Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Composite reliability evaluation using sequential Monte Carlo simulation with maximum and minimum loadability analysis

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ARTICLE INFO

Keywords:
Sequential Monte Carlo simulation
Maximum loadability
Cross-entropy

ABSTRACT

This paper introduces an approach to composite reliability evaluation using sequential Monte Carlo simulation with maximum and minimum loadability analysis. The approach utilizes an estimation of the maximum and minimum loadability of a given system state to avoid linearized power flow (PF) and optimal power flow (OPF) assessments in further states characterized by load transitions. The evaluation of the maximum loadability of a system state is also employed within the cross-entropy (CE) optimization procedure, as an alternative performance function to sampled states. Numerical results highlight the effectiveness of the approach and gains in runtime, in comparison with implementations of benchmark methods, for the IEEE RTS 79 test system.

1. Introduction

In power system reliability evaluations, the Monte Carlo simulation stands out as a powerful tool to evaluate performance indicators [1,2], particularly the ones related to loss of load events. There are several variations of Monte Carlo simulation approaches to reliability evaluation. Among them, the sequential Monte Carlo simulation (SMCS) is the standard choice to represent chronological aspects of power systems and to estimate reliability index probability distributions. In the SMCS, sequences of events are synthetically generated, creating a history of the system operation. As consequence, non-Markovian models may be straightforwardly used to emulate component failures. Moreover, several chronological aspects of the system can be modeled, including correlated space–time load models, fluctuation in capacity of renewable energy sources, cost functions by area or bus and scheduled maintenance.

The reliability assessment of robust composite power systems through SMCS usually requires a considerable amount of computational effort. In robust systems, the states that contribute to the convergence of reliability index estimates are usually associated to rare events. The computational effort is also noteworthy in case reduced coefficient of variations are required. Recent approaches towards reducing the computational effort of Monte Carlo applications to composite reliability evaluation have been focused on addressing two main drawbacks: (a) the large quantity of system state samples required to attain a specified accuracy on the estimation of reliability indices; (b) large amount of runtime needed to evaluate sampled states.

In order to address drawback (a), applications of the CE method [3,4] have recently emerged aiming to estimate reliability indices with adequate accuracy while minimizing computational time. CE-based approaches have been designed to estimate biased

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https://doi.org/10.1016/j.compeleceng.2024.110023
Received 23 September 2024; Received in revised form 10 December 2024; Accepted 18 December 2024

probability density functions and to weight simulation outputs accordingly, reducing the number of system samples required to performance index convergence. Such principle is used in [5–7], where the CE method is applied to the non-sequential Monte Carlo simulation to evaluate reliability indices of composite systems; in [1], where an enhanced CE method is proposed by applying a sequential CE optimization rather than a traditional non-sequential optimization; and in [2], where a novel CE method is introduced by including the usage of the Metropolis–Hastings algorithm to sample states and by defining a novel quantitative stopping criterion. Notwithstanding, other techniques have been exploited to speed up the Monte Carlo simulation process [8,9]. In [8], the authors propose a uniform design algorithm to improve the representativeness of the randomly generated system states combined with a hierarchical sampling technique to reduce sample size and computational times; and in [9] a cluster-based stratified sampling (CBSS) method is proposed as a sampling mechanism for the SMCS. In the latter, generation adequacy evaluation is conducted to create a dataset containing probability, frequency, and severity information of load curtailment events. Subsequently, composite reliability indices are determined via CBSS by utilizing selected meaningful observations from the dataset.

Drawback (b) can be addressed by identifying failed states more efficiently, trying to avoid state evaluations based on PF and/or OPF analysis. This concept can be found, for instance, in [1], where a bisection optimization algorithm is used to rapidly detect system failed states; and in [10], where Lagrange multipliers are applied to determine optimal load shedding solutions in case of injection variations and topology changes. Moreover, several works have employed artificial intelligence based techniques to minimize the need for OPF analysis, including the application of multi label radial basis functions [6]; multi label k-nearest neighbors [11]; convolutional neural networks [7,12–14]; multilayer perceptrons [15] and binary logistic regressions [16]. Within this article, alongside the works [1,6,7], both drawbacks (a) and (b) are exploited with the aim of reducing the computational burden of the reliability evaluation.

In this context, this paper introduces an approach to composite reliability evaluation using SMCS with maximum and minimum loadability analysis. The approach applies an estimation of maximum and minimum loadability of system states aiming to suppress linearized PF and OPF evaluations in further states associated to load transitions. Moreover, the proposed maximum loadability analysis is utilized within a CE optimization procedure in order to optimize system stochastic parameters. Summarily, the main contributions of the work are the following:

(i) a formulation of an optimization problem for computing the maximum and minimum loadability of a system state, aiming to avoid linearized PF and OPF assessments in the SMCS;
(ii) an analytical deduction for solution estimates of the optimization problem stated in (i);
(iii) the application of maximum loadability analysis in the CE method to increase the efficiency of the optimization process.

Contributions (i)–(ii) aim to address drawback (b) by strategically exploiting maximum and minimum loadability analysis within the SMCS. Compared to artificial intelligence-based techniques, the resulting approach is independent of sample data and avoids applying a model-free approach [6,7,11–16], which may lead to inaccuracies in index estimation. Instead, it relies on the actual model of the system to assess the occurrence of success system states. On the other hand, contribution (iii) aims to address drawback (a) and can be applied as an alternative in applications of the CE method [5–7] to composite reliability evaluation. The purpose of these contributions is not to ease the implementation process or to necessarily improve the accuracy of index estimation. Conversely, the objective is to strategically utilize maximum and minimum loadability analysis to reduce simulation time by avoiding time-consuming state evaluations.

