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A new modeling and inference approach for the belief rule base with attribute reliability

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Abstract

A belief rule-based (BRB) model with attribute reliability (BRB-r) has been developed recently, where the systematic uncertainty is regarded as attribute reliability by extending the traditional BRB model. The BRB-r model provides a framework to deal with the systematic uncertainty, but the drawbacks in modeling and inference reduces the accuracy of it. This paper proposed a new modeling and inference approach to improve the effectiveness of the BRB-r. This approach is constituted by two parts: data processing and BRB inference. In the data processing, the attribute reliability is calculated based on the auto regressive model, while the parameters of BRB-r are optimized using the differential evolution algorithm. In the BRB inference, a new attribute reliability fusion algorithm is proposed, which can effectively integrate attribute reliability into the BRB model and ensure the rationality in different situations. A benchmark case about pipeline leak detection and a practical case about condition monitoring are studied to demonstrate the rationality and feasibility of the proposed approach to the BRB-r model.

Keywords Belief rule-based model with attribute reliability (BRB-r) · Attribute reliability · Systematic uncertainty · Auto regressive (AR) model

1 Introduction

In recognition of the need to model and analyze complex systems using hybrid information with uncertainty, the belief rule-based (BRB) model has been proposed in 2006 [29]. The BRB model consists of the belief rule base [30] and the evidential reasoning (ER) algorithm [8, 21]. Since proposed, the BRB model has been successfully used in multiple fields such as fault detection [26, 34, 35], complex system modeling [5, 35] and medical diagnoses [12, 33].

The belief rule base is a collection of multiple belief rules in the same belief structure, which enable the BRB model to describe various types of information and knowledge with uncertainties [5]. Moreover, as the extension of D-S evidence theory, the ER algorithm is applied in the inference of BRB. It has the advantages of processing uncertain information. In recent years, the BRB model often utilized in modeling, analysis and evaluation of complex systems with uncertainty. Uncertainty can be divided into two types: the systematic uncertainty and the random uncertainty [23]. The systematic uncertainty refers to the bias error caused by some fixed reasons in the measurement process, such as degeneration of measuring equipment and influence of external environment (temperature, humidity, electromagnetic field, etc.) [11, 18]. The random uncertainty refers to the compensatory error caused by some unstable causes, such as instability of external environment and improper operation [1]. Take the measurement of *P30* as an example, if the measuring instrument is biased, same systematic uncertainty will exist in the *P30* measured all the time. If the measuring instrument is unbiased, but the environment changes

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dramatically (e.g., earthquake), then the *P30* measured at that moment will have random uncertainty. The existence of the uncertainty brings challenges to the modeling and analysis of complex systems. As a common tool for complex system analysis, traditional BRB model describes the random uncertainty through the belief structure. However, the systematic uncertainty is not taken into account. Since the systematic uncertainty and the random uncertainty are both the constituent of uncertainty, which are difficult to avoid in data acquisition. Integrating systematic uncertainty into the BRB model is significant for improving the modeling accuracy.

The BRB model with systematic uncertainty, called the BRB-r model, has been proposed by Feng et al. in 2018 [9] on the basis of the ER rule [31]. In the BRB-r model, "r" is an abbreviation for attribute reliability, which is used to measure the systematic uncertainty exists in the attributes. Based on the existing work, there are two key problems need to be solved in the modeling and inference of the BRB-r: First, how to obtain attribute reliability from data; Second, how to integrate attribute reliability into the modeling.

To obtain the attribute reliability from data, a statistics algorithm is proposed in the BRB-r (recorded as BRB-r (pre) in this paper) model in [9]. In this algorithm, a tolerance range is constructed to select the outliers in the dataset, and the attribute reliability is defined as the ratio of the amount of these outliers to the size of this dataset. The systematic uncertainty refers to the deviation of all samples in a certain direction caused by some fixed factors, rather than the ratio of outliers. This algorithm does not meet the causes of systematic uncertainty. Therefore, it is not suitable for attribute reliability calculation. Other drawbacks of this method are detailed in Section 3.1.

To integrate the attribute reliability into the traditional BRB model, the meaning of systematic uncertainty needs to be considered. When the systematic uncertainty is equal to zero (i.e., the attribute reliability is equal to one), it means that this attribute data is completely trusted. The influence of systematic uncertainty on the analysis result can be ignored in the inference, and the BRB-r model should degenerate into the BRB model. When the systematic uncertainty is equal to one (i.e., attribute reliability is equal to zero), it means that this attribute data is completely untrusted and is not suitable to be used for further research. In current study of the BRB-r (pre) model, the fusion algorithm is an extension of the ER rule [31]. However, this algorithm is unreasonable in these two extreme cases, which questions the rationality of it in other cases.

In order to overcome the drawbacks of the BRB-r (pre) model, a new modeling and inference approach for the belief rule base with attribute reliability, recorded as the BRB-r (new) model, is proposed in this paper. The BRB-r (new) model contains a new calculation algorithm and a new fusion algorithm for attribute reliability, which overcome the drawbacks of the BRB-r (pre) model. To obtain attribute reliability from data, the new calculation algorithm establishes an auto regressive model (AR model) to quantify the reliability of data. This algorithm is consistent with the meaning of systematic uncertainty and does not rely on expert knowledge. To integrate the attribute reliability into the BRB-r model, the *joint attribute importance factor* is constructed in the new fusion algorithm. The *joint attribute importance factor* guarantees that when the attribute data is completely trusted, the BRB-r model will degenerates into the BRB model; when the attribute data is completely untrusted, the attribute will not affect the further modeling and analysis. A benchmark case and a practical case are studied to prove the feasibility and accuracy of the BRB-r (new) model in practical applications.

The rest of this paper is organized as follows: Section 2 briefly introduces the concepts of the traditional BRB model and the BRB-r model. Sections 3 and 4 account for the drawbacks of the BRB-r (pre) model in the calculation and fusion of attribute reliability respectively, and offer the solutions in the BRB-r (new) model. Section 5 details the steps of the modeling and inference approach of BRB-r (new) model. In Section 6, a benchmark case about pipeline leak detection and a practical case about condition monitoring are studied to illustrate the feasibility and significance of the BRB-r (new) model in practical applications. Some concluding remarks are presented in Section 7.

2 Preliminary

In this section, the basic knowledge of the BRB model and the BRB-r model are briefly introduced as the foundation of this study. Further details on the BRB model and the BRB-r model can be found in references [9, 28, 29] and [30], respectively.

2.1 BRB model

The first step in the modeling and analysis of the BRB model is to build a belief rule base. The belief rule base is composed of a set of belief rules. The influence of the independent variables on the dependent variable recorded through the form of "IF-THEN" in the belief rules, where "IF" represents the independent variables and "THEN" represents the dependent variable. The relationship between the independent variables and the dependent variable in the belief rules is many-to-one. The k-th rule can be described as follows [29]:

$$\begin{aligned} &R_k : IF\ (x_1\ is\ A_1^k) \wedge (x_2\ is\ A_2^k) \wedge \cdots \wedge (x_M\ is\ A_M^k), \\ &THEN\ \{(D_1, \beta_{1,k}), \cdots, (D_N, \beta_{N,k})\}, \\ &with\ rule\ weight\ \theta_k\ and\ attribute\ weight\ \delta_1, \delta_2, \cdots, \delta_M, \end{aligned} \tag{1}$$

where $x_m(m = 1, 2, \cdots, M)$ represents the m-th independent variable, and M denotes the number of them. $A_m^k(k = 1, 2, \cdots, K)$ represents the reference value of the m-th independent variable in the k-th rule, and K records the number of belief rules. When an independent variable is discrete, the reference values are the discrete values of this independent variables; when an independent variable is continuous, some values within the range of it are selected as the reference values. The reference values serve to divide the independent variables. D represents the dependent variable divided into N ranks, while the belief degree of the n-th rank of the dependent variable $D_n(n = 1, 2, \cdots, N)$ in k-th rule is recorded as $\beta_{n,k}$. In the BRB model, the independent variable is called attribute, and the dependent variable is called consequent. Each belief rule builds the relationship between the independent variables and the dependent variable from different perspectives in the form of extended "IF-THEN". The dependent variable usually represents the consequence, while the independent variables represent the attributes associated with it. For example, the fuel flow and pressure ratio (*phi*) is one of the important manifestations of turbofan engine, but it is difficult to measure accurately. Considering that the outlet pressure (*P30*), the static pressure (*Ps30*) and the fan speed (*NRf*) have an influence on *phi*, *P30*, *Ps30*, and *NRf* can be regarded as the independent variables (attributes), and the *phi* is regarded as the dependent variable (consequence) to build several belief rules.

