Investements TA Session

Session 1

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Introduction

Questions

Any questions before we start?

Homework 1

- Overall very good.
- The point of the `R` portion of the homework is to practice coding in `R`.
- You should use `R` to solve the homework questions.
  - Don't just input your answers.
- You should write your own functions and limit the use of packages.
- The following packages will be allowed:
  - `magnittr`
  - `data.table`
  - `xts`
  - `tidyverse` (includes `gglpot2`, `tibble`, etc.)
- Please name the files `hwX-Y-Z.R`.

Send an Email from R

```
library(gmailr)
hiFromR <- mime(
  To = "someone@anderson.ucla.edu",
  From = "me@gmail.com",
  Subject = "hello from R",
  Body = "I am writting you from R."
) %>%
attach_file("./file.R")
send_message(hiFromR)
```

Investments: Basic Valuation

What is an investment?

- At the most general level, an *investment* is a claim to a stream of cash flows.
- Examples:
  - Bonds are a claim to future interest and principal payments.
  - Equities are a claim to future dividends.
- To value investments, we need a way to compare cash flow steams differing in size, timing, and risk.

Net Present Value

- Time value of money: money today is worth more today than in the future.
- **Net Present Value Investment Rule**:
  1. Convert future cash flows into a cash flows today.
  2. Add them up.
  3. Choose the deal with the highest NPV.
- NPV is generally the most robust way to rank projects.
- Disadvantage: you need to know the correct rate to discount future cash flows.

Internal Rate of Return (IRR)

- The internal rate of return (IRR) solves for the discount rate that makes the NPV zero.
- **IRR Investment Rule**: Take any investment opportunity that exceeds the opportunity cost of capital.
  - This only works if all negative cash flows precede the positive cash flows.
  - Example: (1000, -500, -500, -500) -The IRR is 23.38 but the NPV at 10% is negative.
  - Other pitfalls: Multiple IRRs and nonexistent IRR.
- The IRR of different investments cannot be meaningfully compared when
  - they differ in their scale of investment,
  - the timing of cash flows,
  - or their riskiness.

Discount Factors

- Often times we think about discount rates.
  - i.e. dividing cash flows by a gross rate of return.
- However, it is useful to think about valuation as multiplying by discount factor.
- In the most general form, the value V_0 of a future cash flow $X(T)$ is given by

$$V_0 = E\left[M(T)X(T)\middle|\mathcal{F}_0\right]$$

  where $M(t)$ is an object called the *stochastic discount factor*.
- For a zero-coupon bond $V_0 = E_0\left[M(T)\right] \times 1$.
- Therefore, $E_0\left[M(T)\right]$ is the discount factor we apply to a risk-free cash flow at time T.
- The risk-free rate is $1/E_0\left[M(T)\right]^{1/T} - 1$.

Risk-Free Discount Factors

- Let's denote the risk-free discount factor for time t as $D(t) = E_0\left[M(t)\right]$.
  - $D(t)$ is the price of a risk-free zero coupon bond.
  - In practice, it is unclear which bond to choose for $D(t)$, but Treasuries or swaps are common proxies.
- Consider some *risk-free* stream of cash flows $CF_1, CF_2, \ldots, CF_T$ at times $1, 2, \ldots, T$
- The value V_0 of the cash flows is then

$$V_0 = \sum_{t=1}^{T} D(t) CF_t.$$

Compounding Discount Factors

- Consider the discount factors for one, two, and three years: $D(1)$, $D(2)$ and $D(3)$.
- The value of one dollar received at the end of years 1, 2, and 3 is

$$\begin{aligned} V_0 &= D(1) \times 1 + D(2) \times 1 + D(3) \times 1 \\ &= D(1) + D(1)\frac{D(2)}{D(1)} + D(1)\frac{D(2)}{D(1)}\frac{D(3)}{D(2)} \end{aligned}$$

- Let $\delta_t = D(t)/D(t-1)$, i.e. the one-period discount factor.
- Then,

$$V_0 = \delta_1 + \delta_1\delta_2 + \delta_1\delta_2\delta_3.$$

- With the same δ for each period,

$$V_0 = \delta + \delta^2 + \delta^3.$$

What's the point?