Simulation results are acquired using the IEEE RTS 79 test system. Numerical results highlight the suppression of evaluations performed through linearized PF and OPF, consequently diminishing the simulation time required to the convergence of reliability indices. Employing maximum and minimum loadability analysis within the SMCS significantly decreases computation duration, with further time savings achieved by applying maximum loadability analysis in the CE optimization procedure.

The paper is organized as follows. In Section 2, composite reliability evaluation through SMCS is briefly described, highlighting some features exploited in the proposed approach. In Section 3, the proposed approach is described, taking into account two formulations for maximum and minimum loadability assessment. In Section 4, the application of maximum loadability assessments within the CE optimization algorithm is presented. Result analysis and final remarks are outlined in Sections 5 and 6, respectively.

2. Discussions about state evaluation in composite reliability assessments

Composite reliability assessments via SMCS can be generally divided in three main stages which are repeated until the convergence of index estimates: state duration sampling, state evaluation and index estimation. In first stage, named state duration sampling, the time duration in each system state is determined. The principle of the next-event time advance mechanism is applied [17–19], where a clock is advanced to the time instant of the most imminent (first) event, which can be a component (e.g. generator, line, transformer) state transition or load state transition. The most imminent event is determined by computing the minimum value among the scheduled transition times. Hence, in the state evaluation stage, each system state is assessed taking into account its corresponding topology and load level, commomnly using PF and OPF calculations. In the index estimation stage, reliability indices are estimated using test functions to characterize and quantify loss of load events. The simulation is finished when the coefficients of variation of the test functions are inferior to a given threshold. Details and examples of reliability evaluation with SMCS are provided in Sections 3.6 and 3.7 of [20], respectively.

The most time consuming stage is the state evaluation, where the operating conditions and the application of remedial actions must be assessed. Aiming to highlight the designed procedures of the proposed approach, the general steps of the SMCS for composite reliability evaluations are summarized in Algorithm 1, in which the state evaluation is indicated in step 4.

Algorithm 1 SMCS for composite reliability evaluation.

Let n, t_n and N_y be a state counter, time counter and period counter, respectively; Initialize counters as $n \leftarrow 1; t_1 \leftarrow 0;\ N_y \leftarrow 1$.

1: Initialize component and load states to compose the system state $\mathbf{x}_n$;
2: Sample state residence times using stochastic or deterministic functions;
3: Identify the next state transition time t_{n+1}. If t_{n+1} does not overpass the end of the period T, perform the transition to constitute the next system state $\mathbf{x}_{n+1}$. Otherwise, $t_{n+1} \leftarrow T$ and $\mathbf{x}_{n+1} \leftarrow \mathbf{x}_n$.
4: Evaluate the system state $\mathbf{x}_n$;
5: Update test functions $H(\{\mathbf{x}_n\}_{n=1}^{s_y})$, where $\{\mathbf{x}_n\}_{n=1}^{s_y}$ is the sequence of states $\mathbf{x}_n$, with s_y states in period N_y;
6: If a period is completed, update reliability indices using the expected value equation $\mathrm{E}[H(\mathbf{X})] \leftarrow {}^{1}/_{N_y} \sum_{i=1}^{N_y} H\left(\{\mathbf{x}_n\}_{n=1}^{s_i}\right)$. Otherwise, return to step 3;
7: Update coefficients of variation $\beta \leftarrow \sqrt{\mathrm{Var}[H(\mathbf{X})]/N_y}/\mathrm{E}[H(\mathbf{X})]$. If the coefficients of variation are greater than a specified threshold, perform $n \leftarrow 1$, $N_y \leftarrow N_y + 1$ and return to step 3. Otherwise, stop simulation.

The evaluation of system states can be devised assuming either a linear or nonlinear network representation. In long-term evaluations, in which uncertainties in loading, generating capacity and network configuration are considered large, the linear representation is deemed adequate to system representation. The assessments usually target the estimation of loss of load probability (LOLP), expected power not supplied (EPNS) and loss of load frequency/duration (LOLF/LOLD) indices, where each state can be evaluated as a success state or failed state. The latter classification is associated to the existence of curtailed load, whose estimation is required to feed test functions. The index LOLP is used to assess the probability of load curtailment; EPNS represents the average load curtailment in a given period; LOLF denotes the average number of load curtailment events in a given period; and LOLD represents the average duration in a failed state. The expressions for the calculation of each index can be found in [21] (pages 24–25).

In order to evaluate a system state $\mathbf{x}_n$, one can assume a standard two-step procedure: first, if at least a component is in the failed state, the system state is evaluated by analyzing a solution of a PF problem; if the PF solution violates operational limits, a second step promotes the evaluation of the system state by solving an OPF, as formulated in the following equations.

$$\begin{array}{ll} \text{minimize} & c(\widehat{\theta}, \mathbf{P_g}) \\ \text{subject to} & -\overline{\mathbf{B}}\widehat{\theta} + \mathbf{A_g}\mathbf{P_g} = \mathbf{P_d} \\ & -\mathbf{P_l^{max}} \leq \Gamma \widehat{\mathbf{A}}_s \widehat{\theta} \leq \mathbf{P_l^{max}} \\ & \mathbf{P_g^{min}} \leq \mathbf{P_g} \leq \mathbf{P_g^{max}} \end{array} \tag{1}$$

