Hypothesis Testing

1. Hypothesis
   $H_0: \theta = \theta_0$ - null hypothesis
   $H_A: \theta = \theta_1$ - alternative hypothesis (simple)
   $\left.\begin{array}{l}\theta > \theta_1 \\ \theta < \theta_1\end{array}\right\}$ One sided alternatives
   $\theta \neq \theta_0$ - two sided alternatives

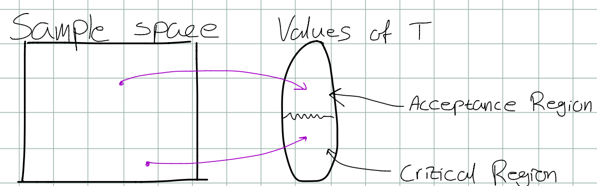
2. T-Statistic

3. Critical region for 2.
   Think about α

Ex

$Bin(n, \frac{1}{2})$ - $\theta_0 = \frac{1}{2}$, H_0 expectation: $\frac{n}{2}$
$Bin(n, \theta)$ - $\theta \neq \frac{1}{2}$, H_A expectation: $\frac{3n}{4}$

Random sample: $X_1, \dots, X_n$
$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ - Test statistic
$\alpha = 0.05$
P-value $= 0.0001$ (unlikely)



	H_0	H_A Reality
H_0	OK	Type II ERROR
H_A	Type I ERROR	OK

Decision

Def

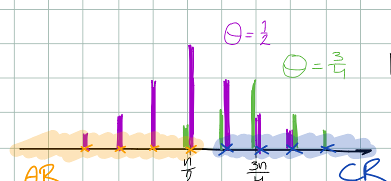
A type I error is an error when the null hypothesis is rejected when it is actually true. The probability of commiting a type I error is called the level of significance of the test.
$\alpha = P[H_A \mid H_0]$

Def

A type II error is an error when the null hypothesis is not rejected when it is actually false
$\beta = P[H_0 \mid H_A]$

Def

Power of the test is $1-\beta$



In order to fix type I => knowing type II $\alpha = 0.05$

Def

P-value is the probability for a test statistic, given H_0 is true, to be at least as extreme as its realization obtained from data.

EX

The survey polled 1010 randomely selected employees.

Def

Consider a random sample $X_1, \dots, X_n$ of size n drawn from a population, where
$X_i = \begin{cases} 1 - \text{if the ith member of the sample has attribute} \\ 0 - \text{otherwise} \end{cases}$
Then $X = \sum_{i=1}^{n} X_i$ gives the number of members with the attribute.

Def

A sample proportion $\hat{p} = \frac{x}{n} = \frac{\text{number in sample with attribute}}{\text{sample size}}$

Sampling distribution of $\hat{p}$:
- $E[\hat{p}] = p$ – population proportion
- $Var[\hat{p}] = \frac{p(1-p)}{n}$
- $\hat{p}$ is approximately Normal for large n (np and $n(1-p)$ are both > 5)

One proportion z-interval procedure

Assumptions:
1. Random sampling
2. The number of successes, x, and number of failures, $n-x$, are both greater than 5.

1. Find $z_{\alpha/2}$ for a given α.
2. Calculate the end points of the CI: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
3. Interpret the confidence interval.

Back to Ex

202 employees answered yes (they are ill when actually not)

$n = 1010$

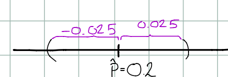
$x = 202$

$n - x = 808$

$\alpha = 0.05$

$\hat{p} = \frac{x}{n} = 0.2$

$z_{\alpha/2} = 1.96 \Rightarrow \begin{cases} L_1 = 0.175 \\ L_2 = 0.225 \end{cases}$



Def

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The margin of error, denoted by E, is given by: $E = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

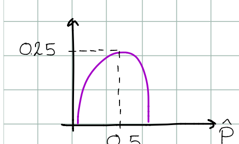
Back to Ex

$n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{E}\right)^2$

How to find n without $\hat{p}$

1. $0 \leq \hat{p} \leq 1$, $f(\hat{p}) = \hat{p}(1-\hat{p}) = \hat{p} - \hat{p}^2$

2. $\hat{p}_g$ - guess $\hat{p}$



Back...

$E = 0.01$, $\alpha = 0.05$, $z_{\frac{\alpha}{2}} = 1.96$

$n = 0.25 \cdot \left(\frac{1.96}{0.01}\right) = 9604$