Assignment 1 (10%)
You will need very few and simple calculation in order to solve the problems of this assignment.
The following questions refer to the matrices

$$A = \begin{bmatrix} 4 & 9 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 7 \\ 1 & 7 & 7 \end{bmatrix}$$

and the vector

$$\boldsymbol{b} = \begin{bmatrix} -1 \\ 2 \\ 8 \end{bmatrix}$$

a) Which of the matrices are in echelon form?
Only vector B
b) Write the system of linear equations that correspond to the matrix equation $Cx = b$.
$x_3 = -1$
$x_2 + 7x_3 = 2$
$x_1 + 7x_2 + 7x_3 = 8$

c) Is the system of linear equations that correspond to the matric equation $Cx = b$ consistent?
Yes, a unique solution x1=-48, x2=9, x3=-1. Rref augmented matrix
d) Explain which of the following five expressions that makes sense.

$$AB, \quad CA, \quad B^2$$
$$\det(B), \quad \det(C)$$

AB makes no sense as # columns of A are not equal to # rows of B
CA makes sense as # columns of C are equal to # rows of A
B^2 makes no sense as you can only square a square matrix
Det(B) does not make sense as determinants are only defined for square matrices
Det C is ok since it is a square matrix.

Assignment 2 (20%)
Given

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

a) Solve the matrix equation $X + A = 2(X - B)$

```
> A=[1 0 0;0 3 0;0 0 -3]

A =

     1     0     0
     0     3     0
     0     0    -3

>> B=[1 1 1;1 1 1;1 1 1]

B =
```

```
     1     1     1
     1     1     1
     1     1     1

>> A+2*B

ans =

     3     2     2
     2     5     2
     2     2    -1
```

b) Determine the rank and the nullity of A.
Since A has pivots in all columns, the nullity is 0 and the rank is 3.

c) Determine the rank and the nullity of B and find a basis for $Col\ B$ and a basis for $Null\ B$.
The rank is 1 and the nullity is 2. Basis for Col space $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and for null space: $\begin{bmatrix}-1\\1\\0\end{bmatrix}\begin{bmatrix}-1\\0\\1\end{bmatrix}$

d) Find the characteristic polynomial of B and use it to find the eigenvalues of B.

```
>> syms x
>> charpoly(B,x)

ans =

x^3 - 3*x^2

>> solve(ans,x)

ans =

 0
 0
 3
```

So the eigenvalues are 3 and 0 where 0 has multiplicity of 2

Assignment 3 (10%)

Let $A = \begin{bmatrix}1 & 1 & 1\\3 & -4 & -1\\-1 & -3 & 2\end{bmatrix}$ and $b = \begin{bmatrix}1\\6\\4\end{bmatrix}$

a) Find the inverse of A using elementary row operations on the augmented matrix $[A \quad I]$

```
>> A=[1 1 1;3 -4 -1;-1 -3 2]

A =

     1     1     1
     3    -4    -1
    -1    -3     2

>> [A eye(3)]
```

```
ans =

     1     1     1     1     0     0
     3    -4    -1     0     1     0
    -1    -3     2     0     0     1

>> C=rref(ans)

C =

    1.0000         0         0    0.3793    0.1724   -0.1034
         0    1.0000         0    0.1724   -0.1034   -0.1379
         0         0    1.0000    0.4483   -0.0690    0.2414

>> Ainv=[C(:,4) C(:,5) C(:,6)]

Ainv =

    0.3793    0.1724   -0.1034
    0.1724   -0.1034   -0.1379
    0.4483   -0.0690    0.2414

>> Ainv=rats(Ainv)

Ainv =

     11/29          5/29          -3/29
      5/29         -3/29          -4/29
     13/29         -2/29           7/29
```

b) Use the inverse of A to solve $Ax = b$

```
>> Ainv*b

ans =

     1
    -1
     1
```

$x_1=1$, $x_2=-1$ and $x_3=1$

Assignment 4 (5%)

