P6)

5. $16x^4-8x^2+1=0$

Sätt $x^2=u$

$16u^2-8u+1=0$

$u^2-\frac{1}{2}u+\frac{1}{16}=0$

$u=\frac{1}{4}\pm\sqrt{\frac{1}{16}-\frac{1}{6}}$

$\left.\begin{array}{l}u_1=\frac{1}{4}\\ u_2=\frac{1}{4}\end{array}\right\}$ $x_{1,2}=\pm\sqrt{\frac{1}{4}}=\frac{1}{2}$ dubbelrötter

Faktorisering av polynomee: $16x^4-8x^2+1=0$

$\Leftrightarrow$

$16\left(x-\frac{1}{2}\right)^2\left(x+\frac{1}{2}\right)^2$

7) $x^3+1=0$

$x^3=-1$

$x=-1$ är en rot

$$\begin{array}{l} \quad x^2-x+1 \\ \quad x^3+1 \;|\; x+1 \\ -(x^3+x^2) \\ \quad -x^2+1 \\ -(-x^2-x) \\ \quad x+1 \\ -(x+1) \\ \quad\quad 0 \end{array}$$

x^2-x+1

$x=\frac{1}{2}\pm\sqrt{\frac{1}{4}-\frac{4}{4}}$

$x=\frac{1}{2}\pm\sqrt{\frac{-3}{4}}=\frac{1}{2}\pm\sqrt{\frac{i\sqrt{3}}{2}}$

$x_1=-1$

$x_2=\frac{1}{2}+\sqrt{\frac{i\sqrt{3}}{2}}$

$x_3=\frac{1}{2}-\sqrt{\frac{i\sqrt{3}}{2}}$

$x^3+1=(x+1)(x^2-x+1)=(x+1)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$

13) $\frac{x^3-1}{x^2-2}$

$$\begin{array}{l} \quad x \\ \quad x^3-1 \;|\; x^2-2 \\ -(x^3-2x) \\ \quad 2x-1 \end{array}$$

$x+\frac{2x-1}{x^2-2}=x+\frac{2x-1}{(x-\sqrt{2})(x+\sqrt{2})}=x+\frac{A}{x-\sqrt{2}}+\frac{B}{x+\sqrt{2}}=x+\frac{A(x+\sqrt{2})+B(x-\sqrt{2})}{(x-\sqrt{2})(x+\sqrt{2})}=\frac{(A+B)x+(A-B)\sqrt{2}}{(x+\sqrt{2})(x-\sqrt{2})}$

$A+B=2$ $B=2-A$ $A=1+\frac{1}{2\sqrt{2}}$

$(A-B)\sqrt{2}=-1$ $2A-2=\frac{1}{\sqrt{2}}$ $B=2-\left(1+\frac{1}{2\sqrt{2}}\right)=1-\frac{1}{2\sqrt{2}}$

$\frac{x^3-1}{x^2-2}=x+\left(1+\frac{1}{2\sqrt{2}}\right)\frac{1}{x-\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right)\frac{1}{x+\sqrt{2}}$

19) $x^2+px+q=0$

$x=\frac{-p}{2}\pm\underbrace{\sqrt{\left(\frac{p}{2}\right)^2-q}}_{D}$ vad händer om $D<0$..? $\Longrightarrow$ $x=\frac{-p}{2}\pm i\sqrt{|D|}$

Om $z = u + iv$, definierar vi konjugatet $\bar{z} = u - iv$

SATS

Om p är ett polynom med reella koefficienter och $p(z)=0$ är även $p(\bar{z})=0$

SATS

$z = x + iy$, $w = u + iv$

$\overline{z+w} = \bar{z} + \bar{w}$

$\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

Bevis

$\overline{z+w} = \overline{x+iy+u+iv} = \overline{(x+u)+i(y+v)} = (x+u) - i(y+v)$

$\bar{z} + \bar{w} = x - iy + u - iv = (x+u) - i(y+v)$

$\overline{z \cdot w} = \overline{(x+iy)(u+iv)} = \overline{xu + iyu + ixv - yv} = xu - yv - i(yu + xv)$