- For each point in time, the discount factor is **unique**.
- In contrast, an interest rate is measured in terms of a compounding frequency and day count convention.
  - For each unique discount factor, there are many different interest rates.
- Example: suppose the one-year discount factor is 0.95.
  - The annual interest rate is 5.26%.
  - The semi-annual interest rate is 5.20%.
  - The monthly interest rate is 5.14%.
  - The monthly interest rate (Actual/360 day count) is 5.06%.
- Whenever you are given an interest rate, you first need to convert it to a discount factor to value a cash flow stream.

Perpetuities

- A perpetuity is an infinite stream of cash flows.
- With a constant discount rate and cash flow C each period:

$$
\begin{aligned}
V_0 &= \sum_{t=1}^{\infty} \delta^t C \\
\delta V_0 &= \sum_{t=2}^{\infty} \delta^t C \\
\rightarrow V_0(1-\delta) &= \delta C \\
\rightarrow V_0 &= \frac{\delta}{1-\delta} C
\end{aligned}
$$

- With $\delta = 1/(1+r)$, $V_0 = C/r$.

Annuity

- With a constant discount rate and cash flow C each period:

$$
\begin{aligned}
V_0 &= \sum_{t=1}^{T} \delta^t C \\
&= \sum_{t=1}^{\infty} \delta^t C - \delta^T \sum_{t=1}^{\infty} \delta^t C \\
&= (1-\delta^T) \sum_{t=1}^{\infty} \delta^t C \\
&= C \frac{\delta}{1-\delta}(1-\delta^T) \\
&= \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}
$$

Fixed Income

Coupon Bond Prices

- A (non-amortizing) bond is an investment that pays a coupon C each period and then it's par value plus C at maturity.
- The price of a bond assumes \$100 face (par) value.
- The price is given by

$$P_0 = \sum_{i=1}^{N} D(t_i)C + 100D(t_N)$$

- Recall that $D(t)$ is the price of a zero-coupon bond with maturity t.
- You can think of a coupon bond as a portfolio of zero-coupon bonds.
  - The **Law of One Price** implies that they have the same price.

Bond Yields

- A yield is an IRR: the constant discount rate that equates the present value and the price.
- A **spot rate** is the yield of zero coupon bond.
- Yields can be tricky to work with because you need to know the quoting convention.
- Each quoting convention involves
  - a compounding frequency and
  - a day count convention.
- Example: suppose the one-year zero coupon bond price is \$95.00.
  - The annual yield is 5.26%.
  - The semi-annual yield is 5.20%.
  - The monthly yield is 5.14%.
  - The monthly yield (Actual/360 day count) is 5.06%.

Bootstrapping

- Bootstrapping is a procedure to obtain spot rates (zero coupon bond prices) from the prices of coupon bonds.
- The bond pricing formula implies

$$\begin{bmatrix} P_0(1) \\ P_0(2) \\ P_0(3) \end{bmatrix} = \begin{bmatrix} C_1 + 100 & 0 & 0 \\ C_2 & C_2 + 100 & 0 \\ C_3 & C_3 & C_3 + 100 \end{bmatrix} \begin{bmatrix} D(1) \\ D(2) \\ D(3) \end{bmatrix}.$$

- Invert the matrix to solve for the zero-coupon bond prices

$$\begin{bmatrix} D(1) \\ D(2) \\ D(3) \end{bmatrix} = \begin{bmatrix} C_1 + 100 & 0 & 0 \\ C_2 & C_2 + 100 & 0 \\ C_3 & C_3 & C_3 + 100 \end{bmatrix}^{-1} \begin{bmatrix} P_0(1) \\ P_0(2) \\ P_0(3) \end{bmatrix}$$