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Project 5

For the course FYS3150

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December 19, 2019
Week XX - 51

Oppgaver å gjøre:

a) Discretize the above differential equations and set up an algorithm for solving these equations using Euler's forward algorithm and the so-called velocity Verlet method

b) Write then a program which solves the above differential equations for the Earth-Sun system using Euler's method and the velocity Verlet method. Dette gjøres uten objekt orientering. Planlegg deretter hva som kan ha objekt orientering.

c) Find out which initial value for the velocity that gives a circular orbit and test the stability of your algorithm as function of different time steps Δt. Make a plot of the results you obtain for the position of the Earth (plot the x and y values and/or if you prefer to use three dimensions the z-value as well) orbiting the Sun. Check also for the case of a circular orbit that both the kinetic and the po- tential energies are conserved. Check also if the angular momentum is conserved. Explain why these quantities should be conserved. Discuss eventual differences between the Verlet algorithm and the Euler algorithm. Consider also the number of FLOPs involved and perform a timing of the two algorithms for equal final times. We will use the velocity Verlet algorithm in the remaining part of the project.

d) Consider then a planet which begins at a distance of 1 AU from the sun. Find out by trial and error what the initial velocity must be in order for the planet to escape from the sun. Can you find an exact answer? How does that match your numerical results? — Endre uttrykket for gravitasjonskraften —. What happens to the earth-sun system when β creeps towards 3? Comment your results.

e) Modify your first-order differential equations in order to accomodate both the motion of the Earth and Jupiter by taking into account the distance in x and y between the Earth and Jupiter. Set up the algorithm and plot the positions of the Earth and Jupiter using the velocity Verlet algorithm. Discuss the stability of the solutions using your Verlet solver. Repeat the calculations by increasing the mass of Jupiter by a factor of 10 and 1000 and plot the position of the Earth. Study again the stability of the Verlet solver.

f) Finally, using our Verlet solver, we carry out a real three-body calculation where all three systems, the Earth, Jupiter and the Sun are in motion. To do this, choose the center-of-mass position of the three-body system as the origin rather than the position of the sun. Give the Sun an initial velocity which makes the total momentum of the system exactly zero (the center-of-mass will remain fixed). Compare these results with those from the previous exercise and comment your results. Extend your program to include all planets in the solar system (if you have time, you can also include the various moons, but it is not required) and discuss your results. Use the above NASA link to set up the initial positions and velocities for all planets.

g) Run a simulation over one century of Mercury's orbit around the Sun with no other planets present, starting with Mercury at perihelion on the x axis. Check then the value of the perihelion angle θp — se SolarSystem.pdf for uttrykk — where xp (yp) is the x (y) position of Mercury at perihelion, i.e. at the point where Mercury is at its closest to the Sun. You may use that the speed of Mercury at perihelion is 12.44AU/yr, and that the distance to the Sun at perihelion is 0.3075AU. You need to make sure that the time resolution used in your simulation is sufficient, for example by checking that the perihelion precession you get with a pure Newtonian force is at least a few orders of magnitude smaller than the observed perihelion precession of Mercury. Can the observed perihelion precession of Mercury be explained by the general theory of relativity?

Ting å gjøre:

- skrive abstract
- skrive introduction
- skrive method
- skrive discussion
- skrive conclusion
- skrive appendix
- fikse kildene

Abstract

An abstract where you give the main summary of your work
The abstract gives the reader a quick overview of what has been done and the most important results. Here is a typical example taken from a scientific article
We study the collective motion of a suspension of rodlike microswimmers in a two-dimensional film of viscoelastic fluids. We find that the fluid elasticity has a small effect on a suspension of pullers, while it significantly affects the pushers. The attraction and orientational ordering of the pushers are enhanced in viscoelastic fluids. The induced polymer stresses break down the large-scale flow structures and suppress velocity fluctuations. In addition, the energy spectra and induced mixing in the suspension of pushers are greatly modified by fluid elasticity.

Contents

1 Introduction

An introduction where you explain the aims and rationale for the physics case and what you have done. At the end of the introduction you should give a brief summary of the structure of the report
What should I focus on? Introduction. You don't need to answer all questions in a chronological order. When you write the introduction you could focus on the following aspects

Motivate the reader, the first part of the introduction gives always a motivation and tries to give the overarching ideas What I have done The structure of the report, how it is organized etc

2 Theory

2.1 Important calculations for this project

Using the section A.2 in the appendix one can calculate the escape velocity of the Earth from the Suns gravity field. The distance from the Earth to the Sun is 1 AU (astronomical unit). G is given as $4\pi^2$. M and m is the mass of the Sun and the planet, which in this case is the Earth. M is given as 1 $M_\odot$ and m is equal to $\frac{1}{333333}M\odot$. Using the theoretical expression (8) one can calculate the earths escape velocity. For this two-body system the escape velocity is

$$v_p = 8.886 \text{ m/s} \tag{1}$$

2.2 Velocity Verlet method

The Velocity Verlet method is a differential equation solver, which was used in this project to solve Newtons second law, given by equation (9). Equation (9) will be used to calculate the next position of the planet as a Taylor expansion. The advantage of using the Velocity Verlet method is that it conserves energy.

Deriving the equations used in this method from Newtons second law of motion is done in section A.3. The method starts by using the old acceleration, velocity and position to calculate the new position. Using equation (2), which is the discretized version of equation (14).

$$x_{i+1} = x(i) + hv(x,t) + \frac{h^2}{2}a(x,t) + (O)h^4(8) \tag{2}$$

The thing that separates this method from the Euler method is that in order to calculate the new velocity the Velocity Verlet method first calculates a new acceleration using the newly gained position, giving $a_{i+1} = a_i(x_{i+1})$.

The newly calculated acceleration is now put into the expression for the new velocity.

$$v_{i+1} = v_i + \frac{h}{2}(a_i + a_{i+1}) \tag{3}$$

The equations (2) and (3) makes up the Velocity Verlet method, and allows for conservation of energy, which is important when working with the solar system.

2.3 Euler method

The Euler method is in many ways similar to the Velocity Verlet method. However, instead of calculating a new acceleration and using that to find the velocity, the method uses the old position to calculate the acceleration and uses that acceleration to find the new velocity. The old velocity is also used to find the new position. Another important difference is that the position is not calculated using the acceleration. Other than that the methods are quite identical.

The equations used are derived from Newtons second law of motion. This was done in the appendix under, see section A.3.

First one calculates the velocity using the old position.

$$v_{i+1} = v_i + a(x_i)\ h \tag{4}$$

Using the newly gained velocity one can now find the new position.

$$x_{i+1} = x_i + v_i(x_i)\ h \tag{5}$$

This method is less precise compared to the Velocity Verlet method and it does not conserve energy.

3 Method

Theoretical models and technicalities. This is the methods section
What should I focus on? Methods sections. Describe the methods and algorithms You need to explain how you implemented the methods and also say something about the structure of your algorithm and present some parts of your code You should plug in some calculations to demonstrate your code, such as selected runs used to validate and verify your results. The latter is extremely important!! A reader needs to understand that your code reproduces selected benchmarks and reproduces previous results, either numerical and/or well-known closed form expressions.

3.1 Euler-Cromer implementation

We use the Euler-Cromer method instead of the Euler method in this project. This is because Euler-Cromer is a better approximation than Euler, and is also easily implemented. Euler-Cromer is a better approximation because it uses the next step in velocity in the current step in position. Because of these advantages we decided to use the Euler-Cromer method instead of the plain Euler method. The Euler-Cromer method is given by the equations (6) and (7) below.

$$x_{i+1} = x_i + v_{i+1}\ h \tag{6}$$

$$v_{i+1} = v_i + a_i\ h \tag{7}$$

Here we can see that the main difference between the two methods are the next velocity step in the equation for the position, (6). However, the Euler-Cromer method does not conserve energy as well.

4 Results

Results
What should I focus on? Results. Present your results An eventual reader should be able to reproduce your calculations if she/he wants to do so. All input variables should be properly explained. Make sure that figures and tables should contain enough information in their captions, axis labels etc so that an eventual reader can gain a first impression of your work by studying figures and tables only.

All plots which do not have the number of iterations specified have 3660 iterations. All plots which do not have the number of years specified have 10 years.

By reading the PDF for the project a little more thoroughly we discovered that in the latter parts from testing the algorithms, it says to only implement the Velocity Verlet method. However, considering how time consuming the making of the data, plots and figures in LaTeXwe will leave the plots in this PDF, but choose not to comment them further than if the results are interesting.

4.1 Sun-Earth solar system

4.1.1 Testing the methods and their stability

HUSK Å LINKE TIL ANIMASJONENE!!!!

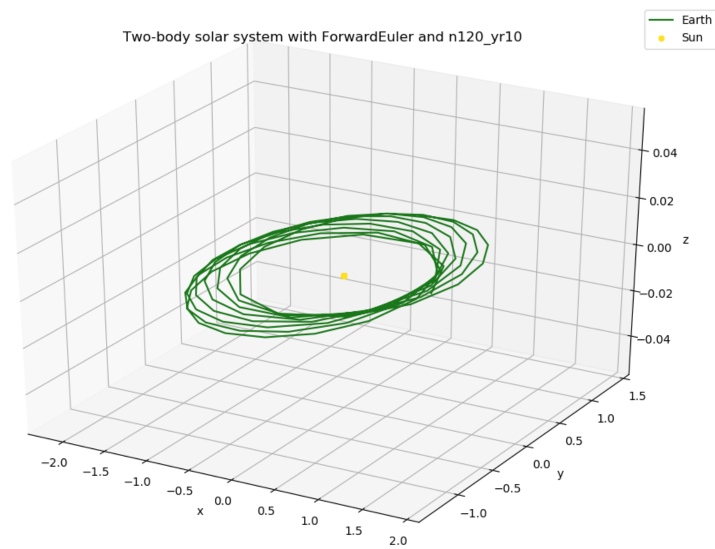

Figure 1: Plot of the Sun-Earth solar system with 120 iterations over 10 years, made with Euler-Cromer.

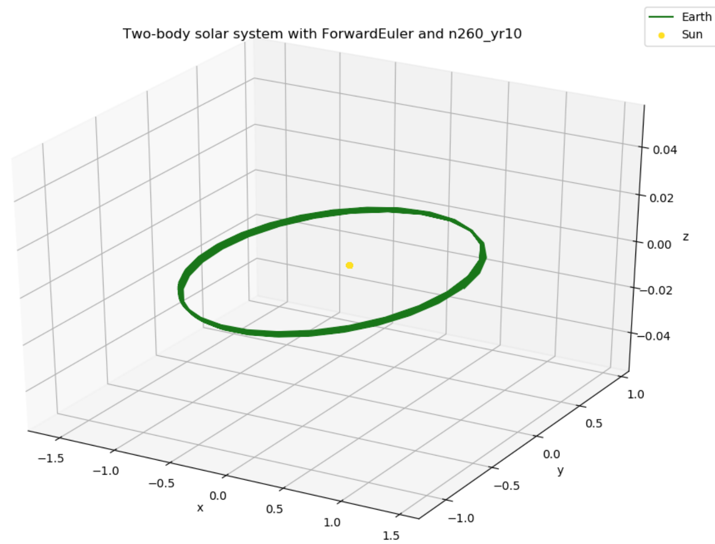

Figure 2: Plot of the Sun-Earth solar system with 260 iterations over 10 years, made with Euler-Cromer.

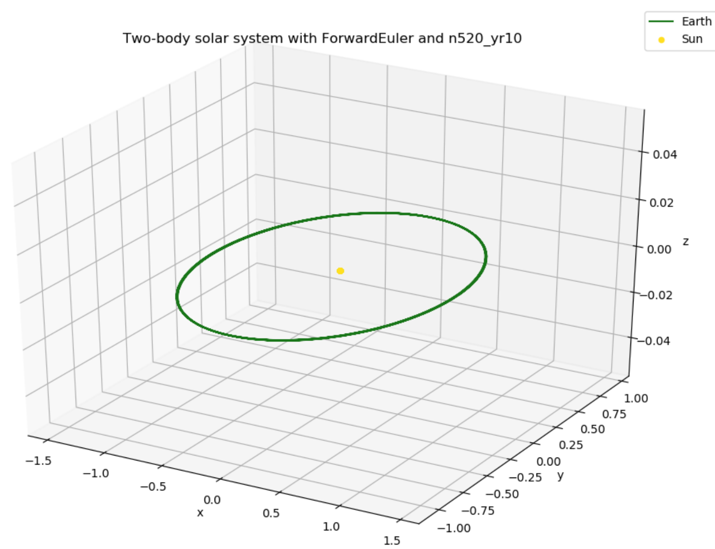

Figure 3: Plot of the Sun-Earth solar system with 520 iterations over 10 years, made with Euler-Cromer.

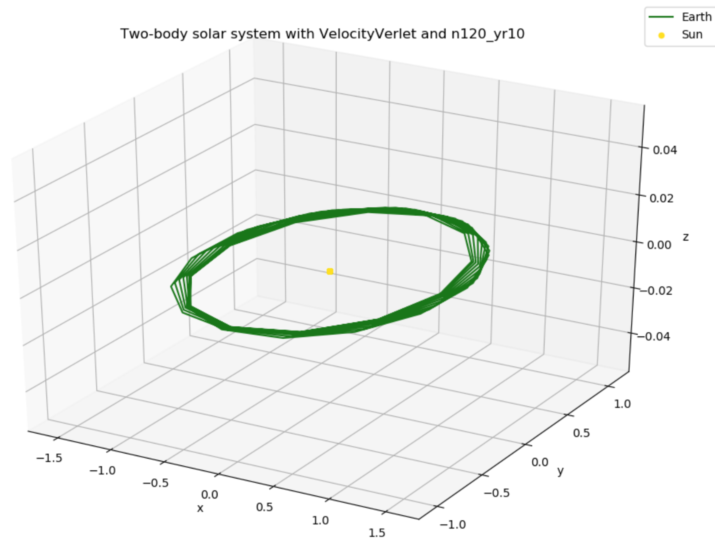

Figure 4: Plot of the Sun-Earth solar system with 120 iterations over 10 years, made with Velocity Verlet.

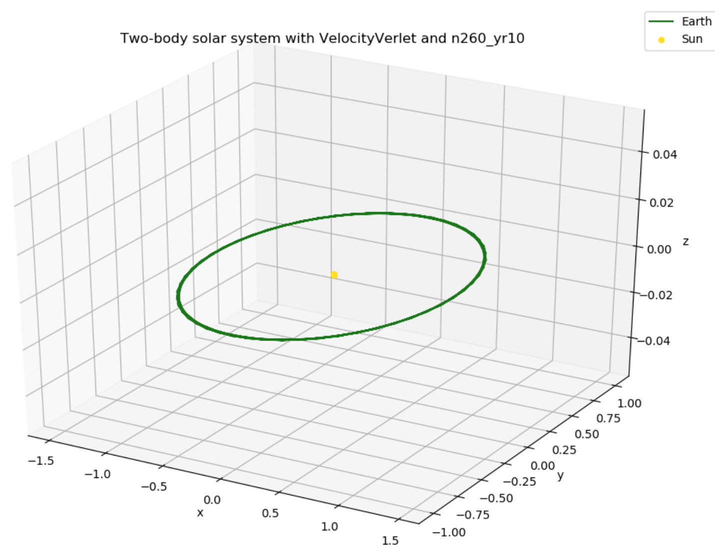

Figure 5: Plot of the Sun-Earth solar system with 260 iterations over 10 years, made with Velocity Verlet.

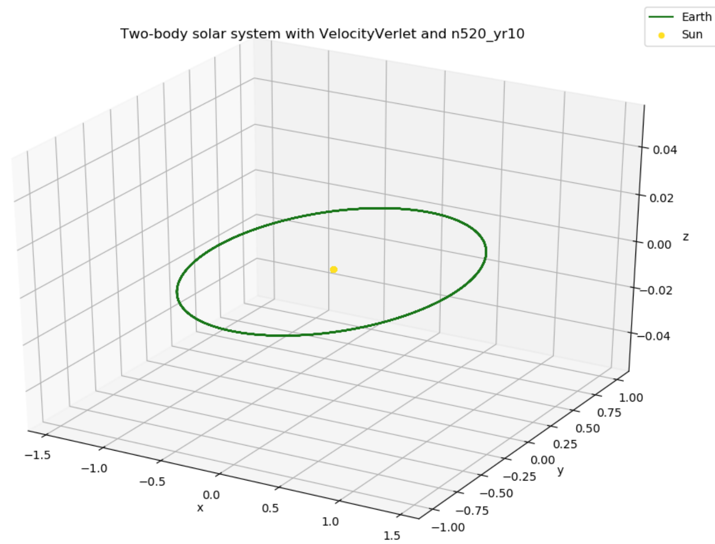

Figure 6: Plot of the Sun-Earth solar system with 520 iterations over 10 years, made with Velocity Verlet.

Euler-Cromer time [s]	Velocity Verlet time [s]
25.8553	34.0578
30.0186	30.5725

Table 1: Timings for the two methods for the ten-body solar system with 3660 iterations over 10 years.

4.1.2 Testing the escape velocity

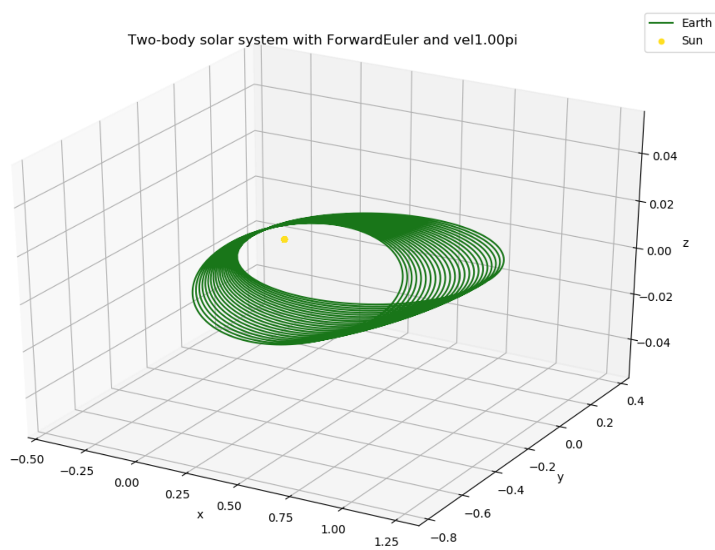

Figure 7: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 1.00\ \pi$ m/s, made with Euler-Cromer.