where $\overline{\mathbf{B}}$ is the susceptance matrix excluding the column corresponding to the reference bus; $\widehat{\theta}$ denotes the vector containing the bus angles, excluding the position corresponding to the reference bus; $\mathbf{A_g}$ is the generation connection matrix where $\mathbf{A_g}(i,j) = 1$ if generator i is connected at bus j and 0 otherwise; $\mathbf{P_g}$ is the vector of produced power in each generating unit, including the effect of fictitious generators located at load buses; $\mathbf{P_d}$ is the vector of bus loads; $\Gamma \triangleq diag\left(\frac{1}{x_1}, \ldots, \frac{1}{x_{nl}}\right)$ where nl is the number of system branches and x_i the reactance of branch i; $\mathbf{A_s}$ the branch-bus adjacency matrix where $\mathbf{A_s}(l,i) = 1$ if i is the branch source bus, -1 if i is the branch end bus, and 0 otherwise; $\widehat{A}_s$ the branch-bus adjacency matrix excluding the column corresponding to the reference bus; $\mathbf{P_l^{max}}$ the vector containing the transmission lines ratings; $\mathbf{P_g^{min}}$ and $\mathbf{P_g^{max}}$ the minimum and maximum generator capacity vectors, and $c(\widehat{\theta}, \mathbf{P_g})$ is an objective function designed to estimate load curtailments by minimizing the production of fictitious generators.

Although the linearized PF/OPF problems are composed of a reduced set of equations in comparison to their nonlinear counterpart, in the assessment of real size power systems, their solutions in composite reliability evaluations might imply on large computational execution times. Notwithstanding, while the time between failures of system components can assume values corresponding to several years, load transitions are more frequent and usually modeled using sampled annual load curves in SMCS approaches. As a consequence, in the sequences $\{\mathbf{x}_n\}_{n=1}^{s_i}$, several consecutive states differ only by a load state transition rather than a component state transition. Moreover, if the system load is modeled with a short time resolution, there is a significant increase in the number of states to be evaluated in the SMCS. In this context, an approach based on the estimation of the maximum and minimum loadability of successful states is proposed to allow verifying if subsequent load values are within a loadability range. Verification is possible within a time range devoid of topology alterations, since bus angles and power flows are assumed linearly related. If the loading is within a loadability range, state evaluation can be performed straightforwardly without requiring solving PF or OPF problems.

3. Proposed state evaluation procedure with maximum and minimum loadability analysis

The proposed approach is developed with the target of avoiding PF and OPF computations within state evaluations by using results from maximum and minimum loadability analysis. Building upon the optimization-based maximum loadability problem presented in [22], a linearized version of this problem is devised, considering all system loads increased or decreased by a proportion ρ, named loadability factor. Fig. 1 illustrates the main steps of the proposed approach. The embedding of the maximum and minimum loadability analysis in the state evaluation stage is performed as follows: the maximum and minimum loadability factors of a system state are calculated whenever a component transition is assigned, at least one component is in the failed state, and the system state is classified as successful; if the loading of a subsequent state originated by load transition belongs to the interval

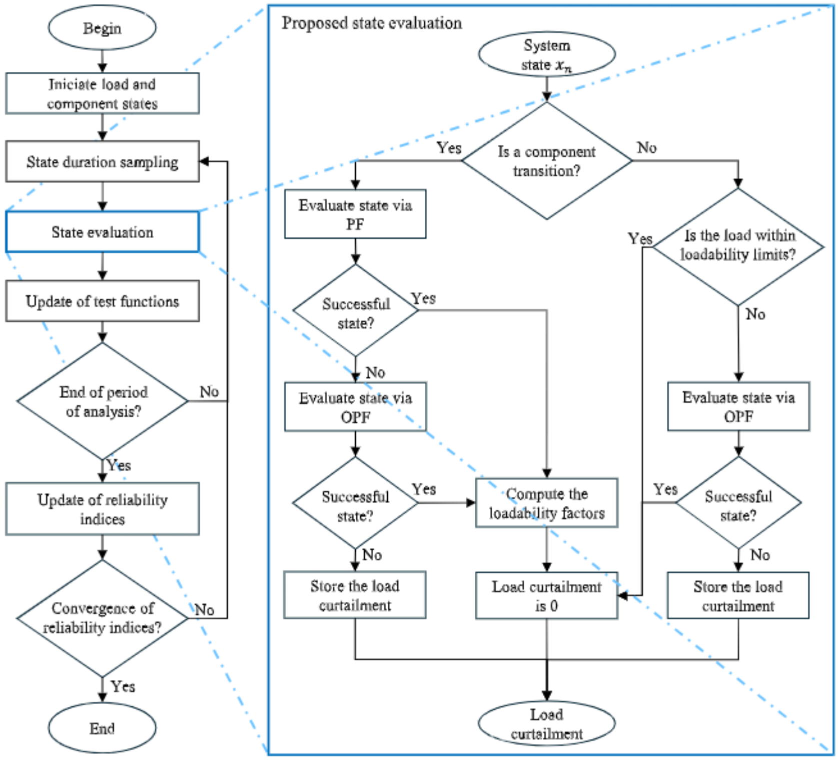

Fig. 1. Proposed state evaluation within the SMCS.

defined by the loadability limits, the state is considered successful and the curtailed load is set to zero. This concept can be verified in Algorithm 2, which summarizes the algorithm procedure to composite evaluation with maximum and minimum loadability analysis.

Algorithm 2 SMCS for composite evaluation with maximum and minimum loadability analysis.

← Definitions and initialization of Algorithm 1.
← Step 1 to 3 of Algorithm 1.
4: If all components modeled in $\mathbf{x}_n$ are in the operating state, $\mathbf{x}_n$ is considered successful. Otherwise, execute the following steps:

i. If $\mathbf{x}_n$ has been constituted by a component state transition, then perform the state evaluation using the procedures described in section Section 2. If the state is considered successful, compute the loadability factors.
ii. If $\mathbf{x}_n$ has been constituted by a load state transition and its corresponding loadability limits have been computed, then classify the state as successful in case the state loading lies within the loadability limits. Otherwise, solve the problem given in (1) to classify the state.

