7.9-20

$y' + \cos(x) \cdot y = 2xe^{-\sin(x)}$, $y(\pi) = 0$

IF: $\int \cos(x)\,dx = \sin(x) + C$

$e^{\sin x} \cdot y' + e^{\sin x} \cdot \cos x \cdot y = 2xe^{\sin x - \sin x} = 2x$

$(e^{\sin x} \cdot y)' = 2x$

$e^{\sin x} \cdot y = \int 2x\,dx$

$e^{\sin x} \cdot y = x^2 + C$

$y = \frac{x^2 + C}{e^{\sin(x)}}$

$y(\pi) = \frac{\pi^2 + C}{1} = 0 \Rightarrow C = -\pi^2$

$y(x) = \frac{x^2 - \pi^2}{e^{\sin(x)}}$

7.9-22

$y(x) = 1 + \int_0^x \frac{(y(t))^2}{1+t^2}\,dt$

$y'(x) = 0 + \frac{y(x)^2}{1+x^2}$

$\frac{dy}{dx} = \frac{y^2}{1+x^2}$

$\frac{1}{y^2}\,dy = \frac{1}{1+x^2}\,dx$

$-\frac{1}{y} = \arctan(x) + C$

$-1 = y(\arctan(x) + C)$

$y = \frac{-1}{\arctan(x) + C}$

$y(0) = 1 + \int_0^0 \frac{y(0)^2}{1+0^2}\,dt = 1$

$\frac{-1}{\arctan(0) + C} = \frac{-1}{C} = 1$

$C = -1$

$y(x) = \frac{-1}{\arctan(x) - 1}$

17.5-15

$x^2 y'' - xy' + 2y = 0$, $x > 0$

Substituera $t = \ln x$

$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = 0$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot \frac{1}{x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dt} \cdot \frac{1}{x}\right) = \left(\frac{d}{dx} \cdot \frac{dy}{dt}\right)\frac{1}{x} - \frac{dy}{dt} \cdot \frac{1}{x^2} = \frac{d^2y}{dt^2} \cdot \frac{1}{x} \cdot \frac{1}{x} - \frac{dy}{dt} \cdot \frac{1}{x^2}$

$x^2 \cdot \frac{d^2y}{dt^2} - x \frac{dy}{dx} + 2y = \frac{d^2y}{dt^2} - \frac{dy}{dt} - \frac{dy}{dt} + 2y = 0$

KE: $r^2 - 2r + 2 = 0 \Rightarrow r = 1 \pm \sqrt{1-2} = 1 \pm i$

$y(t) = e^t(A\cos t + B\sin t)$

$y(x) = x(A\cos(\ln x) + B\sin(\ln x))$

17.6-4

$y'' + y' - 2y = e^x$

Homo: $y'' + y' - 2y = 0$

KE: $r^2 + r - 2 = 0$

$r = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}} = -\frac{1}{2} \pm \frac{3}{2} \Rightarrow \begin{matrix} r_1 = -2 \\ r_2 = 1 \end{matrix}$

Homogena lösn: $y(x) = Ae^x + Be^{-2x}$

Obs att $y = c \cdot e^x$ inte kan vara en partikulär lösning ty den är del av den homogena lösningen.

Sätt $y = cxe^x$

$y = cxe^x$

$y' = ce^x + cxe^x$

$y'' = ce^x + ce^x + cxe^x$

$y'' + y' - 2y = 2ce^x + cxe^x + ce^x + cxe^x - 2cxe^x = e^x$

$3ce^x + xe^x(c + c - 2c) = e^x$

$3ce^x = e^x$

$c = \frac{e^x}{3e^x} = \frac{1}{3} \Rightarrow y(x) = \frac{1}{3}xe^x + Ae^x + Be^{-2x}$

17.6-9

$y''+2y'+2y = e^{x}\sin(x)$

KE: $r^2+2r+2=0$

$r = -1 \pm \sqrt{1-2} = -1 \pm i$

Homo: $y(x) = e^{-x}(A\cos(x) + B\sin(x))$

Partikulärlösning:

Sätt: $y = e^{x}(a\cos x + b\sin x)$

$y' = e^{x}(a\cos x + b\sin x - a\sin x + b\cos x) = e^{x}((a+b)\cos x + (b-a)\sin x)$

$y'' = e^{x}((a+b)\cos x + (b-a)\sin x - (a+b)\sin x + (b-a)\cos x) = e^{x}(2b\cos x - 2a\sin x)$

$y''+2y'+2y = e^{x}(2b\cos x - 2a\sin x + 2(a+b)\cos x + 2(b-a)\sin x + 2a\cos x + 2b\sin x) = e^{x}((4b+4a)\cos x + (4b-4a)\sin x) = e^{x}\sin x$

