MFE R Programming Workshop Lab 1

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Call Options

A European call option on a stock with price S, expiration date T, and strike price K is the right, but not obligation, to buy the underlying asset at time T at price K. If the stock price dynamics follow a geometric Brownian motion, it can be shown that the stock price at time t, S_t, can be written as

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t}$$

where S_0 is the currenct stock price, B_t is a standard Brownian motion which is normally distributed with mean zero and variance t, and μ and σ (expected return and volatility of the stock return) are some fixed values.

Let $N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx$ denote the CDF of the standard normal distribution. The famous Black-Scholes (Black and Scholes, 1973; Merton, 1973), formula (which you will be taught how to derive in your derivatives class) says that a price of a call option on a stock with price S maturing at time T with strike price K is

$$S_0 \times N(d_1) - e^{-rT} K \times N(d_2)$$

where r is the risk-free rate, and

$$\begin{aligned} d_1 &\equiv \frac{\ln(S_0/K) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \\ d_2 &\equiv \frac{\ln(S_0/K) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \end{aligned}$$

Questions

1. Write an R function that takes parameters r, T, K, S_0, and σ and computes the Black-Scholes call price. You will somehow need to evaluate $N(x)$. There are a couple of ways to do this, but all should start with considering how to find the appropriate function.

2. Evaluate the above function for the values $T = 1$, $r = .04$, $\sigma = .25$, $K = 95$, and $S_0 = 100$. You can compare your answer to the `Black_Scholes` function from `qrmtools` package.

3. We amy need to find the price of a call option for many parameters. Compute what the price of a call maturing in $T = 1$ year should be on a stock with current price $S_0 = 100$ and volatility $\sigma = .2$, when the riskless rate of interest is $r = .05$. Write code to do this for every strike $K \in \{75, 76, 77, \ldots, 124, 125\}$, and print the results to the screen. Now suppose that you want to do this for stocks of different maturities also, and that you need to use these prices to conduct some further analysis. For the same S_0, r, and σ, populate a matrix with the prices of an option for strikes and maturities $(K, T) \in \{75, 76, 77, \ldots, 124, 125\} \times \{1/12, 2/12, \ldots 23/12, 2\}$. Again compare your results with the results from the `Black_Scholes` function.

4. The file *optionsdata.csv* contains the parameters for various options. Read in this file and compute the Black-Scholes price for these options. Append a column to the dataset and output the results to its own csv file.

References

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *The journal of political economy* pp. 637–654.

Merton, Robert C, 1973, Theory of rational option pricing, *The Bell Journal of Economics and Management Science* pp. 141–183.