Assignment 1

The answers to the problems of this assignment do not require any calculations but can be inferred from the information given (provided you have the relevant knowledge!)

a) Represent each linear system of equations in matrix form

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 1 \\ 4x_1 + 7x_2 + x_3 = 3 \\ 7x_1 + 10x_2 - 4x_3 = 4 \end{cases}$$

$$\begin{cases} 3x_1 + 3x_2 + x_3 = -4.5 \\ x_1 + x_2 + x_3 = 0.5 \\ -2x_1 - 2x_2 = 5 \end{cases}$$

$$\begin{cases} 2x_2 - 3x_3 = 1 \\ 3x_1 + 6x_2 + x_3 = 13 \\ 4x_1 - 4x_3 = 4 \end{cases}$$

In b) - e) refer to the following coefficient matrix

$$\begin{bmatrix} 1 & 3 & 6 & -1 & 0 \\ 0 & 2 & 5 & 3 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) Determine which columns have pivot positions
c) What is the rank of the matrix?
d) Are there any free variables, and if so which?
e) Explain why the matrix is *not* invertible.

f) Let $det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 5$. Find

$$det\left(\begin{bmatrix} a & b & c \\ d+2a & e+2b & f+2c \\ g & h & i \end{bmatrix}\right)$$

Assignment 2

Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$

a) What value(s) of *k*, if any, will make $AB = BA$?

b) For what value(s) of *k* are the columns of the following matrix linearly dependent?

$$\begin{bmatrix} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & k \end{bmatrix}$$

c) Find the value(s) of *k* for which the matrix $\begin{bmatrix} k^2 & 2k \\ 8 & k \end{bmatrix}$ is singular (i.e. not invertible).

Assignment 3

Find the value(s) of α for which the determinant of the following matrix is zero

$$\begin{bmatrix} a & \sqrt{2} & 0 \\ \sqrt{2} & a & \sqrt{2} \\ 0 & \sqrt{2} & a \end{bmatrix}$$

Assignment 4

Let

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, u_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, and\ v = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \end{bmatrix},$$

a) Show that $\{u_1, u_2, u_3, u_4\}$ is an orthogonal basis for $\mathbb{R}^4$

b) Write *v* as the sum of two vectors, one in $span\{u_1, u_2\}$ and the other in $span\{u_3, u_4\}$.

c) Determine the (shortest) distance between *v* and the subspace spanned by $\{u_1, u_2, u_3\}$

Assignment 5

The following data were measured in an experiment that was aimed at studying the effects of reducing current draw in a magnetic core by electronic means. The researchers measured the current in a magnetic winding with and without the electronics in a paired experiment and the data for the case without electronics is provided in the following table

Supply Voltage	Current Without Electronics (mA)
0.66	7.32
1.32	12.22
1.98	16.34
2.64	23.66
3.3	28.06
3.96	33.39
4.62	34.12
3.28	39.21
5.94	44.21
6.6	47.48

a) Find the best fitted least-squares line to describe the data above
b) Determine the least-squares error of the least squares line found in (a).

Assignment 6

Compute a full singular value decomposition of the following matrix A:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$