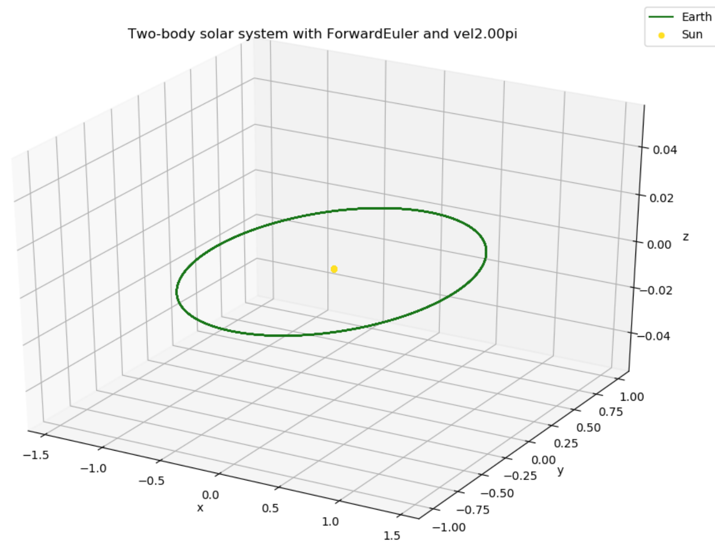

Figure 8: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.00\ \pi$ m/s, made with Euler-Cromer.

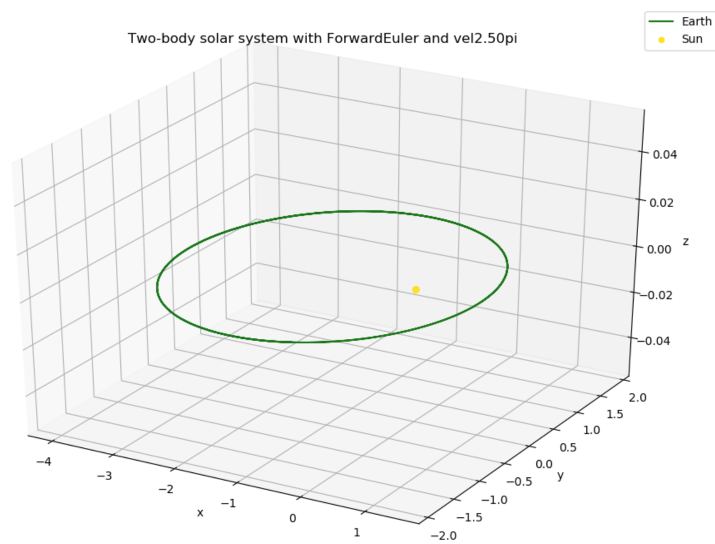

Figure 9: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.50\ \pi$ m/s, made with Euler-Cromer.

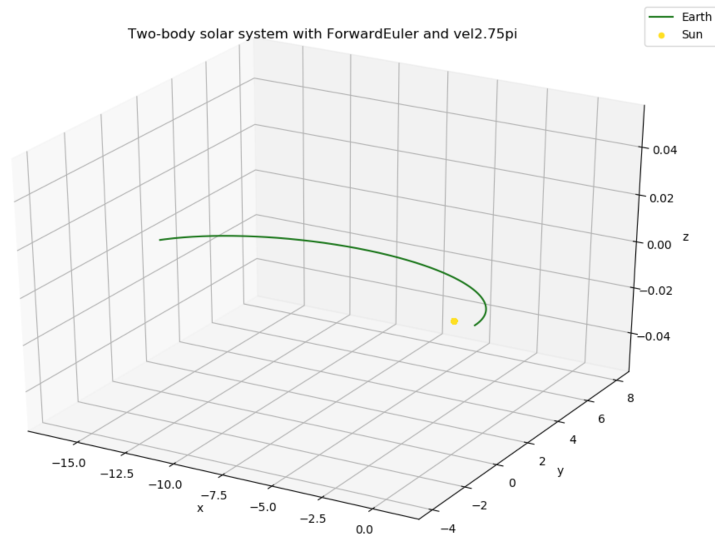

Figure 10: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.75\ \pi$ m/s, made with Euler-Cromer.

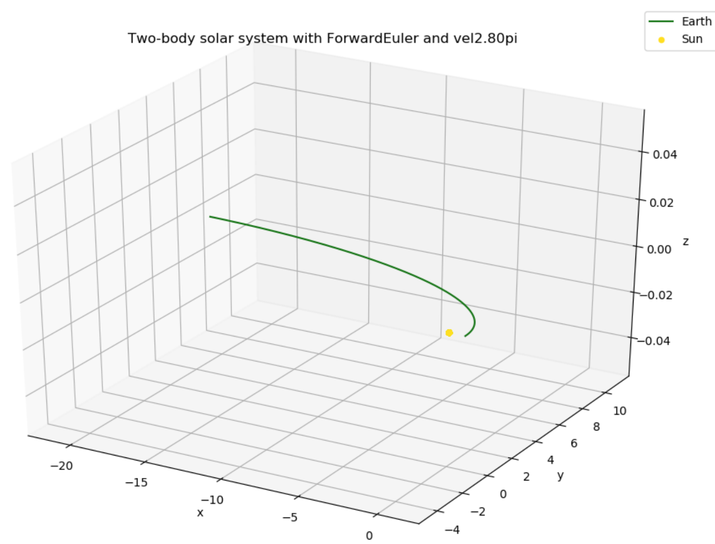

Figure 11: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.80\ \pi$ m/s, made with Euler-Cromer.

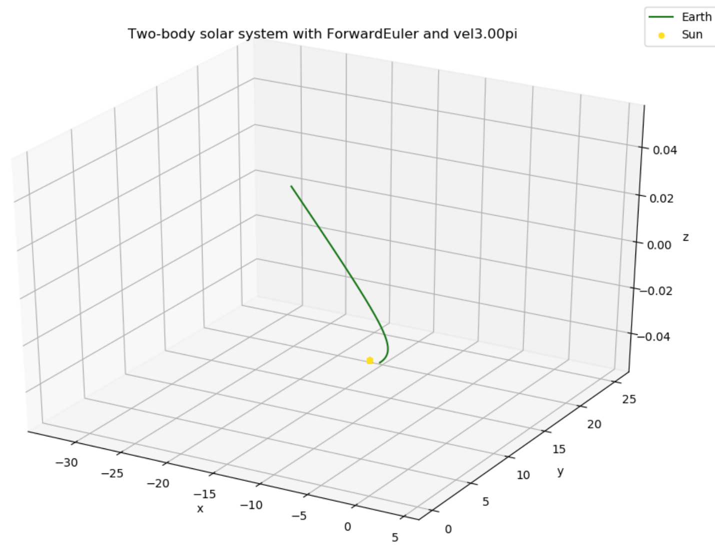

Figure 12: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 3.00\ \pi$ m/s, made with Euler-Cromer.

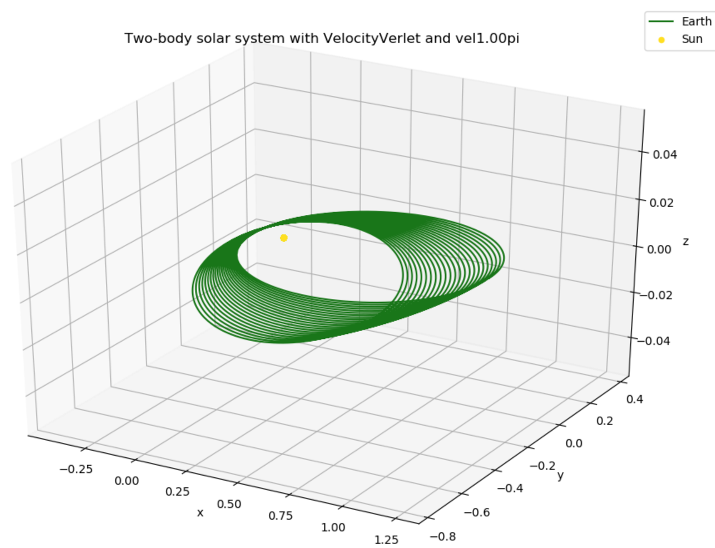

Figure 13: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 1.00\ \pi$ m/s, made with Velocity Verlet.

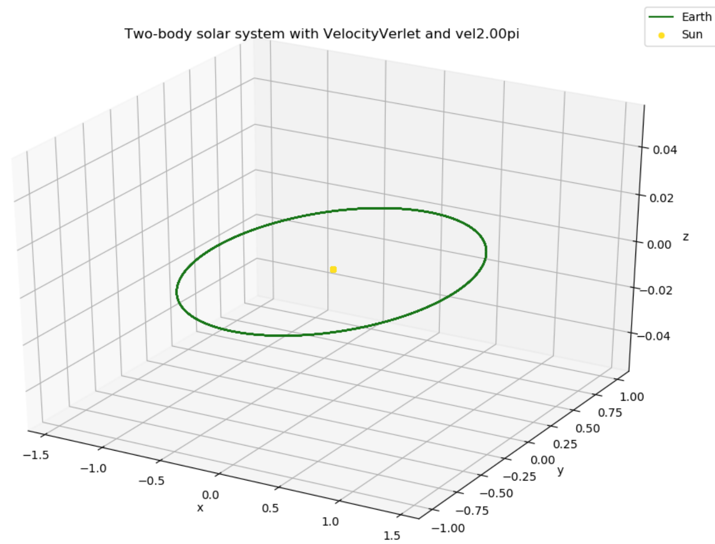

Figure 14: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.00\ \pi$ m/s, made with Velocity Verlet.

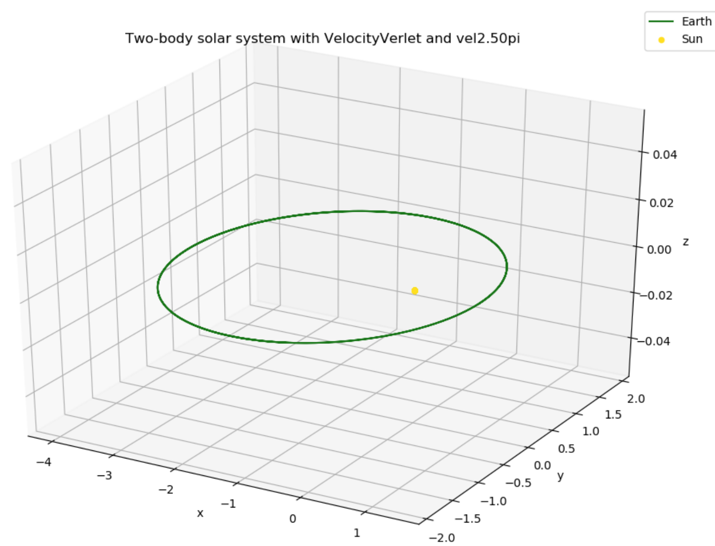

Figure 15: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.50\ \pi$ m/s, made with Velocity Verlet.

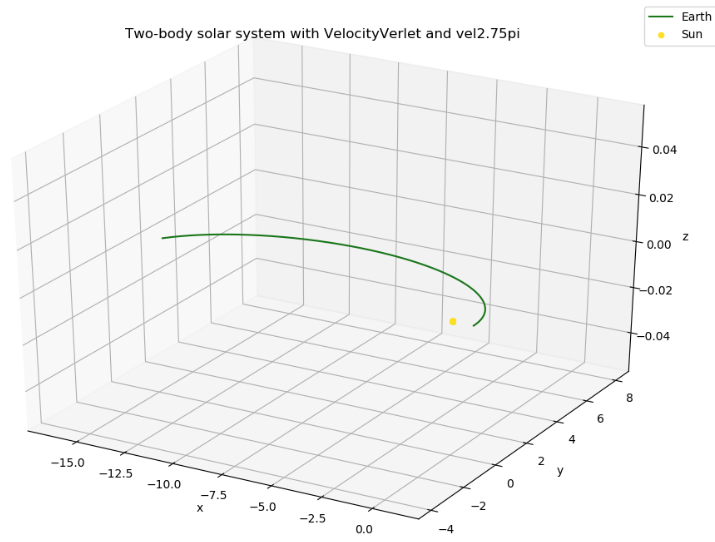

Figure 16: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.75\ \pi$ m/s, made with Velocity Verlet.

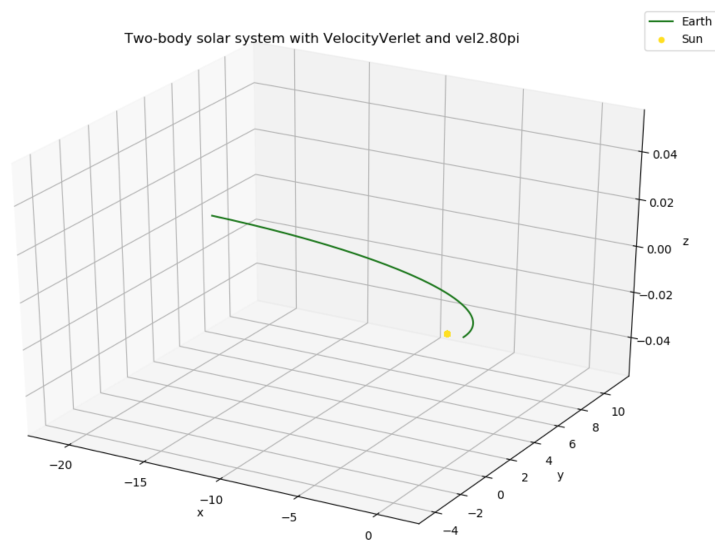

Figure 17: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 2.80\ \pi$ m/s, made with Velocity Verlet.

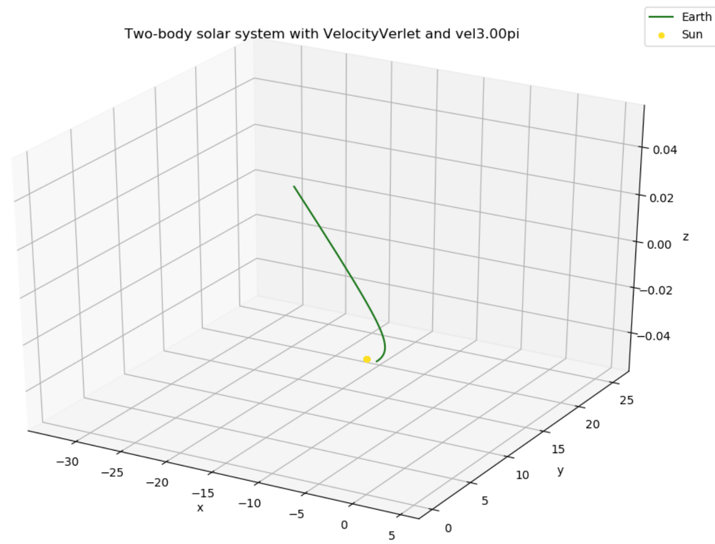

Figure 18: Plot of the Sun-Earth solar system with an escape velocity of $v_p = 3.00\ \pi$ m/s, made with Velocity Verlet.

4.1.3 Testing different values of β

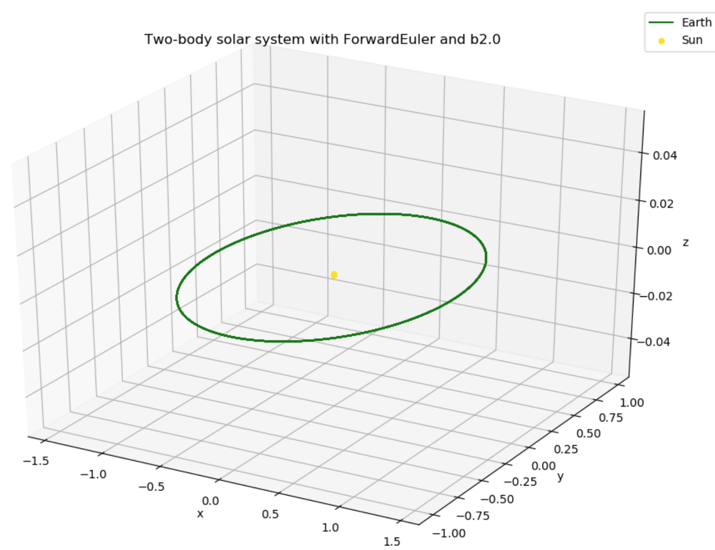

Figure 19: Plot of the Sun-Earth solar system with $\beta = 2.0$, made with Euler-Cromer.

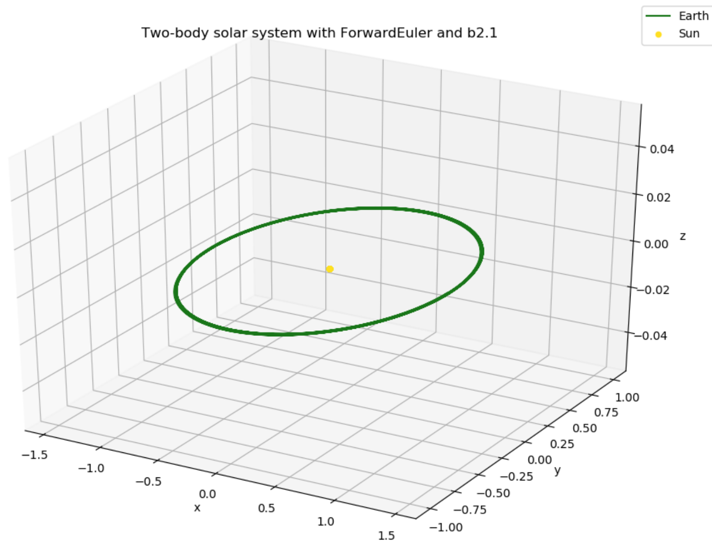

Figure 20: Plot of the Sun-Earth solar system with $\beta = 2.1$, made with Euler-Cromer.