← Step 5 to 7 of Algorithm 1.

Two strategies designed to estimate state loadability limits are presented in the followings.

3.1. Loadability factor estimation for unrestricted dispatch profile (MCSρ)

The linearized version of the maximum loadability problem considering all system loads increased by the proportion ρ in the direction $\boldsymbol{\Delta}\mathbf{P_d}$ can be described as follows.

$$\begin{aligned} \text{minimize} \quad & c(\widehat{\theta}, \mathbf{P_g}, \rho) \\ \text{subject to} \quad & -\overline{\mathbf{B}}\widehat{\theta} + \mathbf{A_g}\mathbf{P_g} - \rho\boldsymbol{\Delta}\mathbf{P_d} = \mathbf{P_d} \\ & -\mathbf{P_l^{max}} \leq \Gamma \widehat{\mathbf{A}}_\mathbf{s} \widehat{\theta} \leq \mathbf{P_l^{max}} \\ & \mathbf{P_g^{min}} \leq \mathbf{P_g} \leq \mathbf{P_g^{max}} \end{aligned} \tag{2}$$

where $c(\widehat{\theta}, \mathbf{P_g}, \rho) = -\rho$, $\boldsymbol{\Delta}\mathbf{P_d}$ is the load vector increment and the other variables have been previously defined.

The formulation presented in (2) has as constraints the active power equation balance, where the weighted load increment is included. Other constraints are designed as in (1) to cover production and transmission limits. The formulation of the minimum loadability problem is analogous to the one presented in (2), differing only by a change in the signal of the objective function.

3.2. Loadability factor estimation for restricted dispatch profile (MCS$\hat{\rho}$)

Although the formulation in (2) might allow reducing significantly the number of states evaluated using PF or OPF through Algorithm 2, it ultimately requires solving two optimization problems with non negligible computational time. An alternative way to estimate the loadability factors can be specified as follows.

Let us define $\mathbf{P}_g$ as the vector of productions that meet the demand $\mathbf{P}_d$, as well as $\boldsymbol{\Delta}\mathbf{P_g}$ as the generation dispatch variation that covers the load variation in a given transition. Then, the power balance equation can be rewritten as

$$-\overline{\mathbf{B}}\hat{\theta} + \mathbf{A_g}\mathbf{P_g} + \rho_g \mathbf{A_g}\boldsymbol{\Delta}\mathbf{P_g} - \rho\boldsymbol{\Delta}\mathbf{P_d} = \mathbf{P_d} \tag{3}$$

Since, in the linearized network representation, the total generation meets the total demand, then $\rho_g = \rho$. Thus, an alternative formulation for the linearized maximum loadability problem can be written as

$$\begin{array}{ll} \text{minimize} & c(\hat{\theta}, \rho) \\ \text{subject to} & -\overline{\mathbf{B}}\hat{\theta} + \mathbf{A_g}\mathbf{P_g} + \rho\mathbf{A_g}\boldsymbol{\Delta}\mathbf{P_g} - \rho\boldsymbol{\Delta}\mathbf{P_d} = \mathbf{P_d} \\ & -\mathbf{P_l^{max}} \leq \Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\theta} \leq \mathbf{P_l^{max}} \\ & \mathbf{P_g^{min}} \leq \mathbf{P_g} + \rho\boldsymbol{\Delta}\mathbf{P_g} \leq \mathbf{P_g^{max}} \end{array} \tag{4}$$

Notice that, by isolating $\hat{\theta}$ in (3) we have

$$\hat{\theta} = \widehat{\mathbf{B}^{-1}}\left[\hat{\mathbf{P}} + \rho\boldsymbol{\Delta}\hat{\mathbf{P}}\right] \tag{5}$$

where $\hat{\mathbf{P}} = \mathbf{A_g}\mathbf{P_g} - \mathbf{P_d}$, $\boldsymbol{\Delta}\hat{\mathbf{P}} = \mathbf{A_g}\boldsymbol{\Delta}\mathbf{P_g} - \boldsymbol{\Delta}\mathbf{P_d}$. Thus, using (5), the power flow constraints can be written as

$$-\mathbf{P_l^{max}} \leq \Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\left[\hat{\mathbf{P}} + \rho\boldsymbol{\Delta}\hat{\mathbf{P}}\right] \leq \mathbf{P_l}^{max} \tag{6}$$

where ρ can be isolated as

$$\underbrace{\oslash\left[\frac{-\mathbf{P_l^{max}} - \Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\hat{\mathbf{P}}}{\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}}}\right]}_{\rho_\mathbf{l}^{\mathbf{left}}} \leq \rho \leq \underbrace{\oslash\left[\frac{\mathbf{P_l}^{max} - \Gamma\hat{A}_s\hat{B}^{-1}\hat{P}}{\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}}}\right]}_{\rho_\mathbf{l}^{\mathbf{right}}} \tag{7}$$

and the operator $\oslash\left[\frac{\mathbf{x}}{\mathbf{y}}\right]$ denotes the element-wise division of vector components of $\mathbf{x}$ and $\mathbf{y}$. Generation capacity constraints must also be verified, a condition modeled by inequalities

$$\underbrace{\oslash\left[\frac{\mathbf{P_g^{min}} - \mathbf{P_g}}{\boldsymbol{\Delta}\mathbf{P_g}}\right]}_{\rho_\mathbf{g}^{\mathbf{left}}} \leq \rho \leq \underbrace{\oslash\left[\frac{\mathbf{P_g^{max}} - \mathbf{P_g}}{\boldsymbol{\Delta}\mathbf{P_g}}\right]}_{\rho_\mathbf{g}^{\mathbf{right}}}. \tag{8}$$

