Calculus Assignment 6

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September 2024

1 Convert the following angles measured in degrees into radians:

$$
\begin{aligned}
90^\circ &= \frac{\pi}{2} \text{ radians} \\
45^\circ &= \frac{\pi}{4} \text{ radians} \\
180^\circ &= \pi \text{ radians} \\
270^\circ &= \frac{3\pi}{2} \text{ radians}
\end{aligned} \tag{1}
$$

2 Find the value of $sin(\frac{\pi}{4})$ using the geometric definition.

Answer: The geometric definition is that $sin(x)$ equals the length of the side of the triangle adjacent to angle, assuming the triangle is defined in the unit circle.

$$
\begin{aligned}
&\frac{\pi}{4} = 45^\circ \\
&\text{length of sides of triangle with degrees (45,90,45)} = (\frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}) \\
&\text{meaning } sin(\frac{\pi}{2}) = \frac{\sqrt{2}}{2}
\end{aligned} \tag{2}
$$

3 The tangent function is defined as:

$$tan(t) = \frac{sin(t)}{cos(t)} \tag{3}$$

Compute the derivative of the tangent function.

Answer:

$$
\begin{aligned}
tan'(t) = (\frac{sin(t)}{cos(t)})' = \frac{sin'(t)cos(t) - sin(t)cos'(t)}{(cos(t))^2} = \\
= \frac{(cos(t))^2 + (sin(t))^2}{(cos(t))^2} = \frac{1}{(cos(t))^2}
\end{aligned}
\tag{4}
$$

4 Find the antiderivative of the following functions.

$$
\begin{aligned}
&x(t) = 3t^2 + 5t \\
&\int x(t)dt = \int 3t^2 + 5tdt = t^3 + \frac{5t^2}{2} + C \\
\\
&x(t) = 6t + 1 \\
&\int x(t)dt = \int 6t + 1dt = 3t^2 + t + C \\
\\
&x(t) = t^4 + t^3 + t^2 \\
&\int x(t)dt = \int t^4 + t^3 + t^2 dt = \frac{t^5}{5} + \frac{t^4}{4} + \frac{t^3}{3} + C \\
\\
&x(t) = e^{2t} \\
&\int x(t)dt = \int e^{2t} dt = \frac{e^{2t}}{2} + C \\
\\
&x(t) = e^{2t} \\
&\int x(t)dt = \int 4^t dt = \frac{4^t}{ln(4)} + C \\
\\
&x(t) = 4^t \\
&\int x(t)dt = \int 4^t dt = \frac{4^t}{ln(4)} + C \\
\\
&x(t) = cos(3t) \\
&\int x(t)dt = \int cos(3t)dt = \frac{sin(3t)}{3} + C \\
\\
&x(t) = 3sin(2t) + 1 \\
&\int x(t)dt = \int 3sin(2t) + 1dt = \frac{-3cos(2t)}{3} + t + C
\end{aligned} \tag{5}
$$

5 The velocity of a car is given by the function:

$$x(t) = t^3 + 1 \tag{6}$$

Find the position $x(t)$ of the car at time t given that $x(0) = 2$.

Answer:

$$x(t) = x(0) + \int_0^t x'(t)dt = 2 + \int_0^t t^3 dt = 2 + \frac{t^4}{4} - 0 = \frac{t^4}{4} + 2 \tag{7}$$

6 Find the area under the function

$$x(t) = t^2 + t^4 \tag{8}$$

from $t = 0$ to $t = 1$.

$$\int_0^1 x(t)dt = \int_0^1 t^4 + t^2 dt = \frac{1^5}{5} + \frac{1^3}{3} + C - 0 - C = \frac{1}{5} + \frac{1}{3} = \frac{8}{15} \tag{9}$$

7 Solve the following integrals:

$$\begin{aligned} \int_{-1}^{1} (5t^6 - 7t^2)dt &= \frac{5(1)^7}{7} - \frac{7(1)^3}{3} + C - \frac{5(-1)^7}{7} - \frac{7(-1)^3}{3} + C = \\ &= \frac{10}{7} - \frac{14}{3} = -\frac{68}{21} \\ \int_0^2 (e^{4t} + t)dt &= \frac{e^8}{4} + 2 + C - \frac{e^0}{4} - 0 - C = \frac{e^8}{4} + \frac{7}{4} \\ \int_0^{\frac{\pi}{3}} sin(3t)dt &= -\frac{cos(\pi)}{3} + C + \frac{cos(0)}{3} - C = -\frac{-1}{3} + \frac{1}{3} = \frac{2}{3} \end{aligned} \tag{10}$$