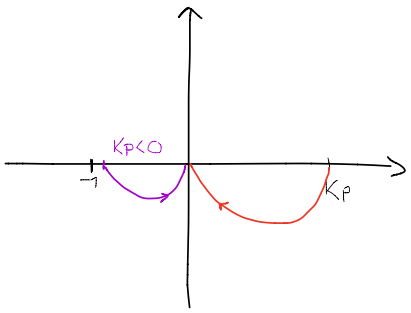
Frekvensanalys

Ex Nyquistdiagram,

$F(s)=K_p$, $G(s)=\frac{1}{1+sT}$ $\Rightarrow$ $L(s)=F(s)G(s)$

$$L(j\omega)=\frac{K_p}{1+j\omega T}=\frac{K_p(1-j\omega T)}{(1+j\omega T)(1-j\omega T)}=\frac{K_p}{1+(\omega T)^2}-j\,\frac{K_p\omega T}{1+(\omega T)^2}$$

ω	0	α	β	γ	∞
Re	K_p	$+a$	$+b$	$+c$	0
Im	0	$-g$	$-f$	$-h$	0



Systemet är stabilt för $K_p>-1$

Bodediagram

Ett annat sätt att studera systemegenskaper där vi även får in frekvensberoende är bodediagram.

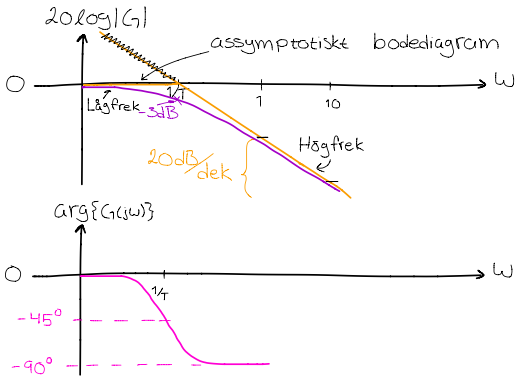
Man studerar i dessa: $\begin{cases}20\log|G(j\omega)| \\ \arg\{G(j\omega)\}\end{cases}$

Ex

$G(s)=\frac{1}{1+sT}$, $G(j\omega)=\frac{1}{1+j\omega T}$

$\omega T\ll 1 \Rightarrow G(j\omega)\approx 1 \Rightarrow \begin{cases}20\log|G(j\omega)|=20\log(1)=0\ dB \\ \arg\{G(j\omega)\}=0\end{cases}$

$\omega T\gg 1 \Rightarrow G(j\omega)\approx\frac{1}{j\omega T} \Rightarrow \begin{cases}20\log|G(j\omega)|=20\log(\frac{1}{\omega T}) \\ \arg\{G(j\omega)\}=-\arctan(\omega T)=-\frac{\pi}{2}\end{cases}$



$\omega=\frac{1}{T}$: Brytfrekvens

$|G(j\frac{1}{T})|=|\frac{1}{1+j}|=\frac{1}{\sqrt{1^2+1^2}}=\frac{1}{\sqrt{2}}$

$20\log|G(j\frac{1}{T})|=-3dB$

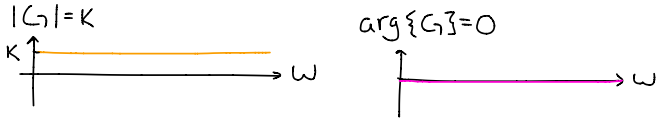
Viktig motivering

Anta $G(s)=\frac{T_1(s)T_2(s)\cdots}{N_1(s)N_2(s)\cdots}$ $\Rightarrow$ 1) $\log|G|=\sum_{k=1}^{}(\log|T_k(j\omega)|-\log|N_k(j\omega)|)$ $\Rightarrow$ summera bidrag i Bodediagram.

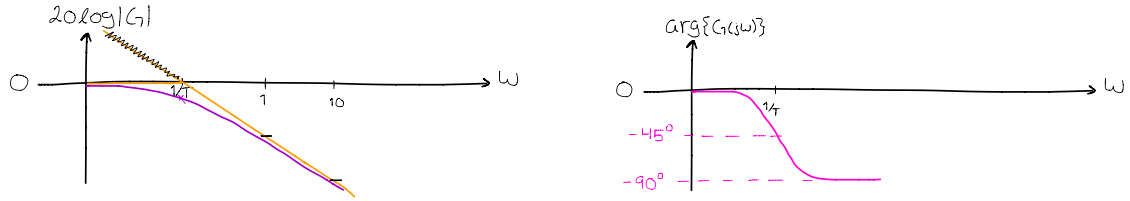
2) $\arg\{G(j\omega)\}=\sum_{k=1}^{}(\arg\{T_k(j\omega)\}-\arg\{N_k(j\omega)\})$ $\Rightarrow$ summera bidrag i Bodediagram.