$\begin{cases} 4b+4a=0 \\ 4b-4a=1 \end{cases}$ $\quad 8b=1 \Rightarrow b=\frac{1}{8} \Rightarrow a=-\frac{1}{8}$

Allmän lösning: $y(x) = e^{x}\frac{1}{8}(\sin x - \cos x) + e^{-x}(A\cos x + B\sin x)$

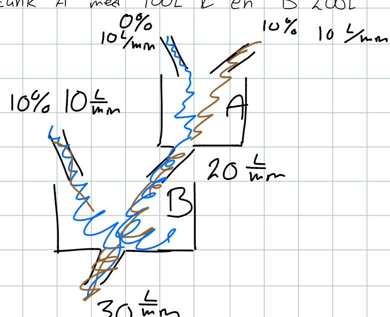
17.6-10

$y''+2y'+2y = e^{-x}\sin x$

Samma VL som innan. Vi vet att vi kommer få resonans $\Rightarrow$ $y = e^{-x}\cdot x(a\cos x + b\sin x)$

Veckoblad

En vattentank A med 100L & en B 200L



Koncentrationerna i A resp B $x(t)$ resp $y(t)$

Smutsbalans i A: $100x'(t) = 10\cdot\frac{1}{10} - 20x$

$x'(t) = \frac{1}{100} - \frac{x}{5}$

$x' + \frac{x}{5} = \frac{1}{100}$

IF: $e^{0,2x}$

$e^{0,2t}x' + 0,2e^{0,2t}x = \frac{e^{0,2t}}{100}$

$(e^{0,2t}x)' = \frac{e^{0,2t}}{100}$

$e^{0,2t}x = \frac{e^{0,2t}}{20} + C$

$x(0)=0 = \frac{1}{20} + C \Rightarrow C = -\frac{1}{20}$

$x = \frac{1}{20} - \frac{e^{-0,2t}}{20}$

$\lim_{t\to\infty} x(t) = \frac{1}{20}$ $\quad \left(\frac{1}{2}\cdot 10\%\right)$

$\lim_{t\to\infty} y(t) = \frac{1}{15}$ $\quad \left(\frac{2}{3}\cdot 10\%\right)$ $\quad \frac{2}{3}\cdot\frac{1}{10} = \frac{1}{15}$

Smutsbalans i B: $200y' = 10\cdot\frac{1}{10} + x\cdot 20 - y\cdot 30$

$200y' + 30y = 1 + 20x = 1 + 1 - e^{-0,2t}$

$200y' + 30y = 2 - e^{-0,2t}$

$y' + \frac{3}{20}y = \frac{1}{100} - \frac{1}{200}e^{-0,2t}$

IF: $e^{\frac{3}{20}t} = e^{0,15t}$

$e^{0,15t}y' + 0,15e^{0,15t}y = \frac{e^{0,15t}}{100} - \frac{1}{200}e^{-0,05t}$

$(e^{0,15t}y)' = \frac{e^{0,15t}}{100} - \frac{1}{200}e^{-0,05t}$

$e^{0,15t}y = \frac{e^{0,15t}}{15} + \frac{e^{-0,05t}}{10} + C$

$y(0)=0 = \frac{1}{15} + \frac{1}{10} + C \Rightarrow C = \frac{-5}{30} = \frac{-1}{6}$

$y = \frac{1}{15} + \frac{e^{-0,2t}}{10} - \frac{e^{-0,15t}}{6}$

Sönderfall

$A \to B + C$ N atomer av A $N(t) = N(0) \cdot e^{-2kt}$

$B \to D + E$ y atomer av B, $y(0) = 0$ B sönderfaller enl. $y' = -ky$

Vi får alltså att $y' = -ky - N'(t)$, $N'(t) = N(0)e^{-2kt}(-2k)$

$y' = -ky + 2k \cdot N(0)e^{-2kt}$

$y' + ky = 2kN(0)e^{-2kt}$

IF: e^{kt}

$(e^{kt} \cdot y)' = 2kN(0)e^{-2kt}$

$e^{kt} \cdot y = \frac{-2kN(0)e^{-th}}{k} + C$

$y(0) = 0 \Rightarrow \quad 0 = -2N(0) + C \quad \Rightarrow \quad y = -2N(0)e^{-2kt} + 2N(0)e^{-kt}$

$y = 2N(0)(e^{-kt} - e^{-2kt})$

y har maximum där $y' = 0$

$-k \cdot e^{-kt} + 2k \cdot e^{-2kt} = 0$

$-1 + 2e^{-kt} = 0$

$e^{-kt} = \frac{1}{2}$

$t = \frac{-(\ln \frac{1}{2})}{k} = \frac{\ln 2}{k}$