Tenta 2015-04-15, uppg 1

$G_c(S) = \frac{K_p}{S^2 + K_d S + K_p}$

1: $K_p = 2$, $K_d = 1$

2: $K_p = 1$, $K_d = 2$

KE: $S^2 + K_d S + K_p = \left(S + \frac{K_d}{2}\right)^2 - \left(\frac{K_d}{2}\right)^2 + K_p = 0 \Rightarrow S = -\frac{K_d}{2} \pm \sqrt{\left(\frac{K_d}{2}\right)^2 - K_p} \Rightarrow$

1: $S = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$

2: $S = -1 \pm \sqrt{1-1} = -1$

2 har reella poler $\Rightarrow$ Plot C

1 stegsvar A: $K_p = K_d = 1$ $\quad S^2 + 2\xi \omega_n S + \omega_n^2 = 0 \Rightarrow \omega_n = \sqrt{K_p} = 1$

$\xi = \frac{K_d}{2\omega_n} = \frac{1}{2 \cdot 1} = \frac{1}{2}$

För 1: $\xi = \frac{K_d}{2\omega_n} = \frac{1}{2\sqrt{2}} \approx 0.35 < 0.5$

Vi har alltså lägre dämpning i 1 än i plot A $\Rightarrow$ 1 matchar B.

Uppgift 4

Givet

Massa m med pos: $y(t)$

$m\ddot{y}(t) = u(t) - ky(t) - b\dot{y}(t)$ $\quad$ k: fjäder, b: friktion, u: yttre kraft

$y(t) = 0$ är massans viloläge då $u(t) = 0$

$m = k = 1$

a) $x_1(t) = y(t)$, $x_2(t) = \dot{y}(t)$ $\Rightarrow$

$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$

$\dot{x}_2(t) = \ddot{y}(t) = \frac{1}{m}(u(t) - ky(t) - b\dot{y}(t)) = u(t) - x_1(t) - bx_2(t)$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

QED

b) $b = 0.5$

$u(t) = -L_u x(t) + K_r \cdot r(t)$

Vi vill ha en dubbelpol i -2. & $L_u = [l_1 \quad l_2]$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = \begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\begin{bmatrix} -l_1 & -l_2 \end{bmatrix} x + K_r \cdot r\right)$$

$$= \left(\begin{bmatrix} 0 & 1 \\ -1 & -b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -l_1 & -l_2 \end{bmatrix}\right) x + \begin{bmatrix} 0 \\ K_r \end{bmatrix} r = \underbrace{\begin{bmatrix} 0 & 1 \\ -1-l_1 & -b-l_2 \end{bmatrix}}_{A-BL_u} x + \underbrace{\begin{bmatrix} 0 \\ K_r \end{bmatrix}}_{BK_r} r$$

$$\det(SI - (A - BL_u)) = 0$$

$$\det\left(\begin{bmatrix} S & -1 \\ 1+l_1 & S+b+l_2 \end{bmatrix}\right) = S(S+b+l_2) - (-1)(1+l_1) = S^2 + (b+l_2)S + 1 + l_1 = 0$$

dubbelpol i -2: $(S+2)^2 = S^2 + 4S + 4 \Rightarrow$

$$\left.\begin{array}{l} b + l_2 = 4 \; = 4 - b = 4 - 0.5 = 3.5 \\ 1 + l_1 = 4 \Rightarrow l_1 = 3 \end{array}\right\} L_u = [3 \quad 3.5]$$

c) $R(s) = \frac{1}{s}$

Vad blir $y(t)$ när massan m ställt in sig i sin nya position?