The belief rule base contains a large number of rules, but not every rule will be used in the aggregation. Therefore, the second step is to activate belief rules based on the inputs. The inputs of the BRB model are the independent variables expressed in the form of a belief structure, which are recorded as follows [29]:

$$(x_1, \varepsilon_1) \wedge (x_2, \varepsilon_2) \wedge \cdots \wedge (x_M, \varepsilon_M), \tag{2}$$

where $x_m(m = 1, 2, \cdots, M)$ is the input of the m-th attribute. $\varepsilon_m(m = 1, 2, \cdots, M)$ denotes the data belief of x_m, which describes the degree of random uncertainty in the input information. The degree of activation for the k-th rule can be calculated as follows:

$$\omega_k = \frac{\theta_k \prod_{i=1}^{T_k} (\alpha_{i,j}^k)^{\overline{\delta}_i}}{\sum_{l=1}^{K} \left[\theta_l \prod_{i=1}^{T_l} (\alpha_{i,j}^l)^{\overline{\delta}_i} \right]} \ and \ \overline{\delta}_i = \frac{\delta_i}{\max_{j=1,2,\cdots,T_k} \{\delta_j\}}. \tag{3}$$

$\alpha_{i,j}^k(i = 1, 2, \cdots, T_k)$, called the *individual matching degree*, is the degree of belief to which the input for the i-th attribute belongs to its j-th referential value $A_{i,j}^k$ in the k-th rule. If $\omega_k \neq 0$, the k-th rule is activated. The *individual matching degree* can be calculated in different ways depending on the situations [28, 30], this paper using (4) to obtain it [29]:

$$\alpha_{i,j}^k = \frac{\varphi(x_i, A_{i,j}^k)\varepsilon_i}{\sum_{l=1}^{T_k} \varphi(x_l, A_{l,j}^k)},$$
$$\varphi(x_i, A_{i,j}^k) = \begin{cases} \frac{A_{i,l+1}^k - x_i}{A_{i,l+1}^k - A_{i,l}^k}, & j = l(A_{i,l}^k \leq x_i \leq A_{i,l+1}^k) \\ \frac{x_i - A_{i,l}^k}{A_{i,l+1}^k - A_{i,l}^k}, & j = l + 1(A_{i,l}^k \leq x_i \leq A_{i,l+1}^k) \\ 0, & else \end{cases}. \tag{4}$$

Thus, the input of the BRB model can be equivalently transformed as belief degrees $S(x_i^*) = (A_{i,j}^k, \alpha_{i,j}^k)$ to describe referential values of k-th rule.

The third step is to aggregate the activated belief rules through the ER algorithm using (5) [29, 30]:

$$\beta_n = \frac{\mu\left[\prod_{k=1}^K (\omega_k\beta_{n,k} + 1 - \lambda - \prod_{k=1}^K (1-\lambda)\right]}{1 - \mu\left[\prod_{k=1}^K (1-\omega_k)\right]},$$
$$\mu = \left[\sum_{n=1}^N \prod_{k=1}^K (\omega_k\beta_{n,k} + 1 - \lambda) - (N-1)\prod_{k=1}^K (1-\lambda)\right]^{-1},$$
$$\lambda = \omega_k \sum_{n=1}^N \beta_{n,k}. \tag{5}$$

Based on the results calculated by (5), we have the following to describe the aggregated results [28]:

$$S = \{(D_n, \beta_n), \ n = 1, 2, \cdots, N\} \tag{6}$$

According to the utility theory, the analysis result can be calculated as $\sum_{n=1}^{N} D_n\beta_n$ [3]

As shown in (2), the random uncertainty in the input data is described by data belief. However, the systematic uncertainty has not been taken into account in the modeling and analysis of the BRB model. In view of that the systematic uncertainty exists widely in data sets, introducing systematic uncertainty is of great significance for enhancing the reliability and robustness of the BRB model.

2.2 BRB-r model

In order to handle the systematic uncertainty, a BRB-r model is proposed by Feng, etc. in 2018 [9], where the systematic uncertainty is described by attribute reliability. Compare with the traditional BRB model, the attribute reliability is added as a new characteristic of the attributes into the belief rules. The k-th rule in the BRB-r can be described as follow:

$$\begin{aligned} &R_k : IF \ (x_1 \ is \ \wedge (x_2 \ is \ A_2^k) \wedge \cdots \wedge (x_M \ is \ A_M^k), \\ &THEN \ \{(D_1, \beta_{1,k}), \cdots, (D_N, \beta_{N,k})\}, \\ &with \ rule \ weight \ \theta_k, \ attribute \ weight \ \delta_1, \delta_2, \cdots, \delta_M \\ &and \ attribute \ reliability \ r_1, r_2, \cdots, r_M \end{aligned} \tag{7}$$

The i-th attribute reliability, recorded as r_i, is used to describe the systematic uncertainty in the i-th attribute.

In the BRB-r model, the attribute reliability is recorded as $r_m(m = 1, 2, \ldots, M)$, the attribute weight is recorded as δ_m, while the data belief is recorded as ε_m. The attribute reliability describes the systematic uncertainty, while the data belief describes of the random uncertainty. The systematic uncertainty and random uncertainty are independent of each other. The systematic uncertainty (attribute reliability) of a set of data for an attribute is the same, but the random uncertainty (data belief) of each of the records can be different. Both attribute weights and attribute reliabilities are the descriptions for attributes, which should affect the attributes in a similar way in the BRB-r model. The former is used to describe the importance of attribute while the latter is used to describe the systematic uncertainty.

There are two key problems need to be solved in the modeling and inference process of the BRB-r: First, how to obtain attribute reliability from data; Second, how to integrate attribute reliability into the modeling.

3 Attribute reliability calculation

The attribute reliability has a direct impact on the analysis results of the BRB-r model. Incorrect attribute reliability will result in incorrect results. Therefore, it is important to obtain attribute reliability from data accurately. This section introduces the calculation algorithm of the BRB-r (pre) model and analyzes the drawbacks of it. A new calculation algorithm for the BRB-r (new) model is proposed to overcome the drawbacks above.

3.1 Existing calculation algorithm and drawbacks

(1) Calculation algorithm

The calculation algorithm of attribute reliability in the BRB-r (pre) model is a statistic based algorithm [9]. This algorithm needs to determine a tolerance range, and the observation data within this tolerance range will be regarded as the reliable data. The attribute reliability is calculated by the ratio of reliable data to total data.

Suppose that the observation data for the i-th attribute are $x_i(1), x_i(2), \cdots, x_i(m_i), (i = 1, 2, \cdots, M)$. m_i is the amount of the observation data of the i-th attribute and M denotes the number of attributes. The tolerance range for the i-th attribute is profiled as $(\bar{x}_i - \varphi\sigma_i, \bar{x}_i + \varphi\sigma_i)$. The $\bar{x}_i$ and the σ_i denote the mean value and the standard deviation of the observation data for the i-th attribute respectively. φ is regarded as the adjustment coefficient to adjust the extent of the tolerance range, which determined by experts. If $x_i(j) \in (\bar{x}_i - \varphi\sigma_i, \bar{x}_i + \varphi\sigma_i)$, the observation data is reliable and $y_{ij} = 1$. The reliability for the i-th attribute can be calculated as (8):

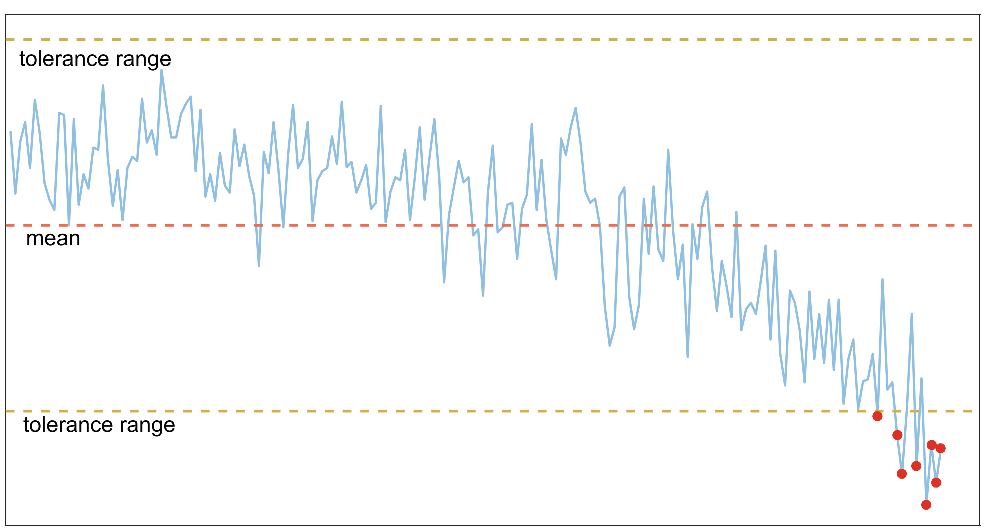
$$r_i = \frac{\sum_{j=1}^{m_i} y_{ij}}{m_i} \tag{8}$$

(2) Drawbacks

The drawbacks of the attribute reliability calculation algorithm in the BRB-r (pre) model will are analyzed in three aspects:

(a) Because the tolerance range is related to the mean and standard deviation of the observation data, the calculation algorithm in the BRB-r (pre) model seems to be a statistic based method. However, as an important parameter in tolerance range, the influence of the *adjustment coefficient* φ in attribute reliability calculation ought to be taken into consideration. It's obvious

Fig. 1 Degradation curve



that the different *adjustment coefficient* will bring different attribute reliability. Giving an accurate *adjustment coefficient* is a difficult task for the experts. If the experts can give the adjustment coefficient accurately, why not directly determine attribute reliability based on expert experience? Essentially, the calculation algorithm of attribute reliability described in Section 3.1 is an expert knowledge based method. The defects in human judgment will introduce uncertainty into the calculated attribute reliability again.

