Purpose

To perform a hypothesis test to compare two population means, μ_1 and μ_2.

Assumptions

1. Random sampling.
2. Independent samples
3. Normal population or a large enough sample.

Steps

1. The null hypothesis: $H_0: \mu_1 = \mu_2$ and the alternative
   $H_1: \mu_1 \neq \mu_2$ - 2 sided
   $H_1: \mu_1 \gtrless \mu_2$ - 1 sided
2. Significance level α.
3. Compute the value of test statistic.
   $T = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$, given $X_1^{(1)}, \dots, X_{n_1}^{(1)}$ - sample from population 1.
   $X_1^{(2)}, \dots, X_{n_2}^{(2)}$ - sample from population 2
   $S_1^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (X_i^{(1)} - \overline{X}_1)$ - sample variance
4. The critical values are: $\pm t_{\frac{\alpha}{2}, \Delta}$ - 2-sided
   $-t_{\alpha, \Delta}$ - left sided ($H_1: \mu_2 < \mu_1$)
   t-student $+t_{\alpha, \Delta}$ - right sided ($H_1: \mu_2 > \mu_1$)
   $\Delta = \frac{\left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right]}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}}$ round up to closest integer
5. If value of test statistic lies within critical region reject H_0. Otherwise: not enough evidence...

Ex - ALPS vs Z-plate

Z-plate (1)

370, 360, 510, 445, 295, 315, 490, 345, 450, 505, 335, 280, 325, 500

$\bar{x}_1 = 394.6$, $n = 14$

$S_1 = 84.7$

ALPS-plate (2)

430, 445, 455, 455, 490, 535

$\bar{x}_2 = 468.3$, $n = 6$

$S_2 = 38.2$

1. $H_0: \mu_1 = \mu_2$
   $H_1: \mu_1 < \mu_2$
2. $\alpha = 0.05$
3. $t = -2.681$
4. $\Delta = 17$ should be 18
   $-t_{0.05,17} = -1.740$
5. Rejecting H_0 in favor of H_1, given $\alpha = 0.05$

CI for difference of means

Same assumptions as above

$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},\Delta}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$ - 2-sided

Calculating 90% CI for $\mu_1 - \mu_2$

$\alpha = 0.1$

$t_{0.05,17} = 1.740$

$(-121.5, -25.9)$ minutes

Generating Functions

Def

The ordinary generating functions (OGF) for the infinite sequence $\langle g_0, g_1, \dots \rangle$ is the power series

$$G(x) = g_0 + g_1 x + g_2 x^2 + \dots = \sum_{i=0}^{\infty} g_i x^i$$

Ex

$$\langle 0, 0, 0, \dots \rangle \longleftrightarrow 0 + 0x + 0x^2 + \dots = 0$$

$$\langle 1, 0, 0, \dots \rangle \longleftrightarrow 1 + 0x + 0x^2 + \dots = 1$$

$$\langle 1, 2, 3, 0, 0, \dots \rangle \longleftrightarrow 1 + 2x + 3x^2$$

Recall

$$1 + z + z^2 + \dots = \frac{1}{1-z}, \quad |z| < 1$$

Ex

$$\langle 1, 1, 1, \dots \rangle \longleftrightarrow 1 + 1x + 1x^2 + \dots = \frac{1}{1-x}$$

$$\langle 1, -1, 1, -1, \dots \rangle \longleftrightarrow 1 - x + x^2 - x^3 + \dots = \frac{1}{1+x}$$

$$\langle 1, 0, 1, 0 \dots \rangle \longleftrightarrow 1 + x^2 + x^4 + \dots = \frac{1}{1-x^2}$$

$$\langle 1, a, a^2, \dots \rangle \longleftrightarrow 1 + ax + a^2x^2 + \dots = \frac{1}{1-ax}$$

Scaling

Ex

$$\langle 1, 0, 1, 0 \dots \rangle \longleftrightarrow \frac{1}{1-x^2}$$

$$\langle 2, 0, 2, 0 \dots \rangle \longleftrightarrow \frac{2}{1-x^2}$$

Theorem-ish

If $\langle g_0, g_1, g_2, \ldots \rangle \longleftrightarrow G(x)$

then $\langle cg_0, cg_1, cg_2, \ldots \rangle \longleftrightarrow cG(x)$

Since: $\langle cg_0, \ldots \rangle \leftrightarrow cg_0 + \ldots = c(g_0 + \ldots) = cG(x)$

