## Rep

$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t}\, dt$, $\omega$ reellt

1) $x(t) = e^{\alpha t} u(t)$, $\alpha > 0 \longleftrightarrow X(j\omega) = \frac{1}{\alpha + j\omega}$
2) $y(t) = (h * x)(t) \longleftrightarrow Y(j\omega) = H(j\omega) X(j\omega)$
3) $x(t) = z(t - t_0) \longleftrightarrow X(j\omega) = e^{-j\omega t_0} Z(j\omega)$
4) $x(t) = z(-t) \longleftrightarrow X(j\omega) = Z(-j\omega)$
5) $x(t) = t\, z(t) \longleftrightarrow X(j\omega) = j \frac{d}{d\omega}(Z(j\omega))$
6) $x(t) = t^2 z(t) \longleftrightarrow X(j\omega) = -1\, Z''(j\omega)$
7) $x(t) = \frac{\sin t}{\pi t} \longleftrightarrow X(j\omega) = \begin{cases} 1 & |\omega| \leq 1 \\ 0 & |\omega| > 1 \end{cases}$
8) $z(t) = x(t) y(t) \longleftrightarrow Z(j\omega) = \frac{1}{2\pi}(X * Y)(j\omega)$

**Ex** $x(t) = e^{5t} u(1-t)$, bestäm $X$.

$x(t) = z(-t)$,

$z(t) = e^{-5t} u(t+1) = e^{-5(t+1-1)} u(t+1) = e^5 \cdot e^{-5(t+1)} u(t+1) = e^5 \cdot w(t+1)$, där $w(t) = e^{-5t} u(t)$

4) $X(j\omega) = Z(-j\omega)$
3) $Z(j\omega) \underset{t_0 = -1}{=} e^5 \cdot e^{-j\omega(-1)} \cdot W(j\omega)$
1) $W(j\omega) = \frac{1}{5 + j\omega}$

$Z(j\omega) = e^5 \cdot e^{j\omega} \cdot \frac{1}{5 + j\omega}$

$\underline{X(j\omega) = e^5 \cdot \frac{e^{-j\omega}}{5 - j\omega}}$

Här blev klockan leet.

Alt: $x(t) = e^{5t} u(-(-t-1)) = e^{5(t-1)} \cdot e^5 u(-(t-1)) = e^5 \cdot z(t-1)$, där $z(t) = e^{5t} u(-t) = w(-t)$
där $w(t) = e^{-5t} u(t)$

3) $X(j\omega) = e^5 \cdot e^{-j\omega} Z(j\omega)$
4) $Z(j\omega) = W(-j\omega)$
1) $W(j\omega) = \frac{1}{5 + j\omega}$

$Z(j\omega) = \frac{1}{5 - j\omega}$

$\underline{X(j\omega) = e^5 \cdot \frac{e^{-j\omega}}{5 - j\omega}}$

**Ex** $x(t) = t^2 e^{-t} u(t-2)$, Bestäm $X$!

5) på sig själv!

$x(t) = t^2 z(t)$, $z(t) = e^{-t} u(t-2)$

$x(t) = t w(t)$, $w(t) = t\, z(t)$ $\Rightarrow$ $W(j\omega) = j \frac{d}{d\omega}(Z(j\omega))$, $X(j\omega) = j \frac{d}{d\omega}(W(j\omega))$ $\Rightarrow \{ \overbrace{x(t) = t^2 z(t) \longleftrightarrow X(j\omega) = -1\, Z''(j\omega)}^{6} \}$

$z(t) = e^{-(t-2+2)} u(t-2) = e^{-2} \underbrace{e^{-(t-2)} u(t-2)}_{s(t-2)}$ där $s(t) = e^{-t} u(t)$

5x2 $X(j\omega) = -Z''(j\omega)$
3 $Z(j\omega) = e^{-2} e^{-2j\omega} S(j\omega)$
1 $S(j\omega) = \frac{1}{1 + j\omega}$

$Z(j\omega) = e^{-2} \frac{e^{-2j\omega}}{1 + j\omega}$

$Z' = e^{-2} \left( \frac{-2j \cdot e^{-2j\omega}(1 + j\omega) - e^{-2j\omega} j}{(1 + j\omega)^2} \right)$

$Z'' = \ldots$

**Ex** Tenta 2015-08-27, upp 3

**Givet**

LTI-system $y(t) = (h * x)(t)$

$h(t) = \left(\frac{\sin(t)}{\pi t}\right)^2$

**Sökt**

a) Stabilt / Kausalt
b) $H$
c) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(t+n)} u(t+n)$ - Bestäm $C_k(x)$
d) Visa att om $x$ från c) är insignal $\Rightarrow$ $Y$ är 1-periodisk.
Bestäm även $C_k(y)$.

