Project 1

For the course FYS3150

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Exercise 1

a) In the exercise we are given the equation

$$-\frac{v_{i+1}+v_{i-1}-2v_i}{h^2}=f_i \qquad \text{for } i=1,2,3,...,n$$

Rewrites the equation to

$$-(v_{i+1}+v_{i-1}-2v_i)=h^2 f_i=\tilde{b}_i$$
$$-v_{i+1}-v_{i-1}+2v_i=\tilde{b}_i$$

where in the exercise we are also given the correlation $\tilde{b}_i = h^2 f_i$, which is implemented here.

Defines the equation for different values of the integer i to get a set of equations. The exercise also gives the boundry conditions $v_0 = v_{n+1} = 0$.

$$\begin{aligned}
i=1: &\quad -v_{1+1}-v_{1-1}+2v_1=-v_2-v_0+2v_1=-0+2v_1-v_2=\tilde{b}_1\\
i=2: &\quad -v_{2+1}-v_{2-1}+2v_2=-v_3-v_1+2v_2=-v_1+2v_2-v_3=\tilde{b}_2\\
i=3: &\quad -v_{3+1}-v_{3-1}+2v_3=-v_4-v_2+2v_3=-v_2+2v_3-v_4=\tilde{b}_3\\
&\vdots\\
i=n: &\quad -v_{n+1}-v_{n-1}+2v_n=-v_{n-1}+2v_n-0=\tilde{b}_n
\end{aligned}$$

Equations can be rewritten as a matrix equation, which gives a matrix A with integers as elements, a vector $\vec{v}=[v_1,v_2,v_3,...,v_n]$ and another vector $\vec{\tilde{b}}=[\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,...,\tilde{b}_n]$.

This gives the matrix equation

$$A\vec{v} = \tilde{\vec{b}}$$

The matrix and the vectors are given as

$$\begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{n-1} \\ v_n \end{bmatrix} = \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \vdots \\ \tilde{b}_{n-1} \\ \tilde{b}_n \end{bmatrix}$$

Therefore the matrix equation has been proved.

b)

c)

d)

e)