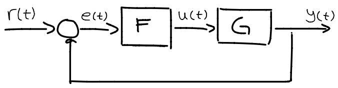
Återkopplade System

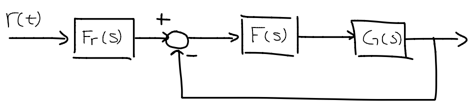


$L(s) = F(s)G(s)$

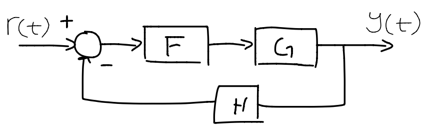
$G_{ry}(s) = \frac{D(s)}{1+L(s)} = \frac{L(s)}{1+L(s)}$

Polerna ges av: $1+L(s)=0$
Detta kallas karaktäristiska ekvationen.

Observera

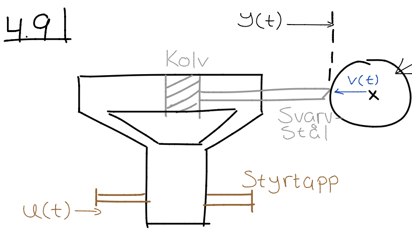


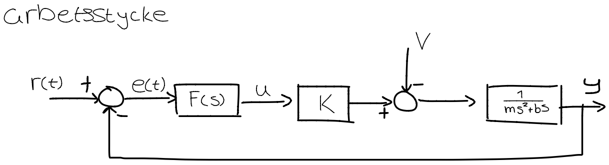
$\Rightarrow \quad G_{ry}(s) = \frac{F_r(s)F(s)G(s)}{1+F(s)G(s)}$



$G_{ry}(s) = \frac{FG}{1+FGH}$

4.9





Sökt

Kvarstående fel vid konstant $v(t) = V_0$ för a) P-reg
b) PI-reg

Lösning

$$G_{ve}(s) = \frac{E(s)}{V(s)} = \frac{\text{Fram}}{1+\text{Krets}} = \frac{D(s)}{1+L(s)} = \frac{\frac{1}{ms^2+bs}}{1+F(s)K\cdot\frac{1}{ms^2+bs}} = \frac{1}{ms^2+bs+F(s)K}$$

$$V(s) = \frac{V_0}{s}$$

a) $F(s) = K_p \Rightarrow E(s) = \frac{1}{ms^2+bs+K_pK} \cdot \frac{V_0}{s}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \cancel{s}\left(\frac{1}{ms^2+bs+K_pK}\cdot\frac{V_0}{\cancel{s}}\right) = \frac{V_0}{K_pK}$$

b) $F(s) = \frac{K_p s + K_i}{s} \Rightarrow E(s) = \frac{1}{ms^2+bs+K\left(\frac{K_ps+K_i}{s}\right)}\cdot\frac{V_0}{s} = \frac{V_0}{ms^3+bs^2+K(K_ps+K_i)}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{sV_0}{ms^3+bs^2+K(K_ps+K_i)} = 0$$

4.24

Givet

$G(s) = \frac{1}{s}$

$F_{PI}(s) = K_p\left(1 + \frac{1}{T_i s}\right) = K\left(1 + \frac{1}{s}\right)$

K bestäms så ~~att~~ det återkopplade systemets relativa dämpning $\zeta = 0.3$.

Sökt

$t_{5\%}$

Lösning

$t_{5\%} = \frac{3}{a}$, $s = -a \pm j\omega_d$

$L(s) = F(s)G(s) = K\left(1 + \frac{1}{s}\right) \cdot \frac{1}{s} = \frac{K(s+1)}{s^2}$

$1 + L(s) = 0 \iff 1 + \frac{K(s+1)}{s^2} = 0 \iff s^2 + Ks + K = 0$ ①

Jmf med: $s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow \omega_n = \sqrt{K}$, $2\zeta\omega_n = K \Rightarrow K = 0.36$

