2016 International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery

Comparative Ananlysis on the Conjunctive and Disjunctive Assumptions for the Belief Rule Base

Leilei Chang, Xiaowei Ma, Liuying Wang
Department of Management
High-Tech Institute of Xi'an
Xi'an, China
leileichang@hotmail.com; xw_ma@hotmail.com

Xiaodong Ling
China Satellite Maritime Tracking and Control Laboratory
Wuxi, China
Jlomac_conf@163.com

***Abstract*—The belief rule base (BRB) is an efficient tool in solving many complex theoretical and practical problems due to its superior performance in nonlinearity modeling under uncertainty. The conventional BRB is constructed following the conjunctive assumption which indicates that all the possible combinations of the referenced values of the attributes must be covered. The conjunctive assumption can ensure that the input space is complete so that any input combination can be handled, which however results in the combinatorial problem since the number of the rules in BRB would grow exponentially along with more attributes and/or more referenced values of the attributes. With this, the disjunctive assumption for BRB construction is introduced to avoid the combinatorial explosion problem by maintaining the complete input space. In this study, the difference and correlation between the two assumptions are discussed and compared. A numerical example is studied to make a further illustration. It shows that, under the new disjunctive assumption, the size of BRB could be significantly reduced and it can be more suggestive for decision-making.**

Keywords-belief rule base; conjunctive assupmtion; disjunctive assupmtion; comparative analysis

I. INTRODUCTION

Based on the traditional "IF-THEN" rule, the Belief Rule Base (BRB) is comprised of multiple rules in the same belief structure [1]. BRB further extends to a more comprehensive manner to include more complex representation of different types of information under uncertainty, including quantitative and qualitative, numeric and linguistic, complete and incomplete information [2]. With this, BRB has been applied in multiple theoretical and practical cases, including product quality assessment [3], hidden behavior prediction of complex system [4], residual life probability prediction [5], etc.

The conventional construction of BRB follows the conjunctive assumption which means that the all the possible combinations of the referenced values of the attributes must be covered. Although the conjunctive assumption can ensure a complete input space which can handle whatever the input is, it could also result in the combinatorial explosion problem because the number of the rules in BRB would grow exponentially along with more attributes and/or more referenced values of the attributes.

Chang has introduced a new rule activation procedure and the corresponding weight calculation procedure (he also uses the conventional matching degree calculation procedure), which has in fact been the core of a new disjunctive assumption for BRB construction. However, he fails to further explain the differences and similarities between the two assumptions, which is the main purpose of this study.

In this study, by reviewing the background and challenge on BRB in Section II, the challenge of following the conventional conjunctive assumption is discussed. The disjunctive assumption is introduced and further explained in Section III. A numeric example is given to compare the difference between the two assumptions in Section IV.

II. BACKGROUND AND CHALLENGE ON BELIEF RULE BASE

A. The belief rule base

BRB is comprised of multiple belief rules in the same belief structure [1]. The *k*th rule in a BRB system is described as:

$$\begin{aligned} R_k : \text{if} \quad & \left(A_1 \text{ is } x_1^k\right) \wedge \left(A_2 \text{ is } x_2^k\right) \wedge \cdots \wedge \left(A_M \text{ is } x_M^k\right), \\ & \text{then } \{(D_1, \beta_{1,k}), \cdots, (D_N, \beta_{N,k})\} \\ & \text{with rule weright } \theta_k \end{aligned} \tag{1}$$

where $A_m (m=1,\cdots M)$ denotes the *m*th attribute, $x_m^k (m=1,\cdots M; k=1,\cdots K)$ denotes the referenced values of the *m*th attribute, M denotes the number of the attributes, $\beta_{n,k} (n=1,\cdots N)$ denotes the belief for the *n*th degree, D_n, N denotes the number of the degrees. The rule described in Eq. (1) follows the conjunctive assumption.

In the "IF" part, the attributes could be either linguistic terms and/or numeric data, which could represent different types of information. In the "THEN" part, the referenced result is derived in the form of belief distribution. Each rule in BRB functions as a piece of evidence which is derived from historic data or experts' knowledge and experience. With BRB, different types of information could be represented in a unified fashion and transformed into the same belief structure for further integration by means of rules.

