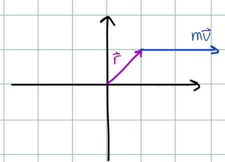
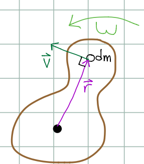
| | Partiklar | Rotationsrörelse |
|---|---|---|
| Läge: | $x$ | $\theta$ |
| Hastighet: | $v = \frac{dx}{dt}$ | $\omega = \frac{d\theta}{dt}$ |
| Acc: | $a = \frac{dv}{dt}$ | $\alpha = \frac{d\omega}{dt}$ |
| | $x_f - x_i = v_0 \cdot t + \frac{1}{2} a t^2$ | $\theta_f - \theta_i = \omega_0 \cdot t + \frac{1}{2}\alpha t^2$ |
| | $v_f^2 - v_i^2 = 2 \cdot a \cdot s$ | $\omega_f^2 - \omega_i^2 = 2\alpha \cdot \Delta\theta$ |
| Tröghet: | $m$ | $I$ |
| | $\frac{1}{2} m v^2$ | $\frac{1}{2} I \omega^2$ |
| | $F = ma$ | $\vec{\tau} = I\vec{\alpha}$ |
| | $p = mv$ | $\vec{L} = I\vec{\omega}$ |
| | $\vec{F} = \frac{d\vec{p}}{dt}$ | $\vec{\tau} = \frac{d\vec{L}}{dt}$ |

## Rörelsemängdsmoment

$\vec{L} \equiv \vec{r} \times \vec{p}$



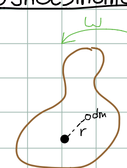

$d\vec{L} = \vec{r} \times dm \cdot \vec{v}$

$|d\vec{L}| = r \cdot dm \cdot v = r \cdot dm \cdot \omega \cdot r = \omega r^2 dm$

$L = \omega \int r^2 dm = \omega \cdot I$

$\frac{dL}{dt} = I \cdot \frac{d\omega}{dt} = I \cdot \alpha = \tau$

## Tröghetsmoment




$I = \int_{\text{hela kroppen}} r^2 dm$

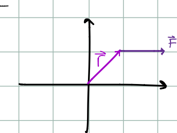
$v = \omega r$

$dK = \frac{1}{2} dm \cdot v^2 = \frac{1}{2} dm\, \omega^2 r^2 = \frac{1}{2} \cdot \omega^2 r^2 \cdot dm \Rightarrow K = \int \frac{1}{2} \omega^2 r^2 dm = \frac{1}{2}\omega^2 \overbrace{\int r^2 dm}^{I}$

rörelse energi för lilla elementets rörelseenergi

## Vridande moment - Torque [M]

$\vec{\tau} \equiv \vec{r} \times \vec{F}$




Här går $\vec{\tau}$ in i tavlan.

$\vec{\tau} \,//\, -\hat{e}_y = -\hat{k}$

Medurs $\equiv$ negativt

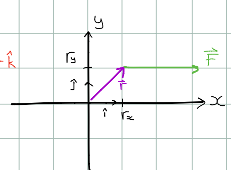
Moturs $\equiv$ positivt

## Linalg

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{i} = -\hat{k}$

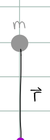
i j k i j k i j k




$\vec{\tau} = r_x \cdot \hat{i} + r_y \cdot \hat{j} = (r_x \hat{i} + r_y \hat{j}) \times F \hat{i} = r_x F (\hat{i} \times \hat{i}) + r_y F (\hat{j} \times \hat{i}) = 0 + r_y F(-\hat{k})$

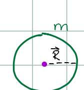
## Räkna

1,



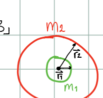

$I = m \cdot r^2$

2,



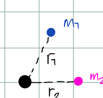

$I = \int r^2 dm = R^2 \int dm = m \cdot R^2$

3,




$I = m_1 R_1^2 + m_2 R_2^2$

4,

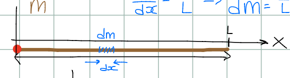



$I = m_1 r_1^2 + m_2 r_2^2$

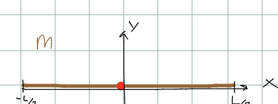
## Pinne

$\frac{dm}{dx} = \frac{M}{L} \Rightarrow dm = \frac{M}{L} dx$

1.

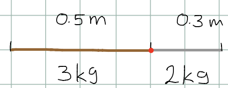



$I = \int r^2 dm = \int_0^L x^2 dm = \int_0^L \frac{M}{L} x^2 dx = \frac{M}{L}\left[\frac{1}{3} x^3\right]_0^L = \frac{M}{L} \cdot \frac{1}{3} \cdot L^3 = \frac{ML^2}{3}$




$I = \frac{M}{L}\left[\frac{1}{3} x^3\right]_{-L/2}^{L/2} = \frac{1}{3}\frac{M}{L}\left(\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right) = \frac{ML^2}{12}$

## Ex



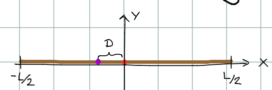

Erik

$I=\frac{3}{0.5}\left[\frac{1}{3}x^3\right]_0^{0.5}+\frac{2}{0.3}\left[\frac{1}{3}x^3\right]_0^{0.3}=\frac{3}{0.5}\left(\frac{1}{3}\cdot 0.5^3\right)+\frac{2}{0.3}\left(\frac{1}{3}\cdot 0.3^3\right)=0.31\ kgm^2$

Åke

$I=\frac{1}{3}\ 2\cdot 0.3^2+\frac{1}{3}\cdot 3\cdot 0.5^2=\frac{1}{3}(0.18+0.75)=0.06+0.25=0.31\ kgm^2$