Find the value(s) of α for which the determinant of the following matrix is -18 by co-expanding on the second row

$$\begin{bmatrix} 1 & 5 & -a \\ a & -a & a \\ 2 & 13 & -7 \end{bmatrix}$$

$-a\begin{vmatrix} 5 & -a \\ 13 & -7 \end{vmatrix} - a\begin{vmatrix} 1 & -a \\ 2 & -7 \end{vmatrix} - a\begin{vmatrix} 1 & 5 \\ 2 & 13 \end{vmatrix} = -18 \Leftrightarrow$

$-a(-35+13a) - a(-7+2a) - a(13-10) = -18 \Leftrightarrow$

$35a - 13a^2 + 7a - 2a^2 - 3a = -18 \Leftrightarrow$

$15a^2 + 39a + 18 = 0 \Leftrightarrow$

$$a = -\frac{2}{5} \; or \; a = 3$$

Assignment 5 (20%)

Let

$$\boldsymbol{u_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \quad \boldsymbol{u_2} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \boldsymbol{u_3} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \; and \; \boldsymbol{u_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

a) Show that $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\}$ is a basis for $\mathbb{R}^4$.

```
> u1=[1 2 0 -1]'

u1 =

     1
     2
     0
    -1

>> u2=[1 0 2 1]'

u2 =

     1
     0
     2
     1

>> u3=[1 2 0 0]'

u3 =

     1
     2
     0
     0

>> u4=[0 0 1 2]'

u4 =

     0
     0
     1
     2

>> U=[u1 u2 u3 u4]

U =

     1     1     1     0
```

```
     2     0     2     0
     0     2     0     1
    -1     1     0     2

>> rref(U)

ans =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

>Since there are pivots in each column, it follows that the set is a basis for $\mathbb{R}^4$

b) Use the Gram-Schmidt process on $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\}$ to obtain an orthogonal basis for $\mathbb{R}^4$.

```
>> v1=u1

v1 =

     1
     2
     0
    -1

>> v2=u2-proj(u2,v1)

v2 =

    1.0000
   -0.0000
    2.0000
    1.0000

>> v3=u3-proj(u3,v1)-proj(u3,v2)

v3 =

   -0.0000
    0.3333
   -0.3333
    0.6667

>> v4=u4-proj(u4,v1)-proj(u4,v2)-proj(u4,v3)

v4 =

   -0.3333
    0.1667
    0.1667
   -0.0000


>> v3=rats(v3)
```

```
v3 =

       0
     1/3
    -1/3
     2/3

>> v4=rats(v4)

v4 =

    -1/3
     1/6
     1/6
       0
```

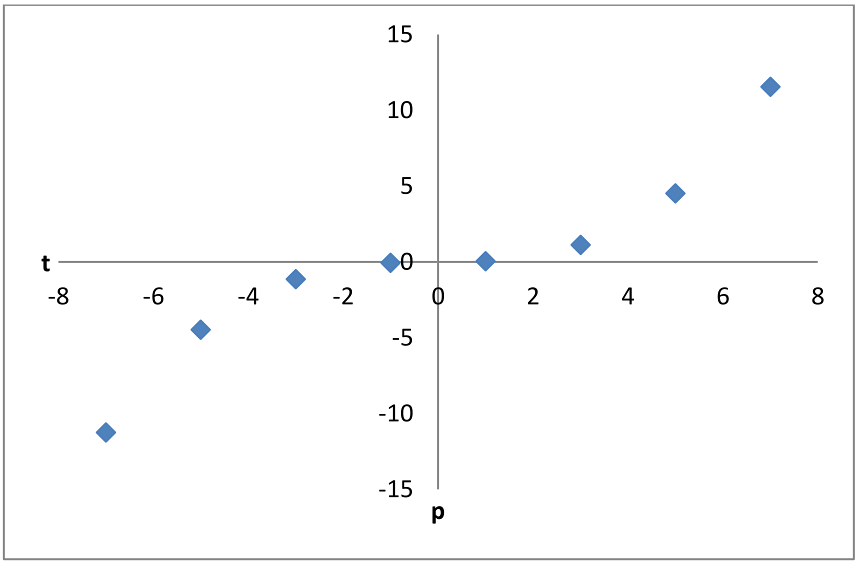
>> So an orthogonal basis would be {v1, v2, v3, v4}

Assignment 6 (15%)

The data

t	p
-7,0	-11,23
-5,0	-4,47
-3,0	-1,12
-1,0	-0,05
1,0	0,06
3,0	1,14
5,0	4,53
7,0	11,56

show some symmetry as can be seen from the following diagram, where the data-points are placed rather symmetrically around origo in the coordinate system.



This suggests that the data can be modelled by an equation of the form

$$p = \beta_0 t + \beta_1 \cdot t^3$$

where β_0 and β_1 are constants.

a) Find the model of this type that produces the least-squares fit of the data.