Vad är ett lämpligt värde på Kr?

$$Y(s) = G(s)R(s)$$

$$\lim_{t\to 0} y(t) = \lim_{s\to 0} s \cdot G_{ry}(s) \cdot R(s) = \lim_{s \to 0} \cancel{s} \cdot G_{ry}(s) \frac{1}{\cancel{s}} = G_{ry}(0)$$

Bestäm Kr så att $G_{ry}(0) = 1$ $\qquad y(t) \xrightarrow[t\to\infty]{} r(t) \Rightarrow y(t) \xrightarrow[t\to\infty]{} 1$ ($r=1$)

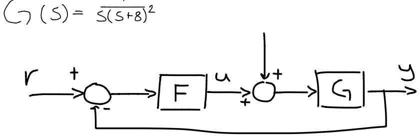
$$G_{ry}(s) = C(sI-(A-BL_u))^{-1} BK_r = \begin{bmatrix} 1 & 0 \end{bmatrix} \left(\frac{1}{\det(sI-(A-BL_u))} \begin{bmatrix} s+b+l_2 & 1 \\ -1-l_1 & s \end{bmatrix} \right) \begin{bmatrix} 0 \\ K_r \end{bmatrix} =$$

$$\frac{1}{s^2+4s+4} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} K_r \\ sK_r \end{bmatrix} = \frac{K_r}{s^2+4s+4}$$

$$G_{ry}(0) = \frac{K_r}{4} = 1 \Rightarrow K_r = 4 \Rightarrow u(t) - [3 \quad 3.5]x(t) + 4r(t)$$

<u>Tentanen 2013-08-22 Uppg 4</u>

$G(s) = \frac{1}{s(s+8)^2}$



<u>Givet</u>

Bort med kvarstående fel efter en stegstörning v.

$\varphi_m = 50°$

a) Sätt $F(s) = K_p\left(1 + \frac{1}{T_i \cdot s}\right)$

$\omega_c = 0.4\,\omega_{150}$, $\angle G(j\omega_{150}) = -150°$

$$\arg\{G(j\omega_{150})\} = -90° - 2\tan^{-1}\left(\frac{\omega_{150}}{8}\right) = -150° \Rightarrow \omega_{150} = 8\tan\left(\frac{150-90}{2}\right) \approx 4.62 \ \frac{rad}{s} \Rightarrow \omega_c \approx 1.85 \ \frac{rad}{s}$$

$$\varphi_m = 180° + \arg\{L(j\omega_c)\} \Rightarrow \arg\{F(j\omega_c)\} = 50° - 180° - \arg\{G(j\omega_c)\} \approx -14°$$

$$\arg\{F(j\omega_c)\} = \arg\left\{\frac{K_p(1+T_i j\omega_c)}{1+T_i j\omega_c}\right\} = \tan^{-1}(T_i\omega_c) - 90° = 14° \Rightarrow T_i = \frac{1}{\omega_c}\tan(90°-14°) \approx 2.17$$

$$|L(j\omega_c)| = 1$$

$$|L(j\omega_c)| = |F(j\omega_c)||G(j\omega_c)| = 1 \Rightarrow |F(j\omega_c)| = \frac{1}{|G(j\omega_c)|} = \frac{1}{\left|\frac{1}{j\omega_c(j\omega_c+8)^2}\right|} = \frac{1}{\frac{1}{\omega_c(\omega_c^2+8^2)}} \approx 124.73$$

$$|F(j\omega_c)| = \left|K_p\left(\frac{1+T_i j\omega_c}{T_i j\omega_c}\right)\right| = \frac{K_p\sqrt{(T_i\cdot\omega_c)^2+1}}{T_i\omega_c} = 124.73 \Rightarrow K_p \approx 121$$

b) $G(s) = \frac{1}{s(s+8)^2} \cdot e^{-T_d s}$ (T_d = fördröjning, bidrar med neg fasvridning)

$$\arg\{e^{-T_d j\omega}\} = -T_d\omega \cdot \frac{180}{\pi}$$

$\varphi_m = 50°$ blir noll om $-T_d\omega_c \cdot \frac{180}{\pi} = -50 \Rightarrow T_d = \frac{50\pi}{\omega_c 180} \approx 0.47s$