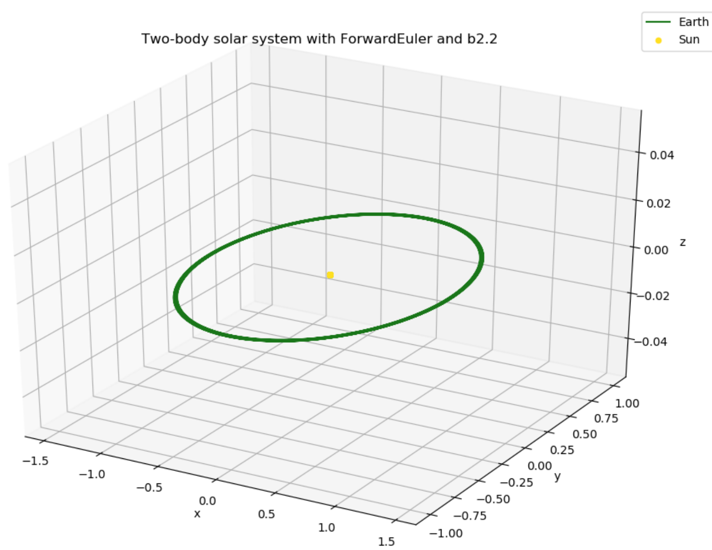

Figure 21: Plot of the Sun-Earth solar system with $\beta = 2.2$, made with Euler-Cromer.

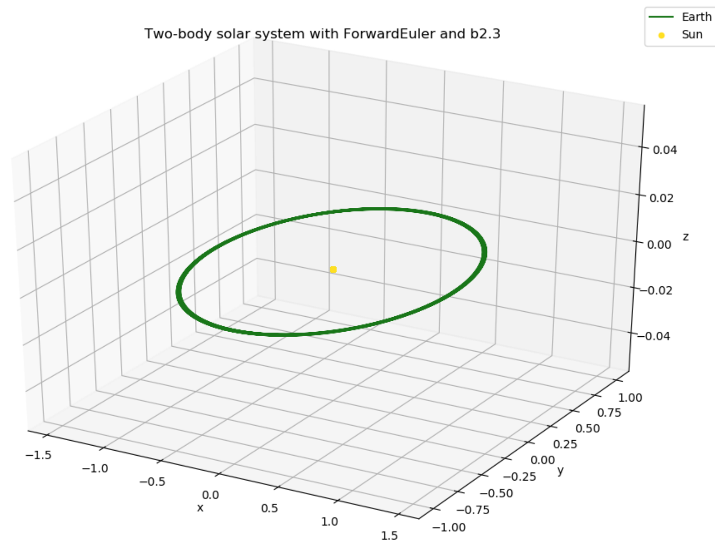

Figure 22: Plot of the Sun-Earth solar system with $\beta = 2.3$, made with Euler-Cromer.

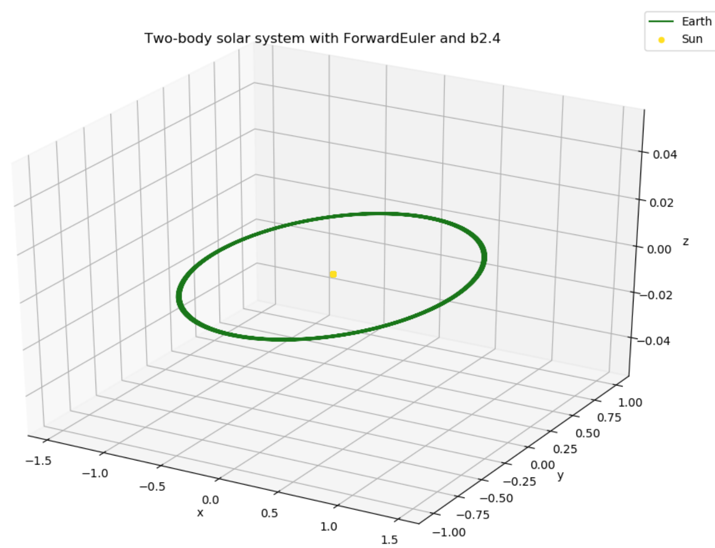

Figure 23: Plot of the Sun-Earth solar system with $\beta = 2.4$, made with Euler-Cromer.

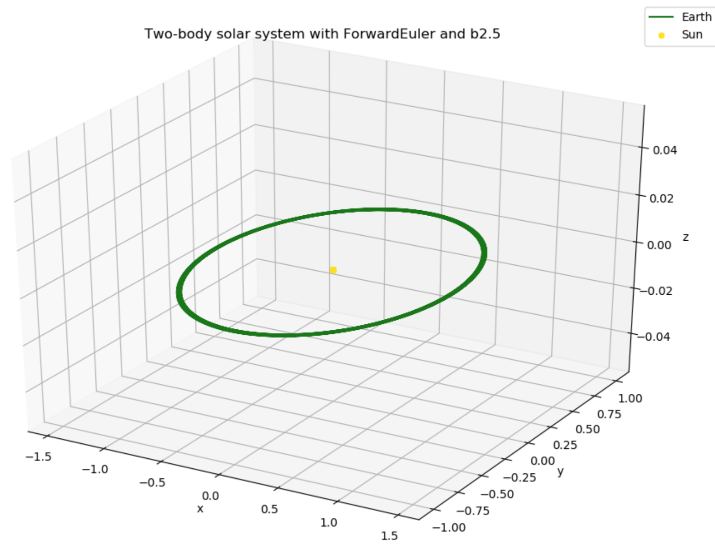

Figure 24: Plot of the Sun-Earth solar system with $\beta = 2.5$, made with Euler-Cromer.

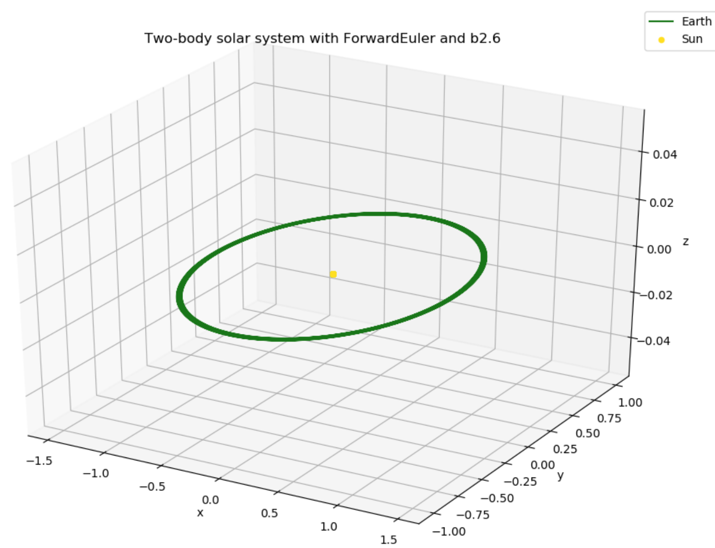

Figure 25: Plot of the Sun-Earth solar system with $\beta = 2.6$, made with Euler-Cromer.

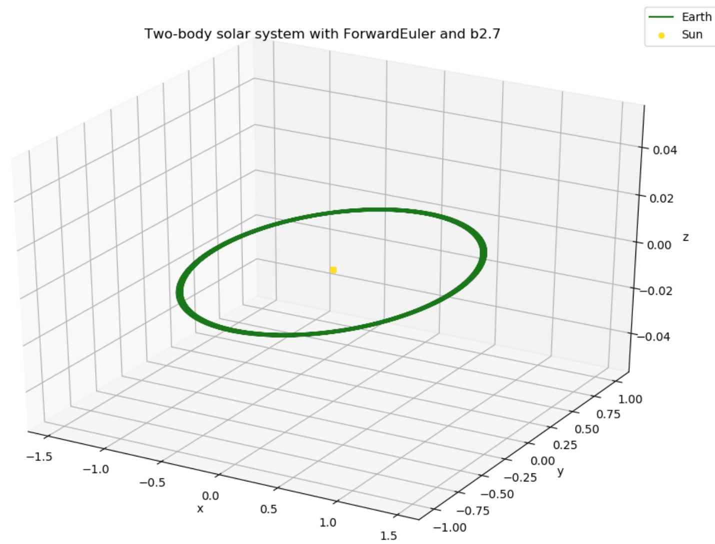

Figure 26: Plot of the Sun-Earth solar system with $\beta = 2.7$, made with Euler-Cromer.

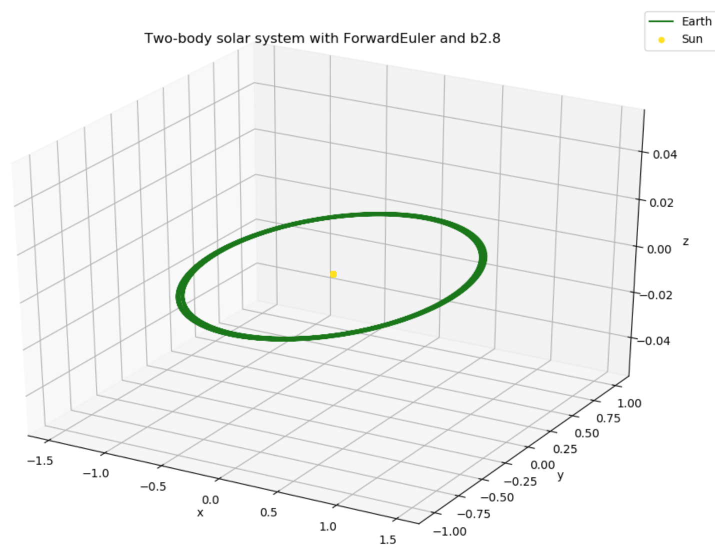

Figure 27: Plot of the Sun-Earth solar system with $\beta = 2.8$, made with Euler-Cromer.

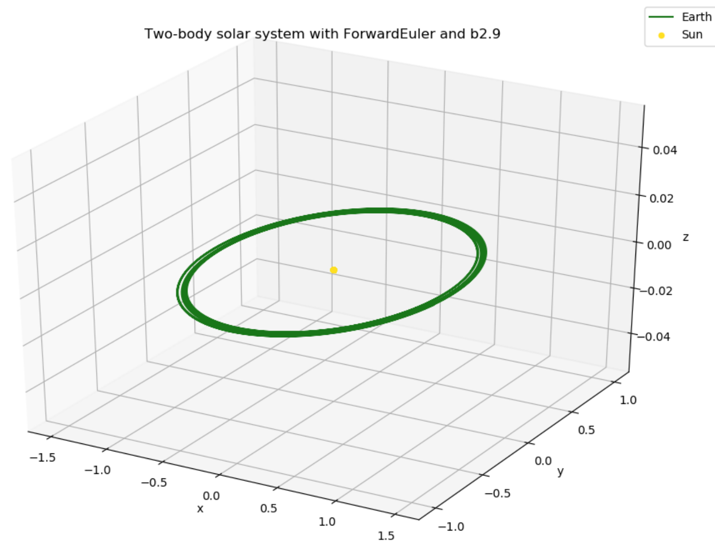

Figure 28: Plot of the Sun-Earth solar system with $\beta = 2.9$, made with Euler-Cromer.

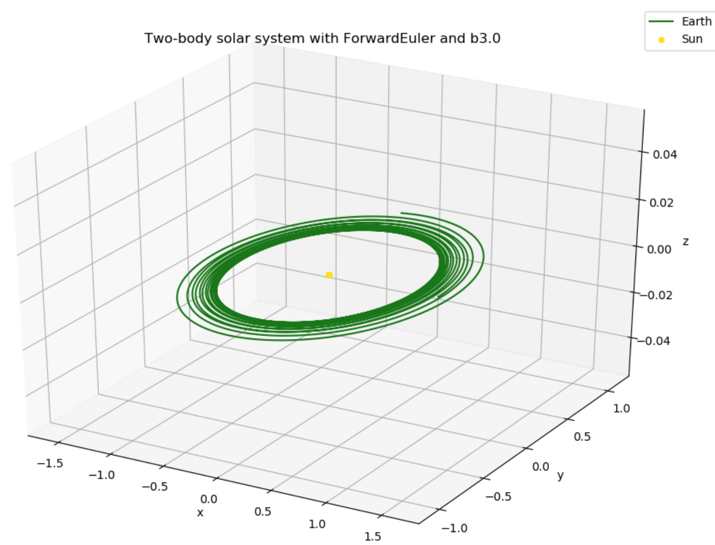

Figure 29: Plot of the Sun-Earth solar system with $\beta = 3.0$, made with Euler-Cromer.

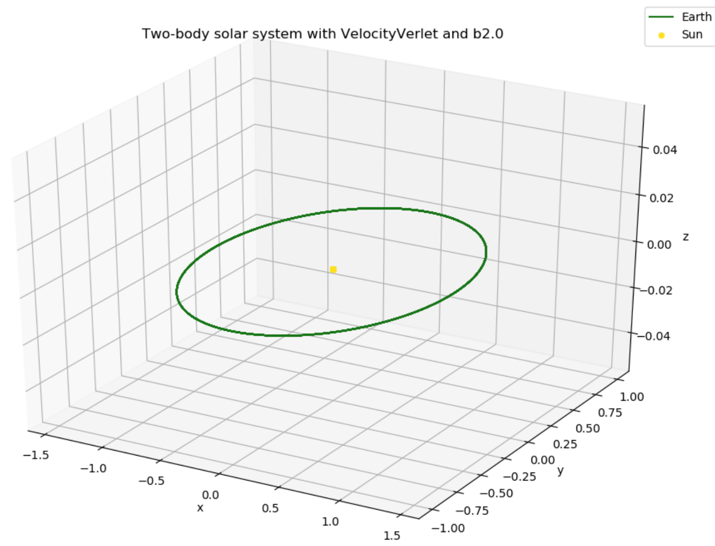

Figure 30: Plot of the Sun-Earth solar system with $\beta = 2.0$, made with Velocity Verlet.

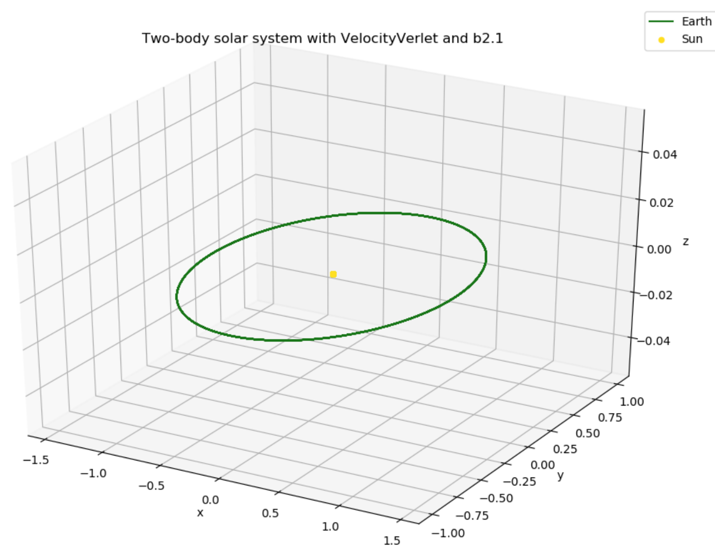

Figure 31: Plot of the Sun-Earth solar system with $\beta = 2.1$, made with Velocity Verlet.

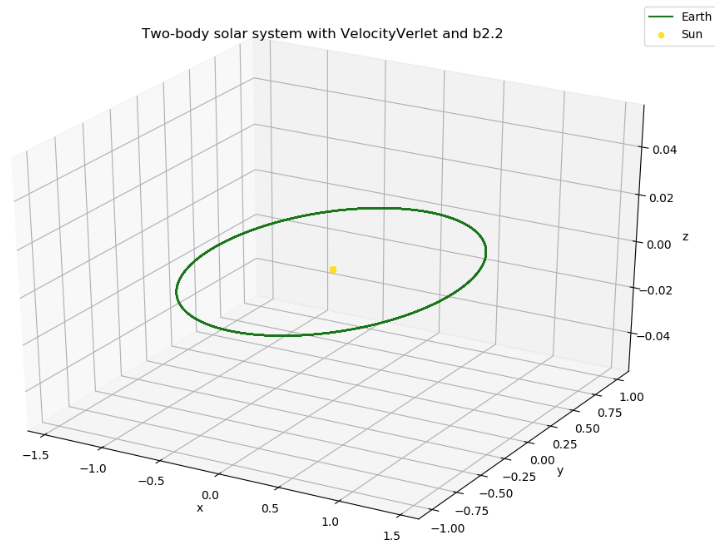

Figure 32: Plot of the Sun-Earth solar system with $\beta = 2.2$, made with Velocity Verlet.

