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Online updating belief rule based system for pipeline leak detection under expert intervention

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ABSTRACT

A belief rule base inference methodology using the evidential reasoning approach (RIMER) has been developed recently, where a new belief rule base (BRB) is proposed to extend traditional IF-THEN rules and can capture more complicated causal relationships using different types of information with uncertainties, but these models are trained off-line and it is very expensive to train and re-train them. As such, recursive algorithms have been developed to update the BRB systems online and their calculation speed is very high, which is very important, particularly for the systems that have a high level of real-time requirement. The optimization models and recursive algorithms have been used for pipeline leak detection. However, because the proposed algorithms are both locally optimal and there may exist some noise in the real engineering systems, the trained or updated BRB may violate some certain running patterns that the pipeline leak should follow. These patterns can be determined by human experts according to some basic physical principles and the historical information. Therefore, this paper describes under expert intervention, how the recursive algorithm update the BRB system so that the updated BRB cannot only be used for pipeline leak detection but also satisfy the given patterns. Pipeline operations under different conditions are modeled by a BRB using expert knowledge, which is then updated and fine tuned using the proposed recursive algorithm and pipeline operating data, and validated by testing data. All training and testing data are collected from a real pipeline. The study demonstrates that under expert intervention, the BRB expert system is flexible, can be automatically tuned to represent complicated expert systems, and may be applied widely in engineering. It is also demonstrated that compared with other methods such as fuzzy neural networks (FNNs), the RIMER has a special characteristic of allowing direct intervention of human experts in deciding the internal structure and the parameters of a BRB expert system.

1. Introduction

In order to handle hybrid information with uncertainty in human decision-making (Walley, 1996), Yang et al. proposed a generic rule base inference methodology using the evidential reasoning approach (RIMER) to establish a nonlinear relationship between antecedent attributes and an associated consequent (Yang, Liu, Wang, Sii, & Wang, 2006). The RIMER approach provides a more informative and realistic scheme than the traditional IF-THEN rule base (Sun, 1995) for knowledge representation and is capable of capturing vagueness, incompleteness, and nonlinear causal relationships.

In the RIMER, a rule, which is designed on the basis of a belief structure, is called belief rule base (BRB). The rules in a BRB may come in different ways, such as extracted from experts, by examining historical data, or through self-learning (Xu et al., 2007; Yang, Liu, Xu, Wang, & Wang, 2007). The process of manually extracting rules may be time consuming and the rules may be approximate, especially for the largely complicated systems. As such, the optimal learning methods for training the BRB have been proposed and investigated by Yang et al. (2007). Because these methods are off-line in nature, if a very large set of data is involved, it can become expensive and impractical when there is a high level of real-time requirement. In order to overcome this problem, the newly optimal methods have been developed for updating parameters of a BRB in such a way that the parameters can be updated recursively once new information becomes available (Zhou, Hu, Yang, Xu, & Zhou, in review). The calculation speed of the recursive algorithms is very high. In order to demonstrate the validity and capability of these optimal methods, the new schemes have been applied to build the expert systems for pipeline leak detection (Xu et al., 2007; Zhou et al., in review).

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Leaks from pipelines may cause immeasurable damage to the environment and losses to the pipeline operating companies. To minimize the damages, many methods and types of systems for pipeline leak detection are developed, such as those based on mass balance and real transit models, statistical analysis and acoustic emission detection (Xu et al., 2007). Real-time systems based on mass balance corrected with pressure are among the very popular ones. The BRB expert system for leak detection which has been studied (Xu et al., 2007; Zhou et al., in review) is also based on the mass balance principle.

However, in the current studies (Xu et al., 2007; Zhou et al., in review), because the proposed methods are locally optimal and there may exist some noise in the real systems, the trained or updated BRB expert systems may violate some certain running patterns that pipeline leak should follow. As such, this paper further investigates how the BRB expert system is updated using the pipeline operating data so that the updated BRB can satisfy the certain running patterns of pipeline leak at the same time. The study demonstrates that compared with fuzzy neural networks (FNNs), the RIMER is designed to allow direct intervention of human experts in deciding the internal structure and the parameters of a BRB (Yang et al., 2007). Since the running patterns of pipeline leak can be determined by the experts, we say that the proposed method in this paper is the online updating algorithm under expert intervention.

This paper is organized as follows. The RIMER approach is briefly reviewed in Section 2. Section 3 proposes the new recursive algorithm for updating the BRB systems under expert intervention. The proposed algorithm is used for pipeline leak detection in Section 4. The paper is concluded in Section 5.

2. Belief rule based expert systems

The RIMER, which is based on the belief rule concept and its inference methodology (Yang and Sen, 1994; Yang and Singh, 1994; Yang, Liu, et al., 2006; Yang et al., 2006; Yang and Xu, 2002) and can reflect the dynamic nature of decision-making problems, was developed on the basis of Dempster–Shafer theory of evidence (Dempster, 1968; Shafer, 1976), decision theory (Huang & Yong, 1981) and fuzzy set theory (Zadeh, 1965). The RIMER consists of the belief rule as the knowledge representations and the evidential reasoning (ER) approach as the inference methodology.

2.1. Belief rule base

In order to capture the dynamics of a system, a belief rule base (BRB) consists of a collection of belief rules defined as follows (Yang, Liu, et al., 2006):

$$R_k: \text{If } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \cdots \wedge x_{M_k} \text{ is } A_{M_k}^k, \text{ Then}\{(D_1,\beta_{1,k}),\ldots,(D_N,\beta_{N,k})\}$$
$$\text{With a rule weight } \theta_k \text{ and attribute weight } \delta_{1,k},\delta_{2,k},\ldots,\delta_{M_k,k} \tag{1}$$

where $x_1, x_2, \ldots, x_{M_k}$ represents the antecedent attributes in the kth rule. A_i^k $(i = 1,\ldots,M_k, k = 1,\ldots,L)$ is the referential value of the ith antecedent attribute in the kth rule and $A_i^k \in A_i$. $A_i = \{A_{i,j}, j = 1,\ldots, J_i\}$ is a set of referential values for the ith antecedent attribute and J_i is the number of the referential values. θ_k $(\in R^+, k = 1,\ldots,L)$ is the relative weight of the kth rule, and $\delta_{1,k}, \delta_{2,k},\ldots,\delta_{M_k,k}$ are the relative weights of the M_k antecedent attributes used in the kth rule. $\beta_{j,k}$ $(j = 1,\ldots,N, k = 1,\ldots,L)$ is the belief degree assessed to D_j which denotes the jth consequent. If $\sum_{j=1}^{N}\beta_{j,k} = 1$, the kth rule is said to be complete; otherwise, it is incomplete. Note that "$\wedge$" is a logical connective to represent the "AND" relationship. In addition, suppose that M is the total number of antecedent attributes used in the rule base.

A BRB given in Eq. (1) represents functional mappings between antecedents and consequents possibly with uncertainty. It provides a more informative and realistic scheme than a simple IF-THEN rule base for knowledge representation. Note that the degrees of belief $\beta_{i,k}$ and the weights could be assigned initially by experts and then trained or updated using dedicated learning algorithms (Yang, Liu, et al., 2006; Yang et al., 2007).

2.2. Belief rule inference using the evidential reasoning algorithm

The evidential reasoning (ER) approach mainly consists of two steps, and we will review the ER approach in this Section.

2.2.1. Calculation of the activation weight

The activation weight of the kth rule at time instant n, $\omega_k(n)$, is calculated by Yang, Liu, et al. (2006)

$$\omega_k(n) = \frac{\theta_k \prod_{i=1}^{M_k}(\alpha_{i,j}^k(n))^{\bar{\delta}_i}}{\sum_{l=1}^{L}\theta_l \prod_{i=1}^{M_k}(\alpha_{i,j}^l(n))^{\bar{\delta}_i}} \text{ and } \bar{\delta}_i = \frac{\delta_i}{\max\limits_{i=1,\ldots,M_k}\{\delta_i\}} \tag{2}$$

where δ_i $(\in R^+, i = 1,\ldots,M_k)$ is the relative weight of the ith antecedent attribute that is used in the kth rule. Because $\omega_k(n)$ will be eventually normalized so that $\omega_k(n) \in [0,1]$ using Eq. (2), θ_k and δ_i can be assigned to any value in R^+. Without loss of generality, however, we assume that $\theta_k \in [0,1]$ $(k = 1,\ldots,L)$ and $\delta_i \in [0,1]$ $(i = 1,\ldots,M_k)$. $\alpha_{i,j}^k(n)$ $(i = 1,\ldots,M_k)$, which is called the individual matching degree, is the degree of belief to its jth referential value $A_{i,j}^k$ in the kth rule at time instant n. $\alpha_k(n) = \prod_{i=1}^{M_k}(\alpha_{i,j}^k(n))^{\bar{\delta}_i}$ is called the normalized combined matching degree. $\alpha_{i,j}^k(n)$ could be generated using various ways, depending on the nature of an antecedent attribute and data available such as a qualitative attribute using linguistic values. The input information can be one of the following types: continuous, discrete, symbolic and ordered symbolic. In order to facilitate data collection, a scheme for handling various types of input information has been summarized by Yang (2001), Yang, Liu, et al. (2006) and Yang et al. (2007).