```
>> t=[-7 -5 -3 -1 1 3 5 7]'

t =

    -7
    -5
    -3
    -1
     1
     3
     5
     7

>> p=[-11.23 -4.47 -1.12 -0.05 0.06 1.14 4.53 11.56]'

p =

  -11.2300
   -4.4700
   -1.1200
   -0.0500
    0.0600
    1.1400
    4.5300
   11.5600

>> X=[t t.^3]

X =

    -7  -343
    -5  -125
    -3   -27
    -1    -1
     1     1
     3    27
     5   125
     7   343

>> XtX=X'*X

XtX =

         168        6216
        6216      268008

>> Xtp=X'*p
```

```
Xtp =

   1.0e+03 *

    0.2114
    9.0031

>> [XtX Xtp]

ans =

   1.0e+05 *

    0.0017    0.0622    0.0021
    0.0622    2.6801    0.0900

>> rref(ans)

ans =

    1.0000         0    0.1094
         0    1.0000    0.0311

>> B=ans(:,3)

B =

    0.1094
    0.0311
```

$p = 0.1094t + 0.0311t^3$

Assignment 7 (20%)
Given a matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 2 \end{bmatrix}$$

a) Compute a singular value decomposition of A in the form

$$A = USV^T$$

```
A =

     1     0
     0     2
     0     2

>> AtA=A'*A

AtA =

     1     0
     0     8
```

```
>> eig(AtA)

ans =

     1
     8

>> AtA-8*eye(2)

ans =

    -7     0
     0     0

>> rref(ans)

ans =

     1     0
     0     0

>> v1=[0 1]'

v1 =

     0
     1

>> AtA-1*eye(2)

ans =

     0     0
     0     7

>> rref(ans)

ans =

     0     1
     0     0

>> v2=[1 0]'

v2 =

     1
     0

>> V=[v1 v2]

V =

     0     1
     1     0
```

```
>> u1=(1/sqrt(8))*A*v1

u1 =

         0
    0.7071
    0.7071

>> u2=(1/sqrt(1))*A*v2

u2 =

     1
     0
     0

>> [u1';u2']

ans =

         0    0.7071    0.7071
    1.0000         0         0

>> rref(ans)

ans =

     1     0     0
     0     1     1

>> u3=[0 -1 1]'

u3 =

     0
    -1
     1
>> u3=u3/norm(u3)

u3 =

         0
   -0.7071
    0.7071

>> dot(u1,u3)

ans =

     0

>> dot(u2,u3)

ans =
```

```
     0

>> U=[u1 u2 u3]

U =

         0    1.0000         0
    0.7071         0   -0.7071
    0.7071         0    0.7071

>> S=[sqrt(8) 0;0 1;0 0]

S =

    2.8284         0
         0    1.0000
         0         0

>> U*S*V'

ans =

     1     0
     0     2
     0     2

>>
```

b) Show that the columns of U are eigenvectors of AA^T and determine the corresponding eigenvalues.

```
>> AAt=A*A'


AAt =


     1     0     0

     0     4     4

     0     4     4
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 8/\sqrt{2} \\ 8/\sqrt{2} \end{bmatrix} = 8 \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

So u1 is an eigenvector with eigenvalue 8.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

So u2 is an eigenvector with eigenvalue 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So u3 is an eigenvector with eigenvalue 0.