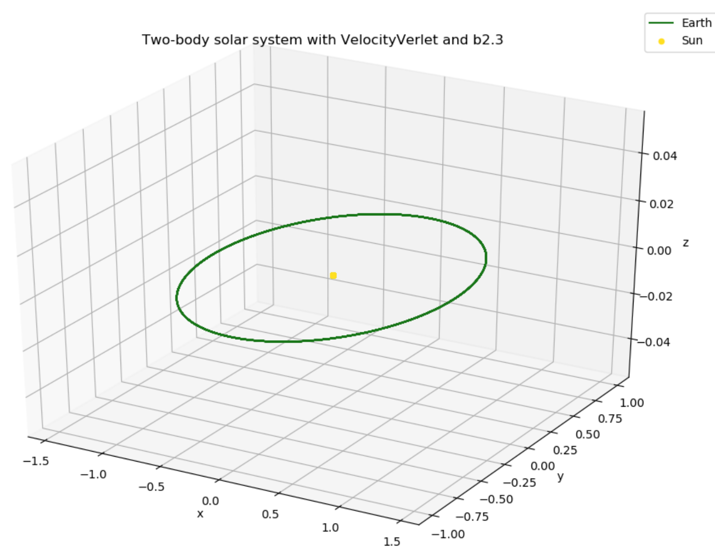

Figure 33: Plot of the Sun-Earth solar system with $\beta = 2.3$, made with Velocity Verlet.

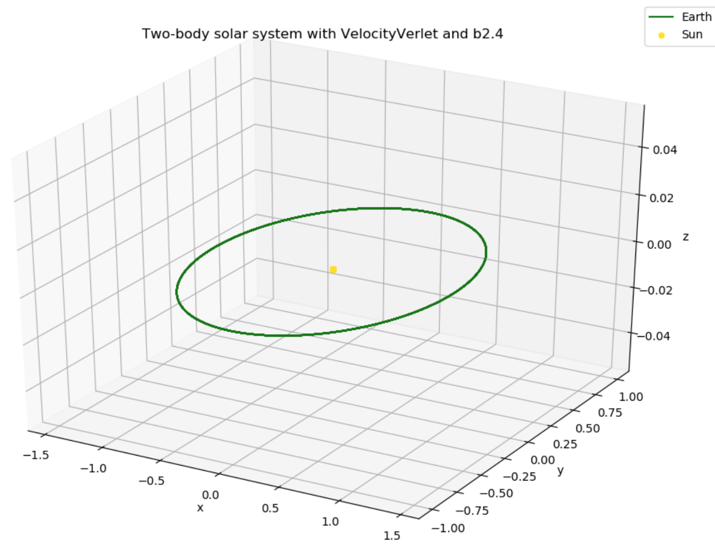

Figure 34: Plot of the Sun-Earth solar system with $\beta = 2.4$, made with Velocity Verlet.

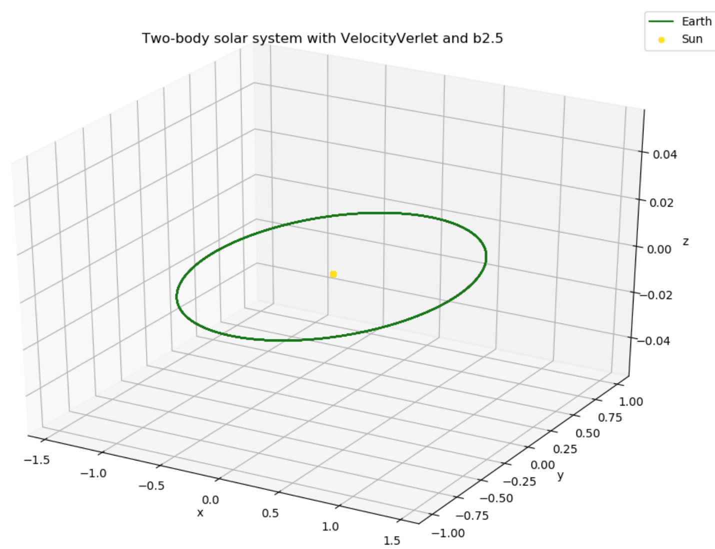

Figure 35: Plot of the Sun-Earth solar system with $\beta = 2.5$, made with Velocity Verlet.

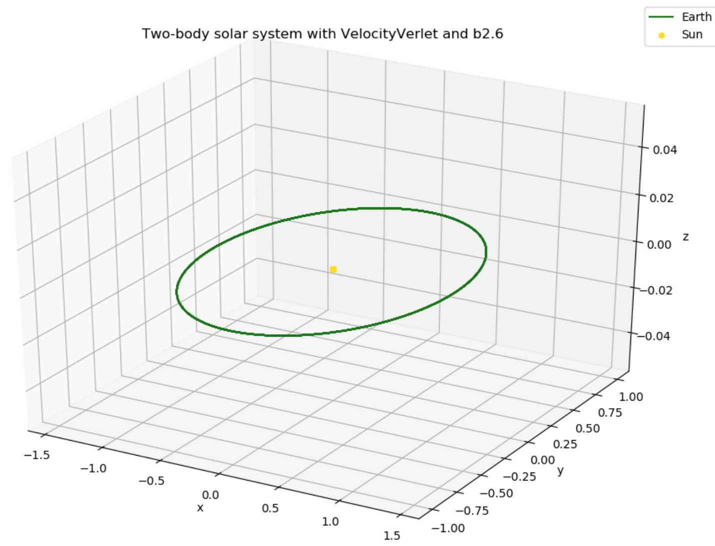

Figure 36: Plot of the Sun-Earth solar system with $\beta = 2.6$, made with Velocity Verlet.

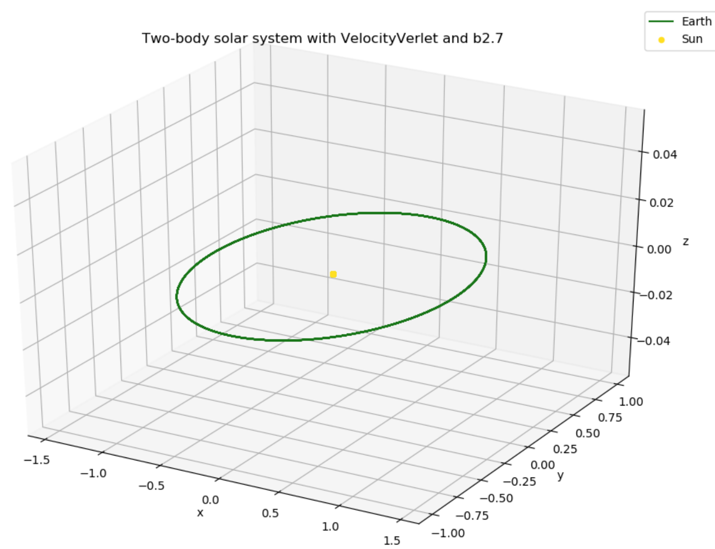

Figure 37: Plot of the Sun-Earth solar system with $\beta = 2.7$, made with Velocity Verlet.

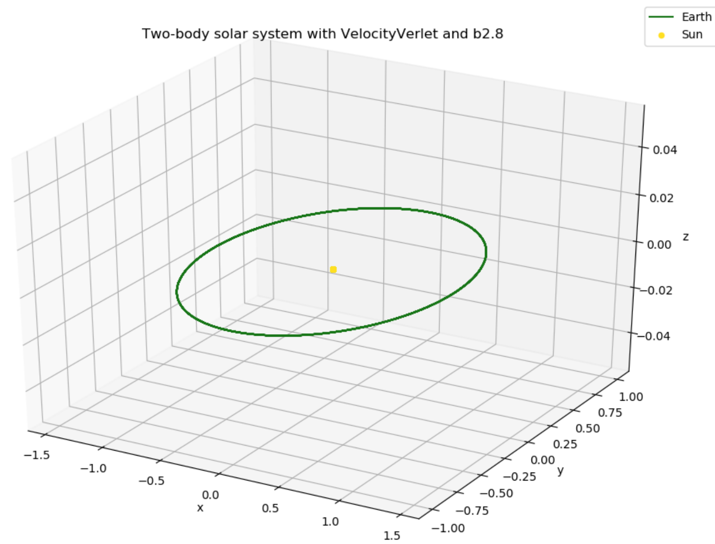

Figure 38: Plot of the Sun-Earth solar system with $\beta = 2.8$, made with Velocity Verlet.

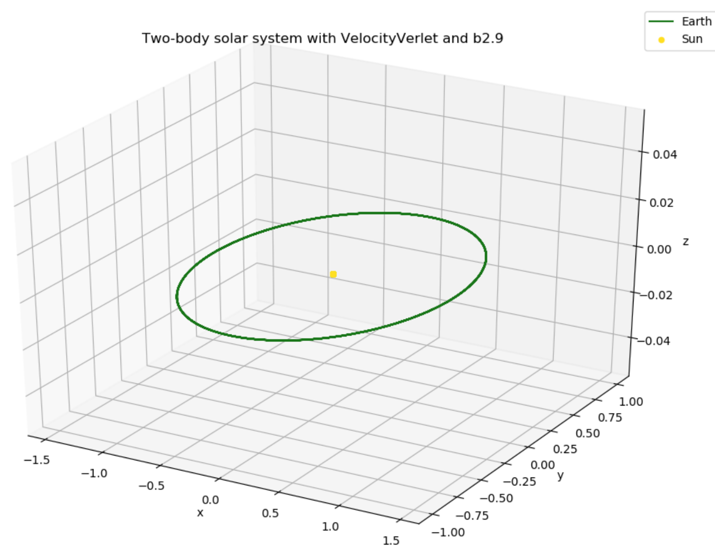

Figure 39: Plot of the Sun-Earth solar system with $\beta = 2.9$, made with Velocity Verlet.

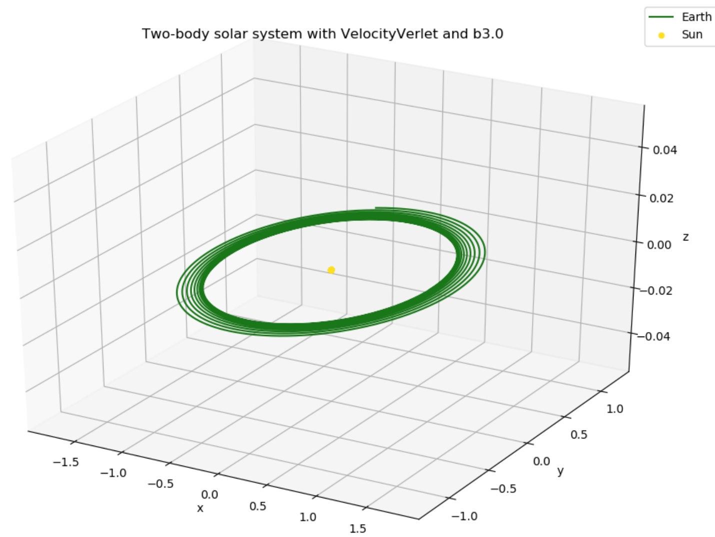

Figure 40: Plot of the Sun-Earth solar system with $\beta = 3.0$, made with Velocity Verlet.

4.2 Sun-Mercury solar system

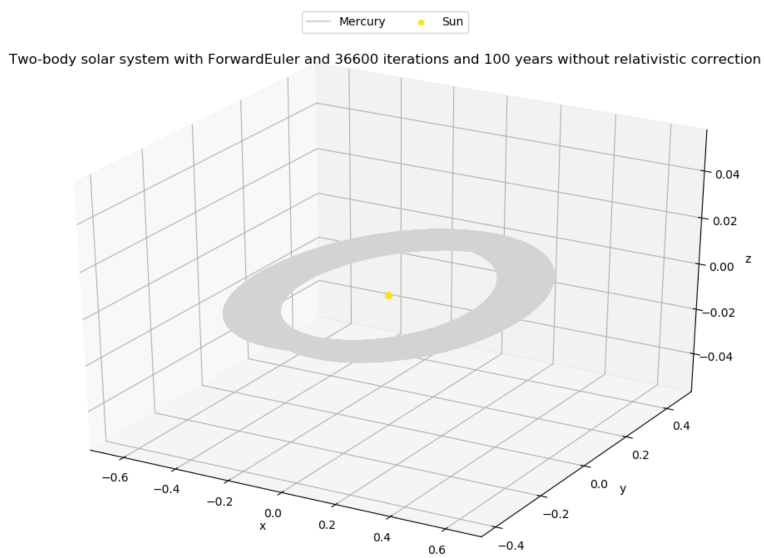

Figure 41: Plot of the Sun-Mercury solar system with 36600 iterations over 100 years without the relativistic correction, made with Euler-Cromer.

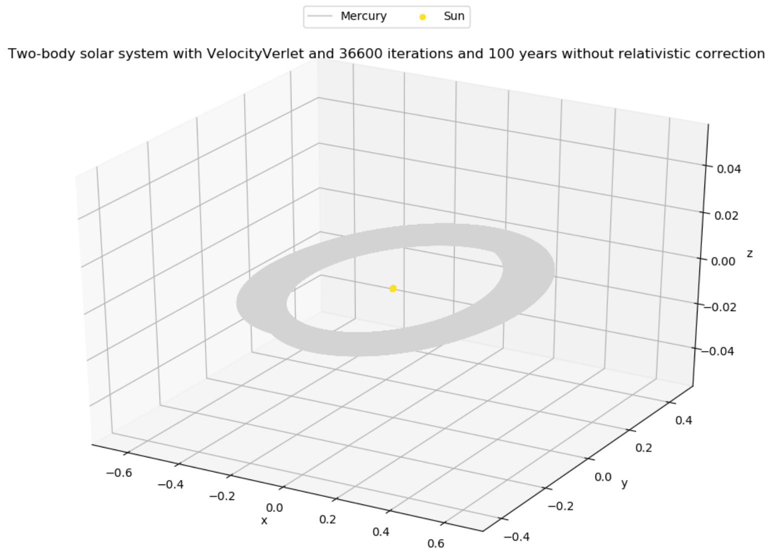

Figure 42: Plot of the Sun-Mercury solar system with 36600 iterations over 100 years without the relativistic correction, made with Velocity Verlet.

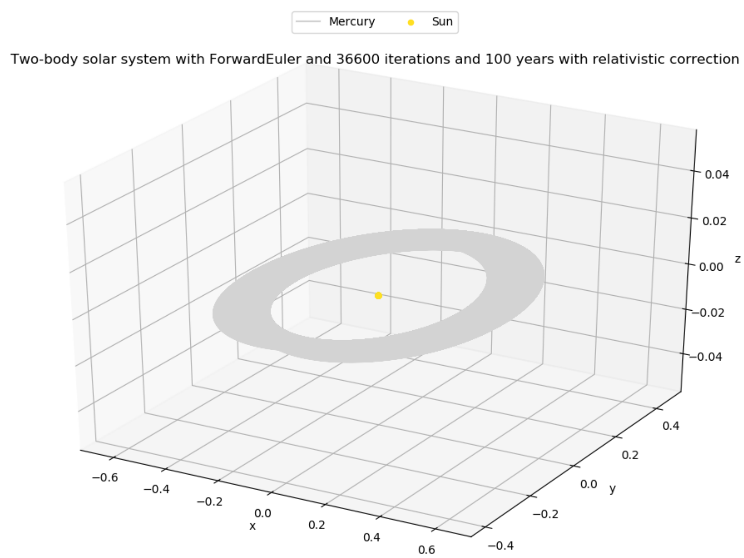

Figure 43: Plot of the Sun-Mercury solar system with 36600 iterations over 100 years with the relativistic correction, made with Euler-Cromer.

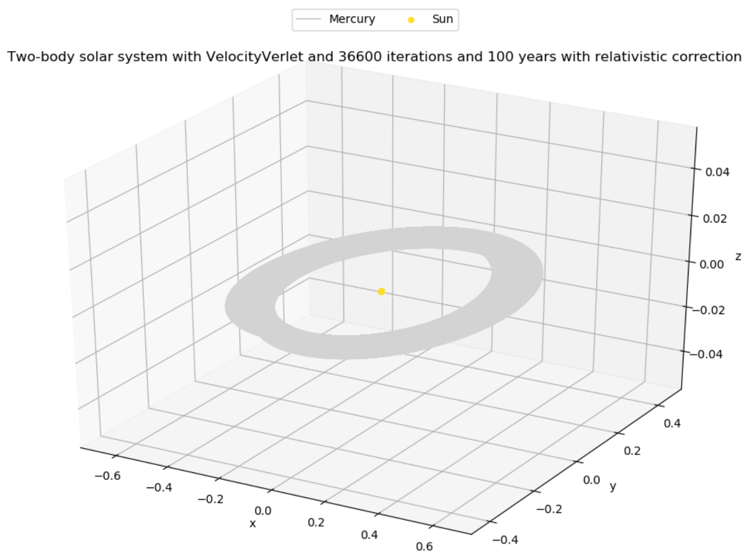

Figure 44: Plot of the Sun-Mercury solar system with 36600 iterations over 100 years with the relativistic correction, made with Velocity Verlet

