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A disjunctive belief rule-based expert system for bridge risk assessment with dynamic parameter optimization model

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ABSTRACT

Bridge risk assessment is an important approach to avoiding the safety accidents of bridges and ensuring the safety of the public. This can be done by investigating the relationship between bridge risks and bridge criteria. However, such relationship usually is highly complicated in actual situations. In this regard, many approaches were proposed to model bridge risks in the past decades. Particularly, four alternative approaches including the artificial neural network (ANN), evidential reasoning with learning (ERL), multiple regression analysis (MRA), and adaptive neuro-fuzzy inference system (ANFIS) were deeply analyzed and compared for bridge risk assessment. However, these approaches are restricted by their shortages. Thus, this paper utilizes the disjunctive belief rule-based (DBRB) expert system to model bridge risks, where the DBRB expert system is one type of the belief rule-based (BRB) expert system by considering disjunctive belief rules (DBRs) rather than conjunctive belief rules (CBRs) in a BRB. Furthermore, the dynamic parameter optimization model and improved differential evolution (IDE) algorithm are proposed to train the parameters of the DBRB expert system, where the model is applied to ensure the completeness of a DBRB and the algorithm is used to get the global optimal solution. For justification purpose, two existing parameter optimization models and nine alternative models developed by the ANN, ERL, MRA, and ANFIS are applied to assess bridge structures. Comparison results indicate that the DBRB expert system with the dynamic parameter optimization model is better than those alternative models and existing parameter optimization models.

1. Introduction

As an important hub for land transports, bridges have a direct influence on economic and social developments. However, the safety accidents of bridges occur frequently due to the reasons of ageing, damage, structural deficiency, and so on (Andric & Lu, 2016). For instance, the safety accident of I-35 W Mississippi River Bridge located in U.S.A. is caused by lacking of maintenance. The safety accident of Jinsha River Nanmen Bridge located in China is due to the damage of boom. The safety accident of Jijiang Rainbow Bridge located in China results from the deficiency of bridge structures.

To avoid these safety accidents, bridge risks have to be assessed periodically so that the bridges can be maintained timely to ensure the safety of the public (Housner & Caughey, 1997). In general, bridge risks are determined in the terms of the risk scores of bridge structures, and these scores are closely associated with different criteria such as *Safety*, *Functionality*, *Sustainability* and *Environment* based on the British Highways Agency (2004). For example, while the level of *Safety* is assessed as *High*, the *Functionality* as *Medium*, the *Sustainability* as *High*, and the *Environment* as *High*, then the bridge structure can be rated as a high risk score.

However, bridge risks are usually being assessed by the experienced or well-trained bridge experts, who are required to provide a numerical rating to bridges in accordance with visual inspections such as what the bridge appearance looks like, whether the bridge structures have crack, or others. Actually, the above process proves to be highly subjective and is not always available, because bridge experts may find it difficult or even impossible to provide a numerical rating due to lack of domain knowledge and measurement data (Yau et al., 2014).

To more effectively assess bridge structures, many prediction models were proposed in the past decades. These models can be divided into two kinds: with and without parameter learning.

For the prediction model without parameter learning, Stewart (2001) put forward a reliability-based approach to assess the ageing bridge structures using risk ranking and life cycle cost decision analyses. This approach provided a meaningful measure of bridge

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performance. Adey, Hajdin, and Bruhwiler (2003) presented a risk-based approach to determine the optimal intervention for the bridge suffered from several hazards. Johnson and Niezgoda (2004) used failure modes and effects analysis (FMEA) to construct a system for choosing the most appropriate bridge scour countermeasures. Their results showed that the FMEA is a relatively simple and systematic technique for assigning relative risks. Andric and Lu (2016) proposed a framework of bridge risk assessment by combining the fuzzy AHP and fuzzy logic techniques into a single integrated approach. Their results show that the integrated approach is an effective tool for quick disaster risk analysis. The similar mechanisms including fuzzy group decision making approach and the integrated AHP-DEA methodology were proposed in Wang and Elhag (2007b) and Wang, Liu, and Elhag (2008) for bridge risk assessment, respectively.

For the prediction model with parameter learning, neural networks were widely used for modeling bridge risks. For example, Li, Shi, and Ososanya (1996) came up with a feasibility study on the use of neural networks in bridge condition evaluation. Cattan and Mohammadi (1997) described an application of neural network system in developing the relationship between subjective ratings and bridge criteria. Kawamura and Miyamoto (2003) developed a concrete bridge rating expert system for deteriorated concrete bridges using multilayer neural networks. Despite the wide application of neural networks, none of these applications included the comparisons of other methods. Therefore, Wang and Elhag (2007a) compared the modeling mechanisms of the artificial neural network (ANN), evidential reasoning with learning (ERL), and multiple regression analysis (MRA) for bridge risk assessment. Their results indicate that the ANN outperforms the other two methods. Another similar comparison was conducted in Elhag and Wang (2007). Furthermore, Wang and Elhag (2008) introduced the adaptive neuro-fuzzy inference system (ANFIS) to compare it with the ANN in modeling bridge risks. Their results showed that the ANFIS was better than the ANN.

Although the above attempts have produced many promising results in modeling bridge risks, the following limitations can be summarized:

(1) The models without parameter learning have to recalculate risk scores for each bridge projects using the original data, which could be computationally expensive as the number of available data increases.
(2) The ERL only determines the relative weight of bridge criteria by using parameter learning, which cannot make full use of available data and their implicit information.
(3) The ANN belongs to a black-box methodology so that it is not easy to explain the meaning of parameters. In particular, whether the negative connection weights are acceptable for modeling bridge risk may need further research.
(4) The application of MRA has to know the concrete functional relationship between inputs and outputs in advance, but the relationship between bridge risks and bridge criteria is complicated and unknown for bridge risk assessment.
(5) The construction of ANFIS must cover all fuzzy numbers of each antecedent attribute. Thus, the number of fuzzy rules is exponential based on the number of fuzzy numbers and/or antecedent attributes.

Due to these limitations, the belief rule-based (BRB) expert system (Yang, Liu, Wang, Sii, & Wang, 2006), which has been widely used for different purposes such as failure prognosis (Jiang, Zhou, Han, Zhang, & Ling, 2014; Liu, Liu, & Lin, 2013; Zhou et al., 2015), oil pipeline leak detection (Xu et al., 2007; Yang, Wang, Lan, Chen, & Fu, 2017), clinical disease diagnosis (Hossain, Ahmed, Fatema, & Andersson, 2017; Zhou et al., 2015; Kong, Xu, Yang, & Ma, 2015), Forex trading prediction (Dymova, Sevastjanov, & Kaczmarek, 2016), consumer preference prediction (Tang, Yang, Chin, Wong, & Liu, 2011; Yang, Fu, Chen, Xu, & Yang, 2016), and basic classification problem (Calzada, Liu, Wang, & Kashyap, 2015; Yang, Wang, Su, Fu, & Chin, 2016), is applied to model bridge risks in this study.

To date, the most common belief rule in a BRB is based on the conjunctive relationship between antecedent attributes, namely conjunctive belief rules (CBRs), which is required to cover all combinations of each referential value for each antecedent attribute. It has been proved to be the combinatorial explosion problem that the number of CBRs would grow exponentially along with the increase of the antecedent attributes and/or the referential values for each antecedent attribute (Chang et al., 2017). Hence, similar to both the modified application model and the original theoretical model, the BRB expert system also needs to be modified for bridge risk assessment.

Recently, Chang, Zhou, You, Yang, and Zhou (2016) proposed the BRB expert system for classification problems, where the BRB consists of disjunctive belief rules (DBRs), so that this type of BRB can efficiently overcome the combination explosion problem of the BRB comprising CBRs. Hence, the present work is based on the BRB expert system which consists of DBRs, namely the DBR-based (DBRB) expert system. However, the existing parameter optimization model (Chang et al., 2016) of the DBRB expert system required that part of DBRs have to keep all lowest and highest referential values of each antecedent attribute during parameter learning. This requirement is subjective and unnecessary for bridge risk assessment. Obviously, many types of DBRs cannot be derived from parameter learning because the DBRB has lost completeness.

As such, a new parameter optimization model for the DBRB expert system is proposed to model bridge risks. In the new model, none of DBRs must include all highest and lowest referential values and these referential values can be distributed on different DBRs owning to two dynamic constraints. Thus, the proposed model is also called the dynamic parameter optimization model. Considering that the conventional optimization toolbox in MATLAB, Excel, and Lingo are not easy to solve the dynamic optimization model, a global optimization algorithm by improving the differential evolution (DE) algorithm (Price, Storn, & Lampinen, 2005), namely IDE algorithm, is then proposed to train the parameters of the DBRB expert system for bridge risk assessment.

