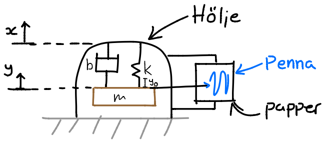
3.2



Givet

x - jordens läge ($x=0$ i jämvikt)

y - massans läge ($y=0$ — || —)

m = massa

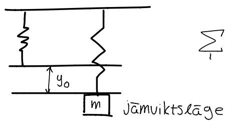
b = dämpkonstant

k = fjäderkonstant

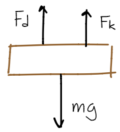
Sökt

$G(s) = \frac{Y(s)}{X(s)}$

Lösning



$\sum_i F_i = ma$, jämvikt $\Rightarrow a = 0$



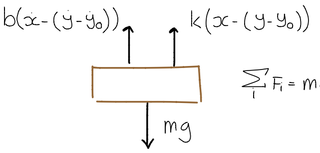
Hooks lag: $F_k = kx$, x = utsträckning $= y_0$.

$F_d = b \cdot x$

$\uparrow: F_d + F_k - mg = 0 \Leftrightarrow 0 + Ky_0 - mg = 0 \Leftrightarrow$

$y_0 = \frac{mg}{k}$

Inte jämvikt



$\sum_i F_i = ma = m\ddot{y}$

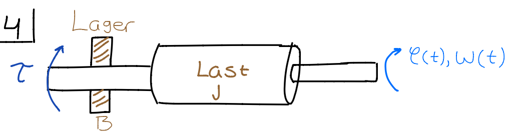
$\uparrow: m\ddot{y} = b(\dot{x}-(\dot{y}-\cancel{\dot{y}_0})) + k(x-(y-\cancel{y_0})) - \cancel{mg}$

$m\ddot{y} = b(\dot{x}-\dot{y}) + k(x-y)$

$Y(s)(ms^2+bs+k) = X(s)(bs+k)$

$G(s) = \frac{bs+k}{ms^2+bs+k}$

3.4



Sökt

$G_1(s) = \frac{\Omega(s)}{\tau(s)}$

$G_2(s) = \frac{\Phi(s)}{\tau(s)}$

Lösning

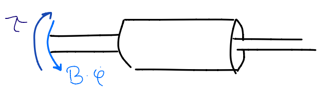
$\sum M = J \cdot \alpha$

α: Vinkelacc

J: Tröghetsmoment

M: Vridande moment

Friläggning

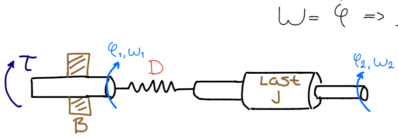


$\circlearrowleft: J \cdot \ddot{\varphi} = \tau - B\dot{\varphi} \overset{\mathcal{L}}{\Longrightarrow} Js^2\Phi(s) = \tau(s) - Bs\Phi(s) \Leftrightarrow \Phi(s)(Js^2+Bs) = \tau(s) \Leftrightarrow$

$G_2(s) = \frac{1}{s(Js+B)}$

$\omega = \dot{\varphi} \Rightarrow \Omega(s) = s \cdot \Phi(s) \Rightarrow \frac{\Omega(s)}{\tau(s)} = s \cdot \frac{1}{s(Js+B)} = \frac{1}{Js+B} = G_1(s)$

b)



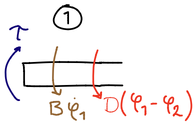
Sökt

$\frac{\Omega_1(s)}{\tau(s)}, \frac{\Omega_2(s)}{\tau(s)}$

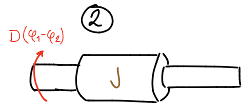
Lösning

Frilägg!

1



2



$J\alpha = \sum M_i \Rightarrow \circlearrowleft: 0 = \tau - B\dot{\varphi}_1 - d(\varphi_1 - \varphi_2)$

$\mathcal{L} \Rightarrow 0 = \tau(s) - B\Omega_1(s) - \frac{d}{s}(\Omega_1(s) - \Omega_2(s))$

$\circlearrowleft: J \cdot \ddot{\varphi}_2 = D(\varphi_1 - \varphi_2)$

$Js\Omega_2(S) = \frac{d}{s}(\Omega_1(s) - \Omega_2(s))$

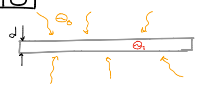
2: $\Omega_2(s)(Js^2+D) = D\Omega_1(s) \Rightarrow \Omega_2(s) = \Omega_1(s)\frac{D}{Js^2+D}$