Lei-lei Chang thanks the partial support by NSFC under Grants 71501182, 71571185 71601180, 71671186 and 61403404, the open funding programme of Joint Laboratory of Flight Vehicle Ocean-based Measurement and Control under Grant FOM2015OF017

978-1-5090-5154-0/16 $31.00 © 2016 IEEE
DOI 10.1109/CyberC.2016.38

B. Combinatorial explosion problem caused by the conjunctive assumption

"$\wedge$" in (1) indicates that the construction of BRB follows the conjunctive assumption which requires to cover all the possible combinations of the referenced values of the attributes. The conjunctive assumption ensures that the input space is complete, denoting that there are corresponding rules being activated regardless whichever the input is.

However, the conjunctive assumption also brings with the combinatorial explosion problem, meaning that the size of BRB (number of the rules in BRB) grows along with more attributes and/or the referenced values of the attributes.

Assume there are n_m referenced value for the mth attributes, $m = 1, \cdots, M$, the size of BRB would be

$$size_{BRB} = \prod_{m=1}^{M} n_m \tag{2}$$

Fig. 1 shows the comparison of the number of rules with different number of the attributes and the referenced values of the attributes.

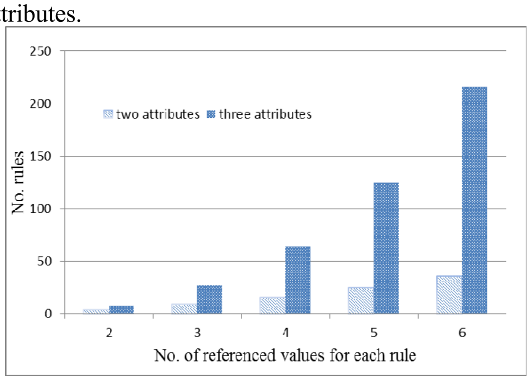

Fig. 1 Comparison of the No. of rules with different number of the attributes and the referenced values of the attributes

As in Fig. 1, with 2 to 6 referenced values for the attributes, there could be 4 to 36 rules when there are only two attributes. When the three attributes, the number of rules would grow up to 9 to 216 rules accordingly.

Take the 216 rules with 6 referenced values for each of the 3 rules as an example. There would be no chances for any expert to give 216 rules at once. Moreover, it would be very hard to further interpret and optimize the 216 rules. This calls for a new BRB construction assumption to downsize BRB.

III. DISJUNCTIVE ASSUMTION FOR BRB CONSTRUCTION

Chang has proposed the following new rule activation and weight calculation procedures, which constitutes of the disjunctive assumption by combining with the conventional matching degrees calculation procedure.

A. Rule activation procedure following the disjuntive assumtion

Consider that there is one classification example with two attributes, namely A and B [6]. Each of the two attributes has three referenced values. In order to cover all the possible conditions, a traversal BRB system which consists of 9(=3×3) rules is constructed. In this condition, suppose that there is one input, namely *I*, as shown in Fig. 2(a), then four adjacent points/rules would be activated by each input data, namely A_2B_3, A_3B_3, A_2B_2 and A_3B_2, and the matching degrees are calculated accordingly by Eq. (A.1) from Appendix A.

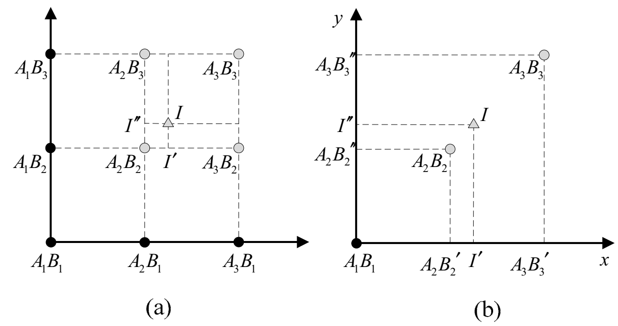

Fig. 2 conventional and new rule activation

Next, simplify the conventional rule activation procedure as in Fig. 2(b), in which only three points/rules are kept, including the two that represent the maximums/minimum values of the attributes (upper right, A_3B_3, and lower left, A_1B_1, in Fig. 2(b)) so as to define the boundaries. With this, the input data (as well as the three points/rules) is projected in the *x*/*y* axis first to determine which (two) points/rules are most adjacent and should be activated.