2.2.2. Rule inference using the evidential reasoning approach

Using the ER analytical algorithms (Wang, Yang, & Xu, 2006; Yang et al., 2007), the final conclusion $O(\mathbf{Y}(n))$ that is generated by aggregating all rules that are activated by the actual input vector $\hat{\mathbf{x}}(n)$ at time instant n can be represented as follows:

$$O(\mathbf{Y}(n)) = F(\hat{\mathbf{x}}(n)) = \{(D_j, \beta_j(n)), \quad j = 1,\ldots,N\} \tag{3}$$

where $\beta_j(n)$ denotes the belief degree in D_j at time instant n, and Eqs. (4) and (5) hold,

$$\beta_j(n) = \frac{\mu(n) \times \left[\prod_{k=1}^{L}\left(\omega_k(n)\beta_{j,k} + 1 - \omega_k(n)\sum_{i=1}^{N}\beta_{i,k}\right) - \prod_{k=1}^{L}\left(1 - \omega_k(n)\sum_{i=1}^{N}\beta_{i,k}\right)\right]}{1 - \mu(n) \times \left[\prod_{k=1}^{L}(1 - \omega_k(n))\right]} \tag{4}$$

$$\mu(n) = \left[\sum_{j=1}^{N}\prod_{k=1}^{L}\left(\omega_k(n)\beta_{j,k} + 1 - \omega_k(n)\sum_{i=1}^{N}\beta_{i,k}\right) - (N-1)\prod_{k=1}^{L}\left(1 - \omega_k(n)\sum_{i=1}^{N}\beta_{i,k}\right)\right]^{-1} \tag{5}$$

where $\omega_k(n)$ is calculated by Eq. (2). Note that $\beta_j(n)$ is the function of the belief degrees $\beta_{i,k}$ $(i = 1,\ldots,N,\ k = 1,\ldots,L)$, the rule weights $\theta_k(k = 1,\ldots,L)$, the attribute weights $\delta_i(i = 1,\ldots,M)$, and the input vector $\hat{\mathbf{x}}(n)$.

The logic behind the approach is that, if the consequent in the k th rule includes D_i with $\beta_{i,k} > 0$ and the kth rule is activated, then the overall output must be D_i to a certain degree. The degree is measured by both the degree to which the kth rule is important to the overall output and the degree to which the antecedents of the kth rule are activated by the actual input $\hat{\mathbf{x}}(n)$ (Yang, Liu, et al., 2006).

3. Online updating belief rule based system under expert intervention

Based on the recursive expectation maximization (EM) algorithm which is a maximum likelihood (ML) algorithm in nature and was proposed by Dempster, Laird,and Rubin (1977) and Titterington (1984), the recursive algorithms for updating the BRB have been developed and the convergence of the algorithms has also been analyzed (Zhou et al., in review). In the proposed recursive algorithms, the observations on the system inputs and outputs are required. We assume that a set of observation pairs $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ is available, where $\hat{\mathbf{x}}$ is a given input vector; $\hat{\mathbf{y}}$ the corresponding observed output vector, either measured using instruments or assessed by experts; and $\mathbf{y}$ is the simulated output that is generated by the BRB system. Therefore, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ can be either numerical or judgmental.

Since the leak size is numerical, we only investigate the case in which $\hat{\mathbf{y}}$ is numerical in this Section. For the judgmental case, the similar results can be obtained.

3.1. Recursive algorithm for online updating belief rule based system

3.1.1. The recursive algorithm

It has been proved that the probabilistic representation of belief is the only appropriate representation of belief which acts correctly under Dempster's combination rule (Halpern & Fagin, 1992). Because the evidence is represented as belief distributions, belief is represented as probability and the Dempster's combination rule is adopted in the ER approach (Yang and Sen, 1994; Yang and Singh, 1994; Yang, Liu, et al., 2006; Yang, Wang, 2006), it has also been proved that when the inputs of the BRB are independent, the true outputs, $\hat{\mathbf{y}}(1), \ldots, \hat{\mathbf{y}}(n)$, can also be assumed to be independent (Zhou et al., in review). Therefore, there is

$$f(\hat{\mathbf{y}}(1), \ldots, \hat{\mathbf{y}}(n)|\hat{\mathbf{x}}(1), \ldots, \hat{\mathbf{x}}(n), \mathbf{Q}) = \prod_{\tau=1}^{n} f(\hat{\mathbf{y}}(\tau)|\hat{\mathbf{x}}(\tau), \mathbf{Q}) \tag{6}$$

where $\hat{\mathbf{y}}$ can be considered as a random variable. $f(\hat{\mathbf{y}}(\tau)|\hat{\mathbf{x}}(\tau), \mathbf{Q})$ is assumed to be the conditional probability density function (pdf) of $\hat{\mathbf{y}}$ at time instant τ and $\mathbf{Q}$ is the unknown parameter vector.

The expectation of the log-likelihood of Eq. (6) at time instant n is defined as

$$L_{n+1}(\mathbf{Q}) \triangleq E\left\{ \sum_{\tau=1}^{n} \log f(\hat{\mathbf{y}}(\tau)|\hat{\mathbf{x}}(\tau), \mathbf{Q}) \middle| \hat{\mathbf{x}}(1), \ldots, \hat{\mathbf{x}}(n), \mathbf{Q}(n) \right\} \tag{7}$$

where $E\{\cdot|\cdot\}$ denotes the conditional expectation at $\mathbf{Q} = \mathbf{Q}(n)$.

The recursive formulation of Eq. (7) can be written as

$$L_{n+1}(\mathbf{Q}) = L_n(\mathbf{Q}) + E\{\log f(\hat{\mathbf{y}}(n)|\hat{\mathbf{x}}(n), \mathbf{Q})|\hat{\mathbf{x}}(n), \mathbf{Q}(n)\} \tag{8}$$

Define

$$\Gamma(\mathbf{Q}(n)) \triangleq \nabla_{\mathbf{Q}} \log f(\hat{\mathbf{y}}(n)|\hat{\mathbf{x}}(n), \mathbf{Q}(n)) \tag{9}$$

$$\Xi(\mathbf{Q}(n)) \triangleq E\left\{ -\nabla_{\mathbf{Q}} \nabla_{\mathbf{Q}}^{\mathrm{T}} \log f(\hat{\mathbf{y}}(n)|\hat{\mathbf{x}}(n), \mathbf{Q}) \middle| \hat{\mathbf{x}}(n), \mathbf{Q}(n) \right\} \tag{10}$$

Based on the recursive EM algorithm (Chung & Bohme, 2005; Titterington, 1984), the maximizing parameter $\mathbf{Q}(n+1)$ is given by Zhou et al. (in review),

$$\mathbf{Q}(n+1) = \mathbf{Q}(n) + \frac{1}{n}[\Xi(\mathbf{Q}(n))]^{-1}\Gamma(\mathbf{Q}(n)) \tag{11}$$

where $\mathbf{Q}$ consists of the belief degrees, rule weights, attribute weights and other possible parameters as given later.

The belief degrees, rule weights and attribute weights must satisfy some equality and inequality constraints as follows (Yang, Liu, et al., 2006; Yang et al., 2007):

(1) A rule weight is normalized, so that it is between zero and one, i.e.,

$$0 \leqslant \theta_k \leqslant 1, \quad k = 1, \ldots, L \tag{11a}$$

(2) An attribute weight is normalized, so that it is between zero and one, i.e.,

$$0 \leqslant \bar{\delta}_m \leqslant 1, \quad m = 1, \ldots, M \tag{11b}$$

(3) A belief degree (subjective probability) must not be less than zeros or more than one, i.e.,

$$0 \leqslant \beta_{j,k} \leqslant 1, \quad j = 1, \ldots, N, \ k = 1, \ldots, L \tag{11c}$$

(4) If the kth belief rule is complete, its total belief degree in the consequent will be equal to one, i.e.,

$$\sum_{j=1}^{N} \beta_{j,k} = 1, \quad k = 1, \ldots, L \tag{11d}$$

Otherwise the total belief degree is less than one.