To verify the effectiveness of the proposed model and algorithm, the process of parameter learning is used to display the model development of the DBRB expert system in modeling bridge risks. Two existing parameter optimization models related with CBRs and DBRs and nine alternative models developed by ANN, ERL, MRA, and ANFIS are applied to compare the performance of the developed DBRB expert system by the proposed model and algorithm.

The rest of the paper is organized as follows. Section 2 briefly reviews the basic knowledge of the DBRB expert system. Section 3 presents the completeness of the DBRB expert system, new parameter optimization model and global optimization algorithm. Section 4 develops a DBRB expert system using 66 bridge maintenance projects and compares its performance with those of other approaches, and Section 5 concludes the paper.

2. DBRB expert system

The BRB expert system was proposed by Yang et al. (2006) based on belief rules and the evidential reasoning (ER)-based inference. Basically, the belief rules can be separated into two types: CBRs and DBRs. This section focuses on DBRs to introduce the BRB expert system and advantages of utilizing the DBRB expert system to model bridge risks.

2.1. Belief distribution and DBR

A belief distribution (BD) was initially applied to describe a subjective assessment with uncertain information. For instance, a bridge risk can be assessed as *High* with risk probability 70%, *Medium* with 20%, and *Small* with 10%. Thus, the assessment of the bridge risk can be described by the following BD:

$$BD(Bridge\ Risk) = \{(High,\ 0.7),\ (Medium,\ 0.2),\ (Small,\ 0.1)\} \tag{1}$$

where the sum of belief degrees is 0.7 + 0.2 + 0.1 = 1.0 which indicates a complete assessment. However, when data, information, and knowledge are lacking, the assessment of bridge risks is often incomplete. For instance, another BD of a bridge risk is represented as:

$$BD(Bridge\ Risk) = \{(High,\ 0.7),\ (Medium,\ 0.1),\ (Small,\ 0.1)\} \tag{2}$$

where the assessment of the bridge risk is incomplete because the sum of belief degrees is 0.7 + 0.1 + 0.1 = 0.9 < 1.0. In addition, the rating such as *High*, *Medium*, and *Small* can be fuzzy number. Therefore, as a general framework, the BD is able to contain several varieties of uncertainties including incomplete, fuzzy, and probabilistic information.

In general, a BD can be described by Eq. (3), where *BD*(*Consequent*) represents the consequent in the term of a BD, D_n is the nth ($n = 1, \ldots, N$) rating and N is the number of ratings, β_n is the belief degree attached to the rating D_n.

$$BD(Consequent) = \{(D_1,\ \beta_1),\ (D_2,\ \beta_2),\cdots,\ (D_N,\ \beta_N)\} \tag{3}$$

By embedding the BD into the THEN part of IF-THEN rules, Yang et al. (2006) proposed the BRB expert system. A belief rule in the form of the disjunctive IF-THEN rule can be written as follows:

$$\begin{aligned} R_l:\ & IF\ (U_1\ is\ A_1^l)\vee(U_2\ is\ A_2^l)\ \vee\cdots\vee(U_M\ is\ A_M^l),\ THEN\ \{(D_1,\beta_1^l),(D_2,\beta_2^l),\cdots,(D_N,\beta_N^l)\},\\ & with\ rule\ weight\ \theta_l\ and\ attribute\ weight\ \delta_1,\delta_2,\ldots,\delta_M \end{aligned} \tag{4}$$

where U_m is the mth ($m = 1, \ldots, M$) antecedent attribute, M is the number of antecedent attributes; A_m^l is the referential value in the mth antecedent attribute for the lth ($l = 1, \ldots, L$) rule, L is the number of rules in a BRB; β_n^l is the belief degree assessed to the nth rating in the lth rule. The attribute weight δ_m and the rule weight θ_l are weight parameters for the mth antecedent attribute and the lth rule, respectively.

Compared with the most common CBRs in the BRB expert system, the construction of DBRs is required to cover each referential value of all antecedent attributes only once at any DBRs, so that it can efficiently reduce the number of rules for a BRB. Taking bridge risk assessment for example, there are four antecedent attributes including *Safety*, *Functionality*, *Sustainability*, and *Environment*, and all these antecedent attributes have three referential values such as *Low*, *Medium*, and *High*. As a result, the number of DBRs is 3 and Table 1 illustrates the generic forms of these DBRs.

2.2. ER-based inference and result explanation

The ER-based inference of the DBRB expert system includes three steps, and these steps are shown in Fig. 1.

Step 1: To calculate individual matching degrees.

Suppose that the tth input $\boldsymbol{x_t} = \{x_{t,m};\ m = 1, \ldots, M\}$ is provided for a DBRB expert system, each input $x_{t,m}$ needs to be transformed into a BD using the utility or rule-based information transformation technique (Yang et al., 2006).

$$S(x_{t,m}) = \{(A_{m,j},\alpha_{m,j});j = 1,\ldots,J_m\}, \tag{5}$$

where

$$\alpha_{m,j} = \frac{u(A_{m,j+1})-x_{t,m}}{u(A_{m,j+1})-u(A_{m,j})}\ and\ \alpha_{m,j+1} = 1-\alpha_{m,j},\ if\ \ u(A_{m,j}) \leqslant x_{t,m} \leqslant u(A_{m,j+1}). \tag{6a}$$

$$\alpha_{m,k} = 0,\ for\ \ k = 1,\ldots,J_m\ \ and\ \ k \neq j,j+1 \tag{6b}$$

where $A_{m,j}$ is the jth referential value of the mth antecedent attribute and J_m is the number of referential values in the mth antecedent attribute, $A_m^l \in \{A_{m,j};\ j = 1, \ldots, J_m\}$ ($l = 1, \ldots, L$), $u(A_{m,j})$ is the utility value of the referential value $A_{m,j}$, $\alpha_{m,j}$ is the individual matching degree to which the input $x_{t,m}$ matches the referential value $A_{m,j}$. Specifically, based on the IF part of DBR illustrated in Eq. (4), the individual matching degree of the lth rule can be denoted as follow:

$$(A_1^l,\alpha_1^l)\vee(A_2^l,\alpha_2^l)\vee\cdots\vee(A_M^l,\alpha_M^l) \tag{7}$$

Step 2: To calculate activated weights.

To calculate the activated weight of the lth DBR, three parameters including rule weights, attribute weights and individual matching degrees should be involved and the calculation formula is expressed as follows:

$$w_l = \frac{\theta_l \sum_{m=1}^{M} (\alpha_m^l)^{\bar{\delta}_m}}{\sum_{k=1}^{L} (\theta_k \sum_{m=1}^{M} (\alpha_m^k)^{\bar{\delta}_m})}, \tag{8}$$

where

$$\bar{\delta}_m = \frac{\delta_m}{\max\limits_{i=1,\ldots,M} \{\delta_i\}}. \tag{9}$$

Remark 1. Compared with the most common BRB expert system which consists of CBRs, the DBRB expert system utilizes cumulative operation (Chang et al., 2016) to calculate activated weights because of the disjunctive behavior used to connect antecedent attributes.

Step 3: To integrate activated rules using ER.

After calculating activated weights, all the activated rules should be integrated using the following analytical ER algorithm (Wang, Yang, & Xu, 2006).

$$\beta_n(\boldsymbol{x}_t) = \frac{\prod\limits_{l=1}^{L} (w_l\beta_n^l + 1-w_l \sum_{i=1}^{N} \beta_i^l) - \prod\limits_{l=1}^{L} (1-w_l \sum_{i=1}^{N} \beta_i^l)}{\sum_{i=1}^{N} \prod\limits_{l=1}^{L} \left(w_l\beta_i^l + 1-w_l \sum_{j=1}^{N} \beta_j^l\right) - (N-1) \prod\limits_{l=1}^{L} \left(1-w_l \sum_{j=1}^{N} \beta_j^l\right) - \prod\limits_{l=1}^{L} (1-w_l)}. \tag{10}$$

Suppose $u(D_n)$ represents the utility value of the nth rating, the output of the DBRB expert system is described as follows:

Table 1
DBRs for bridge risk assessment.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent		
		Safety (1.0)	Functionality (1.0)	Sustainability (1.0)	Environment (1.0)	*Small*	*Medium*	*High*
1	1.0	*Low*	*High*	*Low*	*Low*	0.9	0.1	0.0
2	1.0	*Medium*	*Low*	*Medium*	*Medium*	0.6	0.4	0.0
3	1.0	*High*	*Medium*	*High*	*High*	0.0	0.2	0.8