$\bar{z}\,\bar{w} = (x-iy)(u-iv) = xu - iyu - ixv - yv = xu - yv - i(yu + xv)$

Bevisa att $P(\bar{z}) = 0$

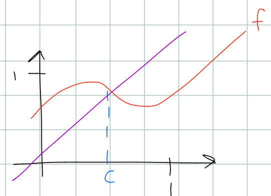
$P(z) = a_n z^n + a_{n-1} z^{n-1} + \dots + a_2 z^2 + a_1 z + a_0 \qquad a_k \in \mathbb{R}$

$\overline{P(z)} = \overline{a_n z^n + \dots + a_0} = \overline{a_n z^n} + \overline{a_{n-1} z^{n-1}} + \dots + \overline{a_2 z^2} + \overline{a_1 z} + \overline{a_0} = \bar{a}_n (\overline{z^n}) + \overline{a_{n-1}} (\overline{z^{n-1}}) + \dots + \bar{a}_1 \bar{z} + \bar{a}_0 = a_n (\bar{z}^n) + a_{n-1} (\bar{z}^{n-1}) + \dots + a_1 \bar{z} + a_0 = P(\bar{z})$

Om nu $P(z_0) = 0$ så är $p(\bar{z}_0) = \bar{0} = 0$

1.4 32)

$f: [0,1] \to [0,1]$ och är kontinuerlig. Då finns $c \in [0,1]$ s.a. $f(c) = c$



Bevis

Betrakta $g(x) = f(x) - x$

$g(0) = f(0) - 0 \geq 0$

$g(1) = f(1) - 1 \leq 0$

Antingen: $g(0) = 0$ (fix punkt)

$g(0) > 0$

då gäller antingen: $g(1) = 0$ (fixpunkt)

$f(1) = 1$

eller: $g(0) > 0$

$g(1) < 0$

Enligt satsen om mellanliggande värden

$\exists c$ s.a. $g(c) = 0$

$f(c) = c$ (fix punkt)

In mathematics, a **fixed point** (sometimes shortened to **fixpoint**, also known as an **invariant point**) of a function is an element of the function's domain that is mapped to itself by the function. A set of fixed points is sometimes called a *fixed set*. That is to say, *c* is a fixed point of the function *f*(*x*) if and only if *f*(*c*) = *c*. This means *f*(*f*(...*f*(*c*)...)) = $f^n(c) = c$, an important terminating consideration when recursively computing *f*. For example, if *f* is defined on the real numbers by

$$f(x) = x^2 - 3x + 4,$$

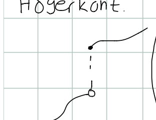
then 2 is a fixed point of *f*, because *f*(2) = 2.

34)

f udda dvs. $f(-x) = -f(x) \ \forall x$ speciellt $x = 0$ $f(0) = -f(0)$, $f(0) = 0$

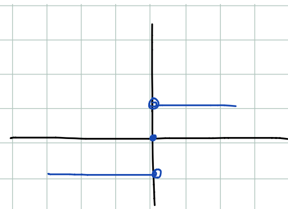
Om f är högerkontinuerlig, dvs $\lim_{\substack{x \to a \\ x > 0}} f(x) = f(a)$ $\left(\lim_{x \to a^+} f(x), \lim_{x \downarrow a} f(x)\right)$ $a = 0$, så är f i själva verket kontinuerlig.

Högerkont.



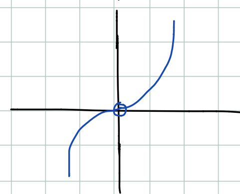
Ty: Låt $x < 0$ $\quad \lim_{x \to 0^-} f(x) \underset{\text{Ty udda}}{=} \lim_{x \to 0} -f(-x) = \lim_{y \to 0^+} -f(y) \underset{\text{Ty högerkont}}{=} -f(0) = 0$

Def $\operatorname{sgn} x = \begin{cases} 1, & x>0 \\ -1, & x<0 \\ (0, & x=0) \end{cases}$



$x = |x| \operatorname{sgn} x$

$g(x) = x^2 \operatorname{sgn} x = \begin{cases} x^2, & x>0 \\ -x^2, & x<0 \\ ?, & x=0 \end{cases}$



g blir kont om $g(0)=0$ dvs. $\operatorname{sgn} 0$ spelar ingen roll

Är g deriverbar i 0?
Vi gissar att $g'(0)=0$

Bevis
$\left|\frac{g(x)-g(0)}{x}\right| = \left|\frac{g(x)}{x}\right| = \frac{x^2}{|x|} = |x| \to 0$ när $x \to 0$