3.1.2. The recursive algorithm under normal distribution of the observation

The output shown in Eq. (3) is represented as a distribution and its average score (Yang, 2001; Yang, Liu, et al., 2006; Yang, Wang, 2006),

$$\mathbf{y}(n) = \sum_{j=1}^{N} \mu_j \beta_j(n) \tag{12}$$

where μ_j denotes the utility (or score) of an individual consequent D_j which can be either given using a scale or estimated using the decision maker's preferences.

We hope that for a given input, $\hat{\mathbf{x}}(n)$, the BRB system can generate an output, represented as Eq. (12), which can be as close to $\hat{\mathbf{y}}(n)$ as possible. Here $\hat{\mathbf{y}}(n)$ is considered as a random variable and $\mathbf{y}(n)$ can be considered as its expectation. So we assume that the pdf of $\hat{\mathbf{y}}(n)$ obeys the following normal distribution,

$$f(\hat{\mathbf{y}}(n)|\mathbf{x}(n), \mathbf{Q}) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{(\hat{\mathbf{y}}(n) - \mathbf{y}(n))^2}{2\sigma} \right\} \tag{13}$$

where $\mathbf{Q} = [\mathbf{V}^{\mathrm{T}}, \sigma]^{\mathrm{T}}$ denotes the parameter vector and σ denotes variance. $\mathbf{V} = [\theta_k, \bar{\delta}_m, \beta_{j,k}]^{\mathrm{T}}$ denotes parameter vector of the BRB and $k = 1, \ldots, L$, $m = 1, \ldots, M$, $j = 1, \ldots, N$.

Due to the independence between the elements of $\mathbf{V}$ and σ, $\Gamma(\mathbf{Q}(n))$ and $\Xi(\mathbf{Q}(n))$ in Eq. (11) can be written as

$$\Gamma(\mathbf{Q}(n)) = \left[\Gamma'(\mathbf{Q}(n))^{\mathrm{T}}, \Gamma''(\mathbf{Q}(n))^{\mathrm{T}}\right]^{\mathrm{T}} \tag{14}$$

$$\Xi(\mathbf{Q}(n)) = \begin{bmatrix} \Xi'(\mathbf{Q}(n)) & \mathbf{0} \\ \mathbf{0} & \Xi''(\mathbf{Q}(n)) \end{bmatrix} \tag{15}$$

where $\Gamma'(\mathbf{Q}(n))$ and $\Xi'(\mathbf{Q}(n))$ are the derivates with respect to the entries of $\mathbf{V}$. $\Gamma''(\mathbf{Q}(n))$ and $\Xi''(\mathbf{Q}(n))$ are the derivates with respect to σ. Obviously, there is

$$[\Xi(\mathbf{Q}(n))]^{-1} = \begin{bmatrix} [\Xi'(\mathbf{Q}(n))]^{-1} & \mathbf{0} \\ \mathbf{0} & [\Xi''(\mathbf{Q}(n))]^{-1} \end{bmatrix} \tag{15a}$$

When we consider only parameter vector $\mathbf{V}$, according to Eqs. (14) and (15a), the recursive algorithm (11) changes into the following form,

$$\mathbf{V}(n+1)=\mathbf{V}(n)+\frac{1}{n}[\Xi'(\mathbf{Q}(n))]^{-1}\Gamma'(\mathbf{Q}(n)) \tag{16}$$

In Eq. (16), $\mathbf{V}(n)$ is known. By definitions (9) and (10), the ath element of the gradient vector $\Gamma'(\mathbf{Q}(n))$ and the entries of $\Xi'(\mathbf{Q}(n))$ at time instant n are

$$[\Gamma'(\mathbf{Q}(n))]_a=\frac{(\hat{\mathbf{y}}(n)-\mathbf{y}(n))}{\sigma(n)}\sum_{j=1}^{N}\mu_j\frac{\partial\beta_j(n)}{\partial V_a}\bigg|_{\mathbf{V}=\mathbf{V}(n)} \tag{16a}$$

$$\begin{aligned}[\Xi'(\mathbf{Q}(n))]_{a,b}&=E\left\{\frac{1}{\sigma}\frac{\partial\mathbf{y}(n)}{\partial V_a}\frac{\partial\mathbf{y}(n)}{\partial V_b}\right.\\&\qquad\left.-\frac{1}{\sigma}\frac{\partial^2\mathbf{y}(n)}{\partial V_a\partial V_b}(\hat{\mathbf{y}}(n)-\mathbf{y}(n))\bigg|\mathbf{Q}(n)\right\}\\&=\frac{1}{\sigma(n)}\left[\sum_{j=1}^{N}\mu_j\frac{\partial\beta_j(n)}{\partial V_a}\right]\left[\sum_{j=1}^{N}\mu_j\frac{\partial\beta_j(n)}{\partial V_b}\right]\Bigg|_{\mathbf{V}=\mathbf{V}(n)}\end{aligned} \tag{16b}$$

where $a=1,\ldots,L+M+L\times N$ and $b=1,\ldots,L+M+L\times N$. The derivatives in Eqs. (16a) and (16b) have been given by Zhou et al. (in review).

In Eqs. (16a) and (16b), $\sigma(n)$ is required. If $\hat{\mathbf{x}}(n)$, $\hat{\mathbf{y}}(n)$ and $\mathbf{V}(n)$ are available, it can be estimated by

$$\begin{aligned}\sigma(n)&=\arg\max_{\sigma}\log f(\hat{\mathbf{y}}(n)|\hat{\mathbf{x}}(n),\mathbf{Q})\Big|_{\mathbf{V}=\mathbf{V}(n)}\\&=(\hat{\mathbf{y}}(n)-\mathbf{y}(n))^2\Big|_{\mathbf{V}=\mathbf{V}(n)}\end{aligned} \tag{17}$$

3.2. Recursive algorithm under expert intervention

3.2.1. The running patterns of the system given by human experts

When a system runs, its behavior will change following some certain patterns. According to some basic physical principles and the historical information, human experts can give these patterns which may be described as the belief rules (Xu et al., 2007). From the BRB of Eq. (1), we can see that whether a rule can satisfy the patterns is decided by the values of belief degrees $\beta_{j,k}$ ($j=1,\ldots,N$, $k=1,\ldots,L$). According to some basic physical principles and the historical information, human experts can give some constraints about $\beta_{j,k}$ to denote the certain running patterns and the constraints include the equalities and inequalities. For example, the equality constraints may denote that a consequence D_j does not occur in a rule, and the inequality ones may denote the relationships between the rules. $\beta_{j,k}=0$ may denote that a consequence D_j in the kth rule cannot occur. $\beta_{j,k}\geqslant\beta_{j,k+1}$ may denote that D_j in the kth rule occurs more possibly than the $(k+1)$th rule.

Without loss of generality, we can define these constraints as follows:

$$h^1_{i_1}(\beta_{j,k})=0,\quad i_1=1,\ldots,I,\ j=1,\ldots,N,\ k=1,\ldots,L \tag{18}$$

$$z^1_{s_1}(\beta_{j,k})\leqslant 0,\quad s_1=1,\ldots,S_1,\ j=1,\ldots,N,\ k=1,\ldots,L \tag{19}$$

where I_1 and S_1 are the numbers of the equality and inequality constraints, respectively.

3.2.2. Recursive algorithm under expert intervention

From the above analysis, we can see that when the BRB system is constructed by human experts, the above constraints (11a)–(11d), (19) should be satisfied. Moreover, if the BRB expert systems are manually tuned, and trained or updated using the optimal methods, $\beta_{j,k}$ must satisfy these constraints. In this Section, we will study the recursive algorithm for updating the BRB under the constraints.

Firstly, let $U=L+M+L\times N$ and $\mathbf{V}=[V_1,\ldots,V_U]^{\mathrm{T}}$. Then the constraints (18) and (19) can be represented as

$$h^1_{i_1}(V_{L+M+1},\ldots,V_U)=0,\ i_1=1,\ldots,I_1 \tag{20}$$

$$z^1_{s_1}(V_{L+M+1},\ldots,V_U)\leqslant 0,\ s_1=1,\ldots,S_1 \tag{21}$$

And the constraints (11a)–(11d) can be written as

$$h^2_{i_2}(V_{L+M+(k-1)\times N+1},\ldots,V_{L+M+(k-1)\times N+N})=\sum_{j=1}^{N}V_{L+M+(i_2-1)\times N+j}-1=0,$$
$$i_2=1,\ldots,L \tag{22}$$

$$z^2_{s_2}(V_{s_2})=-V_{s_2}\leqslant 0,\quad s_2=1,\ldots,S_2 \tag{23}$$

$$z^3_{s_3}(V_{s_3})=V_{s_3}-1\leqslant 0,\quad s_3=1,\ldots,S_3 \tag{24}$$

where $S_2=S_3=U$.