Observe that inequality (7) restricts the load factor ρ to ensure compliance with power flow constraints in transmission lines and transformers. Similarly, inequality (7) confines the load factor ρ to operation points where generation capacities are respected. Using the inequalities (7) and (8), we conclude that a feasible solution for ρ, in the space composed of the constraints in (4), is contained in the range $\left[\rho^{min}, \rho^{max}\right]$ where

$$\rho^{min} = \max\left\{\left[\left[\mathbf{lLim_l}\right]^\mathrm{t}\left[\rho_\mathbf{g}^{\mathbf{left}}\right]^\mathrm{t}\right]^\mathrm{t}\right\} \tag{9}$$

$$\rho^{max} = \min\left\{\left[\left[\mathbf{rLim_l}\right]^\mathrm{t}\left[\rho_\mathbf{g}^{\mathbf{right}}\right]^\mathrm{t}\right]^\mathrm{t}\right\} \tag{10}$$

in which $\mathbf{lLim_l}$ and $\mathbf{rLim_l}$ are column vectors that contain the upper and lower limits for ρ, such that

$$\begin{aligned} \mathbf{lLim_l} = & \left[\left(\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}} > \mathbf{0}\right) \otimes \rho_\mathbf{l}^{\mathbf{left}}\right] \\ & + \left[\left(\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}} < \mathbf{0}\right) \otimes \rho_\mathbf{l}^{\mathbf{right}}\right] \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbf{rLim_l} = & \left[\left(\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}} < \mathbf{0}\right) \otimes \rho_\mathbf{l}^{\mathbf{left}}\right] \\ & + \left[\left(\Gamma\hat{\mathbf{A}}_\mathbf{s}\hat{\mathbf{B}}^{-1}\boldsymbol{\Delta}\hat{\mathbf{P}} > 0\right) \otimes \rho_\mathbf{l}^{\mathbf{right}}\right]. \end{aligned} \tag{12}$$

The operation $\mathbf{X} > \mathbf{Y}$ results in a binary vector, with 1 if $X_i > Y_i$ is a true statement and 0 otherwise. The operator $\mathbf{X} \otimes \mathbf{Y}$ denotes the element-wise product of the components of vectors $\mathbf{X}$ and $\mathbf{Y}$.

The definition of ρ presented in (10) can also be interpreted as a *lower limit of the maximum loadability factor*, assuming a proportion in the increase in generation that follows the increase in loading. Analogously, (9) is the *upper limit of the minimum loadability factor*, assuming the same hypothesis. During a SMCS, a dispatch profile variation $\boldsymbol{\Delta}\mathbf{P_g}$ can be easily built taking $\mathbf{P_g}$ as reference alongside the amount of load in $\boldsymbol{\Delta}\mathbf{P_d}$. While the size of the range provided by (9) and (10) is mathematically inferior than the one retrieved from solving (2), the resulting range can be as well used in Algorithm 2. Moreover, since (9) and (10) can be determined analytically, iterative procedures are not required to its estimation, reducing considerably the computation effort of executing Algorithm 2.

4. Application of maximum loadability analysis to the cross-entropy method

Importance sampling is a variance reduction technique which is based on the concept that certain values of a random variable have a greater impact on the estimation process of a target quantity. Thus, if these important values are sampled more often, the variance of the estimator should be reduced [5]. The CE method provides a simple adaptive procedure for estimating close to optimal reference parameters, by minimizing the distance between the original sampling density and the optimal sampling density, iteratively. Detailed descriptions of the application of the CE method in power system reliabily evaluation can be found in [1,23,24].

Applications of the CE method to reliability evaluation aim to increase the probability of sampling rare states which contribute significantly to performance index convergence. The unavailabilities of components are chosen as reference parameters, which are optimized using an iterative procedure. The bias inserted in the reference parameters affects the state sampling and is corrected during the index estimation stage of the SMCS, shown in Fig. 1. Similarly to the standard SMCS, coefficients of variation of the test functions must be updated to assert the convergence of the stochastic process. The simulation is finished when the coefficients of variation of the test functions are inferior to a given threshold.

The benckmark CE method to composite evaluation is summarized in Algorithm 3 [5]. While optimizing probability density functions of random variables, state evaluations are executed in step 3 to sort states according to their importance.

Algorithm 3 CE method for composite evaluation.

Let N_{CE} be the number of samples, ϱ the rarity parameter, α the smoothing parameter and γ the level parameter.

1: Define $\mathbf{v}_0 \leftarrow \mathbf{u}$, where $\mathbf{u}$ is the original system components unavailability vector; $\gamma \leftarrow L_{MAX}$, where L_{MAX} is the sum of all peak loads of all bus; $k \leftarrow 1$;

2: Generate N_{CE} samples $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N_{CE}}$ based on a probability density function;

3: Evaluate the performance function $S(\mathbf{x}_i)$ for all $\mathbf{x}_i$, $i = 1, \dots, N_{CE}$:

i. Evaluate the state as described in section Section 2. In case of a null load curtailment value, proceed to step ii. Otherwise, proceed to step iii.

ii. Define $S(\mathbf{x}_i)$ as the supply capacity of the system, as follows:

$$S(\mathbf{x}_i) = [C_{i1} \dots C_{in_g}]^t \mathbf{x}_i \tag{13}$$

where $C_{ij}, j = 1, \dots, n_g$ represents the generating capacity of generator j at sample i and n_g is the number of system generators.

iii. Define $S(\mathbf{x}_i)$ as the power supplied to the system, as follows:

$$S(\mathbf{x}_i) = \text{Load Peak} - \text{Load Curtailment} \tag{14}$$

4: Sort results in ascending order such that $\mathbf{S} = \left[S_{[1]}, \dots, S_{[N_{CE}]}\right]$ and $S_{[1]} \le S_{[2]} \le \dots \le S_{[N_{CE}]}$;