(b) When the fluctuation range of the observation data is wide, as shown in Fig. 1, this calculation algorithm [31] cannot effectively find out the unreliable points.

Figure 1 shows the performance degradation curve for a device over its lifetime, and the observation data is declining with fluctuations. The red dotted line indicates the mean of the data, and the yellow dotted line indicates the tolerance range when $\varphi = 1$. It's a high probability that the red points outside the tolerance range is caused by the wear of the device, and these points are not suitable to be regarded as the unreliable data. Therefore, this algorithm is irrational in the attribute reliability calculation when facing the degradation data.

(c) This algorithm seems more suitable for identifying random uncertainties in data. Figure 2 shows a set of random samples to the normal distribution, and the *adjustment coefficient* is set to 2. As shown in Fig. 2, the red points beyond the tolerance range are considered as the unreliable data in the BRB-r (pre) model.

These points obtained from the measurements are more suitable to be regarded as outliers, which are caused by the random uncertainty rather than the systematic uncertainty. The systematic uncertainty (attribute reliability) refers to the bias error caused by some fixed reasons in the measuring process. The difference between the observation value and the actual value is caused by both the systematic uncertainty and the random uncertainty, which can be described using (9) as follows:

$$x_i(j) = x_i\_act(j) + \varepsilon_i(j) + e_i, i = 1, 2, \cdots, M; j = 1, 2, \cdots, m_i \tag{9}$$

where $x_i\_act(j)$ denotes the j-th actual value for the i-th attribute, $\varepsilon_i(j)$ denotes the error caused by the random uncertainty, while e_i denotes the error caused by the systematic uncertainty. The systematic uncertainty is equal for all observation samples of the same attribute, but the random uncertainty is unequal. The relationship between the systematic uncertainty and the random uncertainty is shown in Fig. 3:

In Fig. 3, the blue dots represent observation data and the orange dots represent true data. As shown in Fig. 3, the deviations in the observation data and the actual data are consisted of the systematic uncertainty and the random uncertainty. The black dotted line inside the red dotted line represents the systematic uncertainty, which are equal for all groups of data. The solid black line indicate the random uncertainty, which are unequal for all groups of data. From the deviations exist in the observation data and actual data, the calculation algorithm in the BRB-r (pre) model is not applicable to determine the systematic uncertainty essentially.

3.2 New calculation algorithm

Through the drawbacks of the existing attribute reliability calculation algorithm, the proposed algorithm needs to satisfy: First, conforms to the meaning of systematic

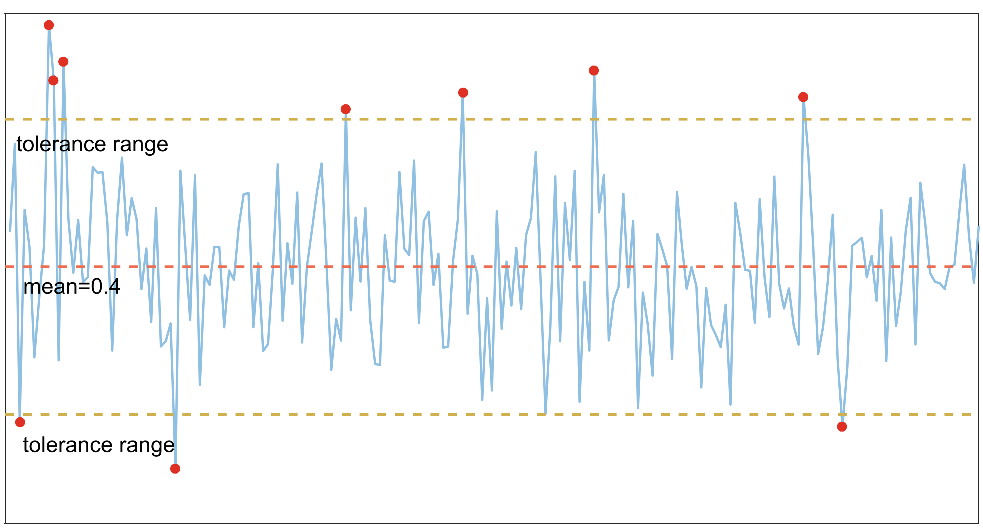


Fig. 2 Unreliable identification in the BRB-r (pre) model

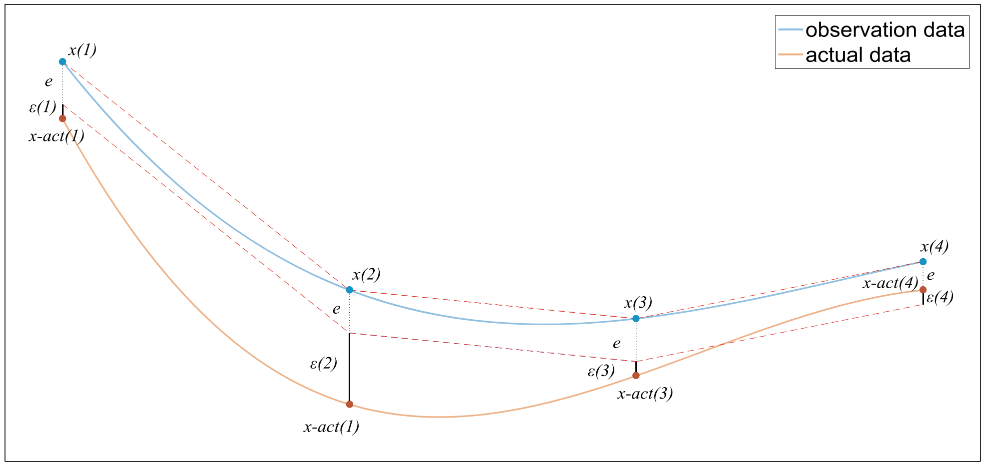


Fig. 3 Relationship between systematic uncertainty and random uncertainty

uncertainty described in (9); second, do not rely on expert knowledge.

In (9), the observation data can be regarded as the combination of the actual data, the systematic uncertainty and the random uncertainty. Under the premise that multiple sets of observation data and actual values are available, the systematic uncertainty and the random uncertainty can be approximately estimated. Due to the actual values are difficult to obtain, this paper decided to introduce the auto regressive model (AR model) [15] in attribute reliability calculation.

The AR model is a statistical method to deal with the processing time series, which uses the $t - p$ observations to predict the value at time t. The p-order AR model is shown in (10):

$$x(t) = \sum_{j=1}^{p} \varphi_j x(t-j) + \varepsilon(t) + e, t = 1, 2, \cdots, m \tag{10}$$

where φ_j are the coefficient for $x(t-j)$.

Compare (10) with (9), the actual values $x_i\_act(j)$ in (9) are approximated by $\sum_{j=1}^{p} \varphi_j x(t-j)$, while the systematic uncertainty is referred by e in (10). Therefore, an appropriate order p to the AR model is important for ensuring the accuracy of the actual values and the systematic uncertainty further. At present, calculate the partial autocorrelation coefficient (*pacf*) [16] is a commonly used method in determining the order of the AR model. If the *pacf* is truncated at the p_1, p_1 will be taken as the order of the AR model.

The systematic uncertainty obtained from (10) is dimensioned, which is related to the value of the attribute x. In order to facilitate the comparison, we define the normalized systematic uncertainty to the interval [0, 1] by $\frac{e}{\bar{x}}$, where $\bar{x}$ is the average of the observation data. The sum of the attribute reliability and the normalized systematic uncertainty is equal to 1. Therefore, the attribute reliability can be calculated using (11):

$$r = 1 - \frac{e}{\bar{x}}. \tag{11}$$

where $\bar{x}$ is the average of the observation data.

The procedure of the new calculation method of attribute reliability is shown in Fig. 4:

Step 1: Judging whether there is systematic uncertainty in data:

- **Step 1.1**: The observation data is randomly divided into two parts.
- **Step 1.2**: Perform U-test [20] on the two parts of data. If the test passes, there is no systematic uncertainty in the observation data, and turn to Step. 3. Otherwise turn to Step. 2.