Define

$$\mathbf{h}(\mathbf{V})\triangleq\left[h^1_{i_1}(\mathbf{V}),h^2_{i_2}(\mathbf{V}),z^1_{s_1}(\mathbf{V}),z^2_{s_2}(\mathbf{V}),z^3_{s_3}(\mathbf{V})\right]^{\mathrm{T}} \tag{25}$$

$$\Omega(\mathbf{V})\triangleq\left\|\left[h^1_1(\mathbf{V}),\ldots,h^1_{I_1}(\mathbf{V}),h^2_1(\mathbf{V}),\ldots,h^2_L(\mathbf{V})\right]^{\mathrm{T}}\right\| \tag{26}$$

$$\tilde{z}^j_{s_j}(\mathbf{V})\triangleq\max\left[0,z^j_{s_j}(\mathbf{V})\right]\ \text{and}\ \Psi_j(\mathbf{V})=\sum_{s_j=1}^{S_j}\left[\tilde{z}^j_{s_j}(\mathbf{V})\right]^2 \tag{27}$$

where i_1, i_2 and s_j ($j=1,2,3$) are same as Eqs. (20)–(24). $\|\cdot\|$ denotes Euclidean norm. $\Omega(\mathbf{V})$ and $\Psi_j(\mathbf{V})$ ($j=1,2,3$) are penalty functions and used to deal with the constraints (Kushner & Kelmanson, 1976).

We assume

$$\boldsymbol{\rho}(\mathbf{V})=\left[\frac{\partial\Omega(\mathbf{V})}{\partial V_1},\ldots,\frac{\partial\Omega(\mathbf{V})}{\partial V_U}\right]^{\mathrm{T}} \tag{28}$$

$$\boldsymbol{\phi}_j(\mathbf{V})=\left[\frac{\partial\Psi_j(\mathbf{V})}{\partial V_1},\ldots,\frac{\partial\Psi_j(\mathbf{V})}{\partial V_U}\right]^{\mathrm{T}},\quad j=1,2,3 \tag{29}$$

Suppose that $\mathbf{I}_u$ is the identity matrix whose dimension is u, then the following algorithm to deal with the constraints (20)–(24) can be obtained (Kushner and Kelmanson, 1976; Kushner and Yin, 1997; Zhou et al., in review),

$$\begin{aligned}\mathbf{V}(n+1)&=\mathbf{V}(n)\\&\quad+\frac{\alpha}{n}\left\{\boldsymbol{\pi}\{\mathbf{V}(n)\}[\Xi'(\mathbf{Q}(n))+\gamma\mathbf{I}_U]^{-1}\Gamma'(\mathbf{Q}(n))-\frac{K}{2}\boldsymbol{\phi}(\mathbf{V})\right\}\end{aligned} \tag{30}$$

where $\boldsymbol{\phi}(\mathbf{V})=\boldsymbol{\rho}(\mathbf{V})+\sum_{j=1}^{3}\boldsymbol{\phi}_j(\mathbf{V})$. $\alpha\geqslant 1$ is the step factor and can change the convergence speed. Because only some rules in the BRB are activated and the matrix $\Xi'(\mathbf{Q}(n))$ is singular at time instant n, $\Xi'(\mathbf{Q}(n))$ is amended using $\gamma\mathbf{I}_U$ so that it becomes positive definite and $\gamma\geqslant 0$. K denotes a positive real number and its value may change from usage to usage. And there is

$$\boldsymbol{\pi}\{\mathbf{V}(n)\}=\mathbf{I}_U-\mathbf{H}(\mathbf{V}(n))^{\mathrm{T}}(\mathbf{H}(\mathbf{V}(n))\mathbf{H}(\mathbf{V}(n))^{\mathrm{T}})^{-1}\mathbf{H}(\mathbf{V}(n)) \tag{30a}$$

where $\mathbf{H}(\mathbf{V})$ denotes the Jacobian matrix of $\mathbf{h}(\mathbf{V})$.

In Eq. (30), the recursive algorithm can also be written as the following general form,

$$\mathbf{V}(n+1)=\prod_{H}\left\{\mathbf{V}(n)+\frac{1}{n}[\Xi'(\mathbf{Q}(n))]^{-1}\Gamma'(\mathbf{Q}(n))\right\} \tag{31}$$

where $\prod_H\{\cdot\}$ is the projection onto the constraint set H which is composed of the constraints (11a)–(11d), (18) and (19). Because the constraints (18) and (19) are determined by human experts, Eq. (30) can also be called the recursive algorithm under expert intervention.

From Eqs. (20)–(24), it can be concluded that the constraint set H is composed of some equality and inequality constraints. According to the convergence theorem provided by Kushner and Yin (1997), it has been proved that when the appropriate initial values of the unknown parameters are given, the recursive algorithm with the equality and inequality constraints can converge to a locally optimal point under some conditions (Zhou et al., in review). Similarly, the recursive algorithm proposed in this paper is also with the constraints, so we can also easily prove its convergence under some conditions.

As a result of the above discussion, the procedure of the recursive algorithm for updating the BRB expert systems under expert intervention may be summarized as the following steps:

Step 1: According to some basic physical principles and the historical information, human experts give some patterns that the system behaviors follow when the system runs. Then the patterns are converted into the corresponding equality constraints (20) and inequality constraints (21).

Step 2: Given the initial values of the parameter vector $\mathbf{V}(0)$ and the variance $\sigma(0)$. $\mathbf{V}(0)$ must satisfy the constraints (11a)–(11d).

Step 3: When the observations $\hat{\mathbf{x}}(0)$ and $\hat{\mathbf{y}}(0)$ are available, the recursive algorithm (30) is used to estimate $\mathbf{V}(1)$ which satisfies the constraints (11a)–(11d), (20) and (21). Then $\sigma(1)$ is estimated using Eq. (17).

Step 4: When $\hat{\mathbf{x}}(n)$, $\hat{\mathbf{y}}(n)$, $\mathbf{V}(n)$ and $\sigma(n)$ are available at time instant n. Step 3 is reused to estimate $\mathbf{V}(n+1)$ and $\sigma(n+1)$. $\mathbf{V}(n+1)$ can satisfy the constraints (20) and (21), i.e., the updated BRB obeys the running patterns of the system.

Step 5: Once the BRB is updated, its knowledge can be used to perform inference from the given inputs.

4. Online updating belief rule based system for pipeline leak detection under expert intervention

4.1. Problem formulation

In this Section, we will also consider a pipeline more than 100 km installed in Great Britain (Xu et al., 2007) and the pipeline leak data will be used to demonstrate the validity of the proposed algorithm.

When a leak develops in a pipeline, flow and pressure in the pipeline will change following certain patterns. According to mass balance principle and historical information, human experts can provide a set of rules to distinguish patterns between operations under normal and leak situations as follows: under normal operations, when inlet flow is larger (or less) than outlet flow, the pressure in the pipeline will build up (or decrease) because the total content in the pipeline is increasing (or decreasing, respectively). However, if the pattern is violated, for example, if the inlet flow is larger than the outlet flow, yet the pressure in the line still decreases, then it is highly likely that there is a leak in the pipeline (Xu et al., 2007).

Therefore, we also choose the difference between inlet flow and outlet flow, the average pipeline pressure change over time and the leak rate, denoted by *FlowDiff*, *PressureDiff* and *LeakSize*, respectively, as the leak data (Xu et al., 2007). During the leak trial, 2008 samples of 4%, 16% and 25% leak data were collected at the rate of 10s per sample, respectively. Figs. 1 and 2 give the *FlowDiff* and *PressureDiff*, respectively, when there is no leak and 25% leak.

4.2. Referential points of the antecedents and consequence

Since *FlowDiff* and *PressureDiff* are the two very important factors in detecting whether there is leak in the pipeline, they are chosen as the antecedent attributes of the rule base. Obviously, *LeakSize* is the consequent attribute of the rule base. The antecedents and consequent in the rule base should be given some referential points. We choose these points as follows.

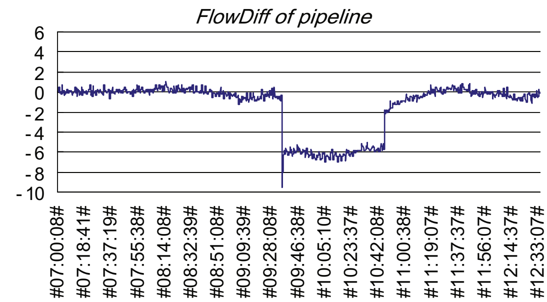

Fig. 1. The *FlowDiff* of the pipeline.