5: If $\mathbf{S}_{[\varrho N_{CE}]} \le \gamma$, then $\gamma_k \leftarrow \gamma$; otherwise, $\gamma_k \leftarrow \mathbf{S}_{[\varrho N_{CE}]}$. Evaluate the test function $H(\mathbf{x}_i)$ for all $\mathbf{x}_i$ such as $H(\mathbf{x}_i) = 1$, if $S(\mathbf{x}_i) < \gamma$, and $H(\mathbf{x}_i) = 0$, otherwise. Estimate the likelihood ratio $\mathbf{W}(\mathbf{x}_i; \mathbf{u}, \mathbf{v}_{k-1})$, $\forall \mathbf{x}_i$, using

$$\mathbf{W}\left(\mathbf{x}_i; \mathbf{u}, \mathbf{v}_{k-1}\right) = \frac{\prod_{j=1}^{(n_g+n_l)} \left(1-u_j\right)^{\mathbf{x}_j} \left(u_j\right)^{1-\mathbf{x}_j}}{\prod_{j=1}^{(n_g+n_l)} \left(1-v_j\right)^{\mathbf{x}_j} \left(v_j\right)^{1-\mathbf{x}_j}}. \tag{15}$$

where n_l is the number transmission elements of the system.

6: Use the same sample $\mathbf{x}_1, \dots, \mathbf{x}_{N_{CE}}$ to evaluate the reference parameters for each element $j = 1, 2, \dots, (n_g + n_l)$, at iteration k, using

$$\hat{v}_{k,j} \leftarrow 1 - \left[\frac{\sum_{i=1}^{N_{CE}} H(\mathbf{X}_i)\mathbf{W}(\mathbf{X}_i; \mathbf{u}, \mathbf{v}_{k-1})X_{ij}}{\sum_{i=1}^{N_{CE}} H(\mathbf{X}_i)\mathbf{W}(\mathbf{X}_i; \mathbf{u}, \mathbf{v}_{k-1})}\right] \tag{16}$$

and update those vectors using the expression

$$v_{k,j} \leftarrow \hat{v}_{k,j}\alpha + v_{k-1,j}(1-\alpha) \tag{17}$$

7: If $\gamma_k = \gamma$, end of the optimization process; otherwise, $k \leftarrow k + 1$ and proceed to step 2;

When the state is defined as successful, the total generation capacity available is used as performance value. Otherwise, the performance value of the state is given by the peak load subtracted by the calculated load curtailment value.

In reliability evaluations, a performance function should allow a state classification focused on the load curtailment events, enabling, whenever possible, a state performance ordering taking into account the proximity to the boundaries that separate the space of successful states from the space of failed states. The performance function defined by expressions (13) and (14) might promote a discontinuity of this premise, mainly in states where component failures do not cause load curtailment. As an example, suppose a system consisting of two generators connected to a load using two transmission lines in parallel. The individual capacities

Table 1
Results for the MCSτ, MCSρ and MCS$\hat{\rho}$ methods.

Method	MCSτ	MCSρ	MCS$\hat{\rho}$
#Simulated years	2 267	2 267	2 267
Runtime [s]	12 126.5	12 216.8	1 996.8
LOLP [$\times 10^{-3}$]	1.1880	1.1880	1.1880
EPNS [MW]	0.1436	0.1436	0.1436
LOLF [occ./year]	2.2268	2.2268	2.2268

of generators and transmission lines are 100 MW, while the load varies from 80 to 100 MW during the year. Now assume that all components are available. In this case, when the load assumes a value of 100 MW, the state performance is given by the total generation capacity of the system of 200 MW. In the event of a transmission line failure, the performance of the new state is identical, even if this state has clearly a larger risk of load curtailment.

Such example motivated the application of the maximum loadability analysis to assess the state performance in the CE method, as described in Algorithm 4. This application has the advantage of allowing state evaluations without the need to link the performance function to a load value, such as the peak load. In the example described above, if all components are available, the maximum loadability of the system is associated with a supplying capacity of 200 MW. In case of a transmission line failure or a generating unit failure, the maximum loadability of the system is associated with a supplying capacity of 100 MW.

Algorithm 4 Modified CE method for composite evaluation.

← Step 1 and 2 of Algorithm 3.
3: Evaluate the performance function $S(\mathbf{x}_i)$ for all $\mathbf{x}_i, i = 1, \ldots, N_{CE}$:

i: Compute the maximum loadability factor considering a restricted dispatch profile $\hat{\rho}$ for the state $\mathbf{x}_i$ using (10);
ii: Define $S(\mathbf{x}_i) = \hat{\rho}$;

← Step 4 to 7 of Algorithm 3.

The inclusion of the determination of the maximum loadability factor considering either the formulation shown in Section 3.2 or in Section 3.1 incurs in minor changes in the CE algorithm. In fact, the modification impacts only in step 3 of Algorithm 3, more specifically, in the calculation of the performance function value. The main idea is, instead of evaluating the system via PF and/or OPF and then calculating the state performance value according to (13) or (14), to estimate and assign as performance value the maximum loadability factor of the state. Particularly, the estimation of the maximum loadability factor using (10) is of interest, since it provides a metric to classify states without resourcing to the application of iterative methods. This design choice is prone to impact considerably the elapsed time required to the convergence of CE-based approaches.

5. Numerical results

The proposed approach has been applied to the IEEE-RTS79 test system [25] in order to illustrate the effectiveness of the proposed algorithms. The IEEE-RTS79 test system has been considered a benchmark test system, commonly used for composite adequacy assessment [1,2,5–16,21,24,26,27]. The SMCS implementation has been validated for coefficients of variation β of 5% and 1% taking into account the results provided in [5,21]. Simulation results are retrieved assuming a minimum coefficient of variation β of 5% and have been performed using an Intel(R) Core(TM) i5-5200U (2.20 GHz, 8 GB RAM).

