Definition av e^{iy}

$e^{iy} = \cos y + i\sin y$

Sant: $e^{iy} \cdot e^{ix} = e^{i(x+y)}$

Def

$e^{z} = e^{x+iy} = e^{x} \cdot e^{iy} = e^{x}(\cos y + i \sin y)$

$e^{-iy} = \cos y - i \sin y$

Ex

$\int \cos^4 x \, dx = \int \left(\frac{e^{ix}+e^{-ix}}{2}\right)^4 dx = \frac{1}{16}\int e^{4ix} + 4e^{(3-1)ix} + 6e^{(2-2)ix} + 4e^{(1-3)ix} + e^{-4ix} dx = \frac{1}{16}\int e^{4ix} + 4e^{2ix} + 6 + 4e^{-2ix} + e^{-4ix} dx = \frac{1}{16}\left[\frac{e^{4ix}}{4i} + \frac{4e^{2ix}}{2i} + 6x + \frac{4e^{-2ix}}{-2i} + \frac{4e^{-4ix}}{-4i}\right] =$

$\frac{1}{16}\left(\underbrace{\frac{e^{4ix}-e^{-4ix}}{4i}}_{\frac{\sin 4x}{2}} + 6x + 4\underbrace{\frac{e^{2ix}-e^{-2ix}}{2i}}_{\sin(2x)}\right) = \frac{1}{32}\sin(4x) + \frac{3}{8}x + \frac{1}{4}\sin(2x) + C$

DEF

$\cos z = \frac{e^{iz}+e^{-iz}}{2}$ (Sant om $z \in \mathbb{R}$)

$\sin z = \frac{e^{iz}-e^{-iz}}{2i}$ — " —

Ex

$\cos z = 2$

$\frac{e^{iz}+e^{-iz}}{2} = 2$

$\left[w = e^{iz} \Rightarrow \frac{w+\frac{1}{w}}{2} = 2, \quad w + \frac{1}{w} = 4, \quad w^2 + 1 = 4w, \quad w^2 - 4w + 1 = 0 \qquad w = 2 \pm \sqrt{4-1} = 2 \pm \sqrt{3}\right]$

$e^{iz} = 2 + \sqrt{3}$

$z = x + iy$

$e^{ix-y} = e^{ix} \cdot e^{-y} = (2+\sqrt{3}) =$

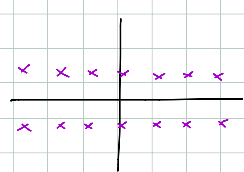
Belopp: $e^{-y} = (2+\sqrt{3})e^{i0}$

Arg : $x = 0 + 2k\pi$

$e^{iz} = 2 - \sqrt{3}$

$e^{-y} = 2 - \sqrt{3}$

$x = 0 + 2k\pi$



$\frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$y = \begin{cases} -\ln(2-\sqrt{3}) \\ -\ln(2+\sqrt{3}) \end{cases}$

$x = 0 + 2k\pi$

Ex

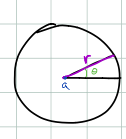
Visa att $w = \frac{1}{z-1}$ avbildar cirkeln $|z| = 2$ på en cirkel med centrum $\frac{1}{3}$.

Bevis

$\left|w - \frac{1}{3}\right|^2 = \left|\frac{1}{z-1} - \frac{1}{3}\right|^2 = \left|\frac{3-z+1}{3(z-1)}\right|^2 = \left[z = 2e^{i\theta}\right] = \frac{1}{9}\left|\frac{4-2e^{i\theta}}{2e^{i\theta}-1}\right|^2 = \frac{4}{9}\left|\frac{2-e^{i\theta}}{2e^{i\theta}-1}\right|^2 = \frac{4}{9}\left|\frac{2-\cos\theta - i\sin\theta}{2\cos\theta - 1 + 2i\sin\theta}\right|^2 = \frac{4}{9}\left(\frac{(2-\cos\theta)^2 + \sin^2\theta}{(2\cos\theta-1)^2 + 4\sin^2\theta}\right) = \frac{4}{9}\left(\frac{4-4\cos\theta+\cos^2\theta+\sin^2\theta}{4\cos^2\theta - 4\cos\theta + 1 + 4\sin^2\theta}\right) =$

$\frac{4}{9}\left(\frac{5-4\cos\theta}{5-4\cos\theta}\right) = \frac{4}{9}.$

$\left|w - \frac{1}{3}\right| = \frac{2}{3}$, cirkel med centrum i $\frac{1}{3}$ och $r = \frac{2}{3}$



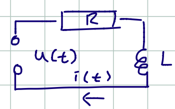
Cirkel med centrum a och radie r.

