V1

Invertering av funktioner

Ex

$y = \frac{e^x - e^{-x}}{2}$ ($= \sinh(x)$)

$[e^x = u]$

$y = \frac{u - \frac{1}{u}}{2}$

$2y = u - \frac{1}{u}$

$u^2 - 2yu - 1 = 0$

$u = y \pm \sqrt{y^2+1}$

$\sqrt{y^2+1} > y$

$u = y + \sqrt{y^2+1}$ $(u>0)$

$x = \ln u = \ln(y + \sqrt{y^2+1})$

Ex

$x = \frac{1+y}{1-y}$

$x(1-y) = 1+y$

$x - yx = 1+y$

$x - 1 = y + yx = y(1+x)$

$y = \frac{x-1}{x+1}$

V2

Derivator

Kedje- och produktreglerna.

Ex

$f(x) = (x^2+1)(2x-1)$

$f'(x) = 2x(2x-1) + (x^2+1)2$

Ex

$f(x) = e^{x^2}$

$f'(x) = e^{x^2} \cdot 2x$

Ex

$f(x) = x \cdot e^{x^2}$

$f'(x) = e^{x^2} + x \cdot 2x \cdot e^{x^2} = e^{x^2} + 2x^2 e^{x^2}$

$f''(x) = 2x \cdot e^{x^2} + 4x \cdot e^{x^2} + 2x^2 \cdot 2x \cdot e^{x^2} = 6x e^{x^2} + 4x^3 \cdot e^{x^2}$

Ex

$f(x) = x \cdot e^{x^2 \cdot \sin(x)}$

$f'(x) = e^{x^2 \cdot \sin(x)} + x \cdot e^{x^2 \cdot \sin(x)} (2x \cdot \sin(x) + x^2 \cos(x))$

Ex

$f(x) = x^x = e^{\ln(x)^x} = e^{x \cdot \ln(x)}$

$f'(x) = e^{x \cdot \ln(x)} (\ln x + x \cdot \frac{1}{x}) = x^x (\ln(x+1))$

Gränsvärden

$\frac{\sin(x)}{x} \to 1$, $\frac{e^x - 1}{x} \to 1$, $x \to 0$

Dessa tolkas som derivator i 0. $\sin 0 = 0$

$e^0 = 1$

eller så nvänder vi Taylors formel:

$\frac{\sin(x)}{x} = \frac{x + O(x^3)}{x} = 1 + O(x^2) \to 1$

$\frac{e^x - 1}{x} = \frac{1 + x + O(x^2) - 1}{x} = 1 + O(x) \longrightarrow 1$

$\lim_{x \to 0} x \cdot \ln(x) = \left[y = \ln(x) \right] = \lim_{y \to -\infty} e^y \cdot y = 0$

Implicit derivering

Ex

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Sökt: $\frac{dy}{dx}$

Derivering map x.

$\frac{2x}{a^2} + \frac{2y \cdot y'}{b^2} = 0$

$\frac{2y \cdot y'}{b^2} = -\frac{2x}{a^2}$

$y' = -\frac{x b^2}{y a^2}$

Primitiva funktioner

Glöm inte:

$\int \frac{1}{x} dx = \ln|x|$

$\int \frac{1}{1+x^2} dx = \arctan x$

$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x$

$\int \frac{1}{x+2} dx = \ln|x+2|$

$\int \frac{1}{1-x^2} dx = \int \frac{1}{(1+x)(1-x)} dx = \int \frac{A}{1+x} + \frac{B}{1-x} dx = \int \frac{A(1-x) + B(1+x)}{(1-x)(1+x)} dx = \int \frac{A - Ax + B + Bx}{(1-x)(1+x)} dx = \left[\begin{matrix} A + B = 1 \\ B - A = 0 \end{matrix} = A = B = \frac{1}{2} \right] = \frac{1}{2} \int \frac{1}{1+x} + \frac{1}{1-x} dx =$

$\frac{1}{2}\left(\ln|1+x| \underset{\uparrow}{-} \ln|1-x| \right)$

inre derivatan

$\int \frac{1}{x^2 - x} dx = \int \frac{1}{x(x-1)} dx = \int \frac{A}{x} + \frac{B}{x-1} dx = \int -\frac{1}{x} + \frac{1}{x-1} dx = -\ln|x| + \ln|x-1| = \ln\left|\frac{x-1}{x}\right|$