Step 2: Constructing and solving an AR model:

- **Step 2.1**: Perform the Augmented Dickey-Fuller test (ADF test) [22] on the observation data. If the tested data is stable, turn to Step 2.3. Otherwise turn to Step 2.2.
- **Step 2.2**: Perform the difference operations on the tested data and turn to Step 2.1;
- **Step 2.3**: Determine the order of the AR model based on the partial correlation coefficient (pacf) [16];
- **Step 2.4**: Build the AR model for the tested value and get the systematic uncertainty e.

Step 3: Obtain the reliability of the attribute data.

Since the AR model is only suitable for stability time series data, Step 2.1 performs an ADF test on the data to check the stability of the data. If the data does not pass the ADF test,

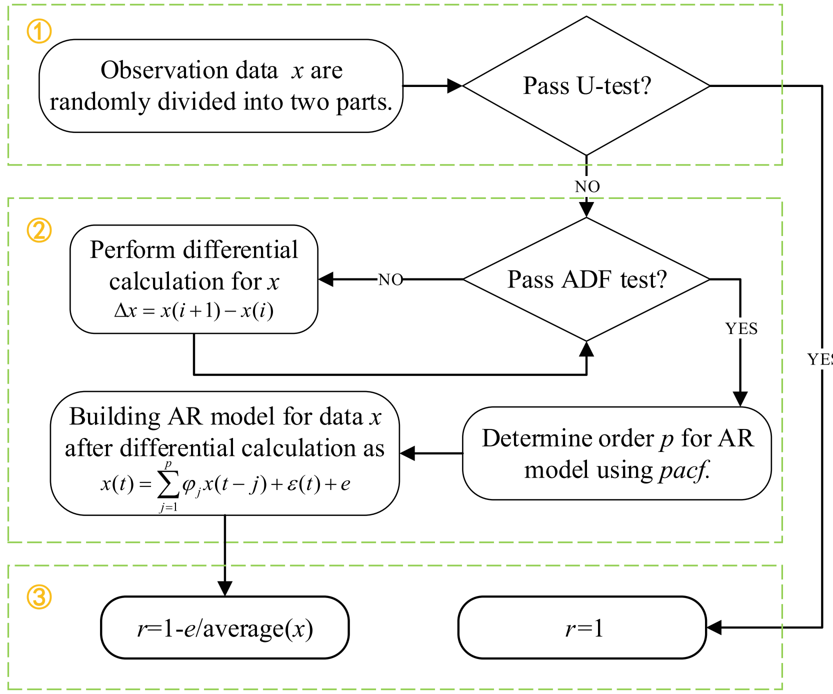


Fig. 4 Calculation algorithm of attribute reliability in the BRB-r (new) model

it is proved that the current data does not meet the stability requirement, and the differentially calculation according to (12) will be taken until the data passes the ADF test.

$$\Delta x = x(i+1) - x(i). \tag{12}$$

The new attribute reliability calculation algorithm proposed in this paper is based on the meaning of systematic uncertainty (as shown in (9)). First, determine the existence of systematic uncertainty in attribute data according to U-test. If the systematic uncertainty does not exist, the reliability of this attribute is $r = 1$. Second, for the attribute data with systematic uncertainty, construct AR model to estimate systematic uncertainty. Third, calculate the attribute reliability based on the normalized systematic uncertainty.

Remark The calculation algorithm of attribute reliability proposed in this paper is only suitable for quantitative data of smooth fluctuations. For the step data, the reliability of the data in each step can be calculated separately, and then weighted according to the amount of data as the reliability of this attribute. The reliability calculation method for qualitative data can refer to [37].

Compare with the attribute reliability calculation algorithm in the BRB-r (pre) model, the new algorithm is more in line with the definition of systematic uncertainty and does not depend on expert knowledge. Accurate calculation of attribute reliability is the basis of BRB-r model.

4 Attribute reliability fusion

From the inference process of the BRB model introduced in Section 2.1, the attribute weight only affect the inference results in the belief rule activation described in (3). Both attribute weight and attribute reliability are the descriptions for attribute, which should affect the attributes in a similar way in the BRB-r model. Therefore, it is appropriate to fuse the attribute reliability in (3).

4.1 Existing fusion algorithm and drawbacks

(1) Fusion algorithm

The BRB-r (pre) model proposed the fusion algorithm of attribute reliability as (13). See [9] for more details.

$$\omega_k = \frac{\theta_k \prod_{i=1}^{T_k} (\alpha_{i,j}^k)^{C_i}}{\sum_{l=1}^{L} \left[\theta_l \prod_{i=1}^{T_l} (\alpha_{i,j}^l)^{C_i} \right]}, \quad C_i = \frac{\overline{\delta}_i}{1 + \overline{\delta}_i - r_i}. \tag{13}$$

In (13), parameter C_i is introduced to describe the joint influence of attribute weight and attribute reliability on the degree of rule activation. Compared with the traditional belief rule activation described in (3), parameter C_i replaces parameter $\overline{\delta}_i$ in the BRB-r (pre) inference.

(2) Drawbacks

Consider two extreme situations: the i-th attribute is fully reliable and fully unreliable, the parameter C_i calculated according to (13) is shown as follow:

$$C_i = \begin{cases} 1, & r_i = 1 \\ \frac{\bar{\delta}_i}{1+\bar{\delta}_i}, & r_i = 0 \end{cases} .$$

As a special case of the BRB model, when attributes are fully reliable (i.e., $r_i = 1$), the BRB-r model should degenerates into the traditional BRB model. But when $r_i = 1$, it has $C_i = 1 \neq \bar{\delta}_i$ according to (13), which violates the consistency between the BRB model and the BRB-r model. When the input data for an attribute is fully unreliable (i.e., $r_i = 0$), this attribute should have no influence on the results. However, when $r_i = 0$, it has $C_i = \frac{\bar{\delta}_i}{1+\bar{\delta}_i}$ according to (13). The C_i -value means although the input data of the i-th attribute is fully unreliable, but there still have rules contain this attribute that will be activated. In summary, the fusion algorithm of attribute reliability in the BRB (pre) model is unreasonable in some extreme situation.

4.2 New fusion algorithm

In order to avoid the aforementioned drawbacks, a new fusion algorithm of attribute reliability in the BRB-r (new) model is proposed as follows:

$$\omega_k = \begin{cases} \dfrac{\theta_k \prod_{i=1}^{T_k} (\alpha_{i,j}^k)^{\bar{C}_i}}{\sum_{k=1}^{K} \left[\theta_k \prod_{i=1}^{T_k} (\alpha_{i,j}^l)^{\bar{C}_i} \right]}, & \bar{C}_i \neq 0 \\ 0, & \bar{C}_i = 0 \end{cases} ,$$
$$\bar{C}_i = \frac{C_i}{\max_{j=1,2,\cdots,T_k} \{C_j\}}, \quad C_i = \delta_i r_i . \tag{14}$$

Parameter $C_i (i = 1, 2, \cdots, M)$, named the *joint attribute importance factor*, is the combination of the attribute weight and the attribute reliability, which is used to measure the importance of attribute. Parameter $\bar{C}_i (i = 1, 2, \cdots, M)$ is called the *normalized joint attribute importance factor*. In the *joint attribute importance factor*, the attribute weight and the attribute reliability are equivalent to each other. It represents that these two characteristics own the same importance in the BRB-r (new) model. When the attribute is fully reliable, the *joint attribute importance factor* will degenerate to the attribute weight. When the attribute is fully unreliable, there are no rules contain this attribute will be activated. The *joint attribute importance factor* can well describe the common influence of attribute weight and attribute reliability, and integrate the attribute reliability into the BRB model reasonably.

In summary, a new attribute reliability fusion algorithm is proposed in this paper in the analysis of the drawbacks of the existing algorithm in the BRB-r (pre) model. The attribute reliability and attribute weight are merged in this algorithm, named *joint attribute importance factor*. The *joint attribute importance factor* guarantees that when the attribute is completely trusted, the BRB-r model will degenerates into the BRB model; when the attribute is completely untrusted, the attribute will not affect further modeling and analysis. Compare with the attribute reliability fusion algorithm in the BRB-r (pre) model, the new algorithm is more reasonable in two extreme cases when the attribute is completely reliable and completely unreliable.