## Lösning

a)

Stabilt $\Leftrightarrow \int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

Jämn funktion

$\int_{-\infty}^{\infty} \frac{\sin^2 t}{\pi^2 t^2}\, dt = \frac{2}{\pi^2} \int_0^{\infty} \frac{\sin^2 t}{t^2}\, dt = \frac{2}{\pi^2} \int_0^1 \frac{\sin^2 t}{t^2}\, dt + \frac{2}{\pi^2} \underbrace{\int_1^{\infty} \frac{\sin^2 t}{t^2}\, dt}_{\leq \int_1^{\infty} \frac{dt}{t^2} = \left[-\frac{1}{t}\right]_1^{\infty} = 1 < \infty} =$

Då $\sin^2 t \leq 1$

$\underline{h \text{ är stabil}}$

Viktigt!

$\int_1^{\infty} \frac{dt}{t^{\alpha}} = \begin{cases} \infty & \alpha \leq 1 \\ < +\infty & \alpha > 1 \end{cases}$

Kausalt $\Leftrightarrow$ $h(t)=0,\ t<0$

I vårt fall $h(-t)=h(t)$    Jämna signaler !kausala

om $h(t)=0,\ t<0 \Rightarrow h(t)=0,\ t>0 \Rightarrow$

$h(t)=0$ för alla $t\neq 0$.

Men $\frac{\sin^2 t}{t^2}\neq 0$ för alla $t\neq 0 \Rightarrow$ Systemet !kausalt

b) 7 & 8

I vårt fall $h(t)=Z(t)^2=z(t)z(t)$ där $z(t)=\frac{\sin t}{t}$ $\Rightarrow \begin{cases} H(j\omega)=\frac{1}{2\pi}(Z*Z)(j\omega) \\ Z(j\omega)=\begin{cases}1, & |\omega|\leq 1\\ 0, & |\omega|0\end{cases}\end{cases}$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty} Z(j(\omega-\eta))Z(j\eta)\,d\eta=\frac{1}{2\pi}\int_{-1}^{1} Z(j(\omega-\eta))\,d\eta$$

$$=\left\{Z(j(\omega-\eta))\right\}=\left\{\begin{matrix}1 & |\omega-\eta|\leq 1\\ 0 & |\omega-\eta|>1\end{matrix},\ -1\leq\eta\leq 1\right\}=\frac{1}{2\pi}(2-|\omega|)\,u(2-|\omega|)=H(j\omega)$$

magi...

c) $x(t)=\sum_{n=-\infty}^{\infty} z(t+n)$,    $z(t)=e^{-t}u(t)$

T=1-periodisk

$C_k(x)=\frac{1}{1}Z(\omega_k),\quad \omega_k=\frac{2\pi k}{T}=2\pi k$

Mer allmänt: $x(t)=\sum_{n=-\infty}^{\infty} z(t+nT)$   T-periodisk

$C_k(x)=\frac{1}{T}Z(\omega_k),\quad \omega_k=\frac{2\pi k}{T}$

I vårt fall: $Z(j\omega)=\frac{1}{1+j\omega}$   1) $\Rightarrow$ $C_k(x)=\frac{1}{1+j2\pi k}$

d) LTI $\Rightarrow$ y 1-per, bevis i föregående

$$C_k(y)=H(j\omega_k)C_k(x)=\underbrace{H(j2\pi k)}\,C_k(x)$$

$H(j\omega)=0$ om $|\omega|>2$

$|2\pi k|\leq 2 \Leftrightarrow \pi|k|\leq 1 \Leftrightarrow k=0$

$$C_k(y)=\begin{cases}0 & k\neq 0\\ \underbrace{\frac{1}{\pi}}_{H(0)}\cdot\underbrace{1}_{C_0(x)} & k=0\end{cases}$$

Detta betyder: $y(t)=\frac{1}{\pi}$, alla $t$

$$H(j\omega)=\frac{1}{2\pi}(2-|\omega|)\underbrace{u(2-|\omega|)}$$

$=0$ om $2-|\omega|<0 \Leftrightarrow |\omega|>2$