4.3 Sun-Earth-Jupiter solar system

4.3.1 Massfactor 1

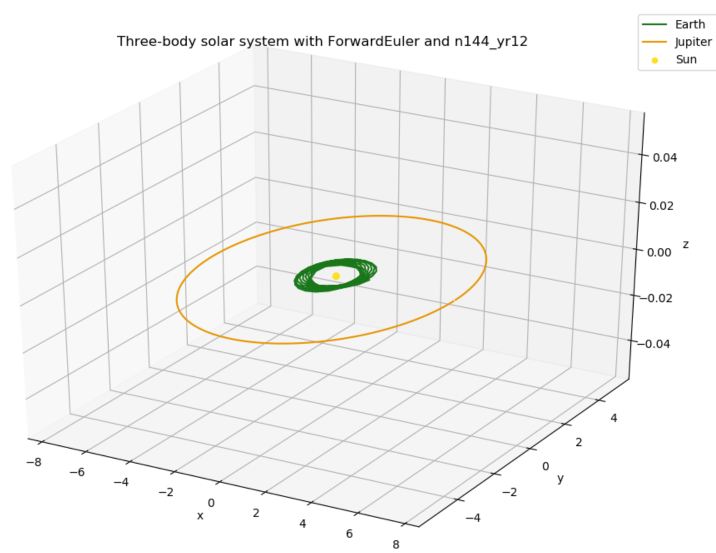

Figure 45: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

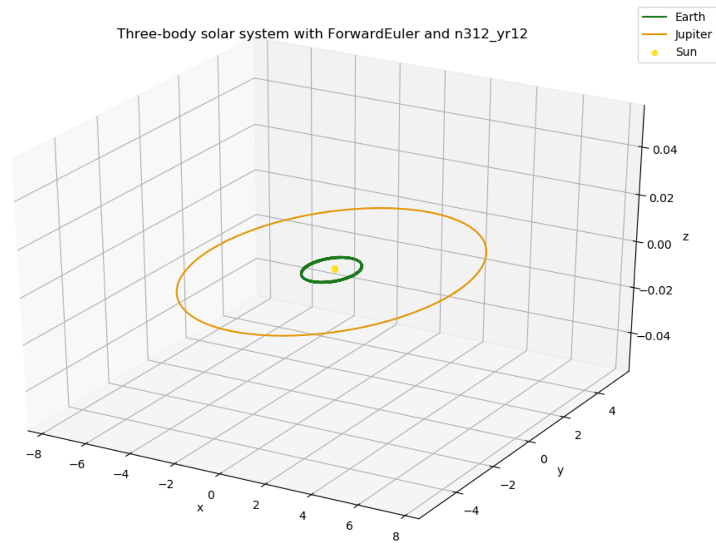

Figure 46: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

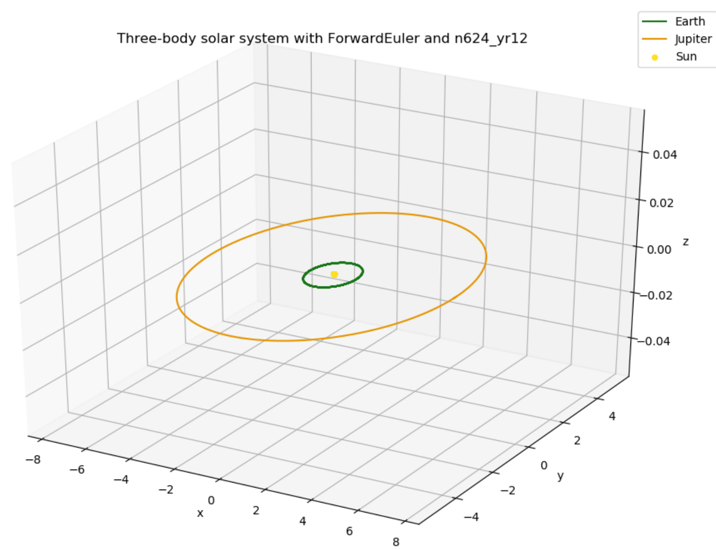

Figure 47: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

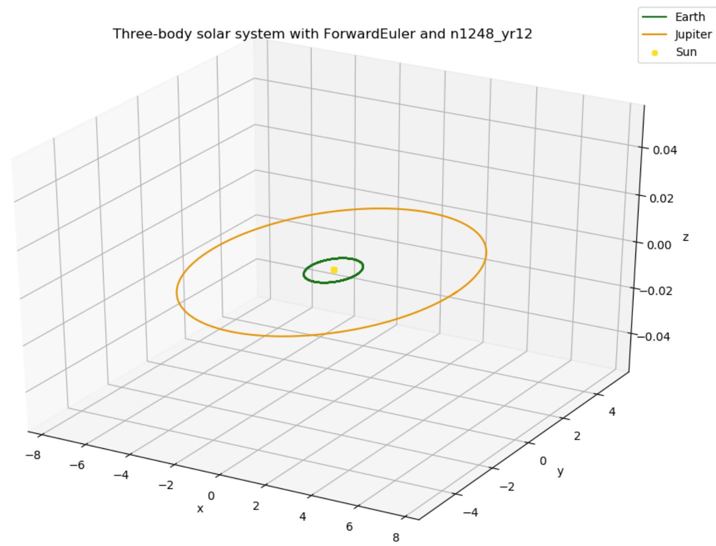

Figure 48: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

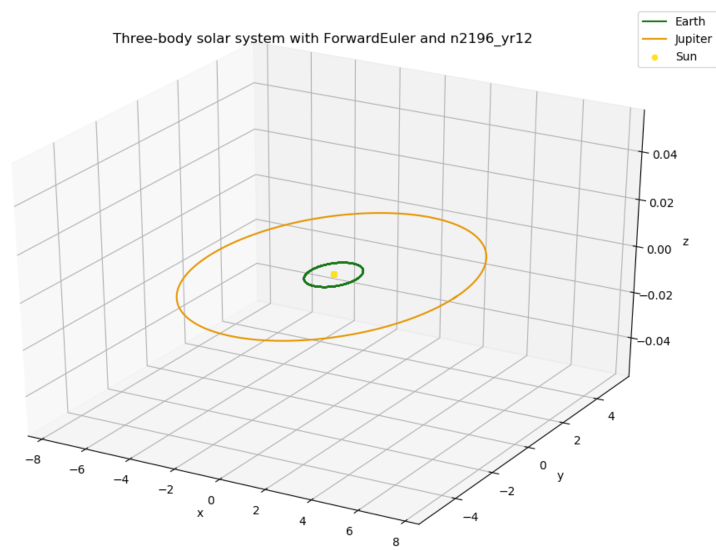

Figure 49: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

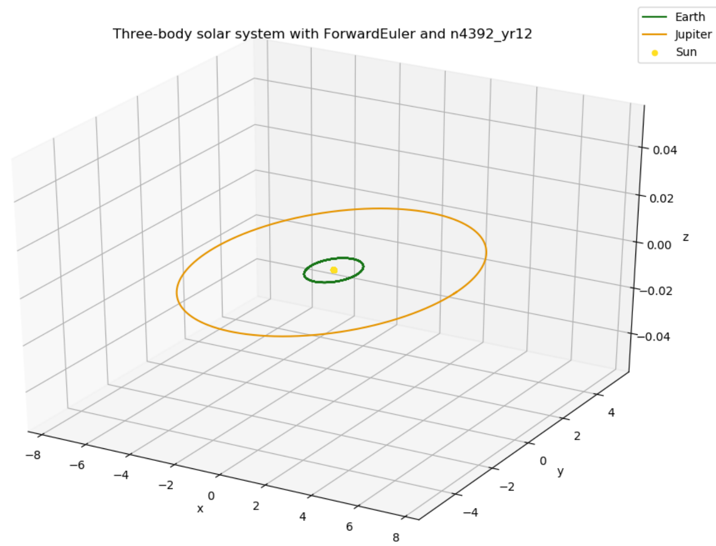

Figure 50: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

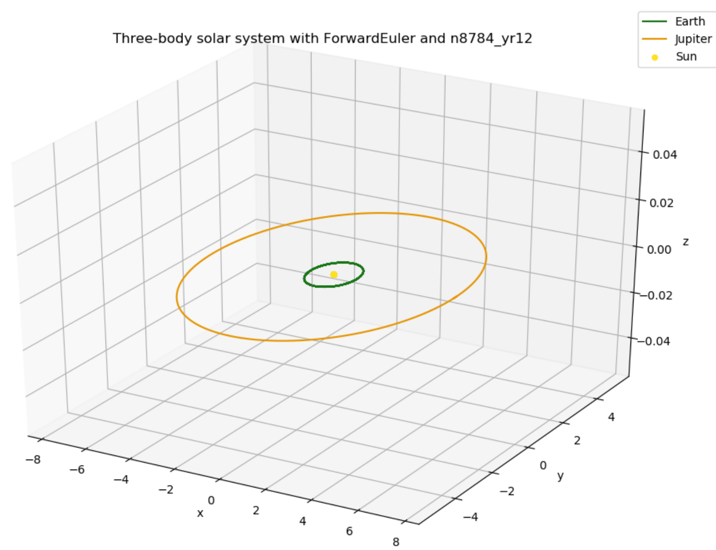

Figure 51: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 1 for Jupiter, made with Euler-Cromer.

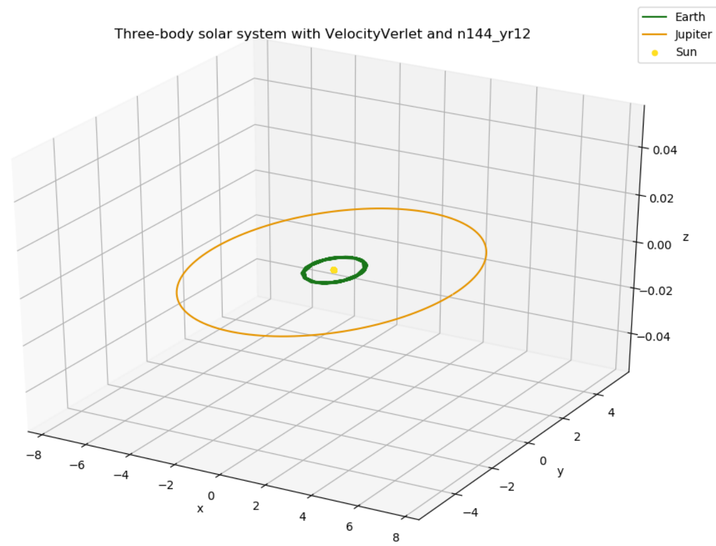

Figure 52: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

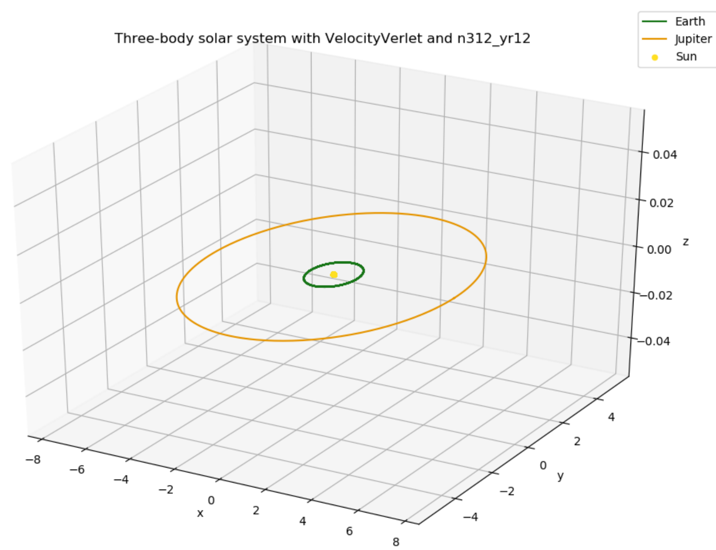

Figure 53: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

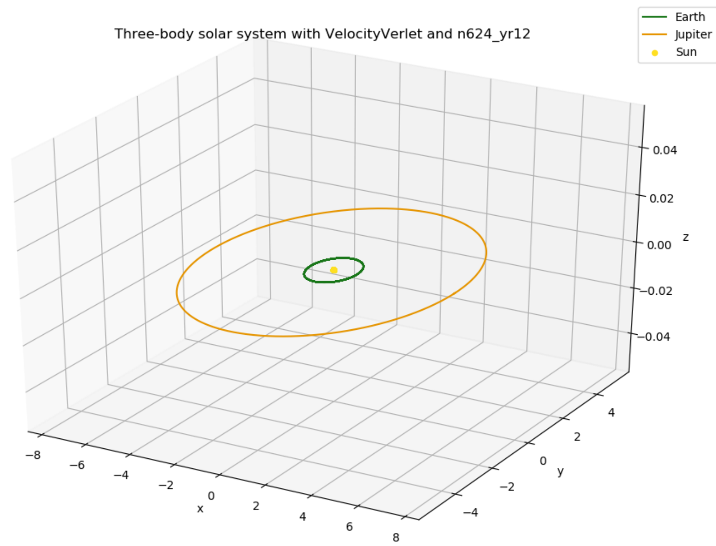

Figure 54: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

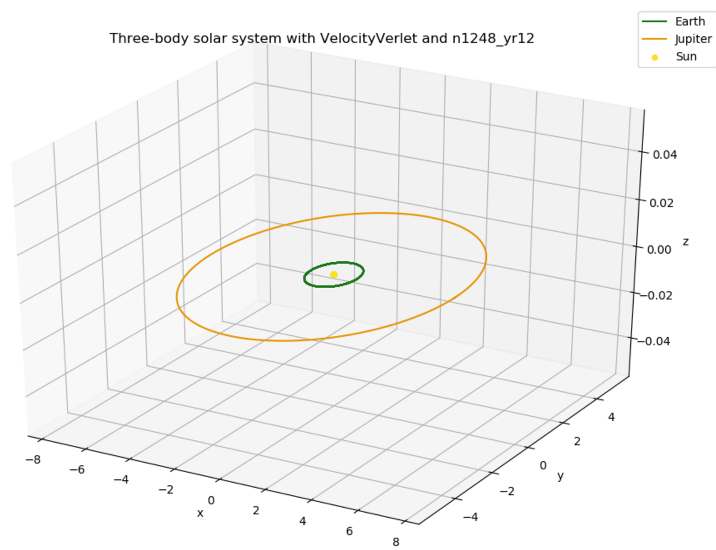

Figure 55: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

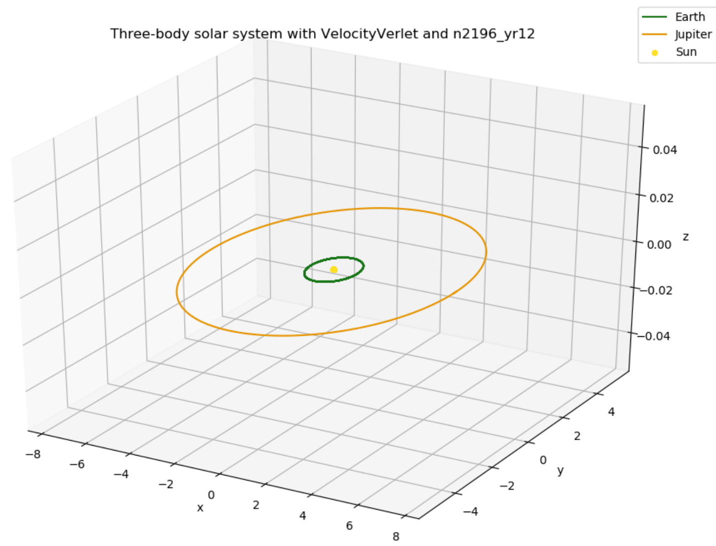

Figure 56: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

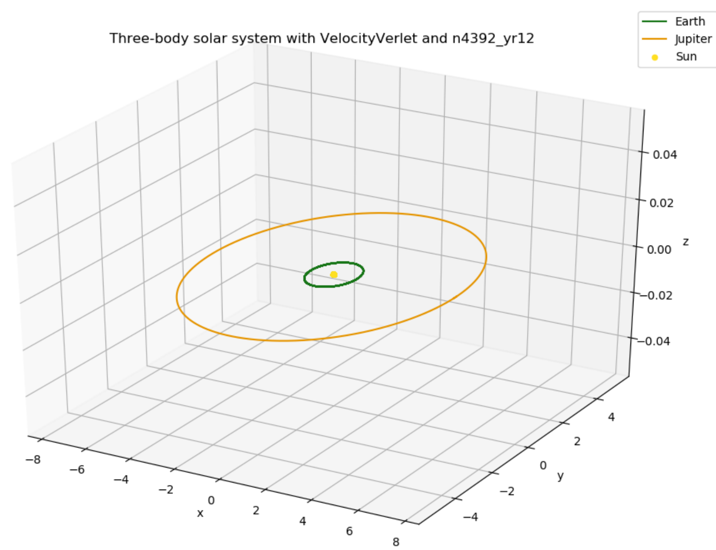

Figure 57: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

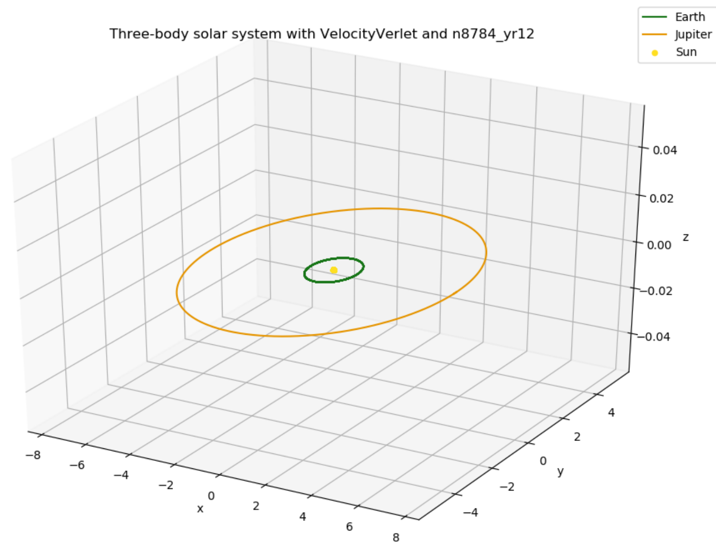

Figure 58: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 1 for Jupiter, made with Velocity Verlet.

