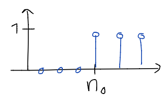
∝ Unit step: $u[n-n_0] = \begin{cases} 1 & , n \geq n_0 \\ 0 & , n < n_0 \end{cases}$ =>



$u(t-t_0) = \begin{cases} 1 & t > t_0 \\ 0, & t < t_0 \end{cases}$

∝ Unit impulse: $\delta[n] = \begin{cases} 1 & , n=0 \\ 0 & , \text{other} \end{cases}$ => $\delta[n] = u[n] - u[n-1]$

$\delta[n-n_0] = \begin{cases} 1 & , n = n_0 \\ 0, & \text{other} \end{cases}$

<u>Shifting property</u>

$x[n] \cdot \delta[n] = x[0] \cdot \delta[n]$

Properties of CT-LTI:
- ∝ Memory less system (Output only depends on input)
- ∝ Invertible (One specific input => one and only one specific output, injective)
- ∝ Causality (Output does not depend on the future)
- ∝ Stability
- ∝ Time invariant
- ∝ Linnear => Additivity
  Homogenuos

2.4 c)

2.4[19] The signals in Figure 3 are zero except as shown.

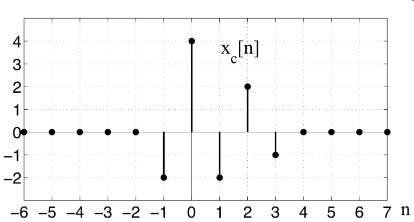
(a) For the signal $x_a[n]$ of Figure 3(a), plot the following

(i) $x_a[-n]\ u[n]$	**(iv)** $x_a[-n]\ u[-2-n]$
(ii) $x_a[n]\ u[-n]$	**(v)** $x_a[n]\ \delta[n-2]$
(iii) $x_a[n]\ u[n+2]$	**(vi)** $x_a[n]\ (\delta[n+1] + \delta[n-1])$

(b) Repeat (a) for the signal $x_b[n]$ of Figure 3(b).

(c) Repeat (a) for the signal $x_c[n]$ of Figure 3(c).

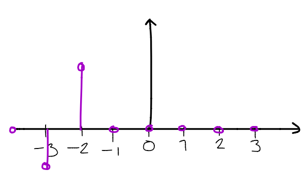
(d) Repeat (a) for the signal $x_d[n]$ of Figure 3(d).



(c)

$x_a[-n] \cdot u[-2-n] = x_a[n] \cdot u[-(n+2)]$

n	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$x_c[n]$	0	0	0	0	0	-2	4	-2	2	-1	0	0	0
$x_c[-n]$	0	0	0	-1	2	-2	4	-2	0	0	0	0	0
$u[-2-n]$	1	1	1	1	1	0	0	0	0	0	0	0	0
$x_c[-n]u[-2-n]$	0	0	0	-1	2	0	0	0	0	0	0	0	0



vi) $x_c[n](\delta[n+1] + \delta[n-1]) = y[n] = x_c[n]\delta[n+1] + x_c[n]\delta[n-1] = x_c[-1]\delta[n+1] + x_c[1]\delta[n-1]$
$= -2\delta[n+1] - 2\delta[n-1] = -2(\delta[n+1] + \delta[n-1])$

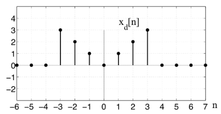
2.11[26] Consider the signals shown in Figure 3.

(a) Write an expression for $x_a[n]$. The expression will involve the sum of discrete impulse functions.

(b) Write an expression for $x_b[n]$.

(c) Write an expression for $x_c[n]$.

(d) Write an expression for $x_d[n]$.



(d)

$n_0=-3$ $\quad n=0 \quad$ $n_0=3$

$$x_d[n]=\{\dots,0,0,3,2,1,0,1,2,3,0,0,\dots\}=$$
$$3\cdot\delta[n+3]+2\delta[n+2]+1\cdot\delta[n+1]+0+1\cdot\delta[n-1]+2\delta[n-2]+3\delta[n-3]$$

1.11[11] Express the following in terms of $x(t)$:

$$y(t)=\frac{1}{2}\int_{-\infty}^{\infty}x(\tau)\Big(\delta(\tau-2)+\delta(\tau+2)\Big)d\tau\ .$$

$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt = f(t_0)$, $f(t)$ continuos at $t=t_0$

$$y(t)=\tfrac{1}{2}\int[x(\tau)\delta(\tau-2)+x(\tau)\delta(\tau+2)]\delta\tau=\tfrac{1}{2}\Big[\int\underbrace{x(\tau)\delta(\tau-2)\,d\tau}_{t_0=2}+\int\underbrace{x(\tau)\delta(\tau+2)\,d\tau}_{t_0=-2}=$$
$$\tfrac{1}{2}[x(2)+x(-2)]$$

1.12[12]

(a) Prove the time-scaling relation from Table 2.3 in PPR. (*Hint*: use a change of variables):

$$\int_{-\infty}^{\infty}\delta(at)dt=\frac{1}{|a|}\int_{-\infty}^{\infty}\delta(t)dt\ .$$