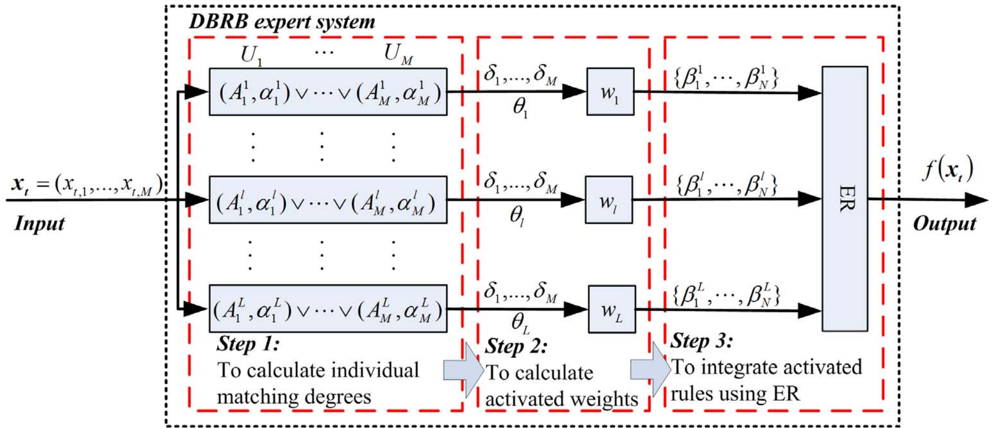

Fig. 1. Illustration of DBRB expert system.

$$f(\boldsymbol{x}_t) = \sum_{n=1}^{N} (u(D_n)\beta_n(\boldsymbol{x}_t)) + \left(1-\sum_{n=1}^{N}\beta_n(\boldsymbol{x}_t)\right)(u(D_1)+u(D_N))/2 \tag{11}$$

In addition, Eq. (11) shows that there is incomplete information in the THEN part of a DBRB. Otherwise, the output of the DBRB expert system can be simplified as:

$$f(\boldsymbol{x}_t) = \sum_{n=1}^{N} (u(D_n)\beta_n(\boldsymbol{x}_t)) \tag{12}$$

2.3. Summary

Modeling bridge risks is very challenging for Highway Agencies because excellent models can save them a considerable amount of cost and time. Therefore, the advantages of utilizing the DBRB expert system to model bridge risks over other approaches are summarized as follows:

Compared with the most common BRB expert system which consists of CBRs, the DBRB expert system can be more concise and more efficient, namely:

(1) At the same condition in constructing belief rules for bridge risk assessment, the number of DBRs is linear relationship with the number of antecedent attributes and/or referential values, so that it can overcome the combinatorial explosion problem of the BRB comprising CBRs. Usually, bridge risks are modeled against various criteria and risk ratings. The number of these criteria and risk ratings may be very large. Even so, the DBRB expert system will certainly be able to model bridge risks, but it is impossible for the most common BRB expert system.
(2) As an important approach to improve the accuracy of the BRB expert system, the parameter learning is usually affected by the number of parameters such as referential values, attribute weights, and rule weights. Owning to the rules fewer than the BRB comprising CBRs, the DBRB can efficiently be trained using parameter learning. As such, with the increasing number of bridge maintenance projects, it is still possible to construct a DBRB expert system for bridge risk assessment within an acceptable cost and time.
(3) The time complexity of the ER-based inference is related to the number of rules, because of the need to calculate activated weights and integrate activated rules. Hence, compared with the most common BRB expert system, the DBRB expert system can efficiently reply to input data. Such the application of the DBRB expert system will obviously be of great benefit to the Highway Agency, which can manage and control bridge risks more easily than before and needs no longer to spend too much time in assessing bridge structures.

Compared with other conventional approaches such as ANN, MRA, and ERL, the DBRB expert system imports all the benefits from the most common BRB expert system which has been proven an advanced methodology. These benefits include:

(1) The DBRB expert system is implemented like a white-box modeling that all parameter have certain meanings and can be explained properly. For instance, the attribute weight denotes the relative importance in the view of different criteria, the belief degree stands for the bridge risk probability in the rating, etc. Therefore, compared with the ANN, the DBRB expert system will provide the decision makers of bridge risk assessment with a complete tool to help them identify risk scores for each bridge maintenance project.
(2) The DBRB expert system is unnecessary to know the specification of the concrete functional relationship between inputs and outputs in advance. Furthermore, the new parameter optimization model illustrated in Section 3 can help the DBRB expert system learn the functional relationship no matter whether it is linear or nonlinear. Therefore, compared with the MRA, the DBRB expert system can learn and memorize the complicated nonlinear relationship between bridge risks and bridge criteria without depending on highly subjective knowledge.
(3) The DBRB expert system can determine the utility value of referential values, rule weights, belief degrees and not just attribute weights by using parameter learning, which make use of available data more effectively. Moreover, the ER algorithm is used as the inference engine of the DBRB expert system so that it inherits the merits of the ER algorithm. Therefore, compared with the ERL, the DBRB expert system can be regarded as an experienced or well-trained bridge expert and has powerful ability to judge what benefits and disbenefits each bridge maintenance project can deliver.

Compared with the ANFIS, which is similar to the most common BRB expert system because of conjunctive rules used for the ANFIS, the DBRB expert system has more concise and efficient parameter learning and rule inference. More importantly, with the increasing number of antecedent attributes and/or fuzzy numbers, the ANFIS may face the combinatorial explosion problem, but not for the DBRB expert system certainly. It is believed that the DBRB expert system can perform better than the ANFIS for bridge risk assessment.

In addition, DBRs are able to capture vagueness (with referential values or ratings), probability (with belief degrees), and incompleteness (the sum of belief degrees being less than one). Also, the ER-based inference can avoid losing information and provide the aggregated result with a BD and numerical value based on the BD. This without doubt improves the accuracy of the DBRB expert system in modeling bridge risks.

3. Dynamic parameter optimization model for DBRB expert system

In order to more effectively model bridge risks, it is necessary to investigate the DBRB expert system. Thus, in this section, the completeness of the DBRB expert system associated with the parameter optimization model is analyzed firstly, followed by a new parameter optimization model and algorithm to ensure the completeness of the DBRB expert system.

3.1. Completeness of DBRB expert system with parameter optimization model

The completeness is an important criterion to reflect the applicability of a rule-based system. In the most common BRB expert system, because of the belief rules constructed by covering each antecedent attribute of all referential values, all types of belief rules can be found in a BRB and the BRB expert system is able to reply to any input data. Hence, the most common BRB expert system is considered to be complete by default.

However, for the DBRB expert system, which is another type of the BRB expert system with fewer belief rules, it is necessary to investigate the completeness. In the previous literatures, Chang et al. (2016) and Chang, Zhou, Liao, Chen, Hua, and Yang (2017) studied the DBRB expert system and proved the completeness of ER-based inference, which can be defined as follows:

Definition 1 (*Completeness of ER-based inference*). The ER-based inference of the DBRB expert system is said to be complete, if and only if no matter what input data are provided, the DBRB expert system can generate concrete outputs.

Clearly, Definition 1 only provides one kind of factors to ensure the completeness of the DBRB expert system, because the DBRB expert system includes two components: a DBRB and the ER-based inference. In the following, a more general investigation on the completeness of the DBRB expert system is provided.

It is obvious that an initial DBRB expert system does not always have a satisfactory performance, because experts may find it difficult or even impossible to determine optimal values for each parameter. To solve this problem, input-output data pairs must be collected to improve the performance of the DBRB expert system, namely parameter learning (Chen, Yang, Xu, Zhou, & Tang, 2011; Yang, Liu, Xu, Wang, & Wang, 2007) which is shown in Fig. 2.

Fig. 2 shows that the essence of parameter learning is to train parameters by using a parameter optimization model. As such, the function of parameter optimization model can be simply regarded as the recombination of referential values for each DBR. For instance, while the DBRB illustrated in Table 1 is unable to accurately assess bridge structures, this DBRB must be trained by using the parameter optimization model for obtaining a satisfactory performance. The trained DBRB can be shown in Table 2.

Compared with two DBRBs shown in Tables 1 and 2, the significant difference is that the same DBR has different referential values. For example, in Table 1, the referential values of the first rule include *Low*, *High*, *Low*, and *Low*. Correspondingly, there are *High*, *Medium*, *Low*, and *High* in the first rule shown in Table 2. Thus, the recombination of referential values for each DBR is an important approach to improving the performance of the DBRB expert system. However, due to some drawbacks of parameter optimization model, it is impossible to obtain all types of DBRs for a DBRB. In this regard, it reflects another factor to ensure the completeness of the DBRB expert system, which can be defined as follow:

Definition 2 (*Completeness of DBRB*). A DBRB is said to be complete, if and only if no matter what types of DBRs in a DBRB can be obtained by using the parameter optimization model.