4.3.2 Massfactor 10

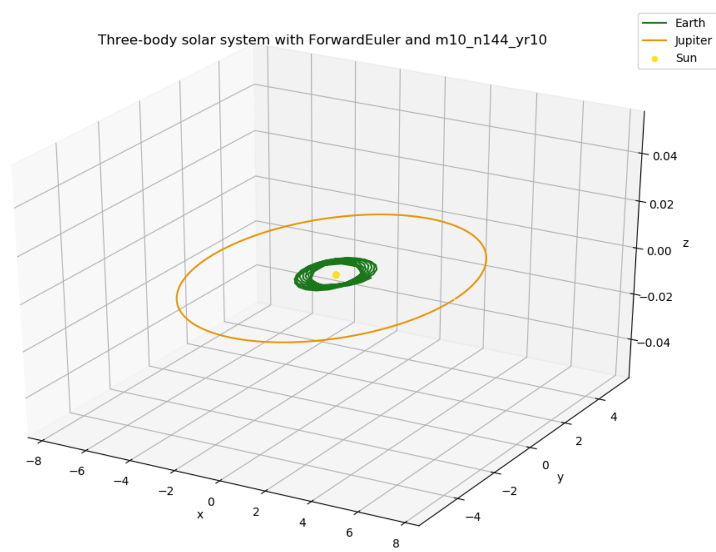

Figure 59: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 10 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

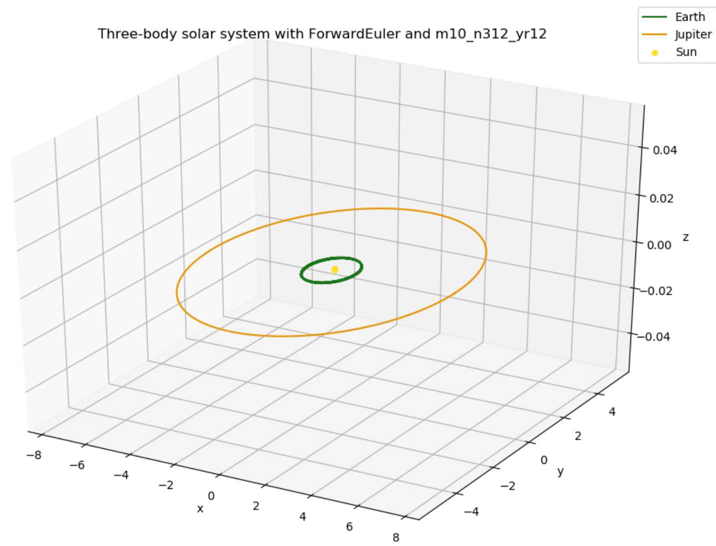

Figure 60: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

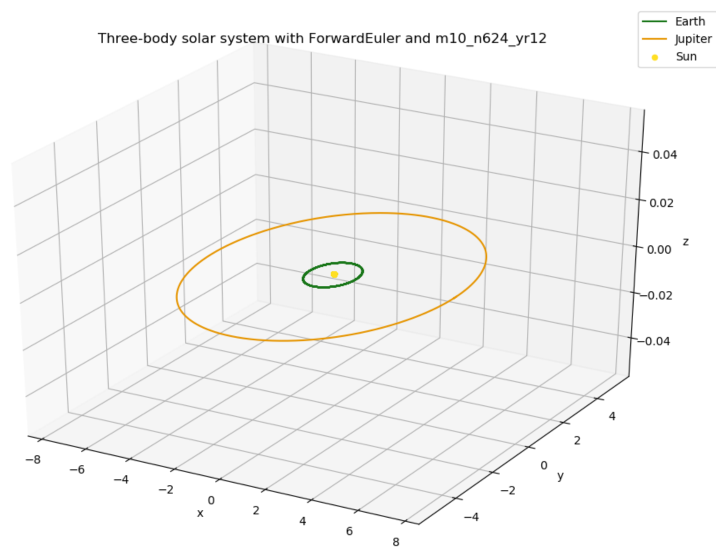

Figure 61: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

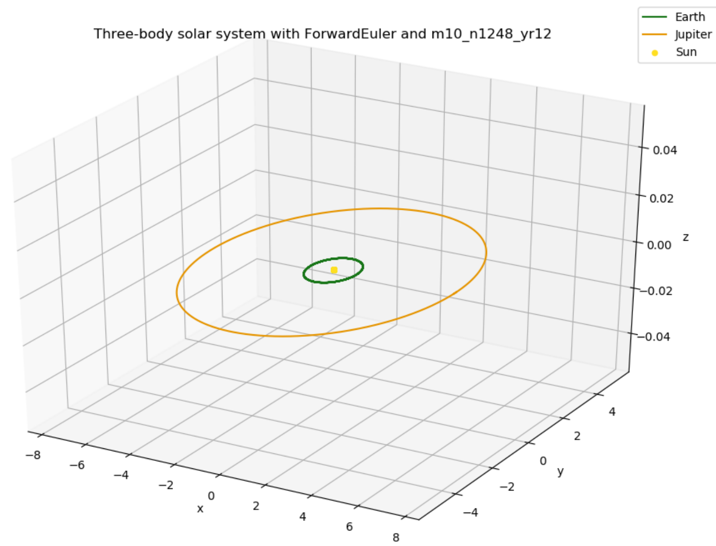

Figure 62: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

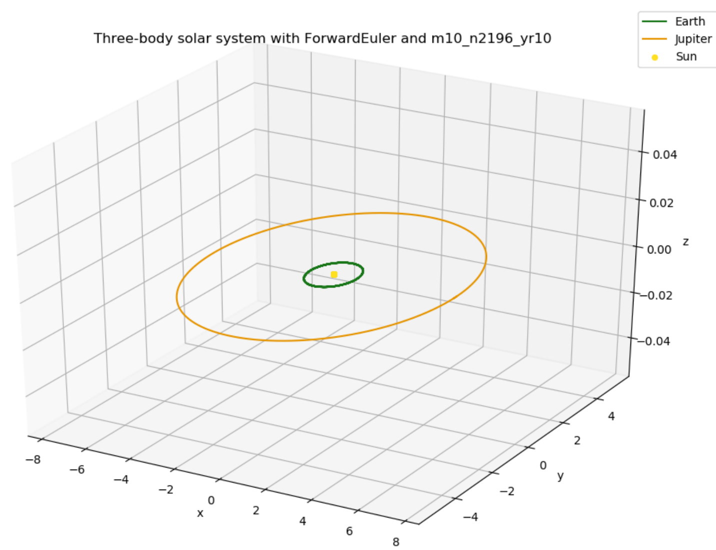

Figure 63: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 10 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

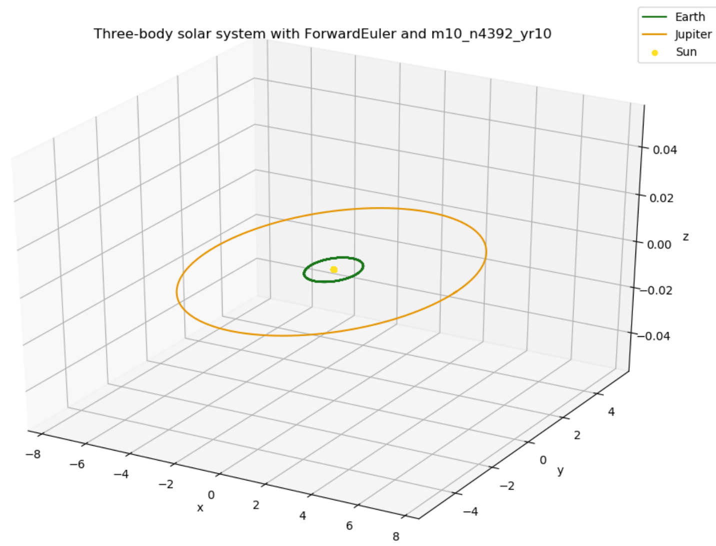

Figure 64: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 10 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

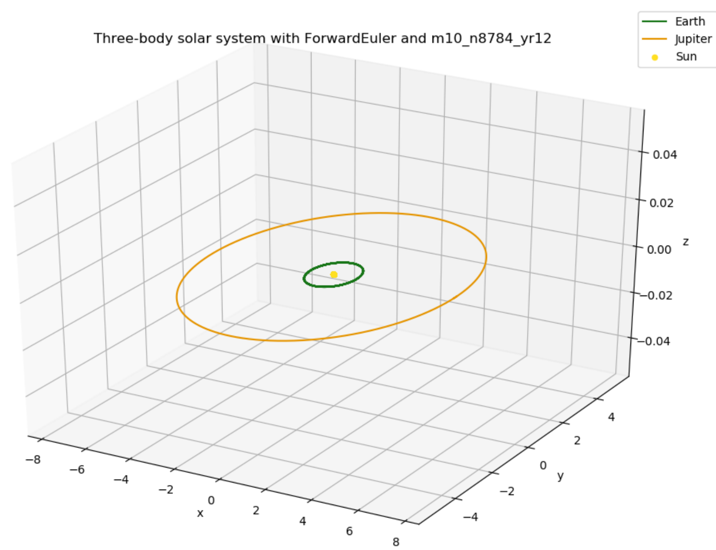

Figure 65: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 10 for Jupiter, made with Euler-Cromer.

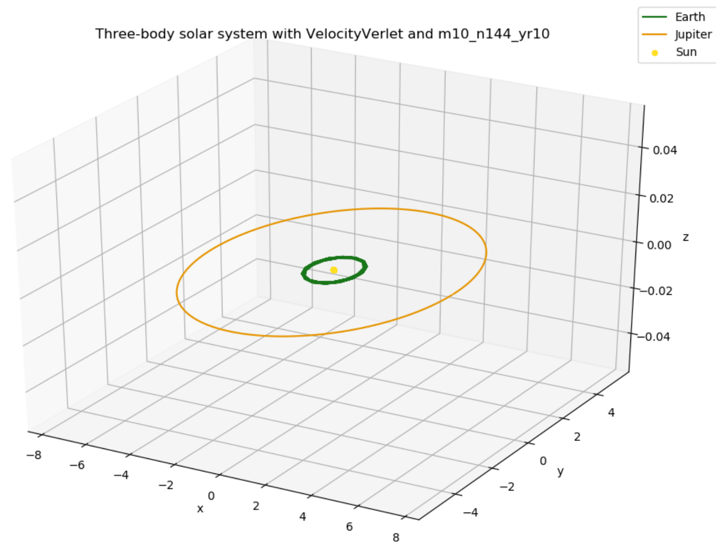

Figure 66: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 10 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

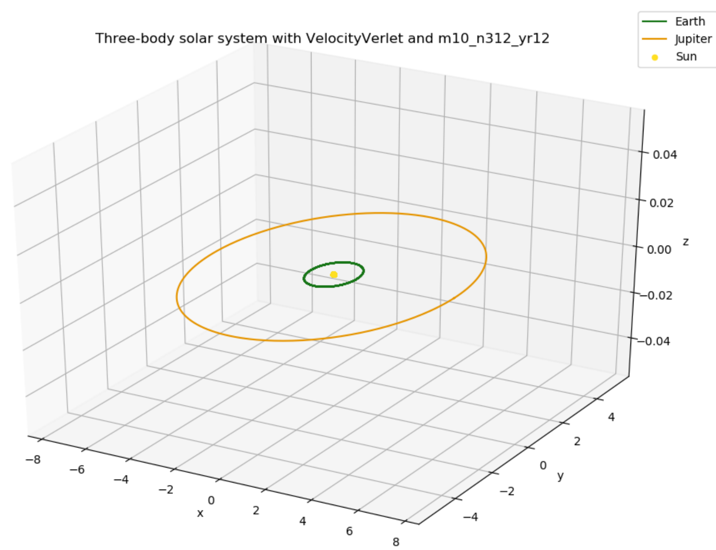

Figure 67: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

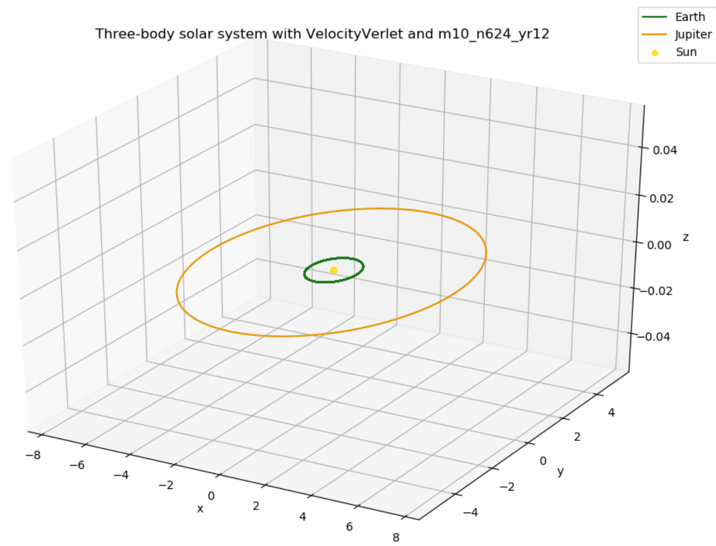

Figure 68: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

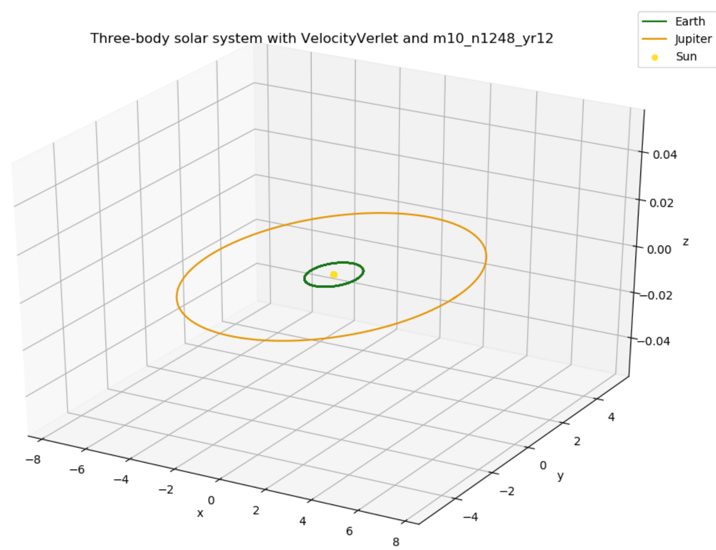

Figure 69: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

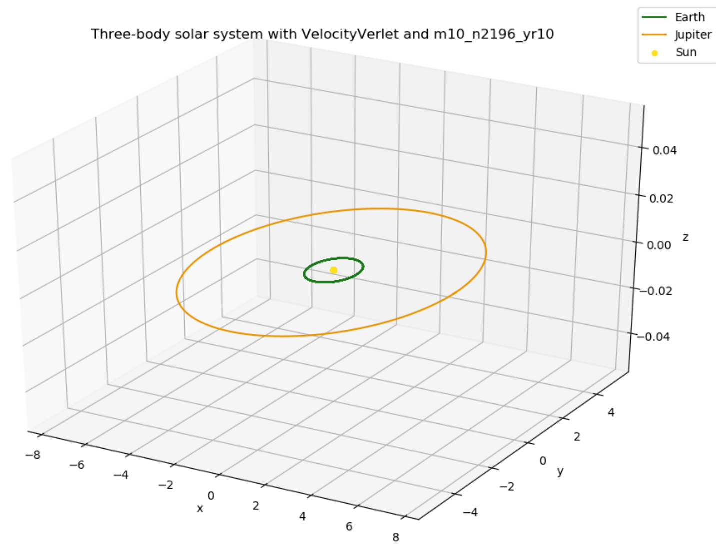

Figure 70: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 10 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

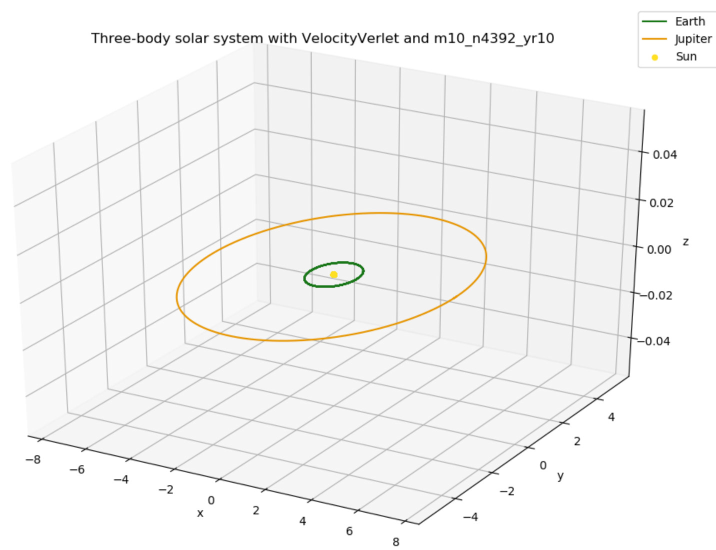

Figure 71: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 10 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

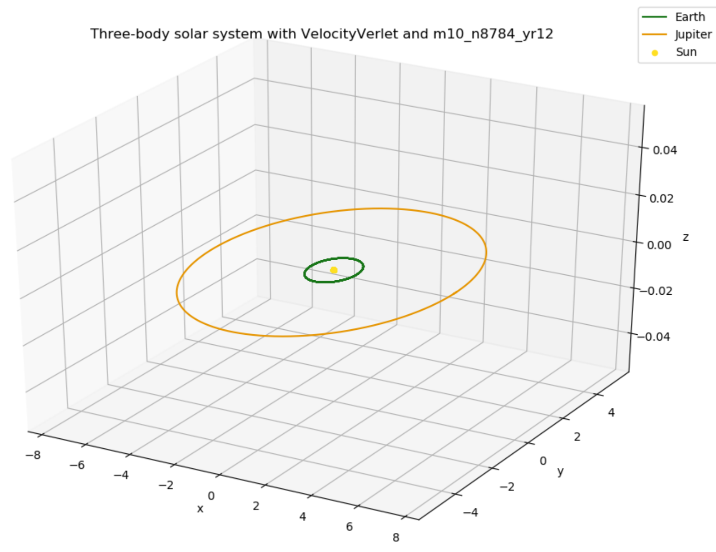

Figure 72: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 10 for Jupiter, made with Velocity Verlet.

