

Assignment 1

The answers to the problems of this assignment do not require any calculations but can be inferred from the information given (provided you have the relevant knowledge!)

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 5 & -7 & -2 \\ 4 & -5 & 3 & 1 & -3 \\ 3 & 7 & -2 & 4 & 5 \\ 2 & 2 & -7 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 9 & 5 & 1 \\ 0 & -3 & 1 & 4 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

- a) Explain why the columns of A are not linearly independent
- b) Explain why $Ax = \mathbf{0}$ has a non-trivial solution
- c) Explain why $Bx = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^3
- d) Explain whether or not AB^T is well-defined

Assignment 2

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & -1 \\ -1 & -3 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

- a) Find the inverse of A using elementary row operations on the augmented matrix $[A \ I]$
- b) Use the inverse of A to solve $Ax = \mathbf{b}$

Assignment 3

Find the value(s) of a for which the determinant of the following matrix is -18 by co-expanding on the second row

$$\begin{bmatrix} 1 & 5 & -a \\ a & -a & a \\ 2 & 13 & -7 \end{bmatrix}$$

Assignment 4

Let

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, u_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \text{ and } v = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \end{bmatrix},$$

- a) Show that $\{u_1, u_2, u_3, u_4\}$ is an orthogonal basis for \mathbb{R}^4 using the inner product.
- b) Write v as the sum of two vectors, one in $\text{span}\{u_1, u_2\}$ and the other in $\text{span}\{u_3, u_4\}$.

Assignment 5

Measurements of the deflection (mm) of particleboard from stress levels of relative humidity are displayed below.

Stress level (%)	Deflection (mm)
54	16.473
54	18.693
61	14.305
61	15.121
68	13.505
68	11.640
75	11.168
75	12.534
75	11.224

- Find the best fitted least-squares line to describe the data above
- Determine the least-squares error of the least squares line found in (a).

Assignment 6

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & -1 \\ -1 & -3 & 2 \end{bmatrix}$, $v_1 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

- Show that v_1 and v_2 are eigenvectors of A with associated eigenvalues λ_1 and λ_2 , respectively
- Determine the eigenspaces of λ_1 and λ_2
- Orthogonally diagonalize A where $A = PDP^{-1}$ and the columns of P are normalized