(b) Prove the following relation from Table 2.3 in PPR:

$$u(t-t_0)=\int_{-\infty}^{t}\delta(\tau-t_0)d\tau\ .$$

(c) Evaluate the following integrals:

- **(i)** $\int_{-\infty}^{\infty}\cos(2t)\delta(t)dt$
- **(ii)** $\int_{-\infty}^{\infty}\sin(2t)\delta(t-\frac{\pi}{3})dt$
- **(iii)** $\int_{-\infty}^{\infty}\cos[2(t-\frac{\pi}{4})]\delta(t-\frac{\pi}{4})dt$
- **(iv)** $\int_{-\infty}^{\infty}\sin(t-1)\delta(t-2)dt$
- **(v)** $\int_{-\infty}^{\infty}\sin(t-1)\delta(2t-4)dt$

III

$$x(t)\cdot\delta(t)=x(0)\delta(0)$$

$$\int_{-\infty}^{\infty}\overbrace{\cos[2(\underbrace{t-\tfrac{\pi}{4}}_{t'})]}^{x(t')}\overbrace{\delta(\underbrace{t-\tfrac{\pi}{4}}_{t'=0=t-\frac{\pi}{4}|_{t=\frac{\pi}{4}}})}^{\delta(t')}dt=\int_{-\infty}^{\infty}x(0)\delta(t')dt'-$$

$$\int_{-\infty}^{\infty}\cos\Big(2\big(\tfrac{\pi}{4}-\tfrac{\pi}{4}\big)\Big)\cdot\delta(t-\tfrac{\pi}{4})dt=\int_{-\infty}^{\infty}\underbrace{\cos(0)}_{=1}\cdot\delta(t-\tfrac{\pi}{4})dt=1$$

$$\int_{-\infty}^{\infty}\delta(t)dt=1$$

V

$$\int_{-\infty}^{\infty}\sin(t-1)\delta(2t-4)dt=\int_{-\infty}^{\infty}\sin(2-1)\delta(2t-4)dt=\sin(1)\int_{-\infty}^{\infty}\delta(2t-4)dt=\left|\begin{matrix}2t=u\Rightarrow t=\frac{u}{2}\\ dt=\frac{du}{2}\end{matrix}\right|=$$

$2t-4=0\Rightarrow 2=t$

$$\sin(1)\int_{-\infty}^{\infty}\delta(u-4)\frac{du}{2}=\frac{\sin(1)}{2}\underbrace{\int_{-\infty}^{\infty}\delta(u-4)\,du}_{=1}=\frac{\sin(1)}{2}$$

1.14[14] Determine whether each system is

- **(i)** memoryless
- **(ii)** invertible
- **(iii)** causal
- **(iv)** stable
- **(v)** time invariant
- **(vi)** linear

The systems are described as

- **(a)** $y(t)=\cos(x(t-1))$
- **(b)** $y(t)=3x(3t+3)$
- **(c)** $y(t)=\ln(x(t))$
- **(d)** $y(t)=e^{tx(t)}$
- **(e)** $y(t)=7x(t)+6$
- **(f)** $y(t)=\int_{-\infty}^{t}x(5\tau)d\tau$
- **(g)** $y(t)=e^{-j\omega t}\int_{-\infty}^{\infty}x(\tau)e^{-j\omega\tau}d\tau$
- **(h)** $y(t)=\int_{t-1}^{t}x(\tau)d\tau$

Memory less: ✓ $\quad y(t)$ depends only on input $x(t)$

Invertible: $y(t)=7x(t)+6 \Rightarrow x(t)=\frac{y(t)-6}{7}$ ✓

Causal: ✓ $\quad y(t)$ depends only post and/or current input.

Stable: Def: $\exists M,N<\infty: |x(t)|\leqslant M, |y(t)|\leqslant N, \forall t \Rightarrow$ BIBO $\quad |x(t)|\leqslant M \Rightarrow |y(t)|\leqslant 7\cdot M+6<\infty$ ✓

Time invariant: $y(t)|_{t-t_0}=y(t)|_{x(t-t_0)}$ $\quad y(t)|_{t-t_0}=7x(t-t_0)+6$ $\quad y(t)|_{x(t-t_0)}=7x(t-t_0)+6$ ✓

Linnear: Additivety

$x_1(t)+x_2(t)\Rightarrow y_1(t)+y_2(t)$

Homogenity $\qquad$ Test: $a_1x_1(t)+a_2x_2(t)\Rightarrow a_1y_1(t)+a_2y_2(t)$

$a\cdot x(t)\Rightarrow a\,y(t)$

Additivety test

$y_1(t)+y_2(t)=7x_1(t)+6+7x_2(t)+6=7(x_1(t)+x_2(t))+12$

Should be the same as: $y(t)|_{x(t)=x_1(t)+x_2(t)}=7\big(x_1(t)+x_2(t)\big)+6$ ✗

4.2[42] Show that, for any function $g[n]$, Convolution: $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$$g[n] \star \delta[n] = g[n] \ .$$

$g[n] * \delta[n] = g[n]$

$$y[n] = g[n] * \delta[n] = \sum_{k=-\infty}^{\infty} g[k] \underbrace{\delta[n-k]}_{=1 \text{ iff } n=k} = g[k]\Big|_{k=n} = g[n]$$

QED