Further, the definition regarding the completeness of the DBRB expert system is given as follow:

Definition 3 (*Completeness of DBRB expert system*). A DBRB expert system is said to be complete, if and only if it satisfies with the completeness of ER-based inference and DBRB.

Remark 2. A DBRB expert system is usually incomplete, because the incompleteness of a DBRB is easily caused by the deficient parameter optimization model which suffers from neglected parameters and inaccurate constraints, i.e., if there is an inaccurate constraint that the first rule does not include the referential value *Medium*, then the trained DBRB shown in Table 2 will have no way to be obtained by the parameter optimization model.

Remark 3. Although a DBRB expert system is incomplete, it is able to have a satisfactory performance, because the deficient parameter optimization model just not considers some sub-optimal solutions of the trained DBRB. For example, while the trained DBRB shown in Table 2 is a sub-optimal solution for the DBRB expert system to have a satisfactory performance, the process of searching for the optimal solution allows the deficient parameter optimization model not to consider that trained DBRB.

3.2. Dynamic parameter optimization model for parameter learning

As a result from earlier investigation in Section 3.1, the parameter optimization model can be regarded as another factor to ensure the completeness of the DBRB expert system. Thus, this section uses the existing parameter optimization model proposed by Chang et al. (2016) to further investigate the completeness of the DBRB expert system.

Suppose T pairs of input $\boldsymbol{x_t} = \{x_{t,m}; m = 1, \ldots, M\}$ $(t = 1, \ldots, T)$ and actual output y_t have been collected. Based on Eqs. (5)–(12) illustrated in Section 2.2, T inferential outputs can be generated using the DBRB expert system while all DBRs in the DBRB are given or known. These inferential outputs may be different from the actual outputs and their deviations are defined as follows:

$$\xi_t = f(\boldsymbol{x}_t) - y_t, \quad t = 1, \ldots, T \tag{13}$$

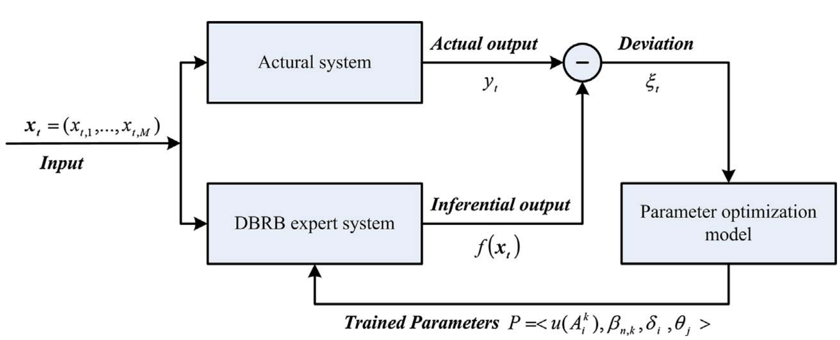

Fig. 2. Parameter learning for DBRB expert system.

Table 2
Trained DBRB for bridge risk assessment.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent		
		Safety (1.0)	Functionality (1.0)	Sustainability (1.0)	Environment (1.0)	*Small*	*Medium*	*High*
1	1.0	*High*	*Medium*	*Low*	*High*	0.9	0.1	0.0
2	1.0	*Low*	*Low*	*High*	*Low*	0.6	0.4	0.0
3	1.0	*Medium*	*High*	*Medium*	*Medium*	0.0	0.2	0.8

It is most desirable that these deviations be kept as small as possible. Based upon this point of view, the parameter optimization model should be used to minimize these deviations. First of all, the parameters to be trained must satisfy linear equality and inequality constraints. By reviewing the model of Chang et al. (2016), the constraints include:

(1) The utility value of referential values used for antecedent attributes. For the mth ($m = 1, \ldots, M$) antecedent attribute in the lth ($l = 1, \ldots, L$) DBR, the utility value $u(A_m^l)$ of the referential value A_m^l must satisfy the following constraints:

$$lb_m \leqslant u(A_m^l) \leqslant ub_m; \quad l = 1,\ldots,L; \quad m = 1,\ldots,M \tag{14a}$$

$$u(A_m^1) = lb_m; \quad m = 1,\ldots,M \tag{14b}$$

$$u(A_m^L) = ub_m; \quad m = 1,\ldots,M \tag{14c}$$

where lb_m and ub_m are the lower and upper bounds of the mth antecedent attribute, respectively.

(2) The rule weight of DBRs. For the lth ($l = 1, \ldots, L$) DBR, the weight θ_l must satisfy the following constraint:

$$0 < \theta_l \leqslant 1; \quad l = 1,\ldots,L \tag{14d}$$

(3) The belief degree of DBRs. For the nth ($n = 1, \ldots, N$) belief degree of the lth ($l = 1, \ldots, L$) DBR, the belief degree β_n^l must satisfy the following constraint:

$$0 \leqslant \beta_n^l \leqslant 1; \quad l = 1,\ldots,L; \quad n = 1,\ldots,N \tag{14e}$$

When there is no incomplete information in the THEN part, the lth DBR must satisfy the following constraint:

$$\sum_{n=1}^{N} \beta_n^l = 1; \quad l = 1,\ldots,L \tag{14f}$$

However, the following shortcomings of the above parameter optimization model must be recognized:

(1) The parameter optimization model fails to train attribute weights for the DBRB expert system. Obviously, attribute weights are key parameters in a DBRB to show the relative importance of different antecedent attributes. For example, for bridge risk assessment, it is necessary to distinguish the importance of antecedent attributes *Safety*, *Functionality*, *Sustainability*, and *Environment*, because these attributes have different influences in modeling bridge risks.
(2) The constraints illustrated in Eqs. (14b) and (14c) are subjective and inaccurate for the application of the DBRB expert system. For bridge risk assessment, due to these constraints, there is only one solution for the trained DBRB and the referential values of each DBR are shown in Table 3. The other solutions such as the DBRBs shown in Tables 1 and 2 are completely impossible to be trained.

Hence, the trained DBRB expert system is incomplete because the existing parameter optimization model suffers from neglected parameters and inaccurate constraints such as the shortcomings (1) and (2). More importantly, it is likely that there is only one solution for the trained DBRB in modeling bridge risks. To ensure the completeness of the DBRB expert system, a dynamic parameter optimization model is proposed as follows:

Table 3
Special trained DBRB for bridge risk assessment.

Rule	Antecedent attributes			
	Safety	Functionality	Sustainability	Environment
1	*Low*	*Low*	*Low*	*Low*
2	*Medium*	*Medium*	*Medium*	*Medium*
3	*High*	*High*	*High*	*High*

$$Minimize\ J = \sum_{t=1}^{T} \xi_t^2 = \sum_{t=1}^{T} (f(\boldsymbol{x}_t) - y_t)^2 \tag{15a}$$

$$Subject\ to\ \sum_{n=1}^{N} \beta_n^l = 1; \quad l = 1,\ldots,L \tag{15b}$$

$$0 \leqslant \beta_n^l \leqslant 1; \quad n = 1,\ldots,N; \quad l = 1,\ldots,L \tag{15c}$$

$$0 \leqslant \theta_l \leqslant 1; \quad l = 1,\ldots,L \tag{15d}$$

$$0 \leqslant \delta_m \leqslant 1; \quad m = 1,\ldots,M \tag{15e}$$

$$lb_m \leqslant u(A_m^l) \leqslant ub_m; \quad m = 1,\ldots,M; \quad l = 1,\ldots,L \tag{15f}$$

$$u(A_m^s) = lb_m, s = \arg\min_{l=1,\ldots,L}\{u(A_m^l)\}; \quad m = 1,\ldots,M \tag{15g}$$

$$u(A_m^s) = ub_m, s = \arg\max_{l=1,\ldots,L}\{u(A_m^l)\}; \quad m = 1,\ldots,M \tag{15h}$$

where Eqs. (15b)–(15d) are based on the constraints (2) and (3), respectively. Eq. (15e) is on the relative importance of attribute weights. Eqs. (15f)–(15h) are on the utility value of referential values used for antecedent attributes in DBRs.