For this specific condition as shown in Fig. 2(b), A_2B_2 and A_3B_3 are activated since their projections in the x axis, A_2B_2' and A_3B_3', are closer to I' which is the x axis projection of I. A_2B_2 and A_3B_3 are also activated since their projections in the y axis, A_2B_2'' and A_3B_3'', are closer to I'' which is the y axis projection of I. To conclude, the two points/rules, A_2B_2 and A_3B_3, would be activated.

B. The conventional weight calculation procedure following the conjunctive assumption

The activated weight for the kth rule is calculated by Eq. (4),

$$w_k = \frac{\theta_k \sum_{i=1}^{M} \alpha_i^k}{\sum_{l=1}^{L} \theta_l \prod_{i=1}^{M} \alpha_i^l} \tag{A.4}$$

where θ_k represents the initial weight of the kth rule, and α_i^k represents the matching degree of the input with the kth rule.

C. The weight calculation procedure following the disjunctive assumtion

The new weight calculation procedure assumes that the attributes are "disjunctive". The kth rule's activated weight is calculated by Eq. (13)

$$w_k = \frac{\theta_k \sum_{i=1}^{M} \alpha_i^k}{\sum_{l=1}^{L} \theta_l \sum_{i=1}^{M} \alpha_i^l} \tag{13}$$

where θ_k represents the initial weight of the kth rule, and α_i^k represents the matching degree of the input with the kth rule.

IV. NUMERIC CASE STUDY

Here we give a numerical example of the disjunctive assumption. Suppose that there is a simple problem with two attributes, namely A and B. For each attribute, there are two referenced values. Following the conventional conjunctive assumption, there would be four rules, as shown in Fig 3(a) while there could be two rules if the new disjunctive assumption is followed, as shown in Fig. 3(b). It is obvious that the two rules in Fig. 3(b) are the right-left and upper-right boundary rules in in Fig. 3(a) so that there would be rules activated whichever the input is.

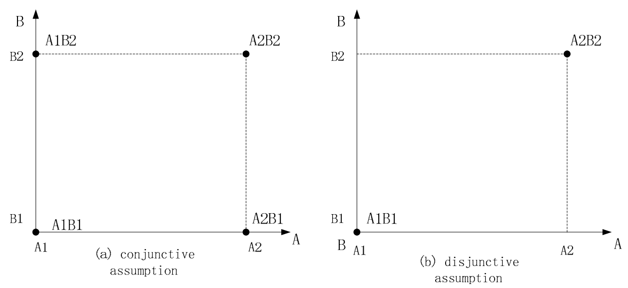

Fig. 3 the difference between the conjunctive and the disjunctive assumptions

Under the disjunctive assumption, it shows that only the referenced values of the upper and lower limits of the attributes are necessary, which constitutes of two rules holding the boundaries of the input space as the A1B1 and A2B2 in Fig. 3(b). Other rules in Fig. 3(a), namely A1B2 and A2B1, can be derived by projecting on the axes of A and B, or more dimensions depending the conditions of the practical problems.

With this, the number of the rules in BRB following the disjunctive assumption is no longer related to the number of attributes but only to the number of the referenced values for the attributes.

Fig. 4 compared the number of rules in BRB following different assumptions.

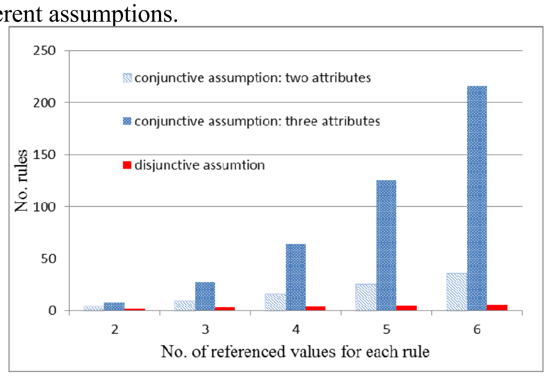

Fig. 4 the number of rules in BRB following different assumptions

Assume that there are two scales in the conclusion part in both the four and two rules, namely G (stands for "good") and B (stands for "bad"). The four rules are given in Table 1.

