## Recap Fourier

$\omega_0$ = fundamental frequency $= \frac{2\pi}{T_0}$   $T_0$ = fundamental period

$x(t)$ is a reel periodic signal, then $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 kt}$   $C_k = \overline{C_{-k}} = C_{-k}^*$

$C_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 kt} dt$   p160

## Periodic signal

$x(t) = x(t+T) = x(t+nT)$

To find a signals common period: $x(t) = x_1(t) + x_2(t)$   $T = m_1 T_1 = m_2 T_2$ integer

Period of $x_1$   Period of $x_2$

## Orthogonality

$g(t), h(t)$ are orthagonal over interval $(a,b)$ if $\int_a^b g(t)h(t) = 0$

$\text{Sinc}(\omega t) := \frac{\text{sinc}(\omega t)}{\omega t}$   $\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2i}$   $\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$

## 5.1 a

**5.1**[57] Consider the Fourier series for the periodic functions given.

**(i)** $x(t) = \sin(4t) + \cos(8t) + 7 + \cos(16t)$
**(ii)** $x(t) = \cos^2(t)$
**(iii)** $x(t) = \cos(t) + \sin(2t) + \cos(3t - \pi/3)$
**(iv)** $x(t) = 2\sin^2(2t) + \cos(4t)$
**(v)** $x(t) = \cos(7t)$
**(vi)** $x(t) = 4\cos(t)\sin(4t)$

a) Find the Fourier coefficients of the exponential form for each signal.

b) Find the Fourier coefficients of the combined trigonometric form for each signal.

Does not depend on t

i) Can be written as: $x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$   $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4} = \frac{\pi}{2}$   $T_3 = \frac{2\pi}{\omega_3} = X$

$\omega_1 = 4$   $\omega_2 = 8$   $\omega_3 = 0$   $\omega_4 = 16$   $T_2 = \frac{2\pi}{\omega_2} = \frac{\pi}{4}$   $T_4 = \frac{2\pi}{\omega_4} = \frac{\pi}{8}$

Common Period $T_0 = m_1 T_1 = m_2 T_2 = m_4 T_4$   $m_1, m_2, m_4 \in \mathbb{N}$

$m_1 = 1 \Rightarrow T_0 = \frac{\pi}{2}$   We were able to find integers that works => periodic signals

$m_2 = 2$   $\omega_0 = \frac{2\pi}{T_0} = 4 \ \frac{\text{rad}}{s}$

$m_4 = 4$

$$\text{i)} = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 kt} = \frac{1}{2j}\left(\underbrace{e^{j\omega_0 t}}_{k=1} - \underbrace{e^{-j\omega_0 t}}_{k=-1}\right) + \frac{1}{2}\left(\underbrace{e^{j2\omega_0 t}}_{k=2} + \underbrace{e^{-j2\omega_0 t}}_{k=-2}\right) + \underbrace{7}_{k=0} + \frac{1}{2}\left(\underbrace{e^{j4\omega_0 t}}_{k=4} + \underbrace{e^{-j4\omega_0 t}}_{k=-4}\right) =$$

| k | 0 | 1 | -1 | 2 | -2 | 3 | -3 | 4 | -4 |
|---|---|---|---|---|---|---|---|---|---|
| $C_k$ | 7 | $\frac{1}{2j}$ | | | | | | | |

$$k = 1 \Rightarrow C_1 e^{j\omega_0 t} = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) = \frac{1}{2i} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} \Rightarrow C_1 = \frac{1}{2j}$$

Handledarn bah: ˜Jag vet inte om jag gjort rätt... återkommer nästa vecka.

iv) $x(t) = 2\sin^2(2t) + \cos(4t)$

$\cos(2t) = 1 - 2\sin^2(t) \Rightarrow 2\sin^2(t) = 1 - \cos(2t) \Rightarrow x(t) = 1 - \cos(4t) + \cos(4t) = 1 = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 kt}$

for $k = 0 \rightarrow C_0 = 1$

all other $k$: $C_k = 0$

5.2 a)

**5.2**[58]

**(a)** Determine whether the following functions can be represented by a Fourier series

**(i)** $x(t) = \cos(3t) + \sin(5t)$

**(ii)** $x(t) = \cos(6t) + \sin(8t) + e^{j2t}$

**(iii)** $x(t) = \cos(t) + \sin(\pi t)$

**(iv)** $x_3(t) = x_1(t) + x_2(t)$ where $x_1(t) = \sin\left(\frac{\pi t}{6}\right)$ and $x_2(t) = \sin\left(\frac{\pi t}{9}\right)$

**(b)** For those signals in part (a) that can be represented by a Fourier series, find the coefficients of all harmonics, expressed in exponential form.

$x(t) = \cos(6t) + \sin(8t) + e^{j2t}$

Is the signal periodic?

| Signal | $\cos(6t)$ | $\sin(8t)$ | $e^{j2t}$ |
|---|---|---|---|
| Freq | $\omega_1 = 6$ | $\omega_2 = 8$ | $2$ |
| Period | $T_1 = \frac{\pi}{3}$ | $T_2 = \frac{\pi}{4}$ | $T_3 = \pi$ |

$T_0 = \underbrace{m_1}_{3} \frac{\pi}{3} = \underbrace{m_2}_{4} \frac{\pi}{4} = \underbrace{m_3}_{1} \pi = \pi$ Periodic signal

Gick i pausen.