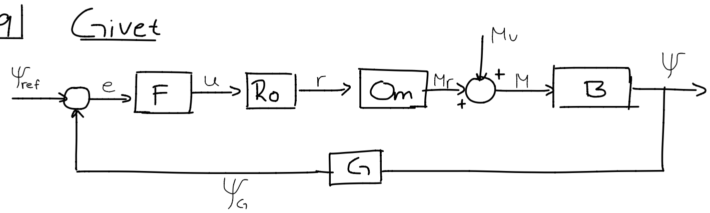
Insättning av K i ① $\Rightarrow$ $s^2 + 0.36s + 0.36 = 0$

$(s + 0.18)^2 = 0.18 - 0.36$

$s = -0.18 \pm \sqrt{0.18^2 - 0.36} = -0.18 \pm j0.57$

$t_{5\%} = \frac{3}{0.18} \approx 17s$

4.19 Givet



F: Fartyg
Ro: Roderservo
Om: Omvandlingsfaktor
B: Fartyg (Båt)
G: Givare
ψ: Kursvinkel
Mv: Vridmoment pga störningar
Mr: — " — roderverkan
M: — " — tot
u: Spänning
e: felsignal
ψ_G: mätsignal från givare
r: rodervinkel

F: $u = 0.5e$

Ro: $5\dot{r} + r = 0.1u$

B: $100\dot{\psi} + \psi = 0.1M$

G: $\psi_G = 0.1\psi$

Om: 10^3

Sökt

$L(s)$, $G_{\psi_r\psi}$, $G_{M_v\psi}$

$L(s) = F(s)R_o(s)Om(s)B(s)G(s)$

$G_{\psi_r\psi}(s) = \frac{D(s)}{1+L(s)} = \frac{F(s)R_o(s)Om(s)B(s)}{1 + F(s)R_o(s)Om(s)B(s)G(s)}$

$G_{M_v\psi}(s) = \frac{D(s)}{1+L(s)} = \frac{B(s)}{1+L(s)}$

4.25

Givet

$F(s) = K\left(1 + \frac{1}{Ts}\right)$

$G(s) = \frac{3}{1+2s}$

Sökt

a) Bestäm K, T s.a. det slutna systemet får en dubbelpol i $s=-1$.

Lösning

Polpolynomet: $P(s) = \prod_{i=1}^{n}(s-p_i) = 0$, n: antalet poler.

Dubbelpol, $s=-1$, $\Rightarrow n=2 \Rightarrow P(s) = (s-(-1))(s-(-1)) = (s+1)^2 = s^2+2s+1=0$

Karaktäristiska ekvationen: $1+L(s)=0 \iff 1+\frac{3}{1+2s} \cdot K\left(\frac{Ts+1}{Ts}\right)=0 \iff \frac{(1+2s)Ts+3K(Ts+1)}{(1+2s)Ts}=0 \iff$

$(1+2s)Ts + 3K(Ts+1) = 0 \iff s^2 2T + s(T+3TK) + 3K = 0 \iff$

$s^2 + s\frac{(1+3K)}{2} + \frac{3}{2}\frac{K}{T} = 0$

Jmf med $P(s)$!

$$\begin{cases} \frac{1+3K}{2} = 2 \Rightarrow K=1 \Rightarrow \\ \frac{3}{2}\frac{K}{T} = 1 \end{cases} \quad T = \frac{3}{2}$$

$F(s) = 1\left(1 + \frac{1}{\frac{3}{2}s}\right)$

b) **Sökt**

Det slutna systemet ska få $\zeta = 0.7$, $\omega_n = 1$

Lösning

Jmf koeff i KE med koeff i $s^2 + 2\zeta\omega_n + \omega_n^2 =$

$$\begin{cases} \frac{1+3K}{2} = 2\zeta\omega_n \Rightarrow K=0.6 \Rightarrow \\ \frac{3}{2}\frac{K}{T} = \omega_n^2 \end{cases} \quad T=0.9$$

$F(s) = 0.6\left(1 + \frac{1}{0.9s}\right)$