Note that rule No.2 and rule No.3 are assumed to be the "averaged" of rule No. 1 and rule No. 4 in order to draw a fair conclusion.

Assume that there is input named as I. For any input, the four rules are activated in the conjunction assumption condition, and two rules are activated following the disjunctive assumption. As shown in Fig. 5.

Table 1 rules for the numeric case

No.	Name	Conclusion
1	A1B1	((B, 100%), (G, 0))
2	A1B2	((B, 50%), (G, 50%))
3	A2B1	((B, 50%), (G, 50%))
4	A2B2	((B, 0), (G, 100%))

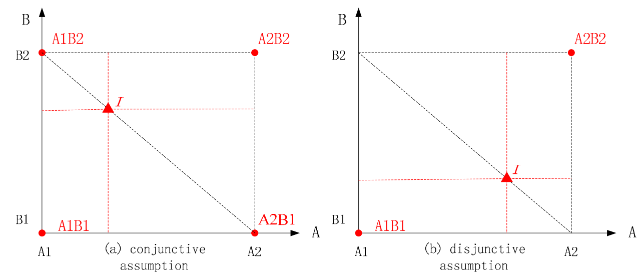

Fig. 5 rule activation following different assumptions

Consider the nine different inputs ranging from ((A, 0.1), (B, 0.1)) to ((A, 0.9), (B, 0.9)). The inference results under both assumptions are shown in Table 2.

Table 2 activation weights and inference results---Condition I

coordinates of inputs		activation weights			
A	B	A1B1	A1B2	A2B1	A2B2
0.1	0.9	0.09	0.81	0.01	0.09
0.2	0.8	0.16	0.64	0.04	0.16
0.3	0.7	0.21	0.49	0.09	0.21
0.4	0.6	0.24	0.36	0.16	0.24
0.5	0.5	0.25	0.25	0.25	0.25
0.6	0.4	0.24	0.16	0.36	0.24
0.7	0.3	0.21	0.09	0.49	0.21
0.8	0.2	0.16	0.04	0.64	0.16
0.9	0.1	0.09	0.01	0.81	0.09
		conclusion (different assumption)			
		conjunctive		disjunctive	
A	B	G	B	G	B
0.1	0.9	0.5	0.5	0.5	0.5
0.2	0.8	0.5	0.5	0.5	0.5
0.3	0.7	0.5	0.5	0.5	0.5
0.4	0.6	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5
0.6	0.4	0.5	0.5	0.5	0.5
0.7	0.3	0.5	0.5	0.5	0.5
0.8	0.2	0.5	0.5	0.5	0.5
0.9	0.1	0.5	0.5	0.5	0.5

Apparently, there would be no difference with these inputs. The inference result would be ((G, 0.5), (B, 0.5)) for each possible input.

It has been proved by Chang et al. (2016) that the matching degree calculation results would be the same under the disjunctive assumption and the conjunctive assumption.

Next, we would consider the inputs ranging from ((A, 0.1), (B, 0.9)) to ((A, 0.9), (B, 0.1)) along the diagonal line from A1B2 to A2B1, as shown in Fig. 6.

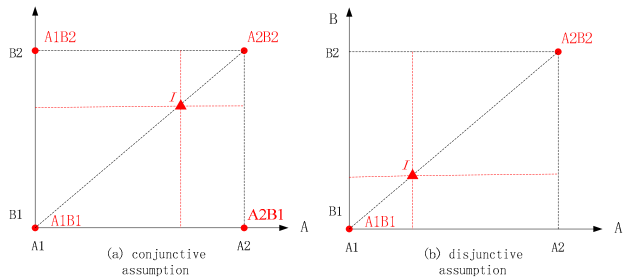

Fig. 6 rule activation following different assumptions---a further discussion

Apparently, the activation condition would be the same by following both the conjunctive and disjunctive assumption: four rules and two rules would be activated, respectively. However, the activation weights and the final inference results, as shown in Fig. 7, are quite different this time.

