Def - Expected value

Let X be a discreeet r.v. with density f. The expected value (average, mean) of X is $E[X]=\sum_{all\,x} x f(x)$.

Ex

Integer	1	2	3	4	5	6	7	8	9	10
Number of divisors	1	2	2	3	2	4	2	4	3	4

Number of div	1	2	3	4
Value of f	0.1	0.4	0.2	0.3

$E[X]=1\cdot0.1+2\cdot0.4+3\cdot0.2+4\cdot0.3=2.7$

Def - Expected value of $H(x)$

Let X be a discrete r.v. with density f. Let H be some function. Expected value of $H(x)$ is given by $E[H(x)]=\sum_{all\,x} H(x)f(x)$

Ex

$H(x)=x^2$, $E[H(x)]=E[X^2]=1^2\cdot0.1+2^2\cdot0.4+3^2\cdot0.2+4^2\cdot0.3=8.3$,

Theorem

Let X be a r.v. Then $E[aX+b]=aE[X]+b$, for any $a,b\in\mathbb{R}$.

Note
$\sum_{all\,x} f(x)=1$

Proof

Consider f is a density of X. $E[aX+b]=\sum_{all\,x}(ax+b)\cdot f(x)=\sum_{all\,x} axf(x)+\sum_{all\,x} bf(x)=aE[X]+b$

Properties of expectations

Let X and Y be r.v. $E[X],E[Y]<\infty$.

1) $E[c]=c$, c is constant
2) $E[cX]=cE[X]$
3) $E[X+Y]=E[X]+E[Y]$

Ex

$X\in\{0,1000\}$

$P(X=0)=0.99$ $\quad$ $E[X]=0\cdot0.99+1000\cdot0.01=10$

$P(X=1000)=0.01$ $\quad$ $Var[X]=9900$, $\sigma\approx99.5$

Def - Variance

Let X be a r.v with mean $E[X]$ (μ). The variance of X, denoted by $Var[X]$ (σ_X^2, $D[X]$) is given by:

$Var[X]=E[(X-E[X])^2]$

Def - Standard deviation

Let X be a r.v. with variance σ_X^2. The st.dev. of X is given by $\sigma_X=\sqrt{\sigma_X^2}=\sqrt{Var[X]}$

Theorem

$Var[X]=E[X^2]-(E[X])^2$

Proof

$Var[X]=E[(X-E[X])^2]=E[X^2-2X\underbrace{E[X]}_{\text{constant}}+(E[X])^2]=E[X^2]-2E[X]E[X]+(E[X])^2=E[X^2]-(E[X])^2$

Ex

$Var[X]=8.3-2.7^2=1.01$, $\sigma=1.005$

Properties of variance

1) $Var[c] = 0$, c is constant
2) $Var[cX] = c^2 \cdot Var[X]$
3) $Var[X+Y] \neq Var[X] + Var[Y]$

but sometimes!

Bionomial distribution

A r.v. X is said to have binomial distribution if its density f is given by $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$, $0 \leq p \leq 1$, $x = 1,2,3 \dots n$

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$

$E[X] = np$, $Var[X] = np(1-p)$

Geometric distribution

$f(x) = (1-p)^{x-1} \cdot p$, $0 \leq p \leq 1$, $x = 1,2,3,\dots,n.$

$E[X] = \frac{1}{p}$, $Var[X] = \frac{1-p}{p^2}$

Poisson distribution

$f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$, $x = 0,1,2 \dots$, $\lambda > 0$

$E[X] = Var[X] = \lambda$

Def - Continuous

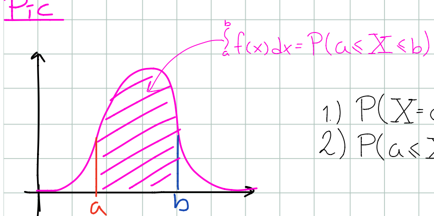
A r.v is continuous if it can assume value in some interval or intervalls of real numbers and the probability that it assumes any particular value is zero.

Def - Continuous density

Let X be a continuous r.v A function f such that:

1) $f(x) \geq 0$ for any x
2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3) $P(a \leq X \leq b) = \int_a^b f(x)\,dx$, this is called "density" of X

Pic



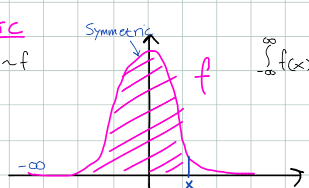
1) $P(X = c) = \int_c^c f(x)\,dx = 0$
2) $P(a \leq X \leq b) = P(a < X < b) = P(a \leq X < b) = P(a < X \leq b)$, if X is continuous

Def

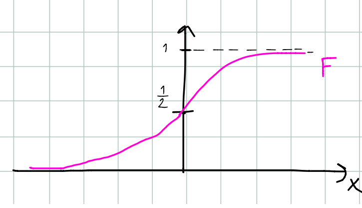
Let X be a continuous r.v with density f The cumulative distribution function (c.d.f) is denoted by F_X, is defined by: $F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$, $x \in \mathbb{R}$

Pic

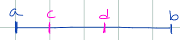
$X \sim f$



$\int_{-\infty}^{\infty} f(x)dx = 1$, $f(x) \geq 0$



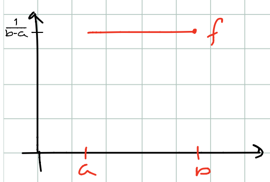
Ex

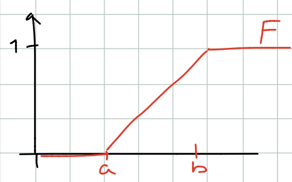
A continuous r.v. X has a uniform ($Uni[a,b]$) distribution on $[a,b]$ if X falls with the same probability in any subset (sub-interval) $(c,d) \subset (a,b)$ 

provided its length is constant

$$f(x) = \begin{cases} \frac{1}{b-a}, & x \in (a,b) \\ 0, & \text{otherwise} \end{cases} \qquad F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & x \in (a,b) \\ 1, & x > b \end{cases}$$

Pic





For math

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$E[H(X)] = \int_{-\infty}^{\infty} H(x) f(x)\,dx$$

$$Var[X] = E[X^2] - (E[X])^2$$