Signalmanipulering (Signaltransformering)

- Amplitudskalning
  $y(t) = a\,x(t) + b$

  $y[n] = a\,x[n] + b$ ← "Skift" (DC-Skift)

  ↑ Förstärkning eller dämpning

  a & b är konstanter

- Ändra tidsskala
  $y(t) = x(at) \quad a \in \mathbb{R}$

  $y[n] = x[kn] \quad x, k \in \mathbb{Z}$

- Spegling
  $y(t) = x(-t) \quad \forall t$

  $y[n] = x[-n]$

- Skift
  $y(t) = x(t - t_0) \quad t_0$ konstant

  $y[n] = x[n - n_0] \quad n_0$ konst

Samtidiga Manipulationer

$x(t)$

Ändra tidsskala $t \to at \Rightarrow x(at)$

Skifta $t \to t - t_0 \Rightarrow x(a(t - t_0))$

Resultat: $x(at - at_0)$

$x(t)$

Skifta $t \to t - t_0 \Rightarrow x(t - t_0)$

Ändra tidsskala $t \to at \Rightarrow x(at - t_0)$

Ordning spelar roll!

Signalmodeller

Kontinuerliga

- Komplex exponential
  $x(t) = C \cdot e^{at} \quad C, a \in \mathbb{C} \Rightarrow x \in \mathbb{C}$

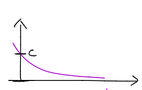
  Komplexa signaler förekommer ej i fysikaliska system men är mycket användbara som matematiska modeller.

  Den fysikaliska signalen kan t.ex. fås som $\mathrm{Re}\{x(t)\}$.

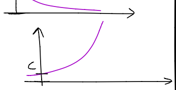
Fall 1 $C, a \in \mathbb{R}$

$x(t) = C \cdot e^{at}$

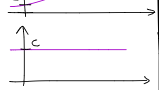
Om $a < 0$



Om $a > 0$



Om $a = 0$



Diskreta

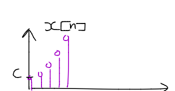
- Komplex exponential

  $x[n] = C \cdot a^n \quad C, a \in \mathbb{C} \Rightarrow x \in \mathbb{C}$
  $n \in \mathbb{Z}$

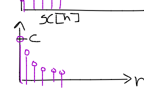
Fall 1 $C, a \in \mathbb{R}$

$x[n] = C \cdot a^n$

Låt $a > 1$



Låt $0 < a < 1$



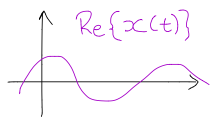
Låt $a < 0$ Teckenväxling

Fall 2 $C, a \in \mathbb{C}$ $\quad \mathrm{Re}\{a\}=0$

Låt $a = j\omega_0$, $C = Ae^{j\Phi}$

$$x(t) = Ae^{j\Phi}\cdot e^{j\omega_0 t} = Ae^{j(\omega_0 t+\Phi)} = A\cos(\omega_0 t+\Phi) + Aj\sin(\omega_0 t+\Phi)$$

"Odämpad sinusformad signal"

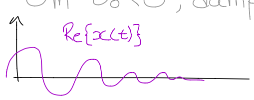


Fall 3 $C, a \in \mathbb{C}$

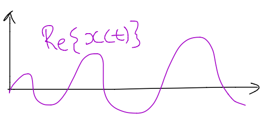
Låt $C = Ae^{j\Phi}$
$a = \sigma_0 + j\omega_0$

$$x(t) = C\cdot e^{at} = Ae^{j\Phi}\cdot e^{(\sigma_0+j\omega_0)t} = Ae^{\sigma_0 t}\cdot e^{j(\omega_0 t+\Phi)} = Ae^{\sigma_0 t}(\cos(\omega_0 t+\Phi) + j\sin(\omega_0 t+\Phi))$$

Om $\sigma_0 < 0$, dämpad sinusformad svängning



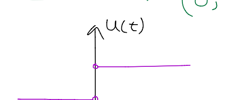
Om $\sigma_0 > 0$, förstärkt sinusformad svängning



Om $\sigma_0 = 0 \Rightarrow$ Fall 2

α Enhetssteg (kontinuerlig)

Def $u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$



Vi låter $u(0)$ vara odef.
Det finns de som definierar $u(0)$ som 0, 1 eller $\frac{1}{2}$.

