Def

Let X be a continuous r.v with density f. Then expected value of X is $E[X]=\int_{-\infty}^{+\infty} x \cdot f(x)dx$

Ex

$X \sim Uni[a,b]$, $f(x)=\begin{cases} \frac{1}{b-a}, & a \leqslant x \leqslant b \\ 0, & \text{otherwise} \end{cases}$

$E[X]=\int_a^b \frac{x}{b-a}dx = \frac{x^2}{2(b-a)}\Big|_a^b = \frac{b+a}{2}$

Def

Let X be a continuous r.v with density f. Then expected value of r.v. $H(x)$, denoted by $E[H(X)]$ is given by $E[H(X)]=\int_{-\infty}^{+\infty} H(x)f(x)dx$.

Ex

$H(x)=x^2$, $X \sim Uni[a.b]$

$E[X^2]=\int_a^b x^2 \cdot \frac{1}{b-a}dx = \frac{x^3}{3(b-a)}\Big|_a^b = \frac{a^2+ab+b^2}{3}$

Def

Let X be a continuous r.v with density f and mean $E[X]$.

$Var[X]=E[X^2]-E^2[X]$

Ex

$Var[X]=E[X^2]-E^2[X]=\frac{a^2+ab+b^2}{3}-\left(\frac{a+b}{2}\right)^2=\frac{a^2-2ab+b^2}{12}=\frac{(a-b)^2}{12}=\frac{(b-a)^2}{12}$

Def

A r.v. X with density $f(x)=\frac{1}{\sqrt{2\pi\cdot\sigma^2}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, where $-\infty<\mu<+\infty$, $\sigma^2>0$ have a normal distribution with parameters μ and σ^2.

Remark

$X \sim N(\mu,\sigma^2)$

$E[X]=\mu$

$Var[X]=\sigma^2$

Pic

Expectations: I: 0 Variations: $\sigma_{II}^2 > \sigma_I^2$

II: 0 $\sigma_{III}^2 = \sigma_I^2$

III: $\mu_3 > 0$

$X_{III} = X_I + \mu_3$

Def

A r.v with parameters $\mu=0$, $\sigma^2=1$ is denoted by Z and is called Standard normal random variable.

Proposition

Let $X \sim N(\mu,\sigma^2)$. The variable $\frac{X-\mu}{\sqrt{\sigma^2}}$ is standard normal.

Theorem

Let $X \sim Bin(n,p)$ $\quad P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}, \quad k=0,1,\dots n$

For large values of n, X is approximately normal with mean np and variance $\underbrace{np(1-p)}_{q}$

Ex

Rolling a dice 108 times. X = number of ones after rolling.

$X \sim Bin(108,\frac{1}{6})$, $\quad E[X]=108\cdot\frac{1}{6}=18$, $\quad Var[X]=108\cdot\frac{1}{6}(1-\frac{1}{6})=15$

1. Exact calculations.

$$P(12 \leq X \leq 20)=\sum_{i=12}^{20}P(X=i)=\sum_{i=12}^{20}\binom{108}{i}\left(\frac{1}{6}\right)^i\left(\frac{5}{6}\right)^{n-i}$$

2. Normal approximation, for $n>25$

approx $N(0,1)$ where F is a cdf of $N(0,1)$

$$P(12 \leq X \leq 20) \approx P(11.5 \leq X \leq 20.5)=P\left(\frac{11.5-18}{\sqrt{15}} \leq \frac{X-18}{\sqrt{15}} \leq \frac{20.5-18}{\sqrt{15}}\right)=F(0.645)-F(-1.678)=0.691$$

Reminder

$$\Phi(x)=P(Z \leq x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du$$

cdf for Z

Kika i VLE-Tables

Def

Let X and Y be discrete r.v's, defined on the same sample space. The ordered pair (X,Y) is called a two-dimensional (bivaritive) discrete random variable.

A function $f_{XY}(x,y)=P(X=x, Y=y)$ is called the joint density for (X,Y).

Properties

1. $f(x,y) \geq 0$, all x,y.
2. $f(x,y)>0$ only in a countable number of points.
3. $\sum_{all\,x}\sum_{all\,y} f(x,y)=1$

EX

Three coin tossings with a fair coin, i.e. $P(Heads)=P(Tails)=\frac{1}{2}$

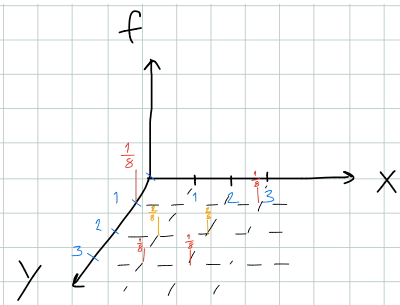
$X=\{\text{number of heads}\}$
$Y=\{\text{number of runs}\}$

"run"
maximal number of consequtive coin flips that are the same.

Sample Point	Number of heads	Number of runs
HHH	3	1
HHT	2	2
HTH	2	3
HTT	1	2
THH	2	2
THT	1	3
TTH	1	2
TTT	0	1

$X \in \{0,1,2,3\}$
$Y \in \{1,2,3\}$

	Number of heads			
	0	1	2	3
Number of runs 1	$\frac{1}{8}$	0	0	$\frac{1}{8}$
2	0	$\frac{2}{8}$	$\frac{2}{8}$	0
3	0	$\frac{1}{8}$	$\frac{1}{8}$	0



Def

Let X and Y be a bivaritive r.v with joint density f_{XY}. The marginal density for X is given by $f_X(x) = \sum_{\text{all } y} f(x,y)$

	Number of heads			
x→	0	1	2	3
Number of runs 1	$\frac{1}{8}$	0	0	$\frac{1}{8}$
2	0	$\frac{2}{8}$	$\frac{2}{8}$	0
3	0	$\frac{1}{8}$	$\frac{1}{8}$	0
P(X= x	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$