Assignment 1 (20%)

You will need very few and simple calculation in order to solve the problems of this assignment.

Let

$$A = \begin{bmatrix} 3 & -1 & 5 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 3 & 5 & -7 & -2 \\ 0 & -5 & 3 & 1 & -3 \\ 0 & 0 & -2 & 4 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

a) Explain why it would be possible to determine all eigenvalues of A without any calculations and then state what these eigenvalues are.

b) State whether or not *A* is invertible and state the reasoning behind your answer.

c) Supply a substantiated answer to whether or not A is diagonalizable.

d) Let B be a $n \times n$ matrix where $\det(B) \neq 0$. What is $rank(B)$ and $dim\ Nul\ B$?

e) Supply a substantiated answer to whether $C\bar{x} = \bar{b}$ is consistent for every $\bar{b}$ in $\mathbb{R}^4$.

f) Does $C\bar{x} = \bar{0}$ have a non-trivial solution? Explain why/why not.

Assignment 2 (10%)

Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } \bar{b} = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}$$

a) Find the inverse of A using elementary row operations and the identity matrix.
b) Use the inverse of A to solve $A\bar{x} = \bar{b}$

Assignment 3 (5%)

Consider the following linear system

$$x_1 + 2x_2 + x_3 = 1$$

$$-2x_1 + x_2 + x_3 = -5$$

$$2x_1 - x_2 - 2x_3 = a$$

For which values of a is the system consistent?

Assignment 4 (25%)

$$A = \begin{bmatrix} a & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$$

a) Express the eigenvalues of A as a function of a
b) Calculate a so that $\lambda = 5$ is an eigenvalue of A
c) Using the value of a found in question (b), determine the remaining eigenvalue(s)
d) Determine the eigenspaces for each of the eigenvalues found in (c)
e) Using the value of a found in (b), find the characteristic equation of A^T and compare it to the characteristic equation of A. What can you conclude about the characteristic equations of A and A^T?

Assignment 4 (15%)

Let

$$\bar{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad and\ \bar{y} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}.$$

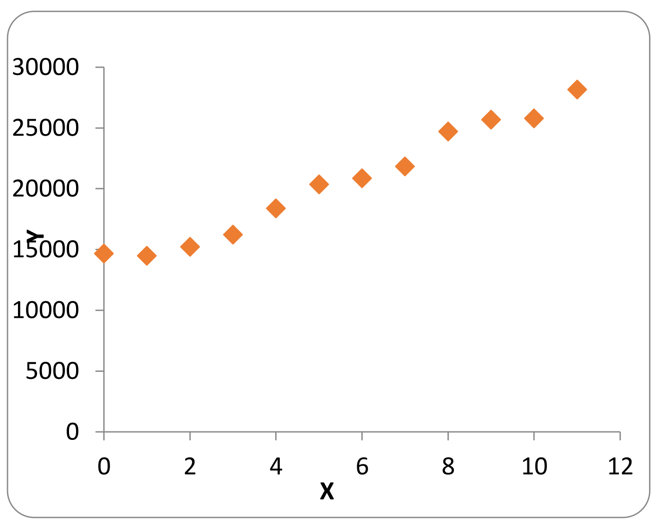
And let $H = span\{\bar{u}_1, \bar{u}_2\}$, let $A = [\bar{u}_1 \quad \bar{u}_2]$, and let $H^\perp$ denote the subspace that is orthogonal to H

a) Show that $\{u_1, u_2\}$ is an orthogonal basis for H.
b) Find a basis for $H^\perp$. Hint: $H = Col\ A \Longrightarrow H^\perp = (Col\ A)^\perp = Nul\ A^T$
c) Calculate the orthogonal projection of $\bar{y}$ onto H

Assignment 5 (15%)

The number of students enrolled at a university study (5+ years) in the period 2005-2016 is as follows (source: DST.dk):

Year	Index year (t)	Number of students (y)
2005	0	14652
2006	1	14480
2007	2	15216
2008	3	16215
2009	4	18378
2010	5	20347
2011	6	20845
2012	7	21822
2013	8	24707
2014	9	25689
2015	10	25787
2016	11	28168



It is believed that the number of students can be modelled by either a linear function or a quadratic function:

$y_1(t) = \beta_0 + \beta_1 t$ or $y_2(t) = \delta_0 + \delta_1 t + \delta_2 t^2$

a) By creating design matrices, find the parameters of both models and state the regression functions
b) Supply a substantiated answer to which model is the best fit for the measured data
c) Use the best fitted model to predict how many students will be enrolled at a university study in 2020.

Assignment 6 (10%)

Compute a full singular value decomposition of the following matrix

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$