5.1. Impact of applying the proposed loadability analysis to state evaluation

Table 1 presents numerical results for the sequential Monte Carlo approach (MCSτ) summarized in Algorithm 1 and the proposed approach described in Algorithm 2. Different outcomes are retrieved from computing the maximum and minimum loadability factors either using the formulation shown in Section 3.1 (MCSρ) or the analytical solution deduced in Section 3.2 (MCS$\hat{\rho}$).

The proposed approach, either using the MCSρ or MCS$\hat{\rho}$ methods, is able to provide the exact same results acquired with MCSτ. Furthermore, the execution time of the MCSτ e MCSρ methods are similar, lasting approximately 3.4 h. The MCS$\hat{\rho}$ method is able to delivery results in approximately 17% of the time required by the MCSτ method, totalizing approximately 33.28 min.

Table 2 highlights, for the MCSτ, MCSρ and MCS$\hat{\rho}$ methods, the number of visited states; number of state evaluations via PF, OPF or comparison with load limiting factors assuming at least one component is in the failed state; number of states straightforwardly evaluated as success via PF; number of states evaluated as success or failure via OPF; number of states with loadability factor estimation; and number of states with avoided PF/OPF evaluations using load limiting factors.

As expected, the number of visited and evaluated states is equal for all methods. The MCSρ and MCS$\hat{\rho}$ methods are able to eliminate around 94% of the PF executions in comparison with the MCSτ method. Using the MCSρ method, the number of states that required OPF assessments reduced around 50%. Indeed, in the MCSρ method, if sampled system states do not satisfy the loadability factor criteria, load curtailments are required and OPFs are executed to quantify load curtailment. Conversely, using the MCS$\hat{\rho}$ method, in case the loadability factor criteria are not satisfied, one cannot assure load curtailment is required, causing the higher number of OPF executions. Notwithstanding, the MCS$\hat{\rho}$ is approximately 6 times faster than MCSτ and MCSρ methods.

Table 2
State evaluations for MCSτ, MCSρ and MCS$\hat{\rho}$ methods.

Counter	MCSτ	MCSρ	MCS$\hat{\rho}$
#Visited states	20 961 532	20 961 532	20 961 532
#Evaluated states	16 235 897	16 235 897	16 235 897
#Success states (PF)	16 172 261	971 308	971 276
#OPF executions	63 636	31 734	63 636
#States with ρ est.	0	977 235	977 235
#Avoided PF/OPF	0	15 232 855	15 200 985

Table 3
Results for the MCS$\hat{\rho}$, CEMCS$\hat{\rho}$ and CE$\hat{\rho}$MCS$\hat{\rho}$ methods.

Method	MCS$\hat{\rho}$	CEMCS$\hat{\rho}$	CE$\hat{\rho}$MCS$\hat{\rho}$
#Simulated years	378 011	1 292	257
CE Runtime [s]	–	251.01	12.78
MCS Runtime [s]	261 539.5	19 305.13	2 645.90
Total Runtime [s]	261 539.5	19 556.14	2 658.68
LOLP [$\times 10^{-6}$]	3.4223	3.5478	3.5509
EPNS [MW$\times 10^{-4}$]	2.5202	2.6230	2.5847
LOLF [(occ./year)$\times 10^{-3}$]	8.5289	8.7745	8.6123

Table 4
State evaluations for the MCS$\hat{\rho}$, CEMCS$\hat{\rho}$ and CE$\hat{\rho}$MCS$\hat{\rho}$ methods.

Counter	MCS$\hat{\rho}$	CEMCS$\hat{\rho}$	CE$\hat{\rho}$MCS$\hat{\rho}$
#Visited states	3 496 457 563	13 114 714	2 553 232
#Evaluated states	2 703 508 100	13 114 255	2 551 478
#Success states (PF)	162 245 676	1 587 281	284 291
#OPF executions	226 628	1 716 609	211 982
#States with ρ est.	162 266 673	1 787 838	309 631
#Avoided PF/OPF	2 541 035 796	9 810 365	2 055 205

As conclusion, the proposed approach in Algorithm 2 allows a considerable avoidance of state evaluations with PF/OPF. The computational effort to estimate the maximum and minimum loadability factors in the MCSρ method can make its implementation unattractive. On the other hand, the application of an analytical solution for an estimation of loadability factors provided in MCS$\hat{\rho}$ has shown to be effective on suppressing evaluations with PF/OPF, reducing computation time required to estimate performance indices. The MCS$\hat{\rho}$ method then permits estimating performance indices with the same precision than its traditional counterpart, but with reduced computation burthen.

5.2. Impact of applying the proposed loadability analysis to the cross-entropy method

This section presents simulation results emphasizing the impact of applying the proposed loadability analysis to the CE method. Numerical results are retrieved for the analysis of the IEEE-RTS79 test system assuming a 25% load reduction with peak load of 2137.5 MW. This condition leads to an increase in the rarity of load curtailment events, allowing a better comparison of the implemented solutions. Results are presented for the following combined 3 approaches: the MCS$\hat{\rho}$ method; the MCS$\hat{\rho}$ method with state sampling and index estimation correction through the CE method in Algorithm 3 (CEMCS$\hat{\rho}$); and the MCS$\hat{\rho}$ method with state sampling and index estimation correction through of the modified CE method in Algorithm 4 (CE$\hat{\rho}$MCS$\hat{\rho}$). Result validation has been performed by verifying the intersection of the confidence intervals of the estimated performance indices for a 95% confidence level. The CE-based methods are applied assuming a sample size of 25000, rarity parameter of 0.1 and smoothing parameter of 0.99.