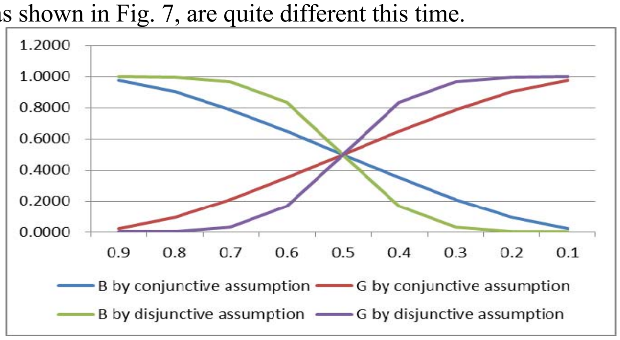

Fig. 7 inference results following different assumptions

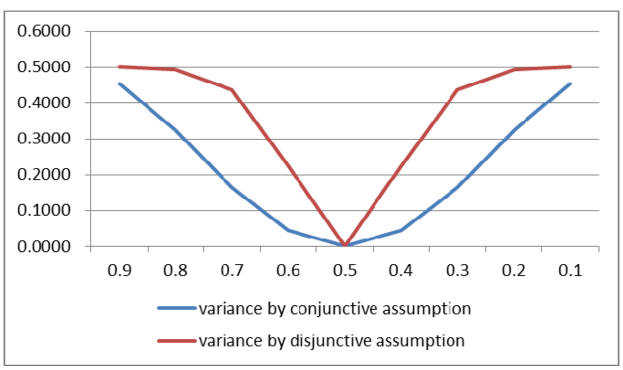

Fig. 8 variances of the results following different assumptions

It shows that the inference results are overall consistent: the inference result following either assumption points to the same result for decision-making.

However, the beliefs, or the confidence in pointing to the result is different. Fig. 8 compared the variances following different assumptions.

Fig. 8 shows that the variances following the disjunctive assumption is significantly bigger than the ones following the conjunctive assumption. The only condition when a tie is derived is when the input is in the middle between the upper and lower levels of the attributes. It denotes that, the disjunctive assumption is more suggestive for the decision-making compared with the conjunctive assumption.

V. Discussion

Although BRB has been successfully applied in many complex system modeling problems, the conventional conjunctive assumption for BRB construction has always been a dilemma when there are over-numbered attribute and/or referenced values of the attributes.

In this study, the disjunctive assumption is further illustrated and further compared with the conjunctive assumption, which is further validated by the numeric example. The case study result shows that, under the conjunctive, the size of BRB could be significantly reduced and it can be more suggestive for decision-making.

For further research, more practical case should be studied to validate the disjunctive assumption. And more characteristics of the disjunctive assumption should be mathematically proved as well.

References

[1] J. B. Yang, J. Liu, J. Wang, H. S. Sii, and H. W. Wang, "Belief rule-base inference methodology using the evidential reasoning approach-RIMER," IEEE Trans. Syst., Man, Cybern., A, Syst., Humans, vol. 36, pp. 266-285, 2006.

[2] Y. W. Chen, J. B. Yang, C. C. Pan, D. L. Xu, and Z. J. Zhou, "Identification of uncertain nonlinear systems: Constructing belief rule-based models," Knowl.-Based Syst., vol. 73, pp.124-133, 2015.

[3] Y. M. Wang, J. B. Yang, D. L. Xu, and K. S. Chin, "The evidential reasoning appraoch for multiple attribute decision analysis using interval belief degrees," Eur. J. Oper. Res., vol. 175, pp. 35-66, 2006.

[4] Z. J. Zhou, C. H. Hu, G. Y. Hu, X. X. Han, B. C. Zhang, and Y. W. Chen, "Hidden behavior prediction of complex systems under testing influence based on semi-quantitative information and belief rule base," IEEE Trans. Cybern., vol. 23, pp.2371-2386, 2015.

[5] L. L. Chang, J. B. Sun, J. Jiang, and M. J. Li, "Parameter learning for the belief rule base system in the residual life probability prediction of metalized film capacitor," Knowl.-Based Syst., vol. 73, pp. 69-80, 2015.

[6] L. L. Chang, Z. J. Zhou, Y. You, L. H., Yang, Z. G. Zhou, "Belief Rule Based Expert System for Classification Problems with New Rule Activation and Weight Calculation Procedures," Infor. Sci., vol. 336, pp. 75-91, 2015.