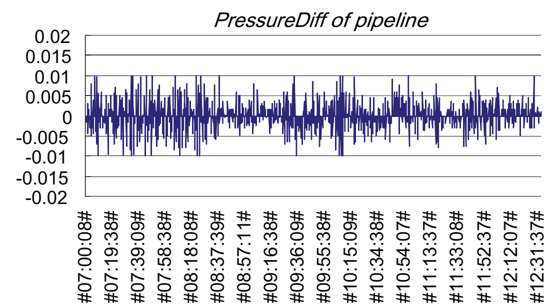

Fig. 2. The *PressureDiff* of the pipeline.

For *FlowDiff*, eight referential points are used and they are negative large (NL), negative medium (NM), negative small (NS), negative very small (NVS), zero (Z), positive small (PS), positive medium (PM), and positive large (PL). That is

$$A_1^k \in \{\text{NL},\ \text{NM},\ \text{NS},\ \text{NVS},\ \text{Z},\ \text{PS},\ \text{PM},\ \text{PL}\} \tag{32}$$

For *PressureDiff*, seven referential points are used and they are NL, NM, NS, Z, PS, PM, PL, i.e.,

$$A_2^k \in \{\text{NL},\ \text{NM},\ \text{NS},\ \text{Z},\ \text{PS},\ \text{PM},\ \text{PL}\} \tag{33}$$

For *LeakSize*, five referential points are used: zero (Z), very small (VS), medium (M), high (H) and very high (VH), i.e.,

$$\mathbf{D} = (D_1, D_2, D_3, D_4, D_5) = (\text{Z},\ \text{VS},\ \text{M},\ \text{H},\ \text{VH}) \tag{34}$$

The referential points defined above for the antecedent and consequent attributes are in linguistic terms and need to be quantified. The quantified results are given in Tables 1–3, respectively.

4.3. Rules

According to the patterns given by human experts, a BRB for the pipeline leak detection can be constructed. A belief rule in the BRB can be represented as

$$\begin{aligned} R_k :\ & \text{If } \textit{FlowDiff} \text{ is } A_1^k \wedge \textit{PressureDiff} \text{ is } A_2^k \\ & \text{Then } \textit{LeakSize} \text{ is } \{(\text{Z},\beta_{1,k}), (\text{VS},\beta_{2,k}), (\text{M},\beta_{3,k}), (\text{H},\beta_{4,k}), (\text{VH},\beta_{5,k})\} \\ & \text{With a rule weight } \theta_k \text{ and attribute weight } \delta_{1,k}, \delta_{2,k}, \\ & \left(\sum_{i=1}^{5} \beta_{i,k} \leqslant 1\right) \end{aligned} \tag{35}$$

where A_1^k and $A_2^k (k = 1, \ldots, 56)$ are the referential values as defined in Tables 1 and 2, respectively. Because *FlowDiff* is divided into 8 terms and *PressureDiff* 7 terms, there are 56 combinations of the 2 antecedents leading to 56 rules in total in the rule base. The initial BRB are given by an expert as shown in Table A-I of Appendix A, but

Table 1
The referential points of *FlowDiff*

Linguistic terms	NL	NM	NS	NVS	Z	PS	PM	PL
Numerical values	−10	−5	−3	−1	0	1	2	3

Table 2
The referential points of *PressureDiff*

Linguistic terms	NL	NM	NS	Z	PS	PM	PL
Numerical values	−0.042	−0.025	−0.01	0	0.01	0.025	0.042

Table 3
The referential points of *LeakSize*

Linguistic terms	Z	VS	M	H	VH
Numerical values	0	2	4	6	8

the belief degrees for *LeakSize* may not be accurate. It is necessary to update the belief degrees so that the performance of the expert system can be improved or optimized in a sense.

4.4. The running patterns of pipeline leak given by human experts

According to the historical information, mass balance principle and the running patterns of pipeline leak as given in Section 4.1, we can see that for the first 7 rules in Table A-I, the leak size should get smaller, i.e., the belief degrees to the linguistic term D_1, which denotes that there is no leak, get larger. These patterns can be described as the following inequalities.

$$\beta_{1,1} \leqslant \beta_{1,2} \leqslant \cdots \leqslant \beta_{1,7} \tag{36}$$

If the whole BRB is divided into 8 groups and each group includes 7 rules, there are similar relationships among 7 rules in each group. Also between adjacent two groups, the leak size should get smaller, i.e.,

$$\beta_{(J_1-1)\times J_2+1,1} \leqslant \beta_{J_1\times J_2+1,1}, \quad J_1 = 1,\ldots,7;\ J_2 = 7 \tag{37}$$

Moreover, the linguistic term D_5 denotes that the leak is most severe. For some rules, according to the historical information and expert knowledge, D_5 in some rules cannot occur, which can be described as follows:

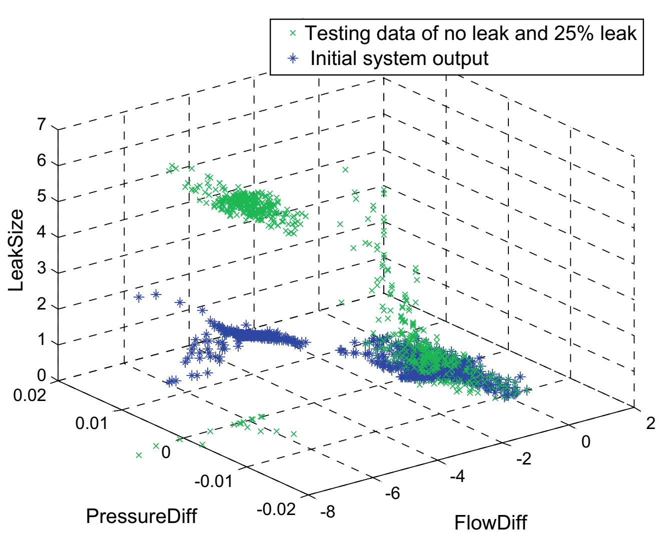

Fig. 3. Testing data of no leak and 25% leak and the output by the initial BRB.

$$\beta_{J_3,5} = 0,\ J_3 = 5,\ldots,56 \tag{38}$$

Obviously, the inequality constraints (36) and (37) reflect the relative relationships between the rules and the equality constraints

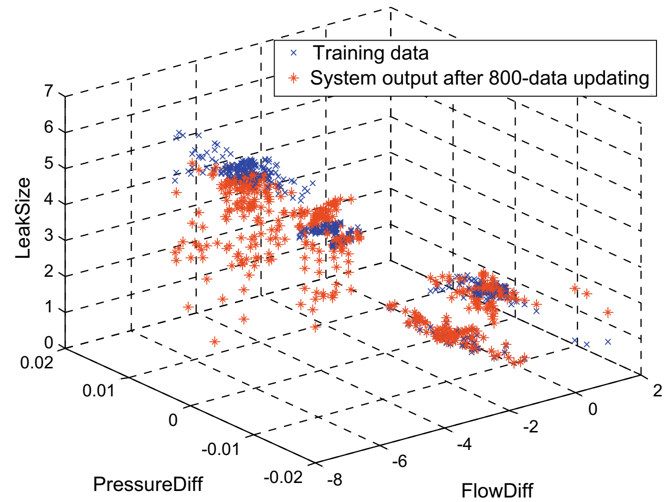

Fig. 4. Training data and the output by the updated BRB under expert intervention.

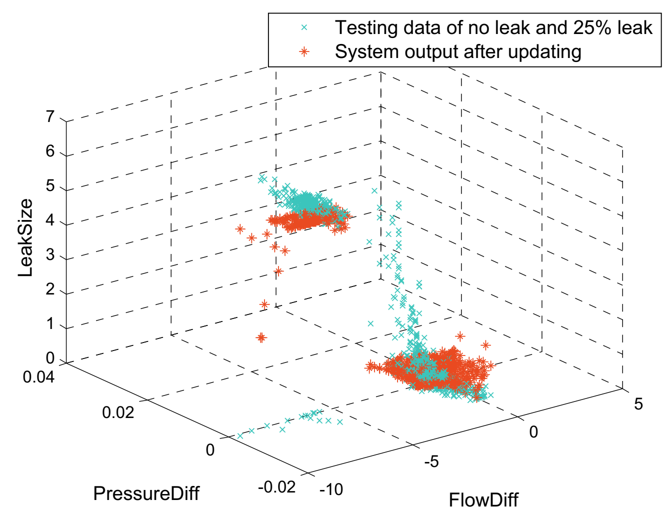

Fig. 5. Testing data of no leak and 25% leak and the output by the updated BRB under expert intervention.