$z - a = re^{i\theta}$

$z = a + re^{i\theta}$

Komplexa impedanser

$U = Ae^{j\omega t}$, $j^2 = -1$



$u(t) = Ri(t) + j\omega L \cdot i(t)$

konstant fasforskjutning

$$i(t) = \frac{u(t)}{R+j\omega L} = \frac{Ae^{j\omega t}}{R+j\omega L} = \frac{Ae^{j\omega t}}{\sqrt{R^2+\omega^2L^2}\cdot e^{j\arctan(\frac{\omega L}{R})}} = \underbrace{\frac{A}{\sqrt{R^2+\omega^2L^2}}}_{\text{Frekvensberoende amplitud}} \cdot e^{j(\omega t - \arctan\frac{\omega L}{R})}$$

Samma frekvens

Taylorpolynom

DEF

Taylorpolynomet p av grad n i a till $f(x)$ är det polynom som uppfyller $p^{(i)}(a) = f^{(i)}(a)$ $i = 0,1,2,\ldots,n$

Ex $n=3$

$P(x) = C_3(x-a)^3 + C_2(x-a)^2 + C_1(x-a) + C_0$

$p(a) = C_0 = f(a)$

$P'(x) = 3C_3(x-a)^2 \cdot 1 + 2C_2(x-a) + C_1$

$P'(a) = C_1 = f'(a)$

$P''(x) = 6C_3(x-a) + 2C_2$

$P''(a) = 2C_2 = f''(a)$

$P'''(x) = 6C_3$

$P'''(a) = 6C_3 = f'''(a)$

$$P(x) = \frac{f'''(a)}{3\cdot 2}(x-a)^3 + \frac{f''(a)}{2}(x-a)^2 + f'(a)(x-a) + f(a)$$

Allmänt:

$$P(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + \ldots + \frac{f^{(k)}(a)(x-a)^k}{k!} + f^{(n)}(a)\frac{(x-a)^n}{n!}$$

Felterm

integrations-konstant

$$f(x) - f(a) = \int_a^x f'(t)\,dt = \int_a^x 1\cdot f'(t)\,dt = \Big[(t-x)f'(t)\Big]_a^x - \int_a^x (t-x)f''(t)\,dt = 0 - (a-x)\cdot f'(a) - \left[\frac{(t-x)^2}{2}\cdot f''(t)\right]_a^x + \int_a^x \frac{(t-x)^2}{2} f'''(t)\,dt =$$

$$(x-a)f'(a) - 0 + \frac{(a-x)^2}{2}f''(a) + \left[\frac{(t-x)^3}{3\cdot 2}\cdot f'''(t)\right]_a^x - \int_a^x \frac{(t-x)^3}{3\cdot 2} f'''(t)\,dt = (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + 0 - \frac{(a-x)^3}{3!}f'''(a) - \left[\frac{(t-x)^4}{4!}f^{IV}(t)\right]_a^x + \int_a^x \frac{(t-x)^4}{4!}f^{IV}(t)\,dt =$$

$$(x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \frac{(x-a)^3}{3!}f'''(a) - 0 + \frac{(a-x)^4}{4!}f^{IV}(a) + \ldots + \frac{(x-a)^k}{k!}f^{(k)}(a) + \underbrace{(-1)^k \int_a^x \frac{(t-x)^k}{k!} f^{(k+1)}(t)\,dt}_{\text{restetermen i integralform}}$$

Vi skall senare visa Lagranges resterm:

$$(-1)^k \int_a^x \frac{(t-x)^k}{k!} f^{(k+1)}(t)\,dt = \frac{(x-a)^k}{(k+1)!} f^{(k+1)}(\xi) \quad , \; a < \xi < x$$