Project 1

For the course FYS3150

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forventer at vi skal sette opp prosjektet på samme måte som man skriver en labrapport

1. gi et sammendrag av arbeidet -> "abstract"
2. en introduksjon som sier hva målet er og hva man har gjort. på slutten et kort sammendrag av strukturen til rapporten
3. teoretiske modeller og teknikaliteter -> metode (noen beregninger/kode-eksempler underveis)
4. resultater og diskusjon
5. konklusjon og perspektiver
6. appendix med ekstra materiale
7. bibliografi/kilder finner bøker/artikler osv ved: universitetsbiblioteket, ”physical review letters of the american physical society”

eksempel på rapport på: https://github.com/CompPhysics/ComputationalPhysics/blob/master/doc/Projec

Exercise 1

a) In the exercise we are given the equation

$$-\frac{v_{i+1}+v_{i-1}-2v_i}{h^2} = f_i \qquad \text{for } i = 1,2,3,...,n$$

Rewrites the equation to

$$\begin{aligned} -(v_{i+1}+v_{i-1}-2v_i) &= h^2 f_i = \tilde{b}_i \\ -v_{i+1}-v_{i-1}+2v_i &= \tilde{b}_i \end{aligned}$$

where in the exercise we are also given the correlation $\tilde{b}_i = h^2 f_i$, which is implemented here.

Defines the equation for different values of the integer i to get a set of equations. The exercise also gives the boundry conditions $v_0 = v_{n+1} = 0$.

$$\begin{aligned}
i = 1: &\quad -v_{1+1} - v_{1-1} + 2v_1 = -v_2 - v_0 + 2v_1 = -0 + 2v_1 - v_2 = \tilde{b}_1 \\
i = 2: &\quad -v_{2+1} - v_{2-1} + 2v_2 = -v_3 - v_1 + 2v_2 = -v_1 + 2v_2 - v_3 = \tilde{b}_2 \\
i = 3: &\quad -v_{3+1} - v_{3-1} + 2v_3 = -v_4 - v_2 + 2v_3 = -v_2 + 2v_3 - v_4 = \tilde{b}_3 \\
&\vdots \\
i = n: &\quad -v_{n+1} - v_{n-1} + 2v_n = -v_{n-1} + 2v_n - 0 = \tilde{b}_n
\end{aligned}$$

Equations can be rewritten as a matrix equation, which gives a matrix A with integers as elements, a vector $\vec{v} = [v_1, v_2, v_3, ..., v_n]$ and another vector $\vec{\tilde{b}} = [\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, ..., \tilde{b}_n]$.

This gives the matrix equation

$$A\vec{v} = \vec{\tilde{b}}$$

The matrix and the vectors are given as

$$\begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{n-1} \\ v_n \end{bmatrix} = \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \vdots \\ \tilde{b}_{n-1} \\ \tilde{b}_n \end{bmatrix}$$

Therefore the matrix equation has been proved.

b) i beregningene til erlend på ark er b diagonalelementene, g er b-vektoren og u er v-vektoren. i pythonkoden er d diagonalelementene, b er b-vektoren og v er v-vektoren. med tilde er algoritmediagonal

For the forward substitution algorithm there are 5 floating point operations.

For the backward substitution algorithm there are 3 floating point operations.

The total number of floating point operations is 8. For one iteration.

For the forward substitution the equations used in the algorithm are

$$\tilde{b}_i = b_i - \frac{a_{i-1}c_{i-1}}{\tilde{b}_{i-1}}$$

$$\tilde{g}_i = g_i - \frac{\tilde{g}_{i-1}a_{i-1}}{\tilde{b}_{i-1}}$$

with the condition that $\tilde{b}_1 = b_1$.

For the backward substitution the equation used in the algorithm is

$$v_i = \frac{\tilde{g}_i - c_i v_{i+1}}{\tilde{b}_i}$$

with the conditions that $v_0 = 0$ and $v_n = \frac{\tilde{g}_n}{\tilde{b}_n}$.

c)

d)

e)