5 Modeling and inference approach of the BRB-r (new) model

Current BRB-r (pre) model introduce the systematic uncertainty into the traditional BRB model, but there are some drawbacks in its attribute reliability calculation and fusion algorithms. Considering these drawbacks, this paper presents new attribute reliability calculation algorithm and fusion algorithm in Sections 3 and 4 respectively. This section synthesizes the content of the Sections 3 and 4, and the modeling and inference approach of the BRB-r (new) model is shown in Fig. 5. The parameter optimization steps of the BRB-r can refer to the research about the BRB

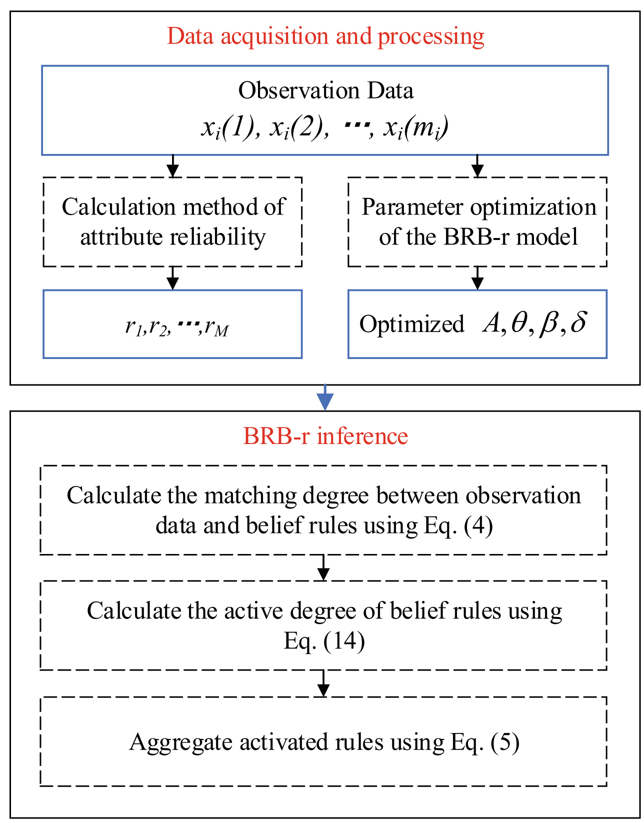


Fig. 5 Modeling and inference approach of the BRB-r (new) model

parameter optimization. See literature [3, 19] and [30]for details.

The modeling and inference approach of the BRB-r (new) model can be divided into two steps: (1) Data acquisition and processing, (2) BRB-r inference. The required data of the BRB-r (new) model are placed in the blue frames. The modeling steps can be detailed as follows:

Step 1: Data acquisition and processing: Obtain the observation data from measuring devices such as sensors, and process these data to support the data requirements of the BRB-r inference:

- **Step 1.1**: Calculate attribute reliability and see Section 3.2 for details;
- **Step 1.2**: Optimize parameter for the BRB-r (new) model and see literature [3, 19] and [30] for details.

Step 2: BRB-r (new) inference:

- **Step 2.1**: Calculate the matching degree between observation data and belief rules using (4);
- **Step 2.2**: Calculate the active degree of belief rules using (14);
- **Step 2.3**: Aggregate activated rules using (5) and the analysis results can be described as (6).

6 Case study

In Section 6.1, a practical case about pipeline leak detection is studied to detail the application of the BRB-r (new) model step-by-step. This case study has been regarded as the benchmark dataset in multiple researches about BRB model and is of great significance to illustrate the impact of attribute reliability on BRB modeling and the rationality of the proposed BRB-r (new) model. Section 6.2 studies another practical case about condition monitoring of turbofan engine. The experiment results are compared with the traditional BRB model, the BRB-r (pre) model and other classical regression models to verify the feasibility of the BRB-r (new) model.

6.1 Benchmark case study: pipeline leak detection

(1) Problem description

A pipeline more than 100km in length is mostly operated in normal condition, with the pressure meters and the mass flow meters at some points to record the flow and pressure. However, a series of leaks happened during a leak trial period estimating the size of leak has been studied in multiple papers so that this case can be used as a benchmark dataset. Considering the uncertainty of flow and pressure in measuring, the BRB-r (new) model is established to estimate the size of leak. The accuracy of estimation is compared with existing algorithm to verify the effective of the proposed model.

(2) Data acquisition and processing

In this case, 2008 samples for a pipeline were collected in every 10s, and each sample contains two independent variables: (1) the average pressure change (denoted as *PressureDiff*) between two observation points, (2) the difference in flow (denoted as *FlowDiff*); and one dependent variable: the size of the leak (denoted as *LeakSize*). The difference in flow and the average pressure change are shown in Fig. 6:

Step 1.1: Calculate attribute reliability

The *PressureDiff* does not pass the U-test, which indicate the existence of systematic uncertainty. According to the new attribute reliability calculation algorithm proposed in Section 3.2, the order of the AR model is defined as $p_1 = 5$. On this basis, the reliability of *PressureDiff* is calculated as $r_1 = 0.9953$.

As shown in Fig. 6b, considering that there are two jumping points in the data of *FlowDiff*, divide the data into three parts and respectively calculate the reliabilities r_{21}, r_{22}, r_{23}, then weigh r_{21}, r_{22}, r_{23} according to the amount of data in each part as the attribute reliability of *FlowDiff*. According to the *pacf*, the orders of each AR model are defined as $p_{21} = 15$, $p_{22} = 2$, $p_{23} = 15$, and the attribute weights are calculated as $r_{21} = 0.9092$, $r_{22} = 0.9930$, $r_{23} = 0.9465$. The reliability of *FlowDiff* is calculated can be calculated as follows:

$$r_2 = \sum_{i=1}^{3} \frac{a_i}{2008} r_{2i} = \frac{934 \times 0.9092 + 426 \times 0.9930 + 648 \times 0.9465}{2008} = 0.9395.$$

where a_i represents the data amount of part i.

Step 1.2: Optimize parameter for the BRB-r (new) model.

The k-th rule in this case can be described as follows:

$$\begin{aligned}
&R_k : IF\ (PressureDiff\ is\ A_1^k) \wedge (FlowDiff\ is\ A_2^k),\\
&THEN\ \{(LeakSize_1, \beta_{1,k}), (LeakSize_2, \beta_{2,k}), (LeakSize_3, \beta_{3,k}),\\
&(LeakSize_4, \beta_{4,k}), (LeakSize_5, \beta_{5,k})\},\\
&with\ rule\ weight\ \theta_k,\ attribute\ weight\ \delta_1, \delta_2, \delta_3\\
&and\ attribute\ reliability\ r_1 = 0.9953, r_2 = 0.9395.
\end{aligned}$$

According to the engineering experience, we define 7 reference values for *PressureDiff* and 8 reference values for *FlowDiff*. Besides, we divide the *LeakSize* into 5 ranks, i.e., $(D_1, D_2, D_3, D_4, D_5) = (0, 2, 4, 6, 8)$. The parameters to be optimized in the BRB-r model include the reference values of attribute, rule weights, belief degrees of consequence and attribute weights, which are recorded as A, θ, β and δ respectively. Totally, there are 353 parameters

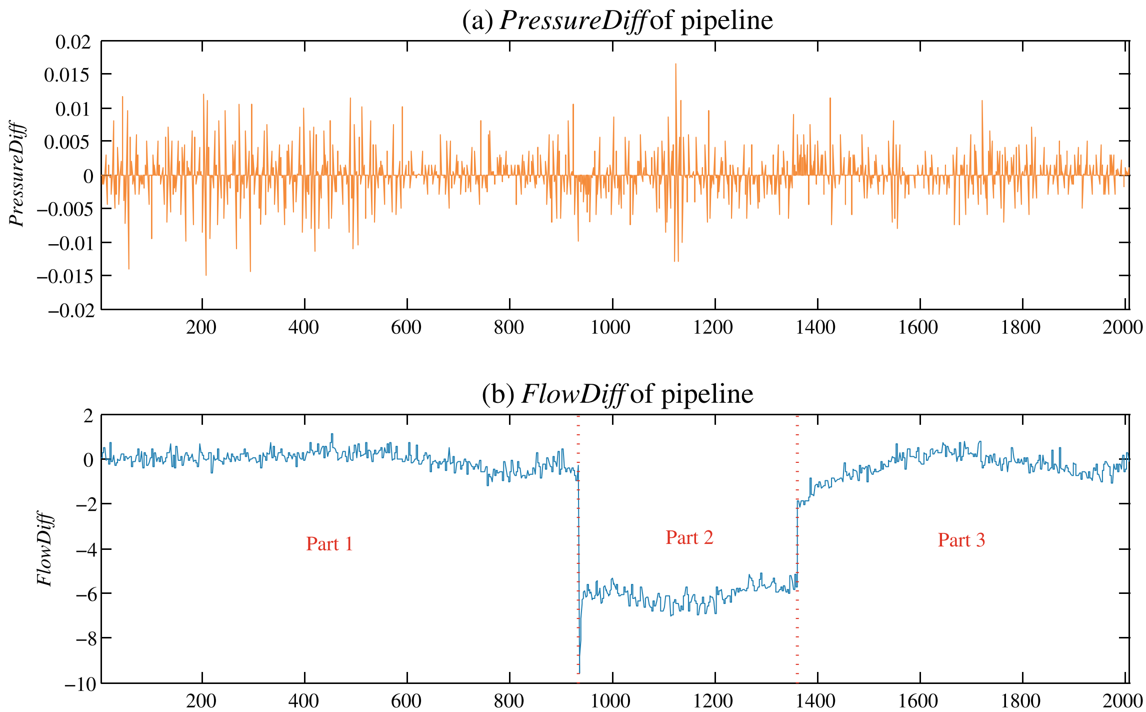


Fig. 6 *FlowDiff* and *PressureDiff* of pipeline

(including 15 reference values, 56 rule weights, 280 belief degrees, and 2 attribute weights) need to be optimized during the parameter learning process. In this case, 500 samples are randomly selected as the training set, and the differential evolution (DE) algorithm is chosen as the engine for parameter optimization. Related steps can be referred in [3]. The population size is set to 50 individuals, and the maximum number of cycles is set to 500 in the experiment to obtain the parameter of the BRB-r (new). The optimized BRB-r is given in Appendix.