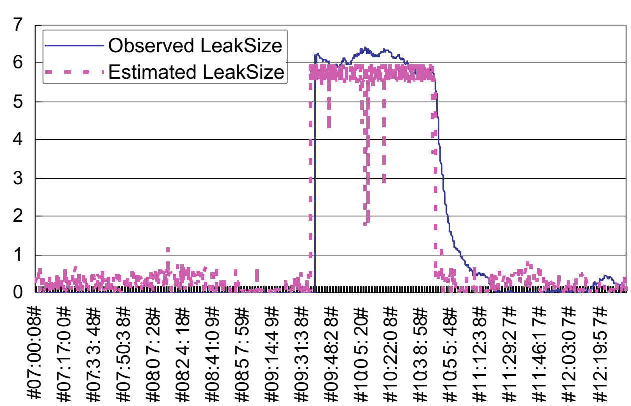

Fig. 6. Testing data of no leak and 25% leak and the output by the updated BRB under expert intervention.

(38) denote some absolute relationships. Compared with the running patterns given in Section 4.1, the above constraints can also be considered as the certain patterns that the pipeline leak should follow, and they are the more accurate reflections of pipeline leak which shows that the more expert knowledge is obtained and used. Likewise, besides the constraints (36)–(38), if other expert knowledge is provided, we can also obtain some other equalities and inequalities.

4.5. Online updating the rule base under expert intervention

In order to update the BRB expert system for pipeline leak detection, 800 data sets are selected, which include 200 from no leak, 200 from 25% leak, 200 from 16% leak and 200 from 4% leak. Then these data are used to update the BRB using the proposed recursive algorithm (30) with the constraints. The process of updating and testing the BRB is implemented using MATLAB.

Step 1: Set initial parameters
The initial belief degrees are given by an expert and listed in Table A-I of Appendix A. θ_k and $\bar{\delta}_j$ are all assumed to be 1, where $k = 1,\ldots,56$ and $j = 1,2$. As shown in Fig. 3, it is obvious that the values of the estimated *LikeSize* calculated by the initial BRB do not match the observed values when the leak is 25%. This means the initial BRB provided by an expert is not good. So it is necessary to update the BRB online.

Table A-I
Initial belief rules for pipeline oil leak detection provided by an expert

Rule number	*FlowDiff* and *PressureDiff*	*LeakSize* distribution $\{D_1,D_2,D_3,D_4,D_5\} = \{0,2,4,6,8\}$
1	NL and NL	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 1)\}$
2	NL and NM	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0.3), (D_5, 0.7)\}$
3	NL and NS	$\{(D_1, 0), (D_2, 0), (D_3, 0.2), (D_4, 0.8), (D_5, 0)\}$
4	NL and Z	$\{(D_1, 0), (D_2, 0), (D_3, 0.8), (D_4, 0.2), (D_5, 0)\}$
5	NL and PS	$\{(D_1, 0.65), (D_2, 0.35), (D_3, 0), (D_4, 0), (D_5, 0)\}$
6	NL and PM	$\{(D_1, 0.85), (D_2, 0.15), (D_3, 0), (D_4, 0), (D_5, 0)\}$
7	NL and PL	$\{(D_1, 0.95), (D_2, 0.05), (D_3, 0), (D_4, 0), (D_5, 0)\}$
8	NM and NL	$\{(D_1, 0), (D_2, 0), (D_3, 0.1), (D_4, 0.9), (D_5, 0)\}$
9	NM and NM	$\{(D_1, 0), (D_2, 0), (D_3, 0.7), (D_4, 0.3), (D_5, 0)\}$
10	NM and NS	$\{(D_1, 0), (D_2, 0.7), (D_3, 0.3), (D_4, 0), (D_5, 0)\}$
11	NM and Z	$\{(D_1, 0), (D_2, 0.9), (D_3, 0.1), (D_4, 0), (D_5, 0)\}$
12	NM and PS	$\{(D_1, 0.8), (D_2, 0.2), (D_3, 0), (D_4, 0), (D_5, 0)\}$
13	NM and PM	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
14	NM and PL	$\{(D_1, 0.99), (D_2, 0.01), (D_3, 0), (D_4, 0), (D_5, 0)\}$
15	NS and NL	$\{(D_1, 0), (D_2, 0), (D_3, 0.4), (D_4, 0.6), (D_5, 0)\}$
16	NS and NM	$\{(D_1, 0), (D_2, 0), (D_3, 0.8), (D_4, 0.2), (D_5, 0)\}$
17	NS and NS	$\{(D_1, 0), (D_2, 0.3), (D_3, 0.6), (D_4, 0.1), (D_5, 0)\}$
18	NS and Z	$\{(D_1, 0.1), (D_2, 0.7), (D_3, 0.2), (D_4, 0), (D_5, 0)\}$
19	NS and PS	$\{(D_1, 0.7), (D_2, 0.3), (D_3, 0), (D_4, 0), (D_5, 1)\}$
20	NS and PM	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 1)\}$
21	NS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
22	NVS and NL	$\{(D_1, 0.02), (D_2, 0.11), (D_3, 0.39), (D_4, 0.48), (D_5, 0)\}$
23	NVS and NM	$\{(D_1, 0.1), (D_2, 0.78), (D_3, 0.12), (D_4, 0), (D_5, 0)\}$
24	NVS and NS	$\{(D_1, 0.36), (D_2, 0.64), (D_3, 0), (D_4, 0), (D_5, 0)\}$
25	NVS and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
26	NVS and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
27	NVS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
28	NVS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
29	Z and NL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
30	Z and NM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
31	Z and NS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
32	Z and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
33	Z and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
34	Z and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
35	Z and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
36	PS and NL	$\{(D_1, 0.39), (D_2, 0.61), (D_3, 0), (D_4, 0), (D_5, 0)\}$
37	PS and NM	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
38	PS and NS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
39	PS and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
40	PS and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
41	PS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
42	PS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
43	PM and NL	$\{(D_1, 0.1), (D_2, 0.9), (D_3, 0), (D_4, 0), (D_5, 0)\}$
44	PM and NM	$\{(D_1, 0.3), (D_2, 0.7), (D_3, 0), (D_4, 0), (D_5, 0)\}$
45	PM and NS	$\{(D_1, 0.85), (D_2, 0.15), (D_3, 0), (D_4, 0), (D_5, 0)\}$
46	PM and Z	$\{(D_1, 0.98), (D_2, 0.02), (D_3, 0), (D_4, 0), (D_5, 0)\}$
47	PM and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
48	PM and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
49	PM and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
50	PL and NL	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
51	PL and NM	$\{(D_1, 0.99), (D_2, 0.01), (D_3, 0), (D_4, 0), (D_5, 0)\}$
52	PL and NS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
53	PL and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
54	PL and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
55	PL and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
56	PL and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$

Step 2: Update the BRB
The input values [$FlowDiff(t), PressureDiff(t)$] need to be transformed and represented in terms of the referential values as defined in Eqs. (32) and (33) at time instant$t$. The detail transformation processes have been developed and discussed (Xu et al., 2007; Yang, 2001; Yang, Liu, et al., 2006). Then the recursive algorithm (30) with constraints (11a)–(11d), (37), (38) is used to update the BRB. The updated BRB and rule weights are listed in Table A-II of Appendix A. Fig. 4 shows that the updated BRB can closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data. It is obvious that the updated BRB in Table A-II can satisfy the constraints (36)–(38), i.e., the certain running patterns of pipeline leak are not violated. Moreover, the calculation speed of the recursive algorithm is very high.

Step 3: Test
For testing the updated BRB, all the 2008 data shown in Figs. 1 and 2 are used. Fig. 5 gives the observed *LeakSize* and the estimated *LeakSize* for the same antecedent values [$FlowDiff(t), PressureDiff(t)$]. It demonstrates that the estimated outcomes match the observed ones very closely. Fig. 6 displays the observed and estimated *LeakSize* on the time scale. It shows that rule base can detect the leak which happened at around 9:38 a.m. and ended at around 10:53 a.m.

