Optimization. Homework 2

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1. The directional derivative $\frac{\partial f}{\partial v}(x_0, y_0, z_0)$ of a differentiable function f are $\frac{3}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$ in the directions of vectors $[0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^T$, $[\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}]^T$ and $[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^T$. Compute $\nabla f(x_0, y_0, z_0)$.

2. Show that the level curves of the function $f(x, y) = x^2 + y^2$ are orthogonal to the level curves of $g(x, y) = \frac{y}{x}$ for all (x, y).

3. Compute the stationary points of $f(x, y) = \frac{3x^4 - 4x^3 - 12x^2 + 18}{12(1+4y^2)}$ and determine their corresponding type (ie: minimum, maximum or saddle point)

4. Compute the gradient $\nabla f(\boldsymbol{x})$ and Hessian $\nabla^2 f(\boldsymbol{x})$ of the Rosenbrock function

$$f(\boldsymbol{x}) = \sum_{i=1}^{N-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2]$$

   where $\boldsymbol{x} = [x_1, \ldots, x_N]^T \in \mathbb{R}^N$

5. Show, without using the optimality conditions, that $f(\boldsymbol{x}) > f(\boldsymbol{x}^*)$ for all $\boldsymbol{x} \neq \boldsymbol{x}^*$ if

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T \mathbf{Q} \boldsymbol{x} - \boldsymbol{b}^T \boldsymbol{x}$$

   $\mathbf{Q} = \mathbf{Q}^T \succ 0$ and $\mathbf{Q}\boldsymbol{x}^* = \boldsymbol{b}$.