n-1}} \cdot \frac{1}{1-n}$

$\frac{x}{a^2+x^2} \Rightarrow \frac{1}{2}\ln(a^2+x^2)$

$\frac{x}{(a^2+x^2)^n} \Rightarrow \frac{1}{2}\cdot\frac{1}{n-1}\ln(a^2+x^2)^{n-1}$

$\frac{1}{a^2+x^2} \Rightarrow \arctan\left(\frac{x}{a}\right)\cdot\frac{1}{a}$

Specialfall $a=1$. Det allmänna fallet klaras sen med variabelsub: $\frac{x}{a}=t$

$$I_{n-1} = \int \frac{1}{(x^2+1)^{n-1}}\,dx = \begin{bmatrix}\text{Part.}\\ \text{int.}\end{bmatrix} = \frac{x}{(x^2+1)^{n-1}} - \int \frac{x(1-n)\cdot 2x}{(x^2+1)^n}\,dx$$

$$= \frac{x}{(x^2+1)^{n-1}} + 2(n-1)\int \frac{x^2+1-1}{(x^2+1)^n}\,dx = \frac{x}{(x^2+1)^{n-1}} + 2(n-1)\int\frac{1}{(x^2+1)^{n-1}}\,dx - \int \frac{1}{(x^2+1)^n}\,dx$$

$$I_{n-1} = \frac{x}{(x^2+1)^{n-1}} + 2(n-1)\cdot I_{n-1} - 2(n-1)\cdot I_n$$

$$2(n-1) = \frac{x}{(x^2+1)^{n-1}} + (2n-3)I_{n-1}$$

$$I_n = \frac{x}{(2n-2)(x^2+1)^{n-1}} + \frac{2n-3}{2n-2}\cdot I_{n-1}$$

### Bestäm $I_3$

$I_1 = \int \frac{1}{1+x^2}\,dx = \arctan(x)$

$I_2 = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$

$I_3 = \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{x}{2(x^2+1)} + \frac{1}{2}\arctan x\right) = \frac{x}{4(x^2+1)^2} + \frac{3x}{8(x^2+1)} + \frac{3}{8}\arctan x$

## 6.3 22

$\int \frac{1}{(4x-x^2)^{3/2}}\,dx$

$$\int_{1/2}^{3/2} \frac{1}{(4x-x^2)^{3/2}}\,dx = \int_{1/2}^{3/2} \frac{1}{(-(x-2)^2+4)^{3/2}}\,dx = \begin{bmatrix} x-2=u, & \frac{1}{2} = -\frac{3}{2},\ \frac{3}{2} = -\frac{1}{2} \\ \frac{du}{dx}=1 & \Rightarrow dx=du \end{bmatrix} = \int_{-3/2}^{-1/2} \frac{1}{(4-u^2)^{3/2}}\,du = \begin{bmatrix} u=2\cdot\sin t \\ \frac{du}{dt} = 2\cos t\end{bmatrix} = \int_{\arcsin(-\frac{3}{4})}^{\arcsin(-\frac{1}{4})} \frac{2\cdot\cos t}{(\sqrt{4-4\sin^2 t})^{3/2}}\,dt = \int_{\mathrm{arc}(-\frac{3}{4})}^{\mathrm{arc}(-\frac{1}{4})} \frac{2\cos t}{(2\cos t)^3}\,dt = \int_{\mathrm{arc}(-\frac{3}{4})}^{\mathrm{arc}(-\frac{1}{4})} \frac{1}{4\cos^2 t}\,dt = \frac{1}{4}\Big(\tan t\Big)\Big]_{\mathrm{arc}\,-\frac{3}{4}}^{\mathrm{arc}\,-\frac{1}{4}}$$

$$= \frac{1}{4}\tan\left(\arcsin\left(-\frac{1}{4}\right)\right) - \frac{1}{4}\cdot\tan\left(\arcsin\left(-\frac{3}{4}\right)\right)$$

## 31

$$\int \frac{1+x^{1/2}}{1+x^{1/3}}\,dx = \begin{bmatrix} t = x^{1/6} \\ \frac{dt}{dx} = \frac{1}{6}x^{-5/6} \Rightarrow dx = dt\cdot 6x^{5/6}\end{bmatrix} = \int \frac{1+t^3}{1+t^2}\cdot 6t^5\,dt = 6\int \frac{t^8+t^5}{t^2+1}\,dt$$

$$\begin{array}{l} t^6 - t^4 + t^3 + t^2 - t - 1 \\ \hline t^8 + t^5 \;\big|\; t^2+1 \\ -(t^8+t^6) \\ \hline -t^6 + t^5 - t^2 \\ \vdots \\ \hline t+1 \end{array}$$

$$= 6\int t^6 - t^4 + t^3 + t^2 - t - 1 + \frac{t+1}{t^2+1}\,dt$$

$$= 6\left(\frac{t^7}{7} - \frac{t^5}{5} + \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} - t + \frac{1}{2}\ln(t^2+1) + \arctan(t)\right), \quad t = x^{1/6}$$

$$\int \frac{t}{t^2+1}\,dt + \int \frac{1}{t^2+1}\,dt = \frac{1}{2}\ln(t^2+1) + \arctan t$$

## 6.5 46

$\Gamma(x) = \int_0^\infty t^{x-1}\cdot e^{-t}\,dt$

a) Konvergent för $x>0$, okej i $\infty$. $e^{-t}$ avtar snabbare än alla potenser. I 0 är $\Gamma$ generaliserad om $x-1<0$, $x<1$ men det är okej om $x-1<1$, $x>0$.

b) $\Gamma(x+1) = \int_0^\infty t^x\cdot e^{-t}\,dt = \begin{bmatrix}\text{Part}\\ \text{int}\end{bmatrix} = -t^x e^{-t}\Big]_0^\infty + \int x\cdot t^{x-1}\cdot e^{-t}\,dt = 0 + x\,\Gamma(x)$

c) $\Gamma(1) = \int_0^\infty t^0 e^{-t}\,dt = -e^{-t}\Big]_0^\infty = -0+1 = 1$

$\Gamma(2) = 1\cdot\Gamma(1) = 1$

$\Gamma(3) = 2\cdot\Gamma(2) = 2$

$\Gamma(4) = 3\cdot\Gamma(3) = 6$

$\Gamma(n) = (n-1)!$