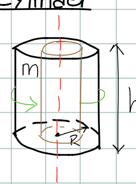
## Parallellaxelförskjutningssatsen



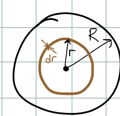

$I=I_{cm}+MD^2$

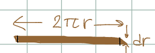
Om $D=L/2 \Rightarrow I=\frac{ML^2}{12}+M\left(\frac{L}{2}\right)^2=ML^2\left[\frac{1}{12}+\frac{1}{4}\right]=\frac{1}{3}ML^2$     Härledning kommer

## Cylinder




Sett ovanifrån




$\rho=\frac{m}{V}=\frac{m}{hR^2\pi}$

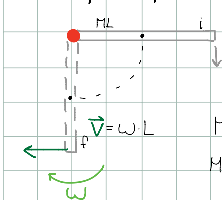
$dV=h\cdot 2\pi r\cdot dr$

$dm=dV\rho=\frac{m}{\cancel{h}\cancel{\pi}R^2}\cancel{h}\cdot 2\cancel{\pi}r\,dr=\frac{2m}{R^2}\cdot r\cdot dr$

$dI=r^2\cdot dm=\frac{2m}{R^2}\cdot r^3\cdot dr$

$I=\int_0^R dI=\frac{2m}{R^2}\int_0^R r^3dr=\frac{2m}{R^2}\left[\frac{1}{4}r^4\right]_0^R=\frac{2m}{R^2}\cdot\frac{1}{4}R^4=\frac{1}{2}mR^2$

## Livet på en pinne




$K_i=0,\quad U_i=Mg\frac{L}{2}$

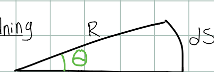
$K_f=\frac{1}{2}I\omega^2,\quad U_f=0$

$I=\frac{1}{3}ML^2,$

Mek. energ. bevaras:

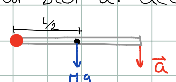
$Mg\frac{L}{2}=\frac{1}{2}\frac{1}{3}ML^2\omega^2 \Rightarrow \cancel{M}g\frac{L}{2}=\frac{1}{6}\cancel{M}\cancel{L^2}\frac{V^2}{\cancel{L^2}} \Rightarrow V=\sqrt{3gL}$

Härledning




$\frac{dS}{dt}=R\frac{d\theta}{dt}$  eller  $V=\omega R$  eller  $a_{ändp}=\alpha\cdot L$

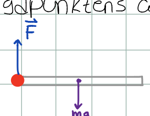
Hur stor är accelerationen av ändpunkten när pinnen släpps?




$$\left.\begin{aligned}\tau&=\frac{L}{2}\cdot Mg=\\ \tau&=I\alpha=\frac{1}{3}ML^2\cdot\alpha\end{aligned}\right\}\ \frac{L}{2}Mg=\frac{1}{3}ML^2\alpha \Rightarrow \alpha=\frac{3}{2}\cdot\frac{g}{L}$$

$a_{änd}=\alpha\cdot L=\frac{3}{2}\cdot\frac{g}{L}\cdot L=\frac{3g}{2}$

Tyngdpunktens acceleration bestäms av summan av de externa krafterna.

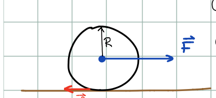



$a_{cm}=\frac{1}{2}a_{änd}=\frac{3}{4}g$

$Mg-F=M\cdot\frac{3}{4}g \Rightarrow F=\frac{Mg}{4}$

## Rullning utan glidning

$I=\frac{1}{2}MR^2$




① $\sum_i \vec{F}_i^{ext}=M\cdot\vec{a}_{cm}$

② $\sum_i \vec{\tau}_i^{ext}=I\vec{\alpha}$

$a_{cm}=R\alpha$

vinkelrät

① $F-f=Ma_{cm}$

② $fR=\frac{1}{2}MR^2\alpha=\frac{1}{2}M\cancel{R^2}\frac{a_{cm}}{\cancel{R}} \Rightarrow f=\frac{1}{2}M\cdot a_{cm}$  (Sätt in i ①)

$F-f=Ma_{cm} \Leftrightarrow F-\frac{1}{2}Ma_{cm}=Ma_{cm} \Rightarrow F=\frac{3}{2}Ma_{cm} \Rightarrow a_{cm}=\frac{2F}{3M}$

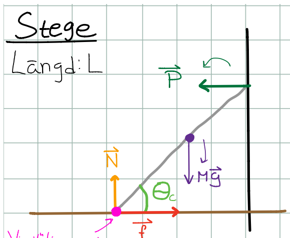
f: $f=\frac{1}{2}Ma_{cm}=\frac{1}{2}M\frac{2F}{3M} \Rightarrow f=\frac{F}{3}$

Om vi drar med en kraft större än $f_{max}$ får vi glidning!

$f_{max}=\mu_s\cdot N$

# Stege

Längd: L




Vi väljer att beräkna utifrån denna punkt.

Villkor för stabilitet

① $\sum_i \vec{F}_i = 0$

② $\sum_i \vec{T}_i = 0$

$1 \Rightarrow$
- $P = f$ (Gränsfallet: $f_{max} = \mu_s \cdot N = \mu_s \cdot Mg$)
- $N = mg$

$$2 \Rightarrow \cancel{Mg}\frac{\cancel{L}}{2}\cos\theta_c = P \cdot L \cdot \sin\theta_c = \mu_s \cdot \cancel{Mg} \cdot \cancel{L} \cdot \sin\theta_c \Rightarrow \frac{\sin\theta_c}{\cos\theta_c} = \tan\theta_c = \frac{1}{2 \cdot \mu_s}$$