1.a. For 2 variables there since there are four states they can take there are 2^4 = 16 unique Boolean functions.

1.b.

- Linearly separable operations: OR, NOR, AND, NAND
- non-Linearly separable operations: XOR, NXOR

1.c.

For to find the value of the weights and bias the following system needs to hold:

f(w0) = 0

f(w0 + w2) = 0

f(w0 + w1) = 0

f(w0 + w1 + w2) = 1

applying the inverse of f:

w0 >= 0

w0 + w2 >= 0

w0 + w1 < 0

w0 + w1 + w2 >= 0

⇨ w2 > -w1 > w0 >= 0

any set of three numbers that satisfy the constraints above would be valid

example: w0 = 0, w1 = -1, w2 = 2

1.d.

First we will specify the conditions for H1 (AND)

f(w10) = 0

f(w10 + w12) = 0

f(w10 + w11) = 0

f(w10 + w11 + w12) = 1

applying inverse of f

w10 < 0

w10 + w11 < 0

$w10 + w12 < 0$

$w10 + w11 + w12 >= 0$

- ⇨ $w11 + w12 > -w10 > w11, w12 >= 0$
  example weights: $w10 = -1.5$, $w11 = 1$, $w12 = 1$

Second we will specify the conditions for H2 (OR)

$f(w20) = 0$

$f(w20 + w22) = 1$

$f(w20 + w21) = 1$

$f(w20 + w21 + w22) = 1$

applying inverse of f

$w20 < 0$

$w20 + w21 >= 0$

$w20 + w22 >= 0$

$w20 + w21 + w22 >= 0$

- ⇨ $w21, w22 >= 0 > w20$

example weights: $w20 = -0.5$, $w21 = 1$, $w22 = 1$

Lastly we will specify the conditions for Y (XOR) using H1(AND) and H2(OR) as our assumed inputs

$f(v0) = 0$

$f(v0 + v2) = 1$

$f(v0 + v1) = 0$

$f(v0 + v1 + v2) = 0$

applying inverse of f

$v0 < 0$

$v0 + v2 >= 0$

$v0 + v1 < 0$

$v0 + v1 + v2 < 0$

- ⇨ $-v1 - v0 > v2 > -v1 > -v0 > 0$

example weights: $v0 = -1$, $v1 = -1.5$, $v2 = 2$