Vilka faktorer är intressanta att studera?

① $G(s)=K$

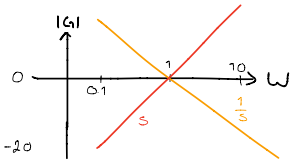


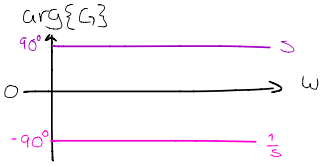
② $G(s)=\frac{1}{1+sT}$ (samma som ovan)



③ $G(s)=\frac{1}{s}$, samt $G(s)=s$

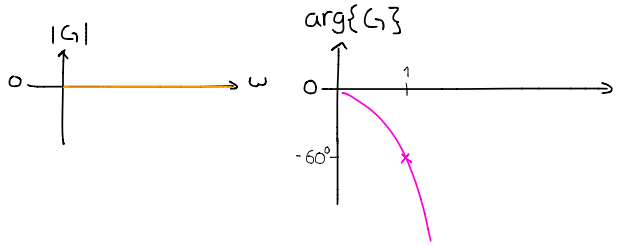
$20\log|G(j\omega)| = -20\log(\omega)$ $\quad \arg\{G(j\omega)\} = -\arctan\left(\frac{\omega}{0}\right) = -\frac{\pi}{2}$





④ $G(s)=e^{-sT}$ (tidsfördröjning av en signal)

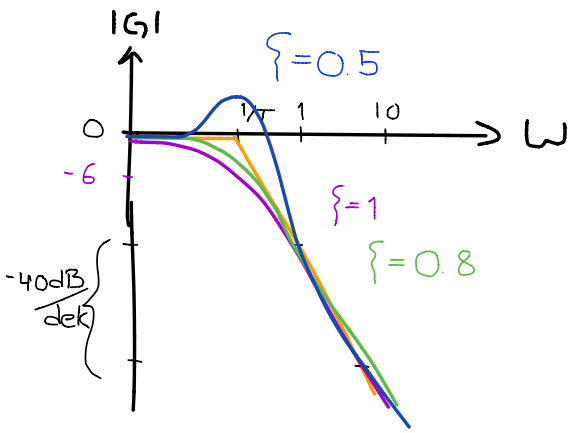
$20\log|G(j\omega)| = 20\log(1) = 0$ $\quad \arg\{G(j\omega)\} = -\omega T$

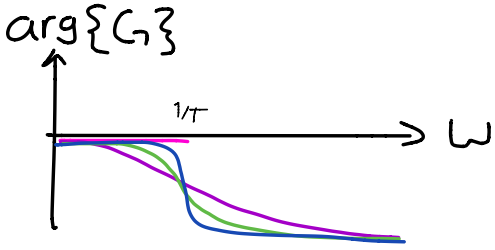


⑤ $G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{1}{1+2\zeta Ts+s^2T^2}$, $T=\frac{1}{\omega_n}$, $0<\zeta<1$

$\omega T \ll 1$: $20\log|G(j\omega)| = 0$, $\arg\{G(j\omega)\} = 0$

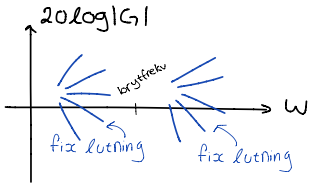
$\omega T \gg 1$: $20\log|G(j\omega)| \approx 40\log\frac{1}{T} - 40\log\omega$, $\arg\{G(j\omega)\} = -180°$

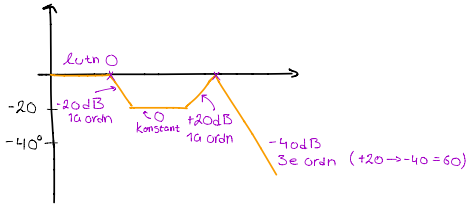




Sammanställning

Alla bidrag ser i princip ut som:





Många brytpunkter: Börja från vänster, gör tabell för argumenten