**Assignment 1 (10%)**

You will need very few and simple calculation in order to solve the problems of this assignment. The following questions refer to the matrices

$$A = \begin{bmatrix} 4 & 9 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 7 \\ 1 & 7 & 7 \end{bmatrix}$$

and the vector

$$\boldsymbol{b} = \begin{bmatrix} -1 \\ 2 \\ 8 \end{bmatrix}$$

a) Which of the matrices are in echelon form?
b) Write the system of linear equations that correspond to the matrix equation $Cx = b$.
c) Is the system of linear equations that correspond to the matric equation $Cx = b$ consistent?
d) Explain which of the following five expressions that makes sense.

$$AB, \quad CA, \quad B^2$$
$$\det(B), \quad \det(C)$$

**Assignment 2 (20%)**

Given

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

a) Solve the matrix equation $X + A = 2(X - B)$
b) Determine the rank and the nullity of $A$.
c) Determine the rank and the nullity of B and find a basis for $Col\ B$ and a basis for $Null\ B$.
d) Find the characteristic polynomial of B and use it to find the eigenvalues of B.

**Assignment 3 (10%)**

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -4 & -1 \\ -1 & -3 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$

a) Find the inverse of *A* using elementary row operations on the augmented matrix $[A \quad I]$
b) Use the inverse of A to solve $Ax = b$

**Assignment 4 (5%)**

Find the value(s) of α for which the determinant of the following matrix is -18 by co-expanding on the second row

$$\begin{bmatrix} 1 & 5 & -a \\ a & -a & a \\ 2 & 13 & -7 \end{bmatrix}$$

**Assignment 5 (20%)**
Let

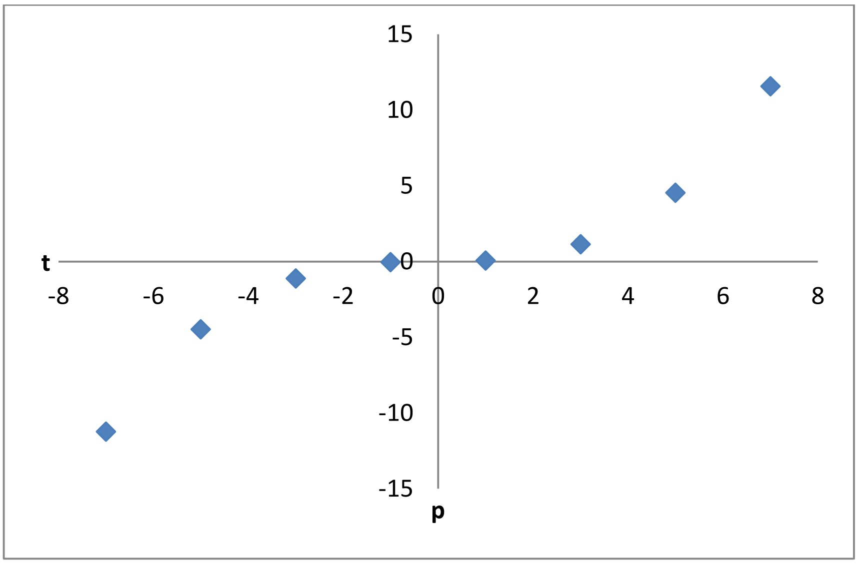
$$\boldsymbol{u_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \quad \boldsymbol{u_2} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \boldsymbol{u_3} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \; and \; \boldsymbol{u_4} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

a) Show that $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\}$ is a basis for $\mathbb{R}^4$.
b) Use the Gram-Schmidt process on $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\}$ to obtain an orthogonal basis for $\mathbb{R}^4$.

**Assignment 6 (15%)**
The data

| t | p |
|---|---|
| -7,0 | -11,23 |
| -5,0 | -4,47 |
| -3,0 | -1,12 |
| -1,0 | -0,05 |
| 1,0 | 0,06 |
| 3,0 | 1,14 |
| 5,0 | 4,53 |
| 7,0 | 11,56 |

show some symmetry as can be seen from the following diagram, where the data-points are placed rather symmetrically around origo in the coordinate system.




This suggests that the data can be modelled by an equation of the form

$$p = \beta_0 t + \beta_1 \cdot t^3$$

where $\beta_0$ and $\beta_1$ are constants.

a) Find the model of this type that produces the least-squares fit of the data.

**Assignment 7 (20%)**

Given a matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 2 \end{bmatrix}$$

a) Compute a singular value decomposition of A in the form

$$A = USV^T$$

b) Show that the columns of U are eigenvectors of $AA^T$ and determine the corresponding eigenvalues.