Table A-II
Updated belief rules and rule weights under expert intervention

Rule number	Updated rule weight	*FlowDiff* and *PressureDiff*	*LeakSize* distribution $\{D_1, D_2, D_3, D_4, D_5\} = \{0, 2, 4, 6, 8\}$
1	1	NL and NL	$\{(D_1, 0.0004), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0.9996)\}$
2	1	NL and NM	$\{(D_1, 0.0004), (D_2, 0), (D_3, 0), (D_4, 0.2999), (D_5, 0.6997)\}$
3	0.9767	NL and NS	$\{(D_1, 0.0007), (D_2, 0.1038), (D_3, 0.1946), (D_4, 0.2980), (D_5, 0.4028)\}$
4	0.022	NL and Z	$\{(D_1, 0.0701), (D_2, 0.0927), (D_3, 0.1851), (D_4, 0.2792), (D_5, 0.3729)\}$
5	0.001	NL and PS	$\{(D_1, 0.6115), (D_2, 0.0317), (D_3, 0.1258), (D_4, 0.2310), (D_5, 0)\}$
6	1	NL and PM	$\{(D_1, 0.8325), (D_2, 0.1648), (D_3, 0), (D_4, 0), (D_5, 0)\}$
7	1	NL and PL	$\{(D_1, 0.9451), (D_2, 0.0549), (D_3, 0), (D_4, 0), (D_5, 0)\}$
8	1	NM and NL	$\{(D_1, 0.0007), (D_2, 0.0010), (D_3, 0.0997), (D_4, 0.8996), (D_5, 0)\}$
9	1	NM and NM	$\{(D_1, 0.0011), (D_2, 0.0009), (D_3, 0..6988), (D_4, 0.2992), (D_5, 0)\}$
10	0.0513	NM and NS	$\{(D_1, 0.0906), (D_2, 0.1046), (D_3, 0.2862), (D_4, 0.5186), (D_5, 0)\}$
11	0.0004	NM and Z	$\{(D_1, 0.6194), (D_2, 0.0403), (D_3, 0.1189), (D_4, 0.2213), (D_5, 0)\}$
12	0.0012	NM and PS	$\{(D_1, 0.6498), (D_2, 0.0500), (D_3, 0.1177), (D_4, 0.1825), (D_5, 0)\}$
13	1	NM and PM	$\{(D_1, 0.8869), (D_2, 0.1101), (D_3, 0.0012), (D_4, 0.0018), (D_5, 0)\}$
14	1	NM and PL	$\{(D_1, 0.9876), (D_2, 0.0112), (D_3, 0.0005), (D_4, 0.0007), (D_5, 0)\}$
15	1	NS and NL	$\{(D_1, 0.0004), (D_2, 0.0009), (D_3, 0.3995), (D_4, 0.5992), (D_5, 0)\}$
16	1	NS and NM	$\{(D_1, 0.0004), (D_2, 0.0009), (D_3, 0.7994), (D_4, 0.1994), (D_5, 0)\}$
17	1	NS and NS	$\{(D_1, 0.0360), (D_2, 0.2893), (D_3, 0.5785), (D_4, 0.0963), (D_5, 0)\}$
18	1	NS and Z	$\{(D_1, 0.4604), (D_2, 0.3851), (D_3, 0.1252), (D_4, 0.0283), (D_5, 0)\}$
19	1	NS and PS	$\{(D_1, 0.6697), (D_2, 0.3294), (D_3, 0.0004), (D_4, 0.0006), (D_5, 0)\}$
20	1	NS and PM	$\{(D_1, 0.8876), (D_2, 0.1101), (D_3, 0.0009), (D_4, 0.0014), (D_5, 0)\}$
21	1	NS and PL	$\{(D_1, 0.9992), (D_2, 0.0001), (D_3, 0.0003), (D_4, 0.0004), (D_5, 0)\}$
22	1	NVS and NL	$\{(D_1, 0.0004), (D_2, 0.1122), (D_3, 0.3978), (D_4, 0.4896), (D_5, 0)\}$
23	1	NVS and NM	$\{(D_1, 0.0114), (D_2, 0.8559), (D_3, 0.1317), (D_4, 0.0010), (D_5, 0)\}$
24	1	NVS and NS	$\{(D_1, 0.2963), (D_2, 0.7017), (D_3, 0.0010), (D_4, 0.0009), (D_5, 0)\}$
25	0.9998	NVS and Z	$\{(D_1, 0.9925), (D_2, 0.0026), (D_3, 0.0025), (D_4, 0.0024), (D_5, 0)\}$
26	1	NVS and PS	$\{(D_1, 0.9969), (D_2, 0.0010), (D_3, 0.0010), (D_4, 0.0010), (D_5, 0)\}$
27	1	NVS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
28	1	NVS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
29	1	Z and NL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
30	1	Z and NM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
31	0.9993	Z and NS	$\{(D_1, 0.9948), (D_2, 0.0020), (D_3, 0.0018), (D_4, 0.0015), (D_5, 0)\}$
32	0.9884	Z and Z	$\{(D_1, 0.9920), (D_2, 0.0010), (D_3, 0.0026), (D_4, 0.0044), (D_5, 0)\}$
33	0.9993	Z and PS	$\{(D_1, 0.9910), (D_2, 0.0018), (D_3, 0.0030), (D_4, 0.0042), (D_5, 0)\}$
34	1	Z and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
35	1	Z and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
36	1	PS and NL	$\{(D_1, 0.0004), (D_2, 0.9996), (D_3, 0), (D_4, 0), (D_5, 0)\}$
37	1	PS and NM	$\{(D_1, 0.4548), (D_2, 0.5416), (D_3, 0), (D_4, 0), (D_5, 0)\}$
38	0.9985	PS and NS	$\{(D_1, 0.5368), (D_2, 0.1568), (D_3, 0.1545), (D_4, 0.1519), (D_5, 0)\}$
39	0.9999	PS and Z	$\{(D_1, 0.6935), (D_2, 0.0548), (D_3, 0.1017), (D_4, 0.1499), (D_5, 0)\}$
40	0.9929	PS and PS	$\{(D_1, 0.9224), (D_2, 0.0120), (D_3, 0.0255), (D_4, 0.0401), (D_5, 0)\}$
41	1	PS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
42	1	PS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
43	1	PM and NL	$\{(D_1, 0.0114), (D_2, 0.9886), (D_3, 0), (D_4, 0), (D_5, 0)\}$
44	1	PM and NM	$\{(D_1, 0.2311), (D_2, 0.7689), (D_3, 0), (D_4, 0), (D_5, 0)\}$
45	1	PM and NS	$\{(D_1, 0.6790), (D_2, 0.3190), (D_3, 0.0010), (D_4, 0.0010), (D_5, 0)\}$
46	1	PM and Z	$\{(D_1, 0.7940), (D_2, 0.0454), (D_3, 0.0634), (D_4, 0.0971), (D_5, 0)\}$
47	0.9941	PM and PS	$\{(D_1, 0.9544), (D_2, 0.0068), (D_3, 0.0150), (D_4, 0.0238), (D_5, 0)\}$
48	1	PM and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
49	1	PM and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
50	1	PL and NL	$\{(D_1, 0.8902), (D_2, 0.1098), (D_3, 0), (D_4, 0), (D_5, 0)\}$
51	1	PL and NM	$\{(D_1, 0.9890), (D_2, 0.0110), (D_3, 0), (D_4, 0), (D_5, 0)\}$
52	1	PL and NS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
53	1	PL and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
54	1	PL and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
55	1	PL and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
56	1	PL and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$

4.6. Comparative studies

In order to demonstrate the validity of the proposed algorithm further, the recursive algorithm without expert intervention (Zhou et al., in review) and the training data in Section 4.5 are used to update the initial BRB in Table A-I. The updated BRB without expert intervention is given in Table A-III of Appendix A, can also closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data and may be used for pipeline leak detection (Zhou et al., in review). However, the rules marked in gray as shown in Table A-III cannot satisfy the constraints (36)–(38). For example, according to the running patterns of pipeline leak, rule 12–14 and 19–21 denote the normal patterns, however, the updated belief degrees to D_5, which denotes consequence when the leak is most severe, are not zero and conflict with the patterns of the pipeline leak. On the contrary, the updated BRB under expert intervention given in Table A-II satisfy the constraints (36)–(38), i.e., the given patterns are not violated.

Therefore, the proposed recursive algorithm under expert intervention can be used to update the BRB online and the updated BRB satisfies the running patterns of the simulated systems.

4.7. Concluding remarks

When the leak size is 16% and 4%, the similar results can also be obtained. From Fig. 4, we can see that there is noise in the 25% leak detected, which may be due to noise in data recorded from instruments and can trigger false leak alarms. Therefore, in a real leak detection system, some kind of noise reduction process to smooth