(3) BRB-r inference

PressureDiff and *FlowDiff* are regarded as the antecedent attributes to construct the BRB-r (new) model, and the *Leaksize* is estimated based on the optimized BRB-r (new) model. The detailed steps are described in Section 5. Taking the first sample as an example to illustrate the BRB-r inference step-by step, the input data is $(PressureDiff, FlowDiff) = (0, 0.05)$.

Step 2.1: Calculate the matching degree between observation data and belief rules.

The optimized reference values for *PressureDiff* and *FlowDiff* are $A_1 = \{-0.02, -0.0192, -0.012, -0.0039, 0.0171, 0.0186, 0.02\}$ and $A_2 = \{-10, -9.7425, -8.3311, -6.7967, -1.9963, -1.8724, 0.3763, 1.50\}$. The matching degrees of the input data for *PressureDiff* and *FlowDiff* are calculated as $\alpha_{1,4} = \varphi_1(0, -0.039) = \frac{0-(-0.0039)}{0.0171-(-0.0039)} = 0.1853$, $\alpha_{1,5} = \varphi_1(0, 0.0171) = \frac{0.0171-0}{0.0171-(-0.0039)} = 0.8147$, $\alpha_{1,6} = \varphi_2(0.05, -1.8724) = \frac{0.05-(-1.8724)}{0.3763-(-1.8724)} = 0.8549$, $\alpha_{1,7} = \varphi_2(0.05, 0.3763) = \frac{0.3763-0.05}{0.3763-(-1.8724)} = 0.1451$ according to (4).

Step 2.2: Calculate the active degree of belief rules.

The optimized attribute weights are $(\delta_1, \delta_2) = (0.4577, 0.4351)$ and the *joint attribute importance factor* are calculated as $(C_1, C_2) = (0.4555, 0.4088)$. The input data activates the 30th, 31st, 37th and 38th rules with the activated weights as $(\omega_{30}, \omega_{31}, \omega_{37}, \omega_{38}) = (0.0075, 0.1315, 0.7005, 0.1605)$ using (14).

Step 2.3: Aggregate activated rules.

Aggregate the 31st, 37th and 38th rules using (5) and get the aggregated result $(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5) = (0.9841, 0.0093, 0.0026, 0.0025, 0.0014)$. The estimated *LeakSize* of the first samples is $\sum_{n=1}^{N} D_n \beta_n = 0.0556$.

Similarly, the estimated *LeakSize* for other samples can be calculated. The comparison of the original *LeakSize* and the estimated *LeakSize* is shown in Fig. 7.

(4) Comparison and analysis

Table 1 compares the modeling accuracy among the BRB-r (new) model with part of present researches. Literature [10] estimates the size of leak based on the ER

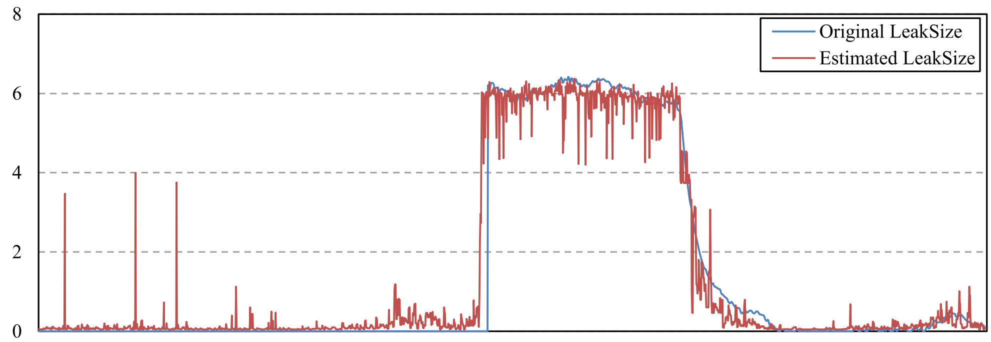


Fig. 7 Comparison on original *LeakSize* and estimated *LeakSize*

rule, and other researche based on the traditional and/or modified BRB model.

As shown in Table 1, in the case that the size of training set size is not larger than other studies, the BRB-r (new) model proposed in this paper produces the smallest MSE comparison with previous studies, which verify the rationality and feasibility of the BRB-r (new) model. Especially, the comparison between the BRB-r (new) model and the traditional BRB model (No.1) [6] shows that the addition of attribute reliability has a certain improvement on the modeling accuracy of traditional BRB in this case. On one hand, the comparison result proves the necessity of introducing attribute reliability into the BRB model. On the other hand, it proves the rationality of the attribute reliability calculation and fusion algorithms proposed in this paper.

6.2 Practical case study: condition monitoring of engine

(1) Problem description

The turbofan engines are often used as the power equipment for aircraft and large ships. The construction of the turbofan engine is complex, resulting that the condition of some operating parameters are difficult to accurately monitor through the equipment. The fuel flow and pressure ratio is one of the manifestations of turbofan engine load, but it's difficult to accurately monitor its real-time conditions based on the sensors. Existing dependency-based study has shown that the outlet pressure (*P30*), the static pressure (*Ps30*) and the fan speed (*NRf*) have an effect on the fuel flow and pressure ratio (*phi*). These three parameters are easier to be observed by the sensors than the phi. However, due to the reasons such as the aging of sensors and the influence of environment, systematic uncertainty may exist in the observation data. Considering the existence of systematic uncertainty, the BRB-r (new) model is used in this paper to monitor the condition of phi through the observation data of *P30*, *Ps30* and *NRf*. The training data is provided by the Prognostics Center of Excellence (PCoE) at Ames Research Center [17], which is developed by the Commercial Modular Aero-propulsion System Simulation (C-MAPSS) [2].

The *P30*, *Ps30* and *NRf* are denoted as the attributes x_1, x_2, and x_3, while *phi* is regarded as the conclusion in the BRB-r model. A belief rule in the BRB-r model for the condition monitoring of the *phi* can be described as follows:

$$\begin{aligned}&R_k : IF\ (P30\ is\ A_1^k) \wedge (Ps30\ is\ A_2^k) \wedge (NRf\ is\ A_3^k),\\ &THEN\ \{(phi_1, \beta_{1,k}), (phi_2, \beta_{2,k}), (phi_3, \beta_{3,k}), (phi_4, \beta_{4,k}), (phi_5, \beta_{5,k})\},\\ &with\ rule\ weight\ \theta_k,\ attribute\ weight\ \delta_1, \delta_2, \delta_3,\ and\\ &attribute\ reliability\ r_1, r_2, r_3.\end{aligned}$$

Each attribute has three reference values and the consequent is divided into five ranks. The referential values for consequence are set as $(phi_1, phi_2, phi_3, phi_4, phi_5) = (516, 518, 520, 522, 524)$. The attribute reliabilities can be calculate using the algorithm described in Section 3.2.

Table 1 Comparison with present researches

No.	Year	Description	MSE	Size (training/test)
1	2007[26]	Traditional BRB	0.4049	500/2008
2	2009[36]	Online updating	0.7880	800/2008
3	2011[6]	Adaptive learning	0.3990	500/2008
4	2015[7]	ER rule	0.3709	–/2008
5	2016[24]	Dynamic rule adjustment	0.4450	900/2008
6	2018[32]	Joint optimization	0.3974	900/2008
7	This study	BRB-r (new)	**0.3466**	500/2008

Bold emphasis refer to the best results in all comparison experiments

(2) Modeling and analysis

There are 20631 records in the observation dataset for 100 engines with different initial attrition rates over the life cycle. The attribute reliabilities can be calculated using the calculation algorithm given in Section 3.2. In this case, the reliabilities for attributes x_1, x_2, and x_3 are calculated as $(r_1, r_2, r_3) = (0.9586, 0.9319, 0.9444)$.