1: $\Omega_1(s)(Bs+D) = \tau(s) \cdot s + D\Omega_2(s) \Rightarrow \Omega_1(s) = \frac{\tau(s)s + D\Omega_2(s)}{Bs+D}$

$\left.\right\} \quad \frac{\Omega_1(s)}{\tau(s)} = \frac{Js^2+D}{JBs^2+JDs+BD}$

$\frac{\Omega_2(s)}{\tau(s)} = \frac{D}{s^2JB+DJs+DB}$

3.161



$q = \sigma(\Theta_0^4 - \Theta_1^4)\left[\frac{W}{m^2}\right]$

Givet

d: tjocklek $[m]$
ρ: densitet $\left[\frac{kg}{m^3}\right]$
c: Specifikt värme $\left[\frac{J}{kg\,K}\right]$
Θ_0: ugntemp $[K]$
Θ_1: Plåt temp $[K]$

Sökt

$\frac{\Delta\Theta_1(s)}{\Delta\Theta_0(s)}$

Lösning

Flödesbalans (s 126)

Energiflöde $\left[\frac{J}{s} = W\right]$: Ändring av upplagrad energi per tidsenhet = {Effekt in − Effekt ut}

Energi i plåten: $d \cdot \rho \cdot c \cdot \Theta_1 = [m]\left[\frac{kg}{m^3}\right]\left[\frac{J}{kg\,K}\right][K] = \left[\frac{J}{m^2}\right]$

$d \cdot \rho \cdot c \cdot \frac{d\Theta_1}{dt} = \left[\frac{J/s}{m^2}\right] = \left[\frac{W}{m^2}\right]$

$$d\rho c \frac{d\Theta_1}{dt} = \sigma(\Theta_0^4 - \Theta_1^4) \Leftrightarrow \frac{d\Theta_1}{dt} = \frac{\sigma}{d\rho c}(\Theta_0^4 - \Theta_1^4) = f(\Theta_0, \Theta_1)$$

Vid arbetspunkten $\bar{\Theta}$ antas jämvikt råda.

$\Theta_{0,0} = \bar{\Theta}$, $\Theta_{1,0} = ?$

$f(\Theta_{0,0}, \Theta_{1,0}) = 0$ ty jämvikt

$$\frac{\sigma}{d\rho c}(\Theta_{0,0}^4 - \Theta_{1,0}^4) = 0 \Rightarrow \Theta_{0,0}^4 = \Theta_{1,0}^4 \Rightarrow \Theta_{0,0} = \Theta_{1,0} = \bar{\Theta}$$

Arb.pkt $(\Theta_{0,0}, \Theta_{1,0}) = (\bar{\Theta}, \bar{\Theta})$

$$\left.\begin{array}{l}\Theta_0 = \bar{\Theta} + \Delta\Theta_0 \\ \Theta_1 = \bar{\Theta} + \Delta\Theta_1\end{array}\right\} \Rightarrow \begin{array}{l}\Delta\Theta_0 = \Theta_0 - \bar{\Theta} \\ \Delta\Theta_1 = \Theta_1 - \bar{\Theta}\end{array} \Rightarrow \Delta\dot{\Theta}_1 = \left.\frac{\partial f}{\partial \Theta_0}\right|_{(\bar{\Theta},\bar{\Theta})}\Delta\Theta_0 + \left.\frac{\partial f}{\partial \Theta_1}\right|_{(\bar{\Theta},\bar{\Theta})}\Delta\Theta_1,$$

$$\frac{\partial f}{\partial \Theta_0} = \frac{4\sigma}{d\rho c}\Theta_0^3\Big|_{(\bar{\Theta},\bar{\Theta})} = \frac{4\sigma}{d\rho c}\bar{\Theta}^3$$

$$\frac{\partial f}{\partial \Theta_1} = \frac{-4\sigma}{d\rho c}\Theta_1^3\Big|_{(\bar{\Theta},\bar{\Theta})} = \frac{-4\sigma}{d\rho c}\bar{\Theta}^3$$

$$\Delta\dot{\Theta}_1 = \frac{\bar{\Theta}^3 4\sigma}{d\rho c}(\Delta\Theta_0 - \Delta\Theta_1)$$

Laplace $\Rightarrow \frac{d\rho c}{4\bar{\Theta}^3\sigma} s\Delta\Theta_1 + \Delta\Theta_1 = \Delta\Theta_0 \Rightarrow \frac{\Delta\Theta_1}{\Delta\Theta_0} = \frac{1}{\frac{d\rho c}{4\bar{\Theta}^3\sigma}s + 1}$ $T = \frac{d\rho c}{4\bar{\Theta}^3\sigma}$, högre $\bar{\Theta} \Rightarrow$ snabbare sys