4.3.3 Massfactor 1000

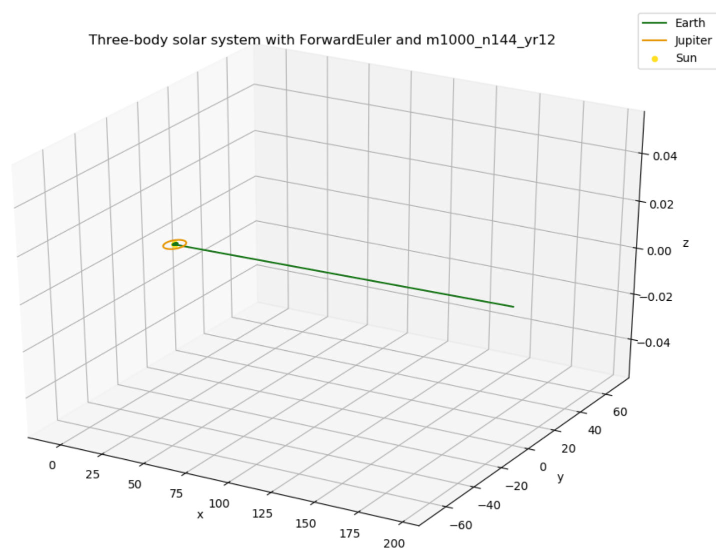

Figure 73: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

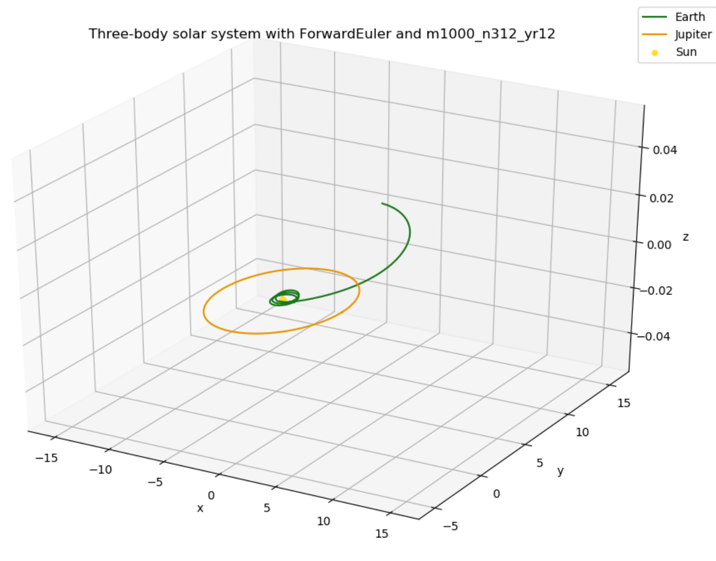

Figure 74: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

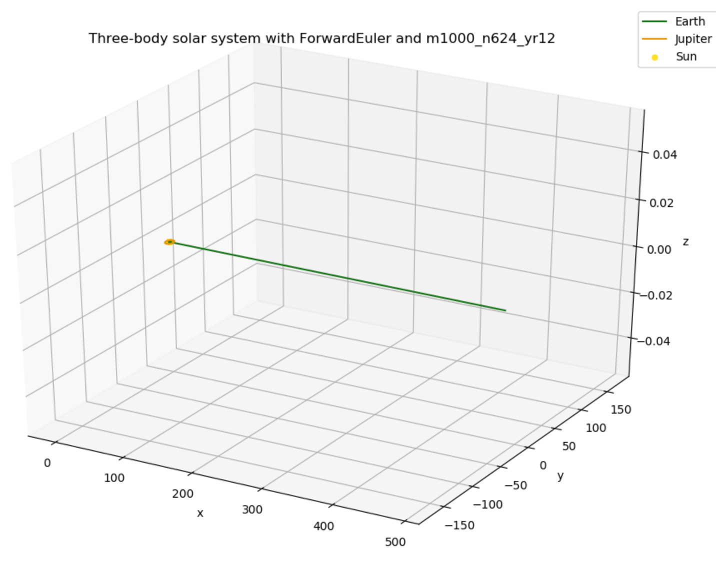

Figure 75: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

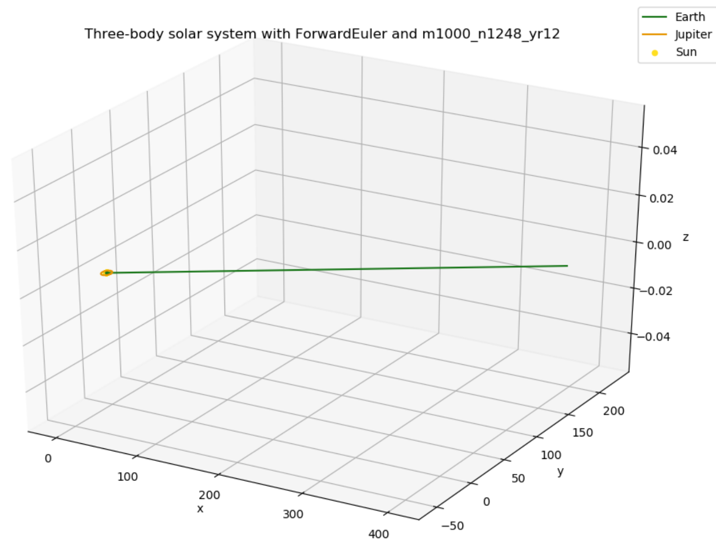

Figure 76: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

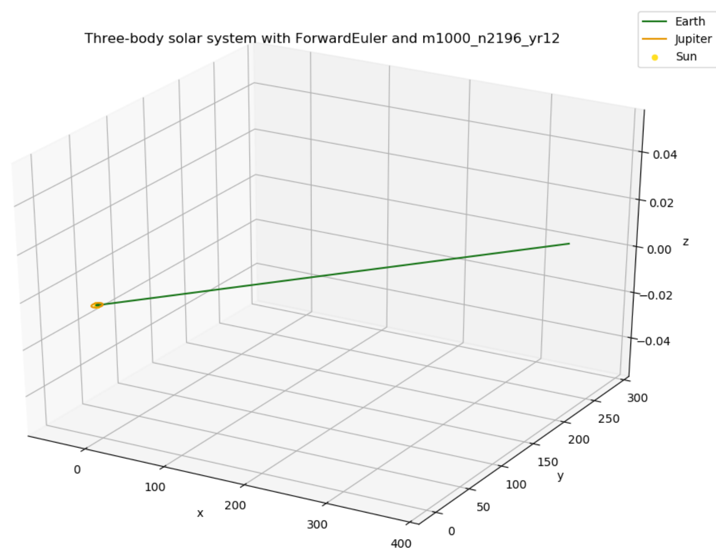

Figure 77: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

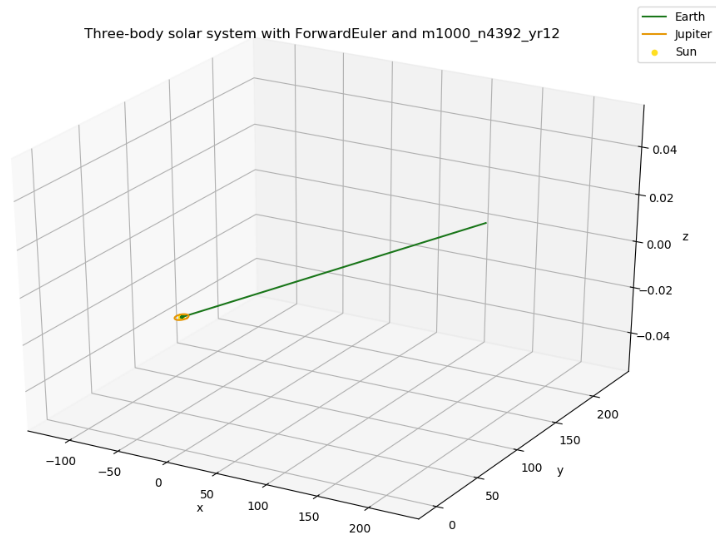

Figure 78: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

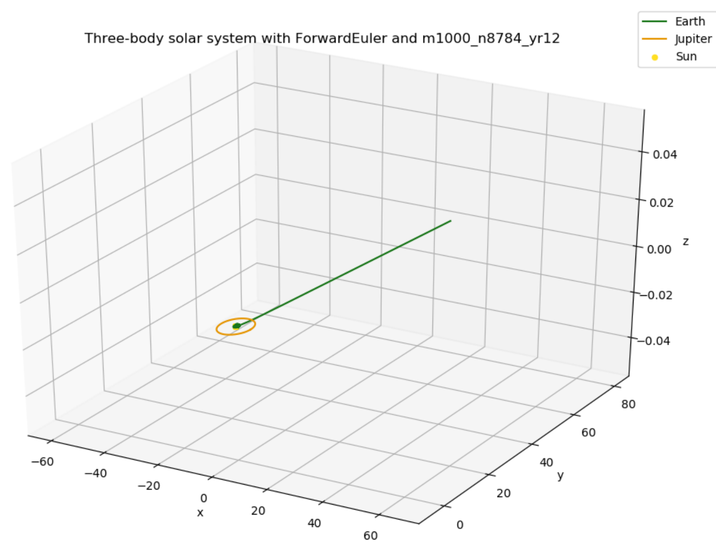

Figure 79: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Euler-Cromer.

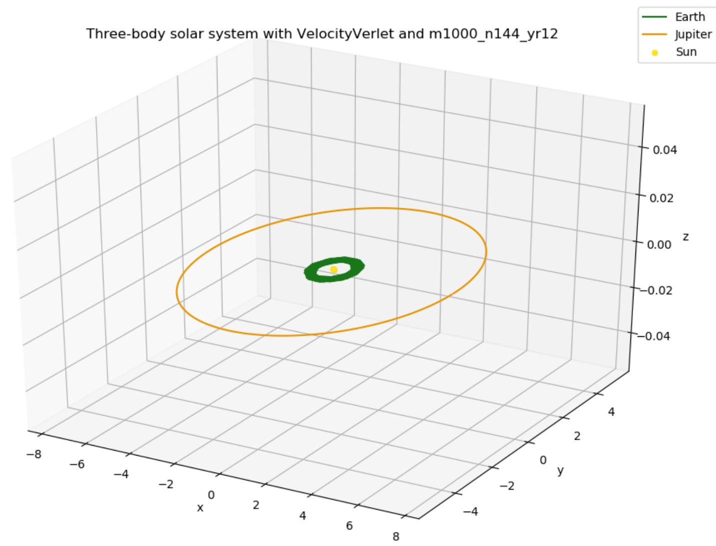

Figure 80: Plot of the Sun-Earth-Jupiter solar system with 144 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

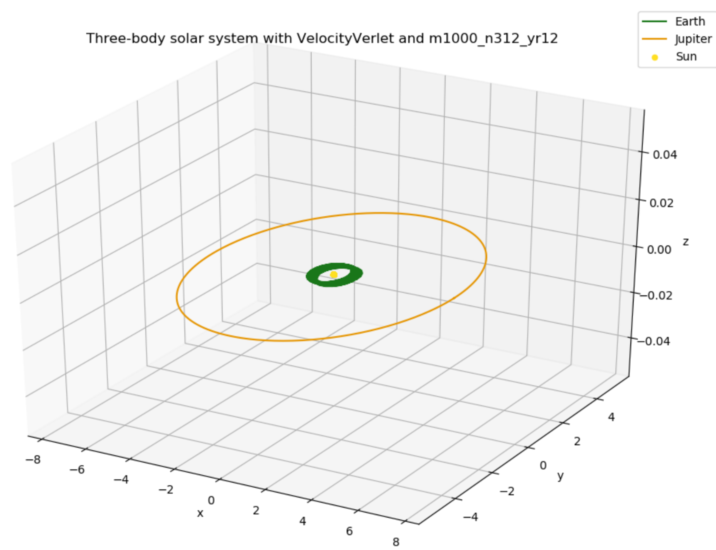

Figure 81: Plot of the Sun-Earth-Jupiter solar system with 312 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

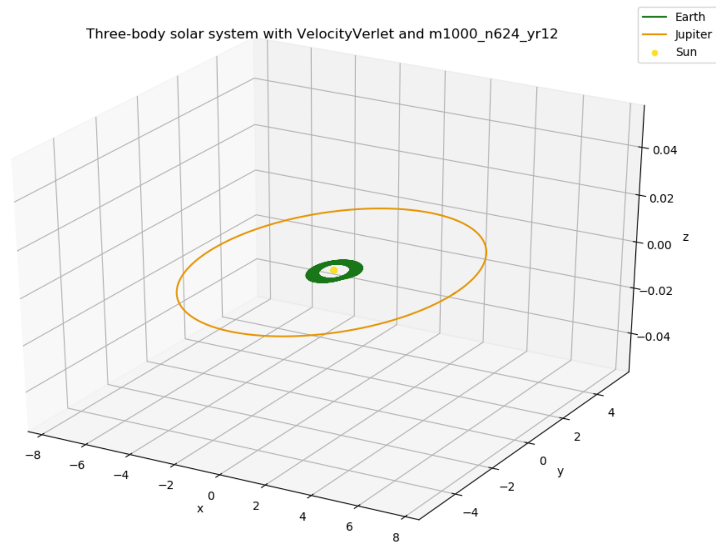

Figure 82: Plot of the Sun-Earth-Jupiter solar system with 624 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

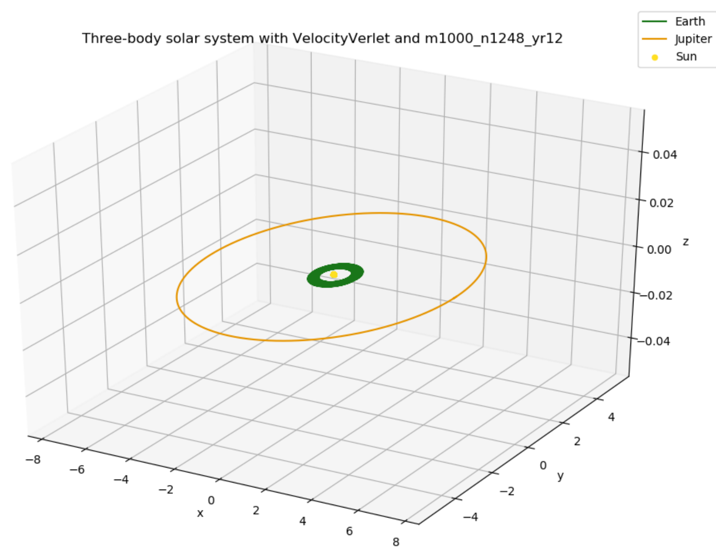

Figure 83: Plot of the Sun-Earth-Jupiter solar system with 1248 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

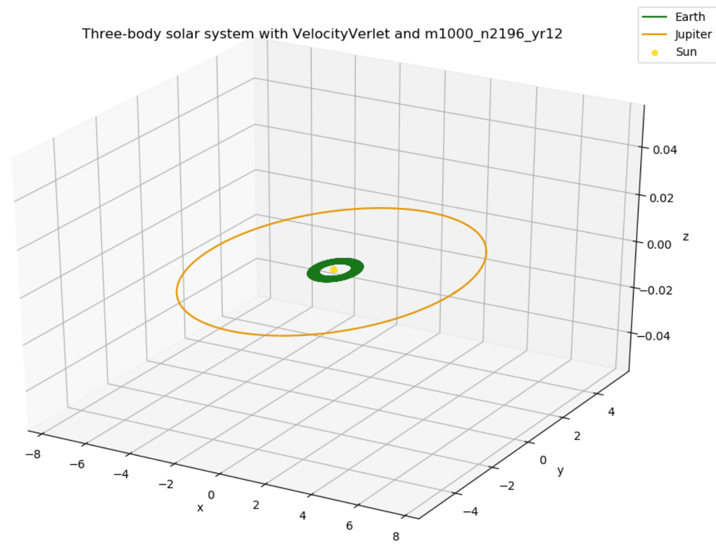

Figure 84: Plot of the Sun-Earth-Jupiter solar system with 2196 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

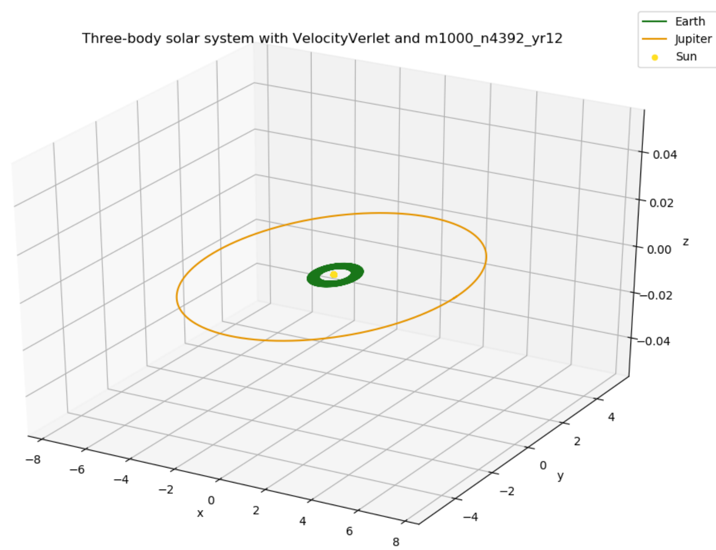

Figure 85: Plot of the Sun-Earth-Jupiter solar system with 4392 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

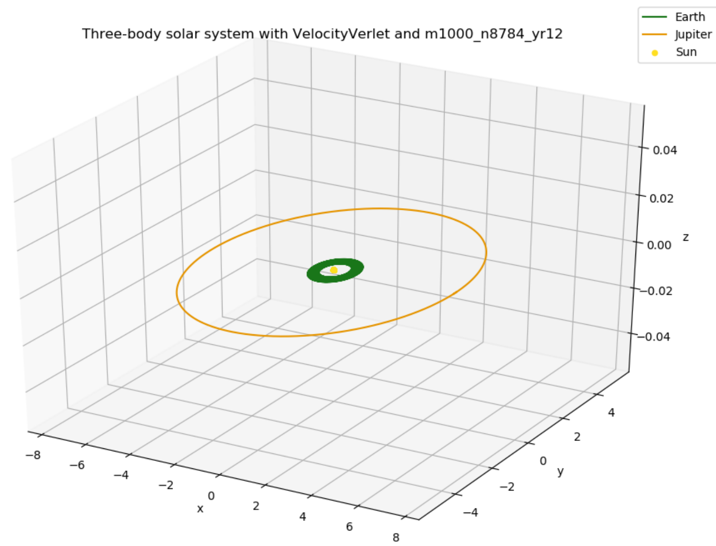

Figure 86: Plot of the Sun-Earth-Jupiter solar system with 8784 iterations over 12 years and a massfactor of 1000 for Jupiter, made with Velocity Verlet.