The BRB-r model in this case contains 27 rules involving 174 $(3 + 3 + 3 + 3 \times 3 \times 3 \times (5 + 1) + 3)$ parameters. It is hard for experts to give these parameters accurately. The introduction of the parameter optimization method is beneficial for both improving the reliability of the rules and the accuracy of the condition monitoring results. In this case, the differential evolution (DE) algorithm is used for parameter learning of the BRB-r (new) in this case, and related steps can be referred to [3]. The population size is set to 50 individuals, and the maximum number of cycles is set to 500 in the experiment to get the parameter of the BRB-r (new).

After obtaining the optimized BRB-r (new), the *P30*, *Ps30* and *NRf* measured at time *t* are taken as input values, and the monitoring value of *phi* at the corresponding time is obtained by the BRB-r (new) model inference described in Section 5, which is recorded as $\widehat{phi}_t$. The mean square error (MSE) is introduced to measure the accuracy of models, which is calculated using (15):

$$MSE = \sum_{t=1}^{T} (phi_t - \widehat{phi}_t)^2, \tag{15}$$

where phi_t represents the observation value of *phi* at time *t*.

In order to illustrate the effectiveness of the BRB-r (new) model, the BRB model and the BRB-r (pre) model [9] are introduced as the comparisons. In the BRB-r (pre) model, the tolerance range is set as $(\bar{x}_i - \sigma_i, \bar{x}_i + \sigma_i)$ and the attribute reliabilities are calculated as $(r_1, r_2, r_3) = (0.8333, 0.6770, 0.7030)$ by (8). In order to ensure the comparability of the results, the DE algorithm is also used to optimize the parameters of the BRB and the BRB-r (pre).

Because the size of test set is too large, it is not easy to observe the inference values in one picture. 200 sets of data are randomly selected from the test set, and Fig. 8 visually shows the performance of these three models. The observed values and the inference values for phi are shown in Fig. 8a, while the MSEs are shown in Fig. 8b.

As shown in Fig. 8, both the BRB model and the BRB-r (new) model have a good performance in phi monitoring, and the monitoring results $\widehat{phi}_t$ are close to each other. The BRB-r (new) model is an extension of the BRB model.

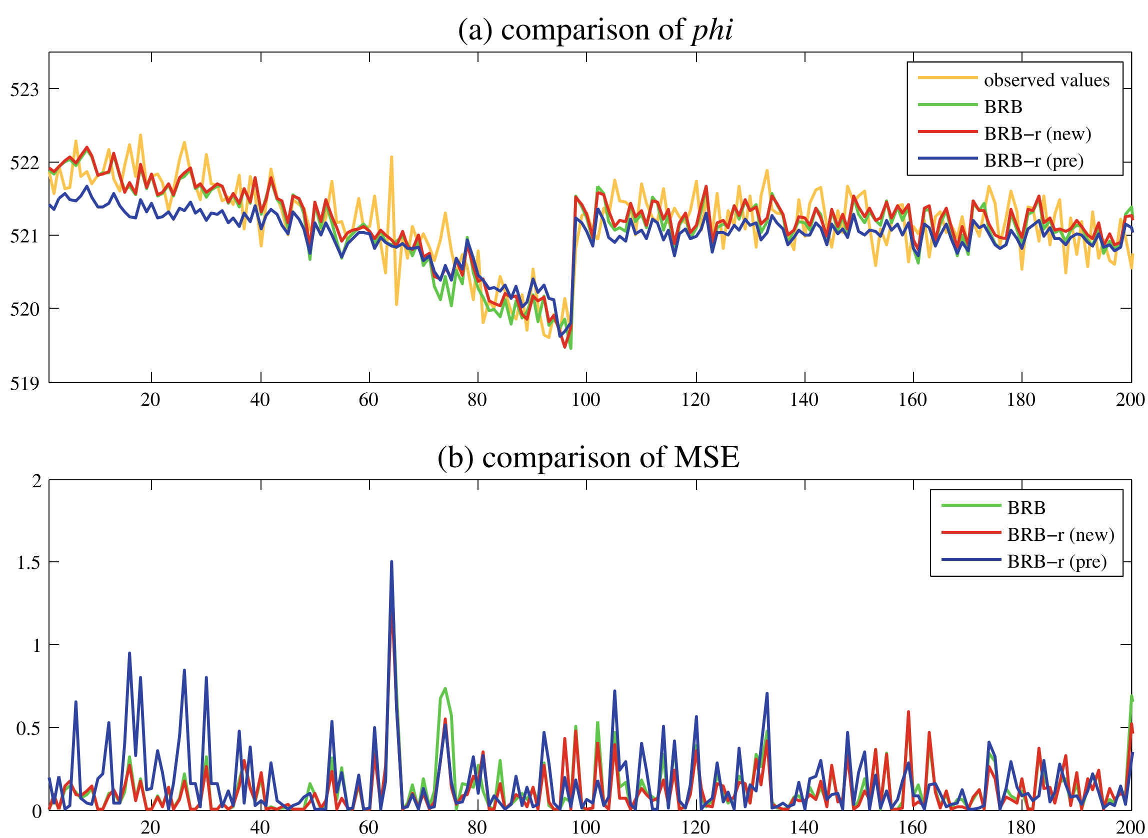


Fig. 8 Comparison on *phi* and MSE of the monitoring results

When the attribute reliability is equal to 1, the BRB-r (new) model will degenerate into a BRB model. In this case, the reliabilities are $(r_1, r_2, r_3) = (0.9586, 0.9319, 0.9444)$. The higher attribute reliabilities make the BRB-r (new) model and the BRB model closer to each other. It is obvious that the accuracy of the BRB-r (pre) is lower than other models, which may be caused by the unreasonable attribute reliabilities or the unreasonable fusion.

10-fold cross-validation is taken to verify the effectiveness of the proposed BRB-r (new) model further, and the MSE is compared with four classical regression models, named Support Vector Machines (SVM) [14], Linear Regression (LR) [13], k-Nearest Neighbor (KNN) [27] and Decision Tree (DT) [4]. The results are shown in Table 2:

In Table 2, the MSEs between the observed values and the inference values are used to measure the accuracy of the model in condition monitoring. The smallest MSE value indicates that the BRB-r (new) model is more accurate than other models in this case, which shows the effectiveness of the BRB-r (new) model in solving the condition monitoring problem.

In order to compare the performance of these seven models in this case more accurately, the Friedman test [25] based on algorithm ordering is introduced. The null hypothesis is that there is no difference in performance between the seven models.

First, sort the performance of models in each fold (i.e., the smaller the MSE, the higher the ordering). The average order of the model performance is denoted as $s_p(p = 1, 2, \cdots, 7)$.

Calculate the value of the statistic τ_F using (16):

$$\begin{aligned} \tau_F &= \frac{(K-1)\tau_{\chi^2}}{K(P-1)-\tau_{\chi^2}}, \\ \tau_{\chi^2} &= \frac{12K}{P(P+1)}\left(\sum_{p=1}^{P} s_p^2 - \frac{P(P+1)^2}{4}\right). \end{aligned} \tag{16}$$

where K denotes the number of folds and P denotes the number of models. τ_F obeys the F distribution with degrees of freedom $k-1$ and $(k-1)(N-1)$.

When what $= {}'s$ 0.05, the F distribution has a critical value of 2.272 at $K = 10$ and $P = 7$. If $\tau_F > 2.272$, reject the null hypothesis and verify there is a significant difference in performance between these models.

According to (16), we have $\tau_F = 24.712 > 2.272$ in this case, and the null hypothesis is rejected. The difference performances of these models are verified by the Friedman test. Introduce of attribute reliability into the BRB model can improve the accuracy of the traditional BRB model in practical applications and enable it analyzing with systematic uncertainty. Since the reliability calculation algorithm and reliability fusion algorithm in the BRB-r (pre) model are both flawed, the accuracy of its inference results cannot be effectively improved.

7 Conclusion

A new modeling and inference approach for the BRB-r, recorded as the BRB-r (new) model, is proposed in this paper to introduce the systematic uncertainty into the traditional BRB model through solving the drawbacks in the BRB-r (pre) model. New calculation algorithm and fusion algorithm for attribute reliability are proposed to fix the drawbacks in the BRB-r (pre) model. Furthermore, the BRB-r (new) model is utilized for monitor the condition of turbofan engines.

To measure the reliability of the input data, a new attribute reliability calculation algorithm is proposed based on the AR model. By analyzing the actual meaning of systematic uncertainty, this calculation algorithm establishes a mathematical model to measure the attribute reliability from data.

