Calculus Assignment 2

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1 Show that the power

$$(t + \Delta t)^4 = (t + \Delta t)(t + \Delta t)(t + \Delta t)(t + \Delta t) \tag{1}$$

can be written as

$$t^4 = 4t^3 \Delta t + (...)(\Delta t)^2 \tag{2}$$

Answer:

$$\begin{aligned} (t + \Delta t)^4 &= t^4 + 4t^3 \Delta t + 6t^2 \Delta t^2 + 4t \Delta t^3 + \Delta t^4 = \\ &= t^4 + 4t^3 \Delta t + (6t^2 + 4t \Delta t + \Delta t^2) \Delta t^2 \end{aligned} \tag{3}$$

2 Use the previous result to show, without using the power rule, that

$$(t^4)' = 4t^3 \tag{4}$$

Answer:

$$\begin{aligned} \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} &= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^4 - t^4}{\Delta t} = \\ &= \lim_{\Delta t \to 0} \frac{t^4 + 4t^3 \Delta t + 6t^2 \Delta t^2 + 4t \Delta t^3 + \Delta t^4 - t^4}{\Delta t} = \\ &= \lim_{\Delta t \to 0} 4t^3 + 6t^2 \Delta t + 4t \Delta t^2 + \Delta t^3 = 4t^3 + 0 + 0 = 4t^3 \end{aligned} \tag{5}$$

The reason the Δt^2 and higher order coefficients don't matter is because $\Delta t = 0$ so once we solve the indeterminate form and replace Δt with the limit all coefficients behind Δt are multiplied by 0.

3 Compute the derivative of the following functions using the power rule, the sum rule and the scaling rule.

$$\begin{aligned} t^5 &= 5t^4 \\ 5t^4 &= 20t^3 \\ 2t + t^5 &= 2 + 5t^4 \\ 4t^2 + 2t^7 &= 8t + 14t^6 \\ 2 + 3t + 4t^2 + 5t^3 &= 3 + 8t + 15t^2 \end{aligned} \tag{6}$$

4 Prove the scaling rule:

$$(cx(t))' = c(x(t))' \tag{7}$$

using the sum rule.

Proof:

$$(cx(t))' = (\underbrace{x(t) + x(t) + ... + x(t)}_{\text{c times}})' = \underbrace{x(t)' + x(t)' + ... + x(t)'}_{\text{c times}} = c(x(t))' \tag{8}$$