Remark 4. Compared with the above two parameter optimization models, the significant differences are that the proposed model not only considers the attribute weights shown in Eq. (15e), but also uses the dynamic constraints shown in Eqs. (15g) and (15h) to keep the highest and lowest referential values in any rules. Therefore, the existing model of Chang et al. (2016) is a special case of the proposed model. For example, Table 3 is the only result of the trained DBRB derived from the existing model. However, Tables 1–3 are the partial results of the trained DBRB derived from the proposed model.

3.3. Global optimization algorithm based on DE algorithm

In order to ensure the completeness of the DBRB expert system, a dynamic parameter optimization model is proposed in Section 3.2. However, the proposed model includes two dynamic constraints shown in Eqs. (15g) and (15h), which are not easily implemented using the conventional optimization toolbox in MATLAB, Excel, and Lingo. Therefore, it is necessary to propose an easy-to-operate optimization algorithm for the DBRB expert system to obtain the optimal value of parameters using the dynamic parameter optimization model.

The DE algorithm (Price et al., 2005) has been proved to be simple and effective for on a large of classic optimization problem and is more

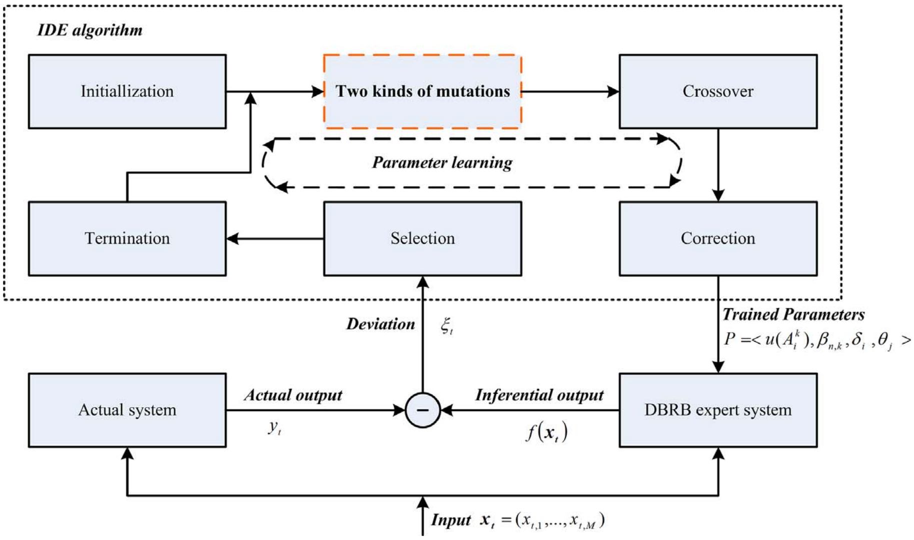

Fig. 3. IDE algorithm for training DBRB expert system.

accurate than several other optimization methods including genetic algorithms, simulated annealing and evolutionary programming (Mohamed, 2015). However, the classical DE algorithm has a shortcoming of easily plunging into a local optimal solution because its mutation operation is based on differential information between individuals. To obtain the global optimal solution of the DBRB expert system, an improved DE (IDE) algorithm is proposed to achieve the parameter learning using the dynamic parameter optimization model, as shown in Fig. 3.

Correspondingly, the detailed steps of the IDE algorithm are shown as follows:

Step 1 (Initialization): Suppose that C is the number of individuals which are actually parameter vectors, and each vector includes the utility value of referential values $u(A_m^l)$, rule weights θ_l, attribute weights δ_m, and belief degrees β_n^l. Thus, the cth ($c = 1, \dots, C$) parameter vector is expressed as follow:

$$\boldsymbol{P_c} = \{p_{c,k}; k = 1,\dots,K\} = \{u(A_m^l)^c, (\beta_n^l)^c, \delta_m^c, \theta_l^c\}; \quad c = 1,\dots,C \tag{16}$$

where $p_{c,k}$ is the kth parameter in the cth parameter vector and K is the total number of parameters in a vector.

For convenience to show the initialization of parameters based on the constraints shown in Eqs. (15b)–(15h), suppose that ub_k and lb_k are the upper and lower bounds of the parameter $p_{c,k}$. Therefore, the initial value of these parameters can be generated by using a random number between 0 and 1:

$$p_{c,k} = lb_k + (ub_k - lb_k) * random(0,1); \quad c = 1,\dots,C; \quad k = 1,\dots,K \tag{17}$$

Step 2 (Two kinds of mutations): For the parameter vector $\boldsymbol{P_c}$, the following two kinds of mutations are simultaneously used to keep its diversity with other parameter vectors:

(1) The first kind of mutation is that while the fitness of the best parameter vector drops less than E during the last IN iterations and $random(0,1) \leqslant MR$, each parameter in the parameter vector $\boldsymbol{P_c}$ should be mutated by using the formula shown in Eq. (17), where MR is a mutation operator.

(2) The second kind of mutation is that a new parameter vector $\boldsymbol{P_{c_0}}$ must be generated using the three different parameter vectors $\boldsymbol{P_{c_1}}$, $\boldsymbol{P_{c_2}}$ and $\boldsymbol{P_{c_3}}$ randomly selected from C parameter vectors. The mutation formula is as follow:

$$p_{c_0,k} = p_{c_1,k} + F * (p_{c_2,k} - p_{c_3,k}); \quad k = 1,\dots,K \tag{18}$$

where F is a mutation operator.

Step 3 (Crossover): For the two parameter vectors $\boldsymbol{P_c}$ and $\boldsymbol{P_{c_0}}$, the following crossover is carried out:

$$p_{c_0,k} = \begin{cases} p_{c,k}, & \text{if } random(0,1) > CR \\ p_{c_0,k}, & \text{otherwise} \end{cases}, \quad k = 1,\dots,K \tag{19}$$

where CR is a crossover operator.

Step 4 (Correction): Clearly, the parameter value in the parameter vector $\boldsymbol{P_{c_0}}$ may not satisfy the constraints shown in Eqs. (15b)–(15h). Hence, it is necessary to correct the value of parameters and dynamic constraints.

(1) For the correction of parameters, based on the derivations shown in Appendix A, each parameter in the parameter vector $\boldsymbol{P_{c_0}}$ must be corrected using the following formula:

$$p_{c_0,k} = \begin{cases} p_{c_0,k}; & \text{if } lb_k \leqslant p_{c_0,k} \leqslant ub_k \\ \frac{p_{c_0,k} + F*(ub_k + lb_k)}{2F+1}; & \text{otherwise} \end{cases}; \quad k = 1,\dots,K \tag{20}$$

In addition, due to the special constraint shown in Eq. (15b), the following correction is carried out for the lth DBR:

$$(\overline{\beta}_n^l)^{c_0} = \frac{(\beta_n^l)^{c_0}}{\sum_{i=1}^N (\beta_i^l)^{c_0}}; \quad l = 1,\dots,L; \quad n = 1,\dots,N \tag{21}$$

(2) For the correction of constraints, based on the two dynamic constraints shown in Eqs. (15g) and (15h), the following corrections are carried out:

$$u(A_m^s)^{c_0} = lb_m, s = \arg\min_{l=1,\dots,L}\{u(A_m^l)^{c_0}\}; \quad m = 1,\dots,M \tag{22a}$$

$$u(A_m^s)^{c_0} = ub_m, s = \arg\max_{l=1,\dots,L}\{u(A_m^l)^{c_0}\}; \quad m = 1,\dots,M \tag{22b}$$

Step 5 (Selection): The parameter vector actually includes all key parameters to generate the inferential output of the DBRB expert system. Therefore, the fitness of the parameter vectors $\boldsymbol{P_c}$ and $\boldsymbol{P_{c_0}}$ can be calculated by using Eq. (15a) and are expressed as $J(\boldsymbol{P_c})$ and $J(\boldsymbol{P_{c_0}})$. In order to update the parameter vector $\boldsymbol{P_c}$, the following selection is carried out:

$$\boldsymbol{P_c} = \begin{cases} \boldsymbol{P_{c_0}}; & \text{if } J(\boldsymbol{P_{c_0}}) < J(\boldsymbol{P_c}) \\ \boldsymbol{P_c} & \text{otherwise} \end{cases} \tag{23}$$

Step 6 (Termination): By performing Steps 2 to 5 repeatedly, the fitness of each parameter vector will decrease. When the number of performing the first mutation reaches *NM* for any parameter vectors, the algorithm is concluded and the parameter vector able to generate the best fitness is selected as the optimal value of parameters for the trained DBRB expert system.

For the proposed IDE algorithm, the following remarks can be given.