Table 2 Comparison with other models

Fold	BRB-r (new)	BRB-r (pre)	BRB	SVM	LR	KNN	DT
1	0.1204	0.1234	0.1341	0.1486	0.1486	0.1724	0.2786
2	0.1208	0.1321	0.1403	0.1490	0.1487	0.1710	0.2762
3	0.1219	0.1977	0.1538	0.1489	0.1486	0.1725	0.2764
4	0.1210	0.2462	0.1773	0.1484	0.1486	0.1724	0.2738
5	0.1205	0.3442	0.1329	0.1483	0.1487	0.1712	0.2723
6	0.1202	0.2445	0.1363	0.1486	0.1486	0.1717	0.2740
7	0.1205	0.2632	0.1451	0.1488	0.1487	0.1720	0.2751
8	0.1229	0.1201	0.1545	0.1488	0.1487	0.1712	0.2774
9	0.1215	0.2525	0.1375	0.1491	0.1486	0.1720	0.2767
10	0.1214	0.2250	0.1367	0.1486	0.1486	0.1726	0.2730
Average	**0.1211**	0.2149	0.1448	0.1487	0.1486	0.1719	0.2754

Bold emphasis refer to the best results in all comparison experiments

To incorporate attribute reliability into the BRB reasoning, a new attribute reliability fusion algorithm is proposed. This algorithm can effectively integrate attribute reliability into the BRB model, which ensures the rationality of the BRB-r (new) model under different situations.

A benchmark case about pipeline leak detection is studied to validate the rationality of the BRB-r (new) model proposed in this paper. The comparison results prove the necessity of introducing attribute reliability into BRB model, and prove the rationality of the attribute reliability calculation and fusion algorithms proposed in this paper further.

A practical case about condition monitoring is studied to validate the feasibility and effectiveness of the modeling and inference approach for the BRB-r. The experimental results show that the BRB-r (new) model proposed in this paper is more reasonable than the BRB-r (pre). The BRB-r (new) model can effectively improve the accuracy and robustness of the traditional BRB model in practical application, especially when faced with uncertain data.

For future research, the ability to process non-stationary series should be added to the attribute reliability calculation method. The application of the BRB-r (new) model in other fields can be further explored.

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Appendix

Table 3 Optimized BRB-r

No.	Rule weight	Attribute		*phi*
		Pressure	*FlowDiff*	
1	0.2074	–0.0200	–10.0000	{0.3603,0.2515,0.0776,0.2112,0.0995}
2	0.7885	0.0171	–10.0000	{0.3579,0.1904,0.1191,0.1100,0.2225}
3	0.6395	–0.0039	–10.0000	{0.1817,0.0023,0.0820,0.5227,0.2114}
4	0.6894	–0.0120	–10.0000	{0.1697,0.3188,0.0580,0.1391,0.3144}
5	0.7607	0.0186	–10.0000	{0.0059,0.2964,0.0629,0.2786,0.3562}
6	0.8107	–0.0192	–10.0000	{0.0137,0.0594,0.1750,0.0061,0.7458}
7	0.3095	0.0200	–10.0000	{0.1167,0.1635,0.0940,0.3310,0.2947}
8	0.1214	–0.0200	–9.7425	{0.0326,0.6582,0.1995,0.0567,0.0531}
9	0.1808	0.0171	–9.7425	{0.3106,0.0587,0.0327,0.1411,0.4569}
10	0.7045	–0.0039	–9.7425	{0.7507,0.1612,0.0526,0.0241,0.0115}
11	0.2966	–0.0120	–9.7425	{0.2911,0.0114,0.3367,0.2695,0.0913}
12	0.4840	0.0186	–9.7425	{0.0302,0.0510,0.3823,0.4432,0.0933}
13	0.7751	–0.0192	–9.7425	{0.2431,0.1788,0.2676,0.0166,0.2941}
14	0.7216	0.0200	–9.7425	{0.0925,0.2337,0.1080,0.1575,0.4083}
15	0.8782	–0.0200	–1.9963	{0.1647,0.2681,0.0195,0.0019,0.5459}
16	0.4299	0.0171	–1.9963	{0.0911,0.2049,0.1472,0.4563,0.1005}
17	0.0456	–0.0039	–1.9963	{0.0230,0.0081,0.8641,0.0002,0.1046}
18	0.4268	–0.0120	–1.9963	{0.1730,0.3421,0.0325,0.0963,0.3561}
19	0.6508	0.0186	–1.9963	{0.0105,0.4049,0.0910,0.0658,0.4277}
20	0.9856	–0.0192	–1.9963	{0.4955,0.0525,0.0600,0.3192,0.0728}
21	0.3884	0.0200	–1.9963	{0.1665,0.1999,0.1401,0.4914,0.0021}
22	0.4299	–0.0200	–8.3311	{0.3580,0.0098,0.0582,0.0669,0.5072}
23	0.9750	0.0171	–8.3311	{0.0915,0.2355,0.0004,0.6367,0.0359}
24	0.6470	–0.0039	–8.3311	{0.2881,0.4241,0.0508,0.1464,0.0906}
25	0.5182	–0.0120	–8.3311	{0.0560,0.0007,0.1097,0.3412,0.4924}
26	0.2305	0.0186	–8.3311	{0.0143,0.3664,0.3161,0.2246,0.0785}
27	0.8158	–0.0192	–8.3311	{0.1799,0.1593,0.2884,0.1475,0.2248}
28	0.0437	0.0200	–8.3311	{0.2086,0.2292,0.4217,0.0174,0.1231}
29	0.8393	–0.0200	0.3763	{0.2314,0.2737,0.0619,0.0354,0.3976}
30	0.0127	0.0171	0.3763	{0.1388,0.3951,0.1537,0.0168,0.2956}

Table 3 (continued)

No.	Rule weight	Attribute		*phi*
		Pressure	*FlowDiff*	
31	0.9822	–0.0039	0.3763	{0.9872,0.0079,0.0020,0.0016,0.0013}
32	0.0623	–0.0120	0.3763	{0.3566,0.1934,0.3120,0.0929,0.0451}
33	0.0719	0.0186	0.3763	{0.2991,0.1285,0.1337,0.2047,0.2339}
34	0.0346	–0.0192	0.3763	{0.1971,0.4277,0.2213,0.0568,0.0972}
35	0.5422	0.0200	0.3763	{0.3353,0.1601,0.4216,0.0320,0.0511}
36	0.4884	–0.0200	–1.8724	{0.0454,0.1889,0.4493,0.2562,0.0602}
37	0.2424	0.0171	–1.8724	{0.0270,0.3063,0.3795,0.1535,0.1337}
38	0.2441	–0.0039	–1.8724	{0.0050,0.4695,0.1682,0.3492,0.0082}
39	0.1837	–0.0120	–1.8724	{0.2578,0.2887,0.0120,0.0457,0.3959}
40	0.1705	0.0186	–1.8724	{0.0403,0.5273,0.0348,0.0068,0.3908}
41	0.7755	–0.0192	–1.8724	{0.0651,0.0514,0.4639,0.3611,0.0585}
42	0.5418	0.0200	–1.8724	{0.0581,0.3469,0.0335,0.1885,0.3730}
43	0.8694	–0.0200	–6.7967	{0.0870,0.2586,0.0767,0.1867,0.3910}
44	0.7970	0.0171	–6.7967	{0.0880,0.0119,0.2371,0.0263,0.6367}
45	0.0464	–0.0039	–6.7967	{0.0492,0.3034,0.4270,0.0403,0.1802}
46	0.7786	–0.0120	–6.7967	{0.0165,0.1381,0.0150,0.3022,0.5282}
47	0.5459	0.0186	–6.7967	{0.2996,0.0281,0.1187,0.2697,0.2838}
48	0.9942	–0.0192	–6.7967	{0.1144,0.0032,0.3579,0.3259,0.1987}
49	0.7954	0.0200	–6.7967	{0.0024,0.1096,0.4080,0.3796,0.1004}
50	0.1436	–0.0200	1.5000	{0.2742,0.3942,0.1357,0.0061,0.1899}
51	0.8928	0.0171	1.5000	{0.1929,0.2644,0.4483,0.0250,0.0693}
52	0.0189	–0.0039	1.5000	{0.2159,0.0935,0.2887,0.1933,0.2087}
53	0.1430	–0.0120	1.5000	{0.2138,0.3686,0.1400,0.1499,0.1277}
54	0.0950	0.0186	1.5000	{0.4533,0.4284,0.0367,0.0120,0.0696}
55	0.9223	–0.0192	1.5000	{0.4904,0.3435,0.0230,0.0590,0.0842}
56	0.2781	0.0200	1.5000	{0.6456,0.0600,0.1528,0.0852,0.0565}

Optimized attribute weights:$(\delta_1, \delta_2) = (0.4577, 0.4351)$

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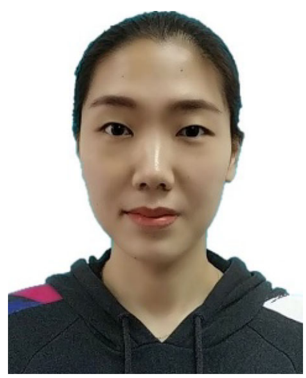

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