Assignment 1 (20%)

You will need very few and simple calculation in order to solve the problems of this assignment.

a) The result of an SVD displays

$$U = \begin{bmatrix} -0.2534 & -0.3430 & 0.9045 \\ -0.4897 & -0.7609 & -0.4257 \\ -0.8343 & 0.5508 & -0.0249 \end{bmatrix}$$

$$V^T = \begin{bmatrix} -0.4132 & 0.4230 & 0.2244 & -0.7746 \\ -0.5756 & -0.2919 & -0.7189 & 0.2582 \\ 0.4397 & 0.8557 & -0.0881 & -0.2582 \\ -0.5519 & -0.0606 & 0.6520 & 0.5164 \end{bmatrix}$$

What is the size of the matrix which has been decomposed?

b) Explain why it would be possible to determine λ_3 of A if λ_1 were 4 and λ_2 were 2 and then state what λ_3 is if this were the case.

$$A = \begin{bmatrix} 8 & 3 & -5 \\ 3 & 3 & -2 \\ -5 & -2 & 1 \end{bmatrix}$$

c) Explain why the following matrix is not invertible

$$\begin{bmatrix} 2 & 4 & 8 & 7 & 1 & 3 \\ 0 & 5 & 3 & 5 & 5 & 6 \\ 0 & 0 & 3 & 3 & 3 & 9 \\ 0 & 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix}$$

d) For what value(s) of k are the columns of the following matrix linearly dependent?

$$\begin{bmatrix} 2 & -10 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 2k-6 \end{bmatrix}$$

e) Explain why $Ax = b$ formed from the following matrix in echelon form has a solution for each b

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Assignment 2 (5%)

Determine a basis for the null space of matrix A by setting up the solution in parametric form

$$A = \begin{bmatrix} -1 & 2 & 1 & 4 \\ 1 & 2 & 2 & 6 \end{bmatrix}$$

Assignment 3 (15%)

A 2 x 2 matrix A is given as

$$A = \begin{bmatrix} a & 2 \\ 1 & 2 \end{bmatrix}$$

a) Express the eigenvalues of A as a function of a
b) Calculate a so that $\lambda = 4$ is an eigenvalue of A
c) Using the value of a found in question (b), determine the remaining eigenvalue(s) and the eigenspaces

Assignment 4 (20%)

Let,

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, and\ x = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix},$$

a) Show that $\{u_1, u_2, u_3\}$ is a linearly independent set
b) Show that $\{u_1, u_2, u_3\}$ is an orthogonal basis for $\mathbb{R}^3$
c) Express x as the sum of two vectors, one in $span\{u_1, u_2\}$ and the other in $span\{u_3\}$.
d) Determine the (shortest) distance between x and the subspace spanned by $\{u_1, u_2, u_3\}$

Assignment 5 (20%)

For a specific sensor, the following corresponding values of time t and output y have been measured

t	y
0	1
1	3
2	7
3	11
4	16
5	24

It is assumed that the system can be approximated by a model of the form $y_1(t) = \beta_0 + \beta_2 t^2$ or of the form $y_2(t) = \delta_0 + \delta_1 t + \delta_2 t^2$

a) Determine the design matrix and observation vector of both models
b) Determine the parameters of both models
c) Supply a substantiated answer to which model is the best fit for the measured data

Assignment 6 (20%)

Let the matrix A be given by

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \end{bmatrix}$$

a) Compute a full singular value decomposition of the matrix A
b) How do the SVDs of A and A^T relate to each other (it is allowed to calculate the SVDs in Matlab in order to answer this question)