Table 3 shows the results acquired with the application of the MCS$\hat{\rho}$, CEMCS$\hat{\rho}$ and CE$\hat{\rho}$MCS$\hat{\rho}$ methods.

As expected, with the decrease in the probability of occurrence of load curtailments, the number of years required to the convergence of the MCS$\hat{\rho}$ method has increased significantly. The MCS$\hat{\rho}$ method required 378011 years to reach convergence, while the CEMCS$\hat{\rho}$ method required only 1292 years, entailing a 99.66% reduction on the simulated years. The CE$\hat{\rho}$MCS$\hat{\rho}$ method reached the desired convergence in just 257 years, which represents less than 0.07% of the years utilized by the MCS$\hat{\rho}$ method. The CE$\hat{\rho}$MCS$\hat{\rho}$ method has been 19.64 times faster than the CEMCS$\hat{\rho}$ method on optimizing probability density functions. Furthermore, the total execution time of the CE$\hat{\rho}$MCS$\hat{\rho}$ method, ranging around 44.31 min, has been much lower in comparison to the other two methods, showing a speed up of 7.36 and 98.37, in comparison to the CEMCS$\hat{\rho}$ and MCS$\hat{\rho}$ methods, respectively.

Table 4 presents a summary of the state evaluation procedures executed in the MCS$\hat{\rho}$, CEMCS$\hat{\rho}$ and CE$\hat{\rho}$MCS$\hat{\rho}$ methods.

The large number of simulated years lead to a large number of assessed states. The proportion of evaluated states within the universe of visited states has been approximately 77.321%, 99.997% and 99.931%, for the MCS$\hat{\rho}$, CEMCS$\hat{\rho}$ and CE$\hat{\rho}$MCS$\hat{\rho}$ methods, respectively. This outcome has been expected, since the idea of applying the CE method in the SMCS method is to increase the probability of sampling states that contribute to the convergence of reliability indices. However, and restricting the analyzes to the universe of states that required evaluation, in the MCS$\hat{\rho}$ method, approximately 6.001% are evaluated via PF, 0.008% are evaluated via OPF and 93.990% of the states have their evaluation via PF/OPF suppressed by the estimation of the loadability factors of the success states. In the CEMCS$\hat{\rho}$ method, approximately 12.103% states are evaluated via PF, 13.090% are evaluated via OPF and 74.807% of the states have their valuation via PF/OPF suppressed by estimating the loadability factors. For the CE$\hat{\rho}$MCS$\hat{\rho}$ method the approximate proportions are 11.142% of states evaluated via PF, 8.308% of states evaluated via OPF, and 80.550% states have their evaluation via PF/OPF suppressed. This shows that, as expected, the suppression of state evaluations via loadability factor estimation is greater when considering the MCS$\hat{\rho}$ method, that is, the optimization of the probability density functions decreases the number of consecutive states where only load transition occurs. For the test system, the proportion of states whose evaluation is suppressed via comparison with loadability factors is higher in the CE$\hat{\rho}$MCS$\hat{\rho}$ method than in the CEMCS$\hat{\rho}$ method.

In the simulations performed with the IEEE-RTS79 test system, the CE$\hat{\rho}$MCS$\hat{\rho}$ method provided improved results in terms of speed up, showing its effectiveness and applicability to composite reliability evaluation. The findings can be used to accelerate the assessment of reliability indices for different planning scenarios, considering varied load forecasts and several configurations of future generation and transmission facilities. Moreover, these findings may serve as a basis for future efforts aimed at verifying the effects of incorporating high-resolution load transition models into the reliability index estimation processes.

6. Discussions and final remarks

This paper introduced an approach to composite reliability evaluation, combining the SMCS with maximum and minimum loadability analysis, aiming to reduce the computational burden of state evaluations on reliability index assessments. Maximum and minimum loadability factors of a given state are evaluated to avoid PF/OPF analysis in further states characterized by load transitions. Two strategies have been introduced: one that calculates loadability factors for each state requiring solving an optimization problem through an iterative method; and other that provides an estimation for the loadability factors using an analytical solution. The strategy based on the estimation of loadability factors has shown to be useful on reducing simulation runtimes without affecting the precision of reliability index estimation. The approach highlights that schemes to assess time connected states can be built to improve applications based on SMCS to composite reliability evaluation.

The estimation of state maximum loadability factors has been also applied as performance function of the CE method. Results illustrate that the maximum loadability factor of a system state can be an adequate choice for performance function to the CE method due to its corresponding reduced computational burden. In our simulations for the IEEE-RTS 79 test system, the combination of the SMCS, maximum loadability analysis and CE method has shown improved results in comparison with its original counterparts, promoting discussions on how to define and measure the efficiency of performance functions to the problem.

The proposed approaches shown herein can be used to speed up the evaluation of reliability indices across various planning scenarios, considering diverse load predictions and multiple configurations for forthcoming generation and transmission facilities. However, it is crucial to emphasize that the proposed approaches are solely applicable to problems where network linear modeling is deemed suitable, and reliability data accurately mirrors real-world conditions. Future works are envisioned to exploit the developed concepts towards the application of maximum loadability analysis in composite reliability assessments considering nonlinear network models.

CRediT authorship contribution statement

Erika Pequeno dos Santos: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Beatriz Silveira Buss:** Writing—Review & Editing, Visualization. **Mauro Augusto da Rosa:** Conceptualization, Resources, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Diego Issicaba:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work has been supported by the Brazilian Coordination for the Improvement of Higher Education Personnel (CAPES).

Data availability

The authors do not have permission to share data.

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