Calculus Assignment 3

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1 Prove the formula

$$(t^2)' = 2t \tag{1}$$

using the product rule and the fact that $t' = 1$

Proof:

$$(t^2)' = (t * t)' = t' * t + t * t' = t + t = 2t \tag{2}$$

2 Using the product rule (and the other rules), take the derivative of the following functions:

Answer:

$$\begin{aligned}
te^t &= e^t + te^t = (t+1)e^t \\
t^2e^t &= 2te^2 + t^2e^t = (t^2+2t)e^t \\
3 + 4^t - t^2 8^t &= 4 - 2t8^t - t^2 8^t ln(8) = 4 - 8^t t(2 + tln(8)) \\
2t^3 2^t 3^t &= (2t^3)' 2^t 3^t + 2t^3 (2^t 3^t)' = 6t^2 2^t 3^t + 2t^3 (2^t 3^t ln(2) + 2^t 3^t ln(3)) = \\
&= (3 + tln(2) + tln(3)) 2t^2 2^t 3^t = (3 + tln(6)) t^2 2^{t+1} 3^t \\
t^4 e^t 2^t &= 4t^3 e^t 2^t + t^4 (e^t 2^t + e^t 2^t ln(2)) = (4 + t + tln(2)) t^3 e^t 2^t
\end{aligned} \tag{3}$$

3 Prove the formula $(t^4)' = 4t^3$ by using the product rule three times.

Proof:

$$\begin{aligned}(t^4)' = (t*t*t*t)' = (t)'t^3 + t(t*t*t)' = t^3 + t((t)'t^2 + t(t*t)') = \\ = t^3 + t(t^2 + t((t)'t + t(t)')) = t^3 + t(t^2 + t(t+t)) = \\ = t^3 + t^3 + t^3 + t^3 = 4t^3\end{aligned} \tag{4}$$

4 Assuming p to be an integer, prove that $ln(t^p) = pln(t)$ using the property $ln(ab) = ln(a) + ln(b)$.

Proof:

$$ln(t^p) = ln(\underbrace{t*t*t*...*t}_{\text{p times}}) = \underbrace{ln(t) + ln(t) + ln(t) + ... + ln(t)}_{\text{p times}} = pln(t) \tag{5}$$

5 Show that $(a^t)' = ln(a)a^t$ assuming that you know that $(e^{bt})' = be^{bt}$ for all values of b

Proof:

$$(a^t)' = (e^{ln(a^t)})' = (e^{ln(a)t})' = ln(a)e^{ln(a)t} = ln(a)e^{ln(a^t)} = ln(a)a^t \tag{6}$$