Assignment 1 (15%)

You will need very few and simple calculation in order to solve the problems of this assignment.

Let

$$A = \begin{bmatrix} -1 & 0 & 2 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 9 & 0 & 2 \\ 4 & -5 & 0 & 1 \\ -7 & 2 & 0 & 3 \\ 3 & 1 & 0 & -4 \end{bmatrix}$$

a) Explain why it is very easy to calculate the determinant of matrix A and state what this determinant is.

Since A is a triangular matrix, the determinant can be computed as the product of the main diagonal. The determinant is thus 2.

b) State whether or not A^5 is invertible and state the reasoning behind your answer.

Since A is triangular, A^5 will also be triangular and the main diagonal will merely consist of each entry raised to the power of 5. That means that the determinant of A^5 is non-zero which means it is invertible.

c) Let B and C be two $n \times n$ matrices. Suppose CBC is invertible. Explain why it follows that $\det(B) \neq 0$ and $\det(C) \neq 0$.

If CBC is invertible, it must be the case that $\det CBC \neq 0$. We also know that $\det CBC = (\det C)(\det B)(\det C)$. It then follows that $(\det C)(\det B)(\det C) \neq 0$ which means that $\det(B) \neq 0$ and $\det(C) \neq 0$

d) Explain whether or not matrix D is invertible.

Matrix D is not invertible since it has a column of zeroes.

e) Supply a substantiated answer to whether or not the columns of D span $\mathbb{R}^4$.

The columns of D do not span $\mathbb{R}^4$ since there are only three vectors that are not the zero-vector. Three vectors cannot span a 4-dimensional space.

Assignment 2 (10%)

Let

$$A = \begin{bmatrix} 4 & 0 & 4 \\ -9 & -2 & -8 \\ 8 & 4 & 6 \end{bmatrix} \text{ and } \bar{b} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$$

a) Determine whether $\bar{b}$ is in $Nul\ A$.

```
A =
    4     0     4
   -9    -2    -8
    8     4     6
>> b=[-2 2 2]';
>> A*b
ans =
   0
  -2

   4
```

So, b is not in Nul A.

b) Determine whether $\bar{b}$ is in $Col\ A$.

```
>> rref([A b])
ans =
   1.0000         0    1.0000         0
        0    1.0000   -0.5000         0
        0         0         0              1.0000
```

So, b is not in Col A.

Assignment 3 (20%)

Consider the following system

$$x - y + 3z = 1$$

$$y = -2x + 5$$

$$9z - x - 5y + 7 = 0$$

a) Write the system in the matrix form $A\bar{x} = \bar{b}$ for $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ -1 & -5 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -7 \end{bmatrix}$$

b) Write out the augmented matrix for this system and state its echelon form

$$\begin{bmatrix} 1 & -1 & 3 & 1 \\ 2 & 1 & 0 & 5 \\ -1 & -5 & 9 & -7 \end{bmatrix}$$

```
B =
    1   -1    3    1
    2    1    0    5
   -1   -5    9   -7
>> B(3,:)=B(3,:)+B(1,:)
B =
    1   -1    3    1
    2    1    0    5
    0   -6   12   -6
>> B(2,:)=B(2,:)-2*B(1,:)
B =
    1   -1    3    1
    0    3   -6    3
    0   -6   12   -6
>> B(3,:)=B(3,:)+2*B(2,:)
B =
    1   -1    3    1
    0    3   -6    3
    0    0    0    0
```

c) Write out the complete set of solutions (if they exist) in parametric vector form.

```
>> rref(B)
ans =
    1    0    1    2
    0    1   -2    1
    0    0    0    0
```

$$\bar{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

d) Calculate the inverse of the coefficient matrix A you found in part (a), if it exists, or show that A^{-1} doesn't exist.

Above we found echelon form of the augmented matrix. If we disregard the last column, we have the echelon form of A. We see that the diagonal contains a zero which means that the determinant is zero. This means that A is not invertible and A^{-1} does not exist.

Assignment 4 (15%)

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

a) Using co-factor expansion, show that $\det(A) = 0$

I use the third column

```
>> D = A(1,3)*det(A([2,3],[1,2]))+A(3,3)*det(A([1,2],[1,2]))

D =

    0
```

b) Find the characteristic polynomial, eigenvalues and eigenvectors of A

```
>> charpoly(A)
ans =
     1    -9    18     0
```
so $\lambda^3 - 9\lambda^2 + 18\lambda = 0$

```
>> [V L] = eig(A)
V =
   -0.6667   -0.3333    0.6667
    0.6667   -0.6667    0.3333
    0.3333    0.6667    0.6667
L =
   -0.0000         0         0
         0    3.0000         0
         0         0    6.0000
>>
```

The columns of V are the eigenvectors and in the corresponding columns of L we find the eigenvalues that correspond to the given eigenvector.

