Applied math assignment 3

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October 2024

Exercise 5.2. Solve the following equations in C:

(b) $\frac{1}{2}x^3 + 4x = 2x^2$.

$$\begin{aligned}
\frac{1}{2}x^3 - 2x^2 + 4x &= 0 \\
x(x^2 - 4x + 8) &= 0 \\
D = 4 - 8 &= -4 \\
x_1 = 2 + \sqrt{-4} = 2 + \sqrt{4 * -1} = 2 + 2\sqrt{-1} &= 2 + 2i \\
x_2 = 2 - \sqrt{-4} &= 2 - 2i \\
x(2+2i)(2-2i) &= 0 \\
x_1 &= 2 + 2i \\
x_2 &= 2 - 2i \\
x_3 &= 0
\end{aligned} \tag{1}$$

Exercise 5.8. Let $z = -4 + 2i$ and $w = 3 - i$. Compute the following:

(a) $\frac{w}{z}$.

$$
\begin{aligned}
I(\frac{w}{z}) &= I(\frac{3-i}{-4+2i}) \\
&= I(\frac{3-i}{-4+2i}\frac{-4-2i}{-4-2i}) \\
&= I(\frac{(-4-2i)(3-i)}{20}) \\
&= I(\frac{-12-2-(6-4)i}{20}) \\
&= I(-\frac{7}{10}-\frac{1}{10}i) \\
&= -\frac{1}{10}
\end{aligned} \tag{2}
$$

Exercise 6.2. Write the following complex numbers in Cartesian form.

(d) $z = cos(\pi + i)$.

$$
\begin{aligned}
z = cos(\pi + i) &= cos(\pi)cos(i) - sin(\pi)sin(i) \\
&= -cos(i) - 0 * sin(i) = -cos(i) \\
&= -\frac{e^{i*i} + e^{-i*i}}{2} \\
&= -\frac{e^{-1} + e}{2}
\end{aligned} \tag{3}
$$

Exercise 6.17. Prove the double-angle formulas using the formula of de Moivre.

Proof:

$$\begin{aligned} z &:= r(cos(a) + sin(a)i) \\ z^2 = r(cos(a) + sin(a)i)^2 &= r(cos(2a) + sin(2a)i) \\ (cos(a) + sin(a)i)^2 &= cos(2a) + sin(2a)i \\ cos^2(a) - sin^2(a) + 2cos(a)sin(a)i &= cos(2a) + sin(2a)i \\ cos^2(a) - sin^2(a) &= cos(2a) \\ 2cos(a)sin(a) &= sin(2a) \end{aligned} \tag{4}$$

Exercise 7.4. Given the function $f(t) = R((1-i)e^{\pi t i})$, with $t \in \mathbb{R}$.

(a) Show that $f(t)$ is a real sinusoid by converting it into standard form $f(t) = Acos(\omega t + \phi)$

Answer:

$$\begin{aligned} R((1-i)e^{\pi t i}) &= R(\sqrt{2}(cos(-\frac{\pi}{4}) + sin(-\frac{\pi}{4})i)e^{\pi t i}) = R(\sqrt{2}e^{-\frac{\pi}{4}i}e^{\pi t i}) = \\ &= R(\sqrt{2}e^{(\pi t - \frac{\pi}{4})i}) = \sqrt{2}cos(\pi t - \frac{\pi}{4}) \end{aligned} \tag{5}$$

(b) Write $f(t)$ as a sum of two complex exponential signals, i.e. determine $A_1, A_2, \omega_1, \omega_2, \phi_1, \phi_2$ such that

$$f(t) = A_1 e^{(\omega_1 t + \phi_1)i} + A_2 e^{(\omega_2 t + \phi_2)i} \tag{6}$$

Answer:

$$\begin{aligned} f(t) &= R((1-i)e^{\pi t i}) = R(e^{\pi t i} - e^{\pi t i}i) = \\ &= R(cos(\pi t) + sin(\pi t)i + sin(\pi t) - cos(\pi t)i) = \\ &= cos(\pi t) + sin(\pi t) = cos(\pi t) + cos(\pi t - \frac{\pi}{2}) \end{aligned} \tag{7}$$

(c) Solve the equation $f(t) = 0$. Give all real solutions.

Answer:

$$\begin{aligned} f(t) = \sqrt{2}cos(\pi t - \frac{\pi}{4}) &= 0 \\ cos(\pi t - \frac{\pi}{4}) &= 0 \\ \pi t - \frac{\pi}{4} &= \pi k + \frac{\pi}{2}, \text{where } k \in \mathbf{Z} \\ \pi t &= \pi k + \frac{3\pi}{4} \\ t &= k + \frac{3}{4} \end{aligned} \tag{8}$$

Exercise 7.7 Given are the following two real sinusoids:

$$\begin{aligned} x_1(t) &= cos(2t + \frac{1}{6}\pi) \\ x_2(t) &= 2cos(2t + \frac{7}{6}\pi) \end{aligned} \tag{9}$$

(c) The sum $x(t) = x_1(t) + x_2(t)$ is again a sinusoid with the same frequency. Write $x(t)$ in standard form.

Answer:

$$\begin{aligned} x(t) &= cos(2t + \frac{1}{6}\pi) + 2cos(2t + \frac{7}{6}\pi) = R(e^{i(2t+\frac{\pi}{6})} + 2e^{i(2t+\frac{7\pi}{6})}) = \\ &= R(e^{\frac{\pi}{6}i}e^{2ti} + 2e^{i\frac{7\pi}{6}}e^{2ti}) = R((e^{\frac{\pi}{6}i} + 2e^{i\frac{7\pi}{6}})e^{2ti}) = \\ &= R((1 + 2e^{\pi i})e^{\frac{\pi}{6}i}e^{2ti}) = R((1-2)e^{\frac{\pi}{6}i}e^{2ti}) = \\ &= R(-e^{\frac{\pi}{6}i+2ti}) = -R(e^{i(2t+\frac{\pi}{6})}) = \\ &= -cos(2t + \frac{\pi}{6}) \end{aligned} \tag{10}$$