Signal: funktion, $x : \mathcal{X} \to \mathbb{C}$ där $\mathcal{X} = (-\infty, \infty)$ kontinuerlig tid
$\mathcal{X} = \{\ldots -1, 0, 1, 2, \ldots\}$ diskret tid
$\mathcal{X} = [0, T]$ periodisk kontinuerlig tid
$\mathcal{X} = \{0, 1, N-1\}$ periodisk diskret tid.

Tack till den stere komikern för anteckningarna!

System, funktion på signaler

$y(t) = (Sx)(t)$ ex $y(t) = x(2t)$ linjärt
$y(t) = x(t-1)$ linjärt
$y(t) = x(t)^2$

<u>Linjärt</u>

$S(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 S(x_1) + \alpha_2 S(x_2)$

<u>Tidsinvariant</u>

$S x_{t_0} = y_{t_0}$ för alla t_0 där $x_{t_0}(t) = x(t - t_0)$

$y(t) = x(2t)$
$(Sx_{t_0})(t) = x_{t_0}(2t) = x(2t - t_0)$
$y_{t_0}(t) = y(t - t_0) = x(2(t-t_0)) = x(2t - t_0)$ ej tidsinv!

<u>Kausalt</u>

$y(t)$ beror ej på $x(s)$ för $s > t$.

$y(t) = x(t-1)$ kausalt
$y(t) = x(t+1)$ EJ kausalt

<u>Studera System</u> $y(t) = (Sx)t$

$y''(t) - ay'(t) - by(t) = x''(t) - cx'(t) - dx(t)$ + varianter

<u>Hur löser man sånt här?</u>

$y'(t) - ay(t) = x(t)$ $y(0)$ känd
Obs "$-a$" e^{-at}

$e^{-at} y'(t) - a e^{-at} y(t) = e^{-at} x(t)$
$\frac{d}{dt}\left(e^{-at} \cdot y(t)\right) = e^{-at} \cdot x(t)$
$e^{-at} \cdot y(t) = y(0) + \int_0^t e^{-a\tau} x(\tau)\, d\tau$
$y(t) = e^{at} \cdot y(0) + \int_0^t e^{a(t-\tau)} x(\tau)\, d\tau$

Fråga? Är $y(t) = (Sx)(t)$ linjär? Tidsinv? Kausalt?
$y_1' - a y_1 = x_1$ $y_2' - a y_2 = x_2$

$\left.\begin{array}{l} y = y_1 + y_2 \\ x = x_1 + x_2 \end{array}\right\}$ $y_1' + y_2' - a(y_1 + y_2) = (y_1' - a y_1) + (y_2' - a y_2) = x_1 + x_2 = x$

$y_{t_0}(t) = y(t - t_0)$ $y_{t_0}'(t) - a y_{t_0}(t) = y'(t - t_0) - a y(t - t_0)$
$x_{t_0}(t) = x(t - t_0)$ $= x(t - t_0)$
$= x_{t_0}(t) \Rightarrow y(t_0) = S x_{t_0} \Rightarrow$ Tidsinv!

Ex

Lös $y'' - 3y' + 2y = x$ $\quad y(0), y'(0)$ känd

Karakteristiska polynomet: $r^2 - 3r + 2 = 0$

$r_1 = 1, \quad r_2 = 2$

$z = y' - r_2 y$

$z(0) = y'(0) - 2y(0)$

$z' - r_1 z = y'' - 2y' - (y' - 2y) = y'' - 3y' + 2y = x$

$\underset{a=1}{z' - z} = x \Rightarrow z(t) = e^{t} z(0) + \int_0^t e^{t-\tau} x(\tau)\, d\tau$

allmän lösning

$\underset{a=2}{y' - 2y} = z \Rightarrow y(t) = e^{2t} \cdot y(0) + \int_0^t e^{2(t-\tau)} z(\tau)\, d\tau$

Differensekvationer

$y[n] = (Sx)[n]$

$y[n] - a y[n-1] - b y[n-2] = e x[n] - f x[n-1] - g x[n-2]$ + variationer

Ex

$\underbrace{y[n] - y[n-1]}_{\text{analogt med } y'} = x[n]$ ← Känd insignal samt $y[0]$.

Känd

$y[1] = y[0] + x[1]$

$y[2] = y[1] + x[2] = y[0] + x[1] + x[2]$

$y[3] = \dots$

$y[n] = y[0] + \sum_{k=1}^{n} x[k]$

Ex

$y[n] - a y[n-1] = x[n]$

$y[1] = a y[0] + x[1]$

.....

$y[n] = a^n y[0] + \sum_{k=1}^{n} a^{n-k} x[k]$

Ex

$y[n] - 3y[n-1] + 2y[n-2] = x[n]$

KE: $r^2 - 3r + 2 = 0 \Rightarrow r_1 = 1, \; r_2 = 2$

$z[n] = y[n] - 2y[n-1]$

$z[n] - 1 \cdot z[n-1] =$
$y[n] - 2y[n-1] - y[n-1] + 2y[n-2] =$
$y[n] - 3y[n-1] + 2y[n-2]$

OBS! $\quad a = 2$

$y[n] - 2y[n-1] = z[n]$

$z[n] - z[n-1] = x[n]$

$z[n] = z[0] + \sum_{k=1}^{n} x[k]$

$y[n] = 2^n \cdot y[0] + \sum_{k=1}^{n} 2^{n-k} z[k]$

Allmän lösning!

$z[0] = y[0] - 2y[-1]$ ← Kända!