Table A-III
Updated belief rules and rule weights without expert intervention

Rule number	Updated rule weight	*FlowDiff* and *PressureDiff*	*LeakSize* distribution $\{D_1,D_2,D_3,D_4,D_5\} = \{0,2,4,6,8\}$
1	1	NL and NL	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 1)\}$
2	1	NL and NM	$\{(D_1, 0), (D_2, 0), (D_3, 0), (D_4, 0.3), (D_5, 0.7)\}$
3	1	NL and NS	$\{(D_1, 0.001), (D_2, 0.001), (D_3, 0.0473), (D_4, 0.4727), (D_5, 0.4780)\}$
4	0.7669	NL and Z	$\{(D_1, 0.0061), (D_2, 0.0075), (D_3, 0.0106), (D_4, 0.5417), (D_5, 0.4341)\}$
5	0.8852	NL and PS	$\{(D_1, 0.0079), (D_2, 0.0092), (D_3, 0.001), (D_4, 0.4399), (D_5, 0.542)\}$
6	1	NL and PM	$\{(D_1, 0.85), (D_2, 0.15), (D_3, 0), (D_4, 0), (D_5, 0)\}$
7	1	NL and PL	$\{(D_1, 0.95), (D_2, 0.05), (D_3, 0), (D_4, 0), (D_5, 0)\}$
8	0.9616	NM and NL	$\{(D_1, 0.1619), (D_2, 0.0771), (D_3, 0.1005), (D_4, 0.6595), (D_5, 0.0009)\}$
9	1	NM and NM	$\{(D_1, 0.1641), (D_2, 0.0865), (D_3, 0..5916), (D_4, 0.1566), (D_5, 0.0011)\}$
10	0.8619	NM and NS	$\{(D_1, 0.0707), (D_2, 0.0392), (D_3, 0.0303), (D_4, 0.3982), (D_5, 0.4615)\}$
11	0.562	NM and Z	$\{(D_1, 0.0678), (D_2, 0.0786), (D_3, 0.0131), (D_4, 0.4946), (D_5, 0.346)\}$
12	0.9196	NM and PS	$\{(D_1, 0.0142), (D_2, 0.0048), (D_3, 0.0079), (D_4, 0.486), (D_5, 0.4871)\}$
13	0.955	NM and PM	$\{(D_1, 0.4595), (D_2, 0.0128), (D_3, 0.0014), (D_4, 0.179), (D_5, 0.3473)\}$
14	0.9981	NM and PL	$\{(D_1, 0.3508), (D_2, 0.001), (D_3, 0.0022), (D_4, 0.2097), (D_5, 0.4363)\}$
15	1	NS and NL	$\{(D_1, 0.2221), (D_2, 1111), (D_3, 0.3248), (D_4, 0.3411), (D_5, 0.001)\}$
16	0.9999	NS and NM	$\{(D_1, 0.1458), (D_2, 0.0762), (D_3, 0.69), (D_4, 0.0853), (D_5, 0.0026)\}$
17	1	NS and NS	$\{(D_1, 0.0554), (D_2, 0.3101), (D_3, 0.5708), (D_4, 0.0623), (D_5, 0.0015)\}$
18	0.9978	NS and Z	$\{(D_1, 0.0479), (D_2, 0.5779), (D_3, 0.1816), (D_4, 0.074), (D_5, 0.1186)\}$
19	0.9939	NS and PS	$\{(D_1, 0.5544), (D_2, 0.2444), (D_3, 0.0008), (D_4, 0.0688), (D_5, 0.1316)\}$
20	0.9521	NS and PM	$\{(D_1, 0.4925), (D_2, 0.0053), (D_3, 0.0009), (D_4, 0.1703), (D_5, 0.3311)\}$
21	0.9597	NS and PL	$\{(D_1, 0.5472), (D_2, 0.001), (D_3, 0.001), (D_4, 0.1434), (D_5, 0.3075)\}$
22	0.9946	NVS and NL	$\{(D_1, 0.0406), (D_2, 0.1176), (D_3, 0.3825), (D_4, 0.4584), (D_5, 0.001)\}$
23	1	NVS and NM	$\{(D_1, 0.1015), (D_2, 0.7771), (D_3, 0.1194), (D_4, 0.001), (D_5, 0.001)\}$
24	1	NVS and NS	$\{(D_1, 0.3590), (D_2, 0.6381), (D_3, 0.001), (D_4, 0.0009), (D_5, 0.0009)\}$
25	0.9674	NVS and Z	$\{(D_1, 0.9496), (D_2, 0.0007), (D_3, 0), (D_4, 0.001), (D_5, 0.001)\}$
26	1	NVS and PS	$\{(D_1, 0.998), (D_2, 0), (D_3, 0), (D_4, 0.001), (D_5, 0.001)\}$
27	1	NVS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
28	1	NVS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
29	1	Z and NL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
30	1	Z and NM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
31	0.9996	Z and NS	$\{(D_1, 0.9948), (D_2, 0.0006), (D_3, 0.001), (D_4, 0.0015), (D_5, 0.002)\}$
32	0.9899	Z and Z	$\{(D_1, 0.9475), (D_2, 0.0011), (D_3, 0.0002), (D_4, 0.0161), (D_5, 0.0351)\}$
33	0.9994	Z and PS	$\{(D_1, 0.9931), (D_2, 0.0001), (D_3, 0.001), (D_4, 0.0022), (D_5, 0.0036)\}$
34	1	Z and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
35	1	Z and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
36	1	PS and NL	$\{(D_1, 0.39), (D_2, 0.61), (D_3, 0), (D_4, 0), (D_5, 0)\}$
37	1	PS and NM	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
38	0.9962	PS and NS	$\{(D_1, 0.9681), (D_2, 0.001), (D_3, 0.0008), (D_4, 0.0101), (D_5, 0.0201)\}$
39	0.9999	PS and Z	$\{(D_1, 0.792), (D_2, 0.0089), (D_3, 0.0024), (D_4, 0.0659), (D_5, 0.1308)\}$
40	0.9933	PS and PS	$\{(D_1, 0.9532), (D_2, 0.0005), (D_3, 0.0006), (D_4, 0.015), (D_5, 0.0307)\}$
41	1	PS and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
42	1	PS and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
43	1	PM and NL	$\{(D_1, 0.1), (D_2, 0.9), (D_3, 0), (D_4, 0), (D_5, 0)\}$
44	1	PM and NM	$\{(D_1, 0.3), (D_2, 0.7), (D_3, 0), (D_4, 0), (D_5, 0)\}$
45	0.9985	PM and NS	$\{(D_1, 0.8354), (D_2, 0.144), (D_3, 0.0009), (D_4, 0.0064), (D_5, 0.0133)\}$
46	1	PM and Z	$\{(D_1, 0.8526), (D_2, 0.0027), (D_3, 0.0009), (D_4, 0.0475), (D_5, 0.0962)\}$
47	0.9955	PM and PS	$\{(D_1, 0.9723), (D_2, 0.0002), (D_3, 0.0008), (D_4, 0.0086), (D_5, 0.0182)\}$
48	1	PM and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
49	1	PM and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
50	1	PL and NL	$\{(D_1, 0.9), (D_2, 0.1), (D_3, 0), (D_4, 0), (D_5, 0)\}$
51	1	PL and NM	$\{(D_1, 0.99), (D_2, 0.01), (D_3, 0), (D_4, 0), (D_5, 0)\}$
52	1	PL and NS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
53	1	PL and Z	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
54	1	PL and PS	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
55	1	PL and PM	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$
56	1	PL and PL	$\{(D_1, 1), (D_2, 0), (D_3, 0), (D_4, 0), (D_5, 0)\}$

data should be included. On the other hand, in order to avoid false leak alarms further, a confirmation period can be used. That is, before a leak alarm can be confirmed, the estimated *LeakSize* needs to stay positive continuously for a specified period. The lengths of the confirmation period vary with some factors such as the quality of the instruments and so on. Thus, another BRB expert system can also be established and updated to determine the confirmation period (Xu et al., 2007).

In summary, the initial belief rules for pipeline leak detection given by an expert are not accurate. When some running patterns about the system are given the new information becomes available, the proposed recursive algorithm can update the BRB expert systems quickly. Moreover, the updated BRB can satisfy the given running patterns.

5. Conclusions

In this paper, a feasible algorithm under expert intervention to online update the belief rule based (BRB) expert system is proposed. Then the recursive algorithm is used to update the BRB expert system for pipeline leak detection. The study demonstrates that by using the recursive algorithm under expert intervention, the BRB system cannot only learn the relationship between leak sizes and the pipeline flow and pressure readings from pipeline operating data, but also can be automatically tuned and satisfy the running patterns of pipeline leak which are converted into some constraints and added into the recursive algorithm. In addition, the more expert knowledge is obtained and used in the process of updating the BRB expert system, the updated BRB can capture the real system more closely.

Compared with the off-line optimal methods for pipeline leak detection (Xu et al., 2007), the proposed method can significantly reduce the parameter tuning time and can be used to update the BRB expert system online in this paper. The optimal parameters may also change with the aging of the pipeline and instruments (Xu et al., 2007), so the quick self-learning capability of the recursive algorithm is very useful to improve the performance of the system.

From this study, we can also see that based on the RIMER, the proposed recursive algorithm in this paper is mainly composed of two parts: the optimal algorithm and the constraints which represent the running patterns and are determined by human experts. Compared with the optimal algorithms (Yang et al., 2007; Zhou et al., in review), there is no sufficient theory to convert the running patterns of the systems into the corresponding constraints. In other words, it is very important to convert the expert knowledge into some feasible constraints, which should be studied in the future.

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Appendix A. The initial and updated BRB expert systems

See Appendix Tables A-I, A-II, A-III.

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