Remark 5. In the step 2, the situation, that the fitness of the best parameter vector drops less than *E* during the last *IN* iterations, indicates that the diversity of parameter vectors has lost. Hence, the function of the first kind of mutation is to increase the diversity of parameter vectors to obtain the global optimal values of parameters for the DBRB expert system.

Remark 6. The classical DE algorithm is a special case of the proposed IDE algorithm, because the first kind of mutation is an additional step in the IDE algorithm comparing to the classical DE algorithm. In other words, while the termination criterion is set as $NM = 1$, the IDE algorithm can be simplified as the classical DE algorithm.

4. Modeling bridge risks using DBRB expert system

The following sections present the model development of a DBRB expert system for bridge risk assessment and test its performance, including the comparisons of other existing parameter optimization models related with CBRs and DBRs, and nine alternative models developed by the ANN, ERL, MRA, and ANFIS.

4.1. Data description and evaluation criteria

The input-output data set used in modeling bridge risks was taken from the British Highways Agency (2004). Fig. 4 shows the hierarchical structure for bridge risk assessment and all criteria are explained in Table 4.

By analyzing the available data set from the British Highways Agency, the input of bridge risk assessment is described by different ratings such as *High*, *Medium*, *Low*, and *None*, and the output of that is characterized by a risk score. According to the data preprocessing strategy from Wang and Elhag (2007a), 66 different bridge structures maintenance projects are extracted from the original 23, 387 projects, which have the same risks including risk ratings and risk scores as one another. Clearly, if a model of bridge risk assessment can learn or fit the risks of these 66 projects very well, then the risk score of any project can be well predicted by the model. Thus, the 66 projects are all utilized to train and test the DBRB expert system.

Some criteria have been widely used in the literature (Wang & Elhag, 2007a, 2008) to evaluate the performance of the proposed model of bridge risk assessment. These criteria include the root mean square error (RMSE), mean absolute percentage error (MAPE), and correlation coefficient (R), and the smallest RMSE and MAPE and the largest R is considered to be the best level for the model of bridge risk assessment. These criteria are defined as follows:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(f(\boldsymbol{x}_t)-y_t)^2} \tag{24}$$

Table 4
Criterion for bridge risk assessment.

Criterion	Explanation
Safety	Effects on the public safety related to bridge structure
Functionality	Effects on the main use and service level of the bridge
Sustainability	Effects on steady state between expenditure and workload, avoiding the built-up of a unavoidable backlog, doing effective and targeted preventative essential maintenance
Environment	Effects on the aesthetic appearance and structure design of the bridge

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{f(\boldsymbol{x}_t)-y_t}{y_t}\right| \times 100 \tag{25}$$

$$R = \frac{\sum_{t=1}^{T}(f(\boldsymbol{x}_t)-\overline{f})(y_t-\overline{y})}{\sqrt{\sum_{t=1}^{T}(f(\boldsymbol{x}_t)-\overline{f})^2\cdot\sum_{t=1}^{T}(y_t-\overline{y})^2}} \tag{26}$$

where

$$\overline{f} = \frac{1}{T}\sum_{t=1}^{T} f(\boldsymbol{x}_t)\ and\ \overline{y} = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{27}$$

4.2. Model development and results

To construct a DBRB expert system for bridge risk assessment, each antecedent attribute is provided five referential values, namely *Very Low* (*VL*), *Low* (*L*), *Medium* (*M*), *High* (*H*), and *Very High* (*VH*). Although the same referential value of different antecedent attributes may have different utility values, all utility values of each referential value for all antecedent attributes fall below a certain value range [−1, 4]. Five ratings are used to define risk scores, namely *Zero* (*Z*), *Small* (*S*), *Medium* (*M*), *High* (*H*), and *Very High* (*VH*), and their utility values are expressed as follows:

$$D = \{Z, S, M, H, VH\} = \{0, 25, 50, 75, 100\} \tag{28}$$

Based on the IDE algorithm, the initial DBRs can be randomly generated under the constraints of parameter optimization model, which is able to overcome the limitation of determining initial values by bridge experts. The initial DBRB shown in Table 5 is provided to illustrate Step 1 in the IDE algorithm.

To solve the dynamic parameter optimization model using the IDE algorithm, assume that $C = 200$, $E = 0.1$, $IN = 500$, $MR = 0.9$, $F = 0.5$, $CR = 0.9$, and $NM = 5$ according to prior expert knowledge. During performing Step 5 in the IDE algorithm, one input of bridge risk assessment, that is {(*Safety*, 3), (*Functionality*, 3), (*Sustainability*, 3), (*Environment*, 3)}, is provided to illustrate the ER-based inference of the DBRB expert system.

Firstly, individual matching degrees can be calculated by using the rule-based transformation technique for the quantitative data transformation. For example, the individual matching degree of *Safety* is $\{(A_1^3, 0.7047), (A_1^4, 0.2953)\}$ (namely {(*H*, 0.7047), (*VH*, 0.2953)}) because $3 = 0.7047 * u(A_1^3) + 0.2953 * u(A_1^4) = 0.7047 * 2.5809 + 0.2953 * 4.0000$. Similarly, all individual matching degrees can be listed as follows:

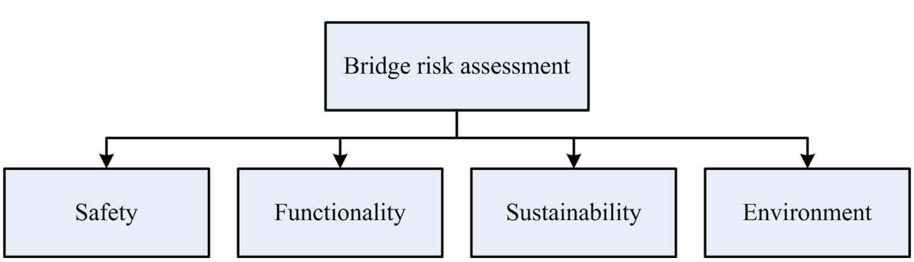

Fig. 4. Hierarchical structure for bridge risk assessment.

Table 5
Randomly initialized DBRB.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.0239)	*Functionality* (0.5044)	*Sustainability* (0.0581)	*Environment* (0.5127)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.9982	−1.0000	0.8327	2.0437	4.0000	0.2028	0.0907	0.1402	0.4353	0.1310
2	0.9038	0.0532	−1.0000	0.8950	1.3322	0.3152	0.1234	0.2228	0.2768	0.0618
3	0.8716	2.5809	4.0000	2.0615	1.6974	0.1531	0.0138	0.0629	0.1748	0.5954
4	0.1012	4.0000	2.7225	−1.0000	2.3377	0.1038	0.2600	0.1097	0.2191	0.3074
5	0.9529	0.0590	−0.3816	4.0000	−1.0000	0.1424	0.1230	0.0663	0.1601	0.5082

$$(A_1^1,0)\vee(A_2^1,0)\vee(A_3^1,0)\vee(A_4^1,0.3984) \tag{29a}$$

$$(A_1^2,0)\vee(A_2^2,0)\vee(A_3^2,0)\vee(A_4^2,0) \tag{29b}$$

$$(A_1^3,0.7047)\vee(A_2^3,0.2172)\vee(A_3^3,0.5159)\vee(A_4^3,0) \tag{29c}$$

$$(A_1^4,0.2953)\vee(A_2^4,0.7828)\vee(A_3^4,0)\vee(A_4^4,0.6016) \tag{29d}$$

$$(A_1^5,0)\vee(A_2^5,0)\vee(A_3^5,0.4841)\vee(A_4^5,0) \tag{29e}$$

Secondly, according to Eq. (8), the activated weight of each DBR can be calculated by using individual matching degrees, rule weights, and attribute weights and these activated weights are $w_1 = 0.1180$, $w_2 = 0$, $w_3 = 0.5517$, $w_4 = 0.0700$ and $w_5 = 0.2603$.

Thirdly, the analytical ER algorithm is employed to integrate the activated rules R_1, R_3, R_4 and R_5. The final result can be denoted as follow.

$$f(\boldsymbol{x}_t) = 0*0.1440 + 25*0.0468 + 50*0.0653 + 75*0.1879 + 100*0.5559 = 74.1210 \tag{30}$$

Finally, the fitness of an individual/parameter vector can be obtained by calculating the deviation between inferential outputs and actual outputs. Fig. 5 shows the change of fitness for the best parameter vector. It is obvious that two significant declines can be found in the iteration ranges [0, 500] and [2500, 5000]. The former decline is due to the poor performance of the initial DBRB expert system, more and more optimal parameter values for the trained DBRB expert system continue to be found. The latter decline is because of the first kind of mutation in the IDE algorithm, which can increase the diversity of parameter vectors and help the DBRB expert system get the global optimal value of parameters. From Fig. 5, three evaluation criteria also tend to the optimal value gradually.