Assignment 5 (15%)

Let

$$\bar{x}_1 = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}, \bar{x}_2 = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}, \quad and\ \bar{y} = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}.$$

And let $H = span\{\bar{x}_1, \bar{x}_2\}$, and let $H^\perp$ denote the subspace that is orthogonal to H

a) Use the Gram-Schmidt process to find an orthogonal basis for H.

```
>> x1=[-1 -2 0]';x2=[-1 3 -1]'; y=[2 -4 1]';
>> v1 = x1;
>> v2 = x2-proj(x2,v1)
v2 =
-2.0000
1.0000
-1.0000
>> Basis = [v1 v2]
Basis =
```

```
  -1.0000   -2.0000
  -2.0000    1.0000
      0     -1.0000
```

b) Decompose $\bar{y}$ as $\bar{y} = \hat{y} + \bar{z}$, where $\hat{y}$ is in H and z is in $H^{\perp}$.

```
>> Hcompl = nulbasis(A')
Hcompl =
   -0.4000
    0.2000
   1.0000
>> z = proj(y,Hcompl)
z =
    0.2000
   -0.1000
   -0.5000
>> yhat = proj(y,v1)+proj(y,v2)
yhat =
    1.8000
   -3.9000
    1.5000
>> y = yhat+z
y =
    2.0000
   -4.0000
    1.0000

>>
```

Assignment 6 (15%)

Concerns about global warming have led to studies of the relationship between global temperature and the concentration of carbon dioxide (CO_2). Listed below are the concentrations (in parts per million) of CO_2 and temperatures (in ºC) for different years.

CO_2 (x)	Temperature (y)
314	13,9
317	14
320	13,9
326	14,1
331	14
339	14,3
346	14,1
354	14,5
361	14,5

369	14,4

It is believed that the global temperature can be modelled by either a linear function or a quadratic function:

$y_1(x) = \beta_0 + \beta_1 x$ or $y_2(x) = \delta_0 + \delta_1 x + \delta_2 x^2$

a) By creating design matrices, find the parameters of both models and state the regression functions

```
>> % Linear function
>> X1 = [ones(length(x),1) x];
>> X1tX1 = X1'*X1;X1ty = X1'*y;
>> b1 = rref([X1tX1 X1ty])
b1 =
    1.0000         0   10.4831
         0    1.0000    0.0109
>> b1=b1(:,end)
b1 =
   10.4831
    0.0109
>> % y1 = 10.4831 + 0.0109*x
>> % Quadratic function
>> X2 = [ones(length(x),1) x x.^2];
>> X2tX2 = X2'*X2;X2ty = X2'*y;
>> b2 = rref([X2tX2 X2ty])
b2 =
    1.0000         0         0    3.8474
         0    1.0000         0    0.0500
         0         0    1.0000   -0.0001
>> b2=b2(:,end)
b2 =
    3.8474
    0.0500
   -0.0001
>> % y2 = 3.8474 + 0.05*x - 0.0001*x^2
```

b) Supply a substantiated answer to which model is the best fit for the measured data

```
>> %Error linear
>> Error1 = norm(y-X1*b1)
Error1 =
    0.3200
>> %Error linear
>> Error2 = norm(y-X2*b2)
Error2 =
```

```
    0.3162
>> min(Error1,Error2)
ans =
    0.3162
>>
```
So the quadratic functions yields the smallest error and is the best fit for the data.

c) Use the best fitted model to predict the global temperature once the CO_2 level reaches 400.

```
>> y400 = b2(1)+b2(2)*400+b2(3)*400
y400 =
   23.8291
```
So the predicted temperature is around 24 degrees Celsius.

Assignment 7 (10%)

Compute a full singular value decomposition of the following matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

```
>> A = [1 1;1 1;0 0]

A =

     1     1

     1     1

     0     0

>> AtA = A'*A

AtA =

     2     2

     2     2

>> [V D] = eig(AtA)

V =

   -0.7071    0.7071

    0.7071    0.7071

D =

     0     0
```

```
     0     4

>> V = fliplr(V)

V =

    0.7071   -0.7071
    0.7071    0.7071

>> AAt = A*A'

AAt =

     2     2     0
     2     2     0
     0     0     0

>> [U D] = eig(AAt)

U =

   -0.7071         0    0.7071
    0.7071         0    0.7071
         0    1.0000         0

D =

     0     0     0
     0     0     0
     0     0     4

>> S = sqrt(fliplr(flip(D(:,2:end))))

S =

     2     0
     0     0
     0     0

>> U = fliplr(U);
>> U

U =

    0.7071         0   -0.7071
    0.7071         0    0.7071
```

```
         0    1.0000         0
>> S
S =
     2     0
     0     0
     0     0
>> Vt = V'
Vt =
    0.7071    0.7071
   -0.7071    0.7071
>> U*S*Vt
ans =
    1.0000    1.0000
    1.0000    1.0000
         0         0
>>
```