Assignment 1 (15%)

You will need very few and simple calculation in order to solve the problems of this assignment.

Let

$$A = \begin{bmatrix} -1 & 0 & 2 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 9 & 0 & 2 \\ 4 & -5 & 0 & 1 \\ -7 & 2 & 0 & 3 \\ 3 & 1 & 0 & -4 \end{bmatrix}$$

a) Explain why it is very easy to calculate the determinant of matrix A and state what this determinant is.

b) State whether or not A^5 is invertible and state the reasoning behind your answer.

c) Let B and C be two $n \times n$ matrices. Suppose CBC is invertible. Explain why it follows that $\det(B) \neq 0$ and $\det(C) \neq 0$.

d) Explain whether or not matrix D is invertible.

e) Supply a substantiated answer to whether or not the columns of D span $\mathbb{R}^4$.

Assignment 2 (10%)

Let

$$A = \begin{bmatrix} 4 & 0 & 4 \\ -9 & -2 & -8 \\ 8 & 4 & 6 \end{bmatrix} \text{ and } \bar{b} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$$

a) Determine whether $\bar{b}$ is in $Nul\ A$.
b) Determine whether $\bar{b}$ is in $Col\ A$.

Assignment 3 (20%)

Consider the following system

$$x - y + 3z = 1$$

$$y = -2x + 5$$

$$9z - x - 5y + 7 = 0$$

a) Write the system in the matrix form $A\bar{x} = \bar{b}$ for $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

b) Write out the augmented matrix for this system and state its echelon form

c) Write out the complete set of solutions (if they exist) in parametric vector form.
d) Calculate the inverse of the coefficient matrix A you found in part (a), if it exists, or show that A^{-1} doesn't exist.

Assignment 4 (15%)

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

a) Using co-factor expansion, show that $\det(A) = 0$
b) Find the characteristic polynomial, eigenvalues and eigenvectors of A

Assignment 5 (15%)

Let

$$\bar{x}_1 = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}, \bar{x}_2 = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}, \qquad and\ \bar{y} = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}.$$

And let $H = span\{\bar{x}_1, \bar{x}_2\}$, and let $H^{\perp}$ denote the subspace that is orthogonal to H

a) Use the Gram-Schmidt process to find an orthogonal basis for H.
b) Decompose $\bar{y}$ as $\bar{y} = \hat{y} + \bar{z}$, where $\hat{y}$ is in H and z is in $H^{\perp}$.

Assignment 6 (15%)

Concerns about global warming have led to studies of the relationship between global temperature and the concentration of carbon dioxide (CO_2). Listed below are the concentrations (in parts per million) of CO_2 and temperatures (in ºC) for different years.

CO_2 (x)	Temperature (y)
314	13,9
317	14
320	13,9
326	14,1
331	14
339	14,3
346	14,1
354	14,5
361	14,5
369	14,4

It is believed that the global temperature can be modelled by either a linear function or a quadratic function:

$y_1(x) = \beta_0 + \beta_1 x$ or $y_2(x) = \delta_0 + \delta_1 x + \delta_2 x^2$

a) By creating design matrices, find the parameters of both models and state the regression functions
b) Supply a substantiated answer to which model is the best fit for the measured data
c) Use the best fitted model to predict the global temperature once the CO_2 level reaches 400.

Assignment 7 (10%)

Compute a full singular value decomposition of the following matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$