After about 7500 iterations for parameter learning, the trained DBRB can be obtained and is shown in Table 6. Three evaluation criteria are $RMSE = 2.4742$, $MAPE = 6.2408$, and $R = 0.9964$ derived from the trained DBRB expert system, which are significantly better than the initial DBRB expert system and its evaluation criteria are $RMSE = 20.8991$, $MAPE = 62.2268$ and $R = 0.7659$. Fig. 6 compares the predicted risk scores of the initial and trained DBRB expert systems. It is clear that the predicted risk scores of the initial DBRB expert system do not match the actual output of bridge risks well. However, the trained DBRB expert system can closely replicate the relationship among *Safety*, *Functionality*, *Sustainability*, *Environment* and *Risk scores*.

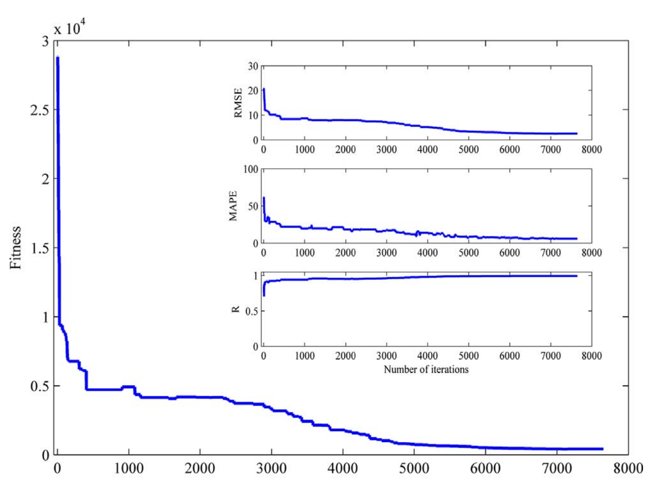

Fig. 5. Process of parameter learning for DBRB expert system.

4.3. Performance analysis and comparison

To further verify the validity of the DBRB expert system, the result derived from 10 runs are further compared with other existing parameter optimization models related with CBRs and DBRs, ANN, ERL, MRA, and ANFIS. Note that the results of 10 runs can be found in Tables 6 and B1–B9 from Appendix B.

4.3.1. Comparative analysis with existing parameter optimization models

Three alternative models are constructed and tested for bridge risk assessment based on the existing parameter optimization models and different variations of the DE algorithm. These models are described as follows:

Model 1. This model is based on the most common BRB expert system comprising CBRs and adaptive parameter optimization model proposed by Chen et al. (2011), where the model of Chen et al. is an advanced parameter optimization model for the BRB expert system. To obtain the optimal value of parameters, the DE and IDE algorithm are used to train the parameter of CBRs. In addition, a variation of the IDE algorithm is also used here and the utility value of referential values used for antecedent attributes is initialized by the normal method, namely:

$$u(A_{m,j}) = lb_m + (j-1)\frac{ub_m - lb_m}{J_m - 1}; \quad m = 1,\ldots,M; \quad j = 1,\ldots,J_m \tag{31}$$

where $u(A_{m,j})$ is the utility value of the referential value $A_{m,j}$, $A_m^l \in \{A_{m,j};\, j = 1, \ldots, J_m\}$ ($l = 1, \ldots, L$), L is the number of rules in a BRB, J_m is the number of referential values in the mth antecedent attribute, lb_m and ub_m are the lower and upper bounds of the mth antecedent attribute, respectively. For convenience, this model is abbreviated as CBR-Chen, and the IDE algorithm related to the normal initialization method as N-IDE.

Model 2. This model is based on the DBRB expert system and parameter optimization model proposed by Chang et al. (2016), where the model of Chang et al. is the only existing parameter optimization model for the DBRB expert system. To obtain the optimal value of parameters, the DE, N-IDE, and proposed IDE algorithm are used to train the parameters of DBRs, respectively. For convenience, this model is abbreviated as DBR-Chang.

Model 3. This model is based on the DBRB expert system and proposed dynamic parameter optimization model. To obtain the optimal value of parameters, the three variations of the DE algorithm are used to train the parameters of DBRs, respectively. For

Table 6
Trained DBRB derived from IDE algorithm.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.8717)	*Functionality* (0.6518)	*Sustainability* (0.5577)	*Environment* (0.7643)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (1 0 0)
1	0.0281	−0.3901	−1.0000	−0.8058	0.3614	0.9855	0.0004	0.0041	0.0088	0.0012
2	0.0047	−0.6125	0.9972	1.1484	−1.0000	0.1312	0.3814	0.1830	0.1906	0.1138
3	0.0419	−1.0000	2.0346	2.9740	3.7538	0.3791	0.1021	0.0327	0.0621	0.4239
4	0.9793	4.0000	4.0000	4.0000	4.0000	0.0022	0.0034	0.0021	0.0113	0.9810
5	0.1676	1.9240	2.6761	−1.0000	3.9121	0.0793	0.0348	0.5964	0.1191	0.1705

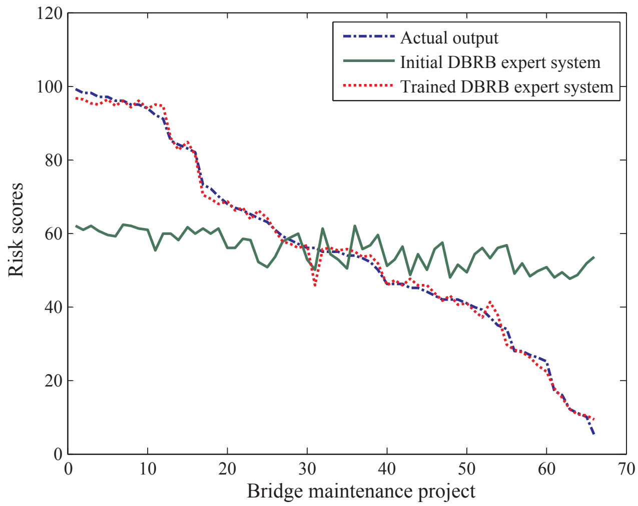

Fig. 6. Comparison of initial and trained DBRB expert system.

convenience, this model is abbreviated as DBR-Dynamic.

The results of three kinds of models are illustrated in Table 7 and Fig. 7.

Based on Table 7 and Fig. 7, Model 1 performs the best in the MAPE, RMSE, and R. However, some advantages of Model 3 which is actually the proposed dynamic parameter optimization model have to be highlighted as follows:

(1) Model 3 is more concise than Model 1 while modeling bridge risks, i.e. there is an average of 0.08 rules per bridge maintenance project in Model 3 but 9.5 rules per bridge maintenance project in Model 1. Furthermore, while more referential values are used to describe each antecedent attribute, the number of rules in Model 1 may be overlarge. For example, there are 10 referential values used for each antecedent attribute, the resulting BRB in Model 1 has 10^4 rules. Correspondingly, in the same case, there are 10 rules in the DBRB of Model 3.
(2) Model 3 is more efficient than Model 1 while training the parameters of a rule base. Obviously, the more rules a rule-based expert system involves, the more time parameter learning needs. The total of time taken to execute the DE, N-IDE, and IDE algorithms implemented in Microsoft Visual C++ 6.0 on Intel (R) Core (TM) i5-4570 CPU 3.20 GHz can easily illustrate differences. For example, for the three variations of the DE algorithm, the total of time for Model 1 are 362.5, 808.2, and 553.8 s but those for Model 3 are only 54, 77.3, and 118.7 s.
(3) Model 3 is more accurate than Model 1 while considering the same number of rules in a rule base. Based on Table 7, the significant difference of two models is that Model 1 imposes 625 rules for bridge risk assessment and it is far more than 5 rules in Mode 3. To minimize this difference, as shown in Fig. 8, four BRBs comprising 625, 256, 81, and 16 CBRs, namely CBR-625, CBR-256, CBR81, and CBR-16, are used to compare with this study, where the four BRBs are generated by using 5, 4, 3, and 2 referential values, respectively. The comparisons of these models show that the MAPE, RMSE, and R of CBR-81 and CBR-16 are worse than those of this study, even though these two models still have more rules than Model 3.