Addition

$\langle 1, 1, 1 \ldots \rangle \longleftrightarrow \frac{1}{1-x}$

$\langle 1, -1, 1, -1, \ldots \rangle \overset{+}{\longleftrightarrow} \frac{1}{1+x}$

$\langle 2, 0, 2, 0 \ldots \rangle \leftrightarrow \frac{2}{1-x^2} = \frac{1}{1-x} + \frac{1}{1+x}$

Theorem-ish

$\langle g_0, g_1, g_2, \ldots \rangle \longleftrightarrow G(x)$

$\langle f_0, f_1, f_2, \ldots \rangle \overset{+}{\longleftrightarrow} F(x)$

$\langle g_0 + f_0, g_1 + f_1, \ldots \rangle \leftrightarrow G(x) + F(x)$

Right Shifting

$\langle 1, 1, 1, \ldots \rangle \longleftrightarrow \frac{1}{1-x}$

$\langle \overbrace{0, 0, \ldots, 0}^{k \text{ zeroes}}, 1 \ldots \rangle \longleftrightarrow x^k + x^{k+1} + \ldots = x^k \overbrace{(1 + x + x^2 \ldots)}^{\frac{1}{1-x}} = \frac{x^k}{1-x}$

Theorem-ish

If $\langle g_0, g_1, g_2, \ldots \rangle \longleftrightarrow G(x)$

then $\langle \overbrace{0, \ldots 0}^{k \text{ zeroes}}, g_0, g_1, g_2 \ldots \rangle \leftrightarrow x^k G(x)$

Differentiation

$$\frac{d}{dx}\left(1+x+x^2+\dots\right) = \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\Longleftrightarrow$$

$$1+2x+3x^2+\dots = \frac{1}{(1-x)^2}$$

$$\Longleftrightarrow$$

$$\langle 1, 2, 3, \dots \rangle \longleftrightarrow \frac{1}{(1-x)^2}$$

Theorem-ish

If $\langle g_0, g_1, g_2, \dots \rangle \longleftrightarrow G(x)$

then $\langle g_1, 2g_2, 3g_3, \dots \rangle \longleftrightarrow \frac{d}{dx} G(x)$

Ex

$\langle 0, 1, 4, 9, 16 \dots \rangle \longleftrightarrow ?$ sequence of squares

$\langle \underset{0}{1}, \underset{1}{1}, \underset{2}{1}, \dots \rangle \longleftrightarrow \frac{1}{1-x}$

derivative

$\langle 1, 2, 3, \dots \rangle \longleftrightarrow \frac{1}{(1-x)^2}$

right shift

$\langle 0, 1, 2, 3 \dots \rangle \longleftrightarrow \frac{x}{(1-x)^2}$

derivative

$\langle 1, 4, 9, \dots \rangle \longleftrightarrow \frac{1+x}{(1-x)^3}$

right shift

$\langle 0, 1, 4, 9, \dots \rangle \longleftrightarrow \frac{x(1+x)}{(1-x)^3}$

Product rule

If $\langle g_0, g_1, g_2, \dots \rangle \longleftrightarrow G(x)$

and $\langle f_0, f_1, f_2, \dots \rangle \longleftrightarrow F(x)$

then $\langle c_0, c_1, c_2, \dots \rangle \longleftrightarrow G(x)\,F(x)$

$$c_n = g_0 \cdot f_n + g_1 \cdot f_{n-1} + \dots + g_{n-1} \cdot f_1 + g_n \cdot f_0$$

	g_0	g_1x	g_2x^2
f_0	g_0f_0	g_1f_0x	$g_2f_0x^2$
f_1x	g_0f_1x	$g_1f_1x^2$	$g_2f_1x^3$
f_2x^2	$g_0f_2x^2$	$g_1f_2x^3$	$g_2f_2x^4$