4.4 Dynamic sun

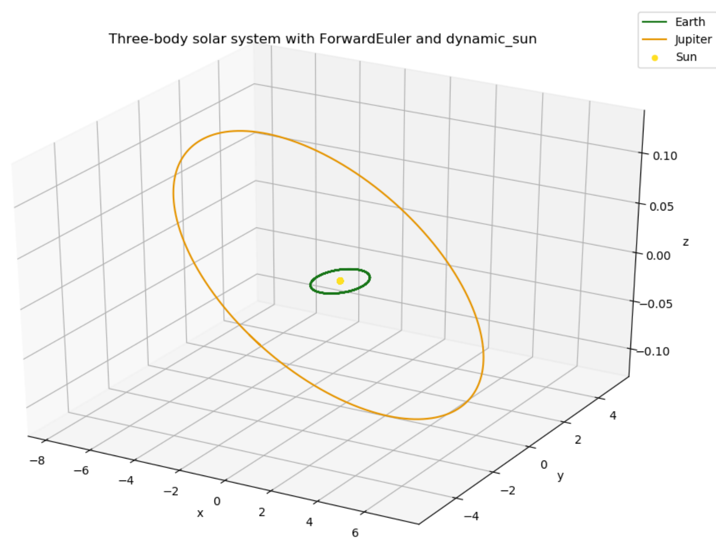

Figure 87: Plot of the Sun-Earth-Jupiter solar system with a dynamic sun, made with Euler-Cromer.

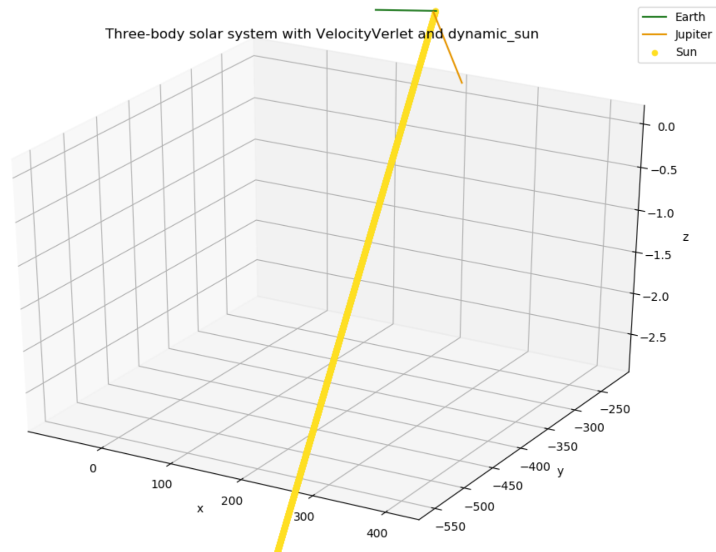

Figure 88: Plot of the Sun-Earth-Jupiter solar system with a dynamic sun, made with Velocity Verlet.

4.5 Ten-body solar system

4.5.1 Static sun

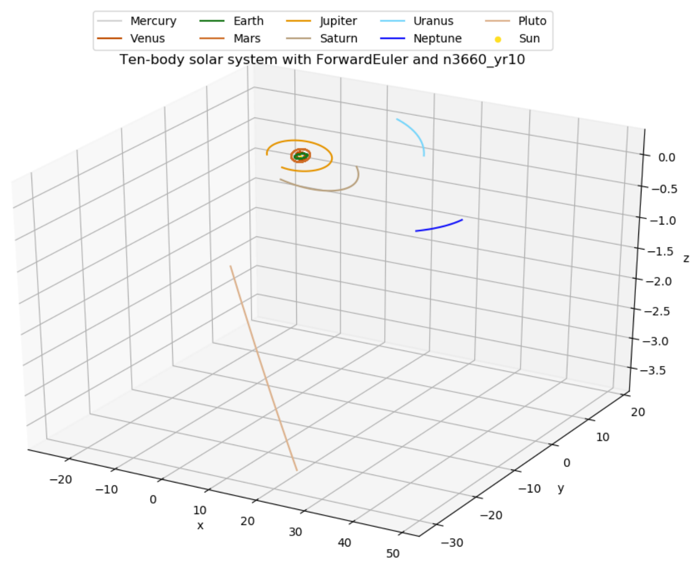

Figure 89: Plot of the ten-body solar system with 3660 iterations over 10 years with a static sun, made with Euler-Cromer.

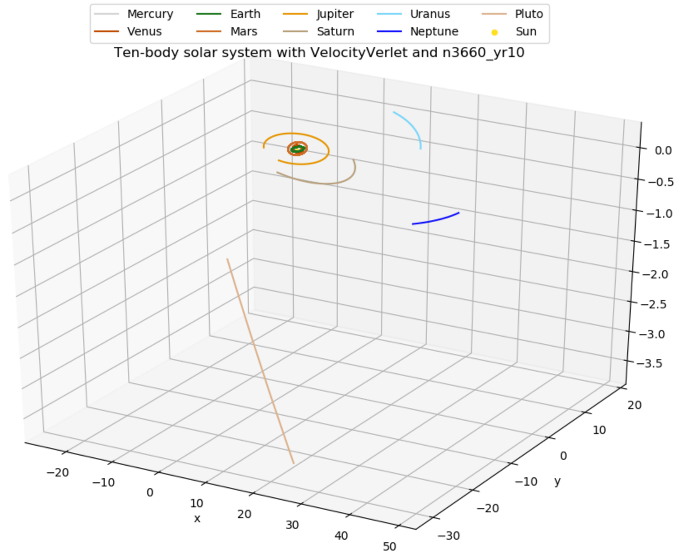

Figure 90: Plot of the ten-body solar system with 3660 iterations over 10 years with a static sun, made with Velocity Verlet.

4.5.2 Dynamic sun

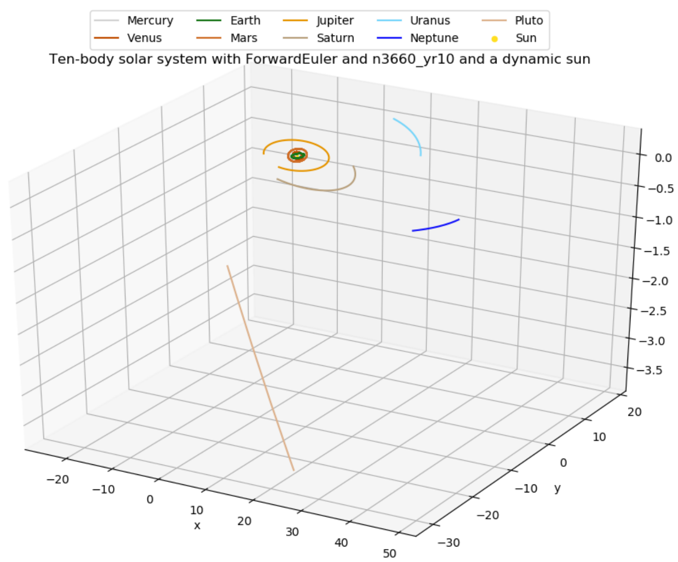

Figure 91: Plot of the ten-body solar system with 3660 iterations over 10 years with a dynamic sun, made with Euler-Cromer.

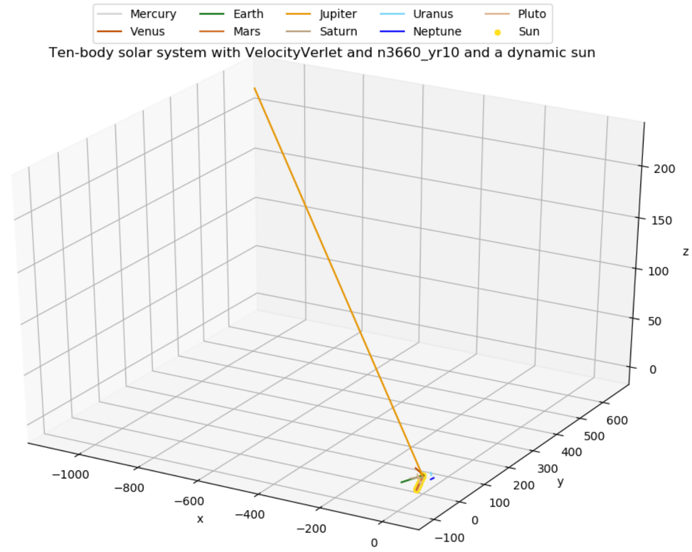

Figure 92: Plot of the ten-body solar system with 3660 iterations over 10 years with a dynamic sun, made with Velocity Verlet.

5 Discussion

and discussion
Give a critical discussion of your work and place it in the correct context. Relate your work to other calculations/studies

5.1 Stability of the Sun-Earth solar system giving by the Euler-Cromer method and the Velocity Verlet method

Figures (1) through (6) plots the Sun-Earth solar system for different iterations. Here it is possible to see that an increasing number of iterations will improve the stability of Earth's orbit around the Sun. For figure (1) and (4) the Earth's orbit is not stable, given by that the orbit changes position drastically. The Velocity Verlet method demonstrates how it is a better approximation than Euler-Cromer given by that the orbit changes less and is therefore more stable. By increasing the number of iterations to $n = 260$ the orbit has stabilized by a lot for both methods, though mostly for Velocity Verlet which is almost indistinguishable from the next iteration step with $n = 520$. With further increase to $n = 520$ both methods have stabilized compared to a higher number of iterations. Figures (3) and (6) illustrates this. The plot given by the Euler-Cromer method may have a small increase in stability by a higher number of iterations, but given by the figures for the Velocity Verlet method this should be very small. Originally this test of the two methods have data for other larger numbers of iterations, but seeing as the orbits have seemingly stabilized we deemed the plotting of these datas as unnecessary. The stability should increase by an increasing number of iterations because a larger number of iterations means that more changes in the bodies in the solar system can be accounted for, giving a more accurate representation of the solar system. The stabilizing of the Earth's orbit therefore implies the stability of the Euler-Cromer method and the Velocity Verlet method.

The number of FLOPs for one iterations is for Euler-Cromer $(2n+8)_{+}4$ and for Velocity Verlet $(2n+8)_{+}6$, where n is the number of iterations and is the number of planets. Therefore, the Velocity Verlet should use longer time to compute, because it has more operations to compute. The timings is listed in the table (1) and supports the theory of FLOPs. The table shows that Euler-Cromer used 26 s whereas Velocity Verlet used 34 s. Euler-Cromer used fewer seconds because it had fewer FLOPs. However, the approximation of Velocity Verlet is better than Euler-Cromer, hence the time difference is easy to overlook to instead have better accuracy of the solar system. When looking at all iterations performed, the FLOPs above should be multiplied with n for the number of iterations.

When looking at the conservation of energy for the two different methods, it is possible to see that Euler-Cromer does not conserve energy, whereas Velocity Verlet does. This is because plots of the Sun-Earth system with Euler-Cromer has an orbit of the Earth which moves around the Sun, while the orbit for the Earth with Velocity Verlet just rotates around the Sun. Figures (1) and (4) illustrates this the best. The conservation of energy is important in simulations of the solar system because energy has an important effect on the orbit of the planets. This is shown by the plots with the Velocity Verlet method, which conserves energy, and how the orbits are rounder and more consistent.

5.2 The escape velocity of the Earth in the Sun-Earth solar system

The plots of the escape velocity is given in figures (13) through (18). From the theory and appendix, equation (1) and section A.2, the theoretical escape velocity of the Earth is $v_p = 8.886$ m/s, which is a little smaller than 2.8π. Therefore, it is expected that the Earth will not escape its orbit around the Sun until the initial velocity of the planet is around 2.8π. In figure (13) the Earth keeps its orbit, which is expected as the velocity is 1.0π. The same applies for figures (14) and (15), with respectively an initial velocity of 2.0π and 2.5π. For figure (16) it is possible to see that Earth escapes. This is not in agreement with the theoretical escape velocity. However, around $x = -15$ Earth starts to move back to the Sun. The Earth does not escape successfully, and with more iterations, which gives more positions, the Earth should return to its orbit around the Sun. For figure (17), with an initial velocity of 2.8π, the Earth has successfully escaped, as expected from the theoretical escape velocity. Figure (18) further illustrates the escape with initial velocity over the theoretical escape velocity, where the initial velocity is 3.0π.

5.3 Varying β

The plots of the varying of β is given in figures (30) through (40). β is the number which denotes the

6 Conclusion and perspective

Conclusions and perspectives
What should I focus on? Conclusions. State your main findings and interpretations Try as far as possible to present perspectives for future work Try to discuss the pros and cons of the methods and possible improvements

7 References

[1] Morten H. Jensen (2019), Project 5, Departement of Physics, University of Oslo, Norway

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[3] Morten H. Jensen (2015), Lecture slides for FYS3150, Department of Physics, University of Oslo, Norway

[4] Jon. D. Giorgini (2019), Physical data of the planets and the sun., Solar System Dynamics Group, Horizons On-Line Ephemeris System, Jet Propulsion Laboratory, Pasadena, California, USA

A Appendix

Appendix with extra material
What should I focus on? additional material. Additional calculations used to validate the codes Selected calculations, these can be listed with few comments Listing of the code if you feel this is necessary You can consider moving parts of the material from the methods section to the appendix. You can also place additional material on your webpage.

A.1 Mass-conversion

Celestial Body	Mass [kg]	Mass [$M_\odot$]	Distance to sun [AU]
Sun	$2 \cdot 10^{30}$	1	0
Mercury	$3.3 \cdot 10^{23}$	$1.65 \cdot 10^{-7}$	0.39
Venus	$4.9 \cdot 10^{24}$	$2.45 \cdot 10^{-6}$	0.72
Earth	$6 \cdot 10^{24}$	$3.0 \cdot 10^{-6}$	1
Mars	$6.6 \cdot 10^{23}$	$3.3 \cdot 10^{-7}$	1.52
Jupiter	$1.9 \cdot 10^{27}$	$9.5 \cdot 10^{-4}$	5.20
Saturn	$5.5 \cdot 10^{26}$	$2.75 \cdot 10^{-4}$	9.54
Uranus	$8.8 \cdot 10^{25}$	$4.4 \cdot 10^{-5}$	19.19
Neptune	$1.03 \cdot 10^{26}$	$5.15 \cdot 10^{-5}$	30.06
Pluto	$1.31 \cdot 10^{22}$	$6.55 \cdot 10^{-9}$	39.53

Table 2: Astronomical data retrieved from [4].

A.2 Explaining the calculation of a planets escape velocity

As some of these problems are soluble by hand there are some calculations that can be done prior to the simulations in order to have something to compere results with. For instance one can quite easily calculate a planets escape velocity from the sun. One can use that that the escape velocity would be the lowest energy required to move the planet out of the gravitational pool from the sun. In that case the planet would be left with zero kinetic energy. Because the planet is moved out of the gravitational field the potential energy would also bee 0. Therefore the expression for the system will be given as. This is with the condition that there is no other forces acting on the system except for the gravitational force.

$U_i + K_i = U_f + K_f$

Due too the criteria we have sett for the model and the need for conservation of energy the expretion becomes:

$U_i + K_i = 0 + 0$

The potential energy is given by, the gravitational potential which is:

$P = -\frac{GMm}{r}$

The kinetic energy is given by the planets kinetic energy:

$k = \frac{1}{2} m v_p$

Therefore the escape velocity can be calculated as the kinetic energy being equal to the potential energy.

$$v_p = \sqrt{\frac{2GM}{r}} \tag{8}$$

A.3 Rewriting Newtons second law of motion

Newtons second law of motion is stated in equation (1).

$$F = ma(1) \tag{9}$$

Equation (1) can be written as a differential equation that will give the force as a function of position (2).

$$F(x,t) = m\frac{d^2x}{dt^2}(2) \tag{10}$$

Since we know that the velocity,v, is given as a function of change in position over change in time. This gies us equation (3).

$$v(x,t) = \frac{dx}{dt}(3) \tag{11}$$

Using equation (1),(2) and (3), one can now created two coupled differential equations. The first one is (3) and the second one is (4). (4) is a combination of (1) , (2) and (3).

$$\frac{dv}{dt} = F(x,t)/m = a(x,t)(4) \tag{12}$$

Solving this system fro x with a tailor expantion one gets (5).

$$x(t+h) = x(t) + hx^1(t) + \frac{h^2}{2}x^2(t) + Oh^4(5) \tag{13}$$

From equation (3) and (4) we have that $a(x,t) = x^2(t)$. Using equation (3), $x^1(t) = v(x,t)$ This can now be substituted in too equation (5). And gives equation (6).

$$x(t+h) = x(t) + hv(x,t) + \frac{h^2}{2}a(x,t) + (O)h^4(6) \tag{14}$$

As one can see from the equation the system has a troncation error of $(O)h^4$. Using a Tayler expansion for the velocity as well as for the position.

$$v(t+h) = v(t) + hv^1(x,t) + \frac{h^2}{2}v(t)(7) \tag{15}$$

It is important to discretize the expressions as they are the ones that will be used in the program. This changes x(t+h) too x(i+1).

A.4 Adjusting Newtons method for relativity