MFE R Programming Workshop Lab 1

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Call Options

A European call option on a stock with price S, expiration date T, and strike price K is the right, but not obligation, to buy the underlying asset at time T at price K. If the stock price dynamics follow a geometric Brownian motion, it can be shown that the stock price at time t, S_t, can be written as

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t}$$

where S_0 is the currenct stock price, B_t is a standard Brownian motion which is normally distributed with mean zero and variance t, and μ and σ (expected return and volatility of the stock return) are some fixed values.

Let $N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx$ denote the CDF of the standard normal distribution. The famous Black-Scholes (Black and Scholes, 1973; Merton, 1973), formula (which you will learn how to derive in your derivatives class) says that a price of a call option on a stock with price S maturing at time T with strike price K is

$$S_0 \times N(d_1) - e^{-rT} K \times N(d_2)$$

where r is the risk-free rate, and

$$d_1 \equiv \frac{\ln(S_0/K) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_2 \equiv \frac{\ln(S_0/K) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Questions

1. Write an R function that takes parameters r, T, K, S_0, and σ and computes the Black-Scholes call price. You will somehow need to evaluate $N(x)$. There are a couple of ways to do this, but all should start with considering how to find the appropriate function.

2. Evaluate the above function for the values $T = 1$, $r = .04$, $\sigma = .25$, $K = 95$, and $S_0 = 100$. You can compare your answer to the one obtained using the `Black_Scholes()` function from the `qrmtools` package.[1]

3. We may need to find the price of a call option for many parameters. Compute what the price of a call maturing in $T = 1$ year should be on a stock with current price $S_0 = 100$ and volatility $\sigma = .2$, when the riskless rate of interest is $r = .05$. Write code to do this for every strike $K \in \{75, 76, 77, \ldots, 124, 125\}$, and print the results on R console. Now suppose that you want to do this for stocks of different maturities also, to use these call prices to conduct some further analysis. For the same S_0, r, and σ, populate a matrix with the prices of an option for strikes and maturities $(K, T) \in \{75, 76, 77, \ldots, 124, 125\} \times \{1/12, 2/12, \ldots 23/12, 2\}$. Again compare your results with the results using the `Black_Scholes()` function.

4. The file *optionsdata.csv* contains the parameters for various options. Read in this file and compute the Black-Scholes price for these options. Append a column to the dataset for the call price and write the results to its own csv file.

[1] If you use a Mac and have issues loading the `qrmtools` package, you can alternatively use the `BS_EC` function from the `OptionPricing` package to calculate the price of a European call option.

References

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *The journal of political economy* pp. 637–654.

Merton, Robert C, 1973, Theory of rational option pricing, *The Bell Journal of Economics and Management Science* pp. 141–183.