α Enhetsimpuls
(Ingen vanlig funktion, den sorterar under distributionen.)

Beskrivning
$\delta(t) = 0, \quad t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

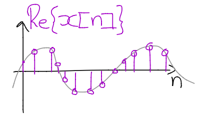
Amplituden vid $t=0$ är obegränsad

Fall 2 $C, a \in \mathbb{C}$

Låt $|a| = 1$
Låt $a = e^{j\Omega_0}$
$C = Ae^{j\Phi}$

$$x[n] = Ae^{j\Phi}\cdot e^{j\Omega_0 n} = Ae^{j(\Omega_0 n+\Phi)} = A\cdot\cos(\Omega_0 n+\Phi + jA\sin(\Omega_0 n+\Phi)$$

Diskret odämpad sinusformad signal



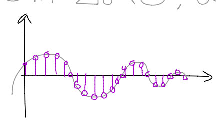
Fall 3 $C, a \in \mathbb{C}$

Låt $C = Ae^{j\Phi}$
$a = e^{\Sigma_0}\cdot e^{j\Omega_0} = e^{\Sigma_0 + j\Omega_0}$

$$x[n] = Ae^{j\Phi}\cdot e^{(\Sigma_0+j\Omega_0)n} = Ae^{\Sigma_0 n}\cdot e^{j(\Omega_0 n+\Phi)} = A\cdot e^{\Sigma_0 n}(\cos(\Omega_0 n+\Phi) + j\sin(\Omega_0 n+\Phi))$$

Diskret sinusformad svängning med n-beroende amplitud

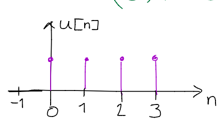
Om $\Sigma_0 < 0$, amplitud minskar



Om $\Sigma_0 > 0$, amplitud ökar

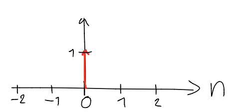
α Enhetssteg (diskret)

$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n<0 \end{cases}$, $\quad u[n] = (\ldots, 0, 0, \underline{1}, 1, \ldots)$
$n=0$



α Diskret enhetsimpuls

$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

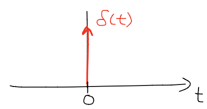


Samband

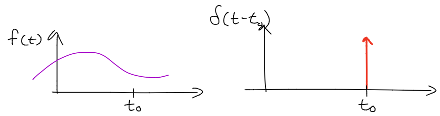
$u[n] = \sum_{-\infty}^{n} \delta[k]$

$\delta[n] = u[n] - u[n-1]$

Grafisk beskrivning



Enhetsimpulsen definieras genom sina egenskaper. Låt f(t) vara en godtycklig funktion (signal) som är kontinuerlig för $t=t_0$.



$\Rightarrow$

$$f(t)\cdot\delta(t-t_0)=f(t_0)\cdot\delta(t-t_0)$$

vidare gäller även

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt=\int_{-\infty}^{\infty} f(t_0)\,\delta(t-t_0)\,dt=f(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)\,dt$$

$$=f(t_0)\cdot 1=f(t_0)$$

Samband: $u(t)=\int_{-\infty}^{t}\delta(\tau)\,d\tau$

$\delta(t)=\frac{du(t)}{dt}$

Repetition

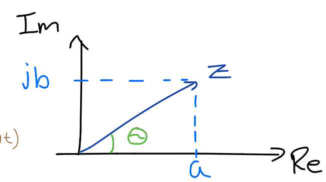
Eulers formel.

Polar form: $e^{j\theta}=\cos(\theta)-j\sin(\theta)$

$Ae^{j\omega t}=A\cos(\omega t)-jA\sin(\omega t)$

$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}$

$j\sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2}$



$Z=\sqrt{a^2+b^2}$

$\theta=\arctan\left(\frac{b}{a}\right)$ om $a>0$

$\theta=\arctan\left(\frac{b}{a}\right)+\pi$ om $a<0$