Kvadratkomplettering

$\int \frac{1}{x^2 + 2x + 2} dx = \int \frac{1}{(x+1)^2 + 2 - 1} dx = \int \frac{1}{(x+1)^2 + 1} dx = \arctan(x+1)$

$\int \frac{1}{2x+3} dx = \frac{1}{2} \ln|2x+3|$

$\int \frac{1}{(2x+3)^2} dx = \int (2x+3)^{-2} dx = \frac{(2x+3)^{-1}}{-1} \cdot \frac{1}{2} = -\frac{1}{2(2x+3)}$

V3

Integraler

Glöm inte inre derivatan vid variabelsubstitution.

Skriv ut dx, du och $\frac{du}{dx}$.

Ex

S: tid mätt i sek

h: tid mätt i tim

$h = \frac{s}{3600}$

$v(s)$ = hastigheten i $\frac{m}{s}$.

Tillryggalagd sträcka: $\int_0^T v(s)\, ds = \left[\begin{matrix} h = \frac{s}{3600} \\ dh = \frac{1}{3600} ds \end{matrix} \right] = \int_0^{\frac{T}{3600}} v(h \cdot 3600)\, 3600\, dh$

$\left[\begin{matrix} u = s^2 \\ du = 2s\, ds \end{matrix} \right] \Rightarrow \int_0^T v(s)\, ds = \int_0^{T^2} v(\sqrt{u}) \frac{1}{2s} du = \int_0^{T^2} \frac{v(\sqrt{u})}{2\sqrt{u}} du$

Ex

$I = \int \sin(2x) \cdot \cos(3x) dx = \frac{\sin(2x) \cdot \sin(3x)}{3} - \int \cos(2x) \cdot \sin(3x) \cdot \frac{2}{3} dx = \frac{\sin(2x)\sin(3x)}{3} + \frac{2}{9} \cos(2x)\cos(3x) + \frac{2}{9} \int \sin(2x)\cos(3x) \cdot 2 dx^* =$

$\frac{\sin(2x)\sin(3x)}{3} + \frac{2}{9} \cos(2x)\cos(3x) + \frac{4}{9} I$

$\frac{5}{9} I =$ —— || ——

Huvudsatsen

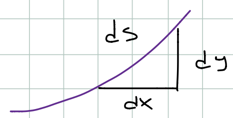
$\int_a^b f(t) dt = F(b) - F(a)$

$$\frac{d}{dx}\int_x^{x^2}(y(x))^3\,dx = \frac{d}{dx}\left(F(x^2)-F(x)\right) = \frac{d}{dx}F(x^2)\cdot 2x - \frac{d}{dx}F(x) = (y(x^2))^3\cdot 2x - (y(x))^3$$

V4

Tillämpn.

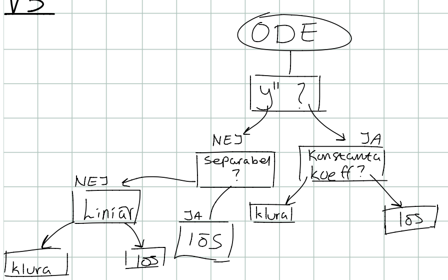
Kom ihåg



$$ds = \sqrt{dx^2+dy^2} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2} = dt\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$$

dt, om du har en parametrisering av kurvan

V5



Komplexa tal

Polär form: $z = |z|e^{i\theta} = |z|(\cos\theta + i\sin\theta)$

Ex

$$1 - i = \sqrt{1^2+(-1)^2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \sqrt{2}\,e^{-i\frac{\pi}{4}}$$

Ex

$$-2+3i = \sqrt{13}\left(\frac{-2}{\sqrt{13}} + \frac{3i}{\sqrt{13}}\right) = \sqrt{13}\,e^{i(\arctan(-\frac{3}{2})+\pi)}$$

$$\tan\theta = \frac{\frac{3}{\sqrt{13}}}{\frac{-2}{\sqrt{13}}} = -\frac{3}{2}$$