For the comparisons of Models 2 and 3, the proposed dynamic parameter optimization model with the IDE algorithm shows the best performance. Some insights can be derived based on these comparison results:

(1) By comparing with the results of the IDE algorithm, the proposed dynamic parameter optimization model is proved to be able to ensure the completeness of DBRB, which can achieve the parameter learning of the DBRB expert system within more number of feasible solutions. Also, the global optimal solution for bridge risk assessment is found by using the proposed IDE algorithm.
(2) By comparing with the results of the N-IDE algorithm, the random initialization method of the IDE algorithm is proved to be better than the normal initialization method to generate utility values which are evenly distributed within the definition interval of an input variable, because the former is useful for the IDE algorithm to keep the diversity of parameter vectors.
(3) By comparing with the results of the DE algorithm, it is indicated that the DBRB expert system is easily trapping in local optimal solutions using the proposed dynamic parameter optimization model, because of two dynamic constraints in the model. Clearly, the proposed IDE algorithm is effective in increasing the diversity of parameter vectors, so that it can significantly improve the performance of the DBRB expert system for bridge risk assessment.

Table 7
Comparisons of three models for bridge risk assessment.

Performance criterion	Model 1 (CBR-Chen)			Model 2 (DBR-Chang)			Model 3 (DBR-Dynamic)		
	DE	N-IDE	IDE	DE	N-IDE	IDE	DE	N-IDE	IDE (This paper)
MAPE (%)	3.4932	1.3913	3.2068	7.5241	11.0657	6.4870	10.0989	11.1552	4.9645
RMSE	2.0164	1.2508	1.8708	3.5682	4.6958	2.9662	4.4690	4.6900	2.5111
R	0.9968	0.9988	0.9973	0.9891	0.9832	0.9927	0.9821	0.9833	0.9952
Number of rules	625	625	625	5	5	5	5	5	5
Time (s)	362.5	808.2	553.8	59.9	74.1	106.9	54.0	77.3	118.7

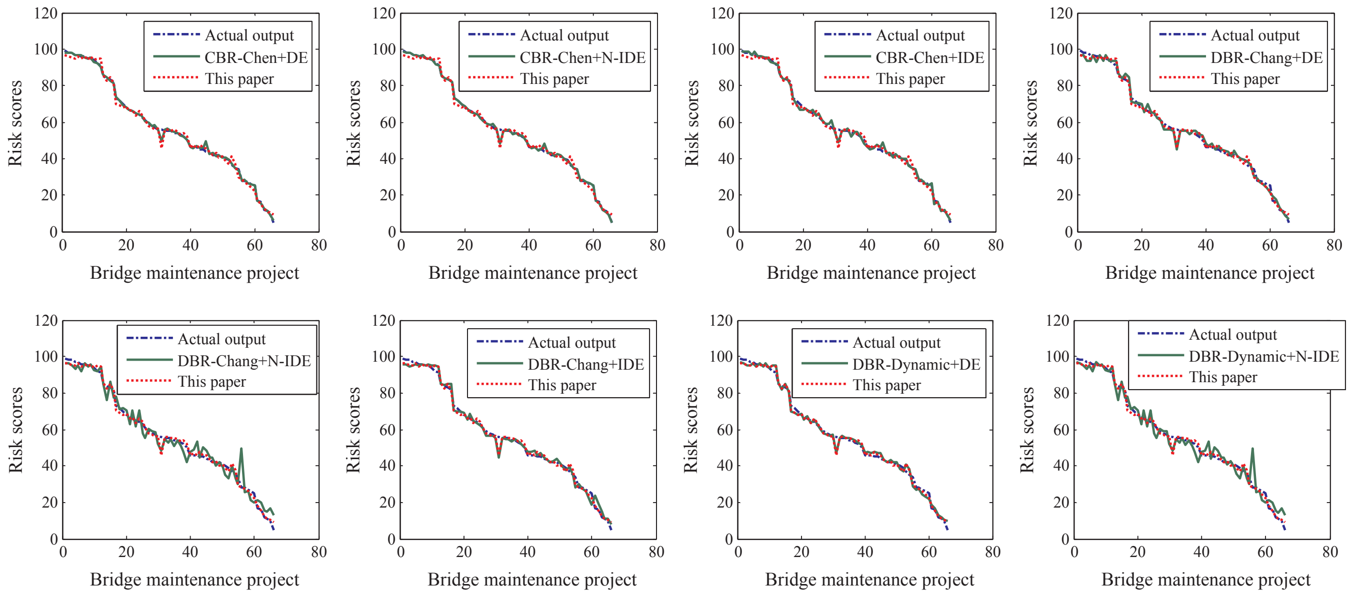

Fig. 7. Comparison of three kinds of parameter optimization models and algorithms.

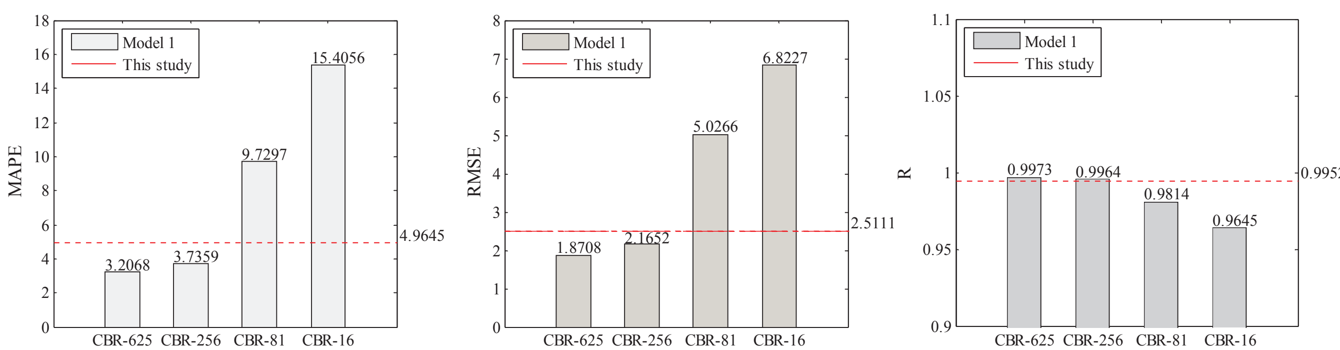

Fig. 8. Comparison of four kinds of Model 1 with different rules.

4.3.2. Comparative analysis with conventional approaches

Next, four conventional approaches, including the ANN, ERL, MRA, and ANFIS, are used to compare with the DBRB expert system trained by the proposed dynamic optimization model and IDE algorithm.

Firstly, seven alternative models developed by the ANN, ERL, and MRA are used to assess bridge structures, denoted by BP-ANN, ER1, ER2, MRA3, MRA7, MRA8, and MRA9 based on Wang and Elhag (2007a), where four MRA models are obtained by stepwise regression technique, the model of ER1, MRA3 and MRA8 all use zero to represent the minimum risk score of each bridge structure, and the model of ER2, MRA7 and MRA9 all take no account of zero to stand for the minimum risk score.

By comparing the results of seven alternative models and the DBRB expert system in Table 8, it is validated that the DBRB expert system can produce satisfactory results that the DBRB expert system achieves better results in the MAPE, RMSE and R, which are 4.9645, 2.5111, and 0.9952 (these values are marked as bold in Table 8), respectively. Fig. 9 shows the predicted risk scores of the DBRB expert system and other seven models for 66 bridge maintenance projects. It is clear that the DBRB expert system can generate accurate risk scores for these bridge maintenance projects better than the ANN, ERL, and MRA. Particularly, the relatively worse results obtained from the ERL and MRA indicate that the DBRB expert system can make the effective use of available data and is unnecessary to know the relationship between inputs and outputs in advance.

Secondly, as an effective approach that has been claimed to be better than the ANN and MRA based on Wang and Elhag (2008), the ANFIS continued to be used to compare with the DBRB expert system for bridge risk assessment. Hence, two alternative models are developed from the ANFIS, denoted as ANFIS1 and ANFIS2, where ANFIS1 includes 16 fuzzy rules constructed by 2 fuzzy numbers in each antecedent attribute and ANFIS2 consists of 81 fuzzy rules constructed by 3 fuzzy numbers. In addition, the trapezoidal, triangular, and Gauss membership functions are used to implement the fuzzy number for

Table 8
Comparison of DBRB expert system and other approaches.

Performance criterion	Models							
	BP-ANN	ER1	MRA3	MRA8	ER2	MRA7	MRA9	This paper
MAPE (%)	10.21	22.71	20.89	21.19	19.82	17.03	20.16	**4.9645**
RMSE	4.7843	8.6448	8.9953	9.0118	11.1141	10.7328	12.1520	**2.5111**
R	0.9823	0.9408	0.9357	0.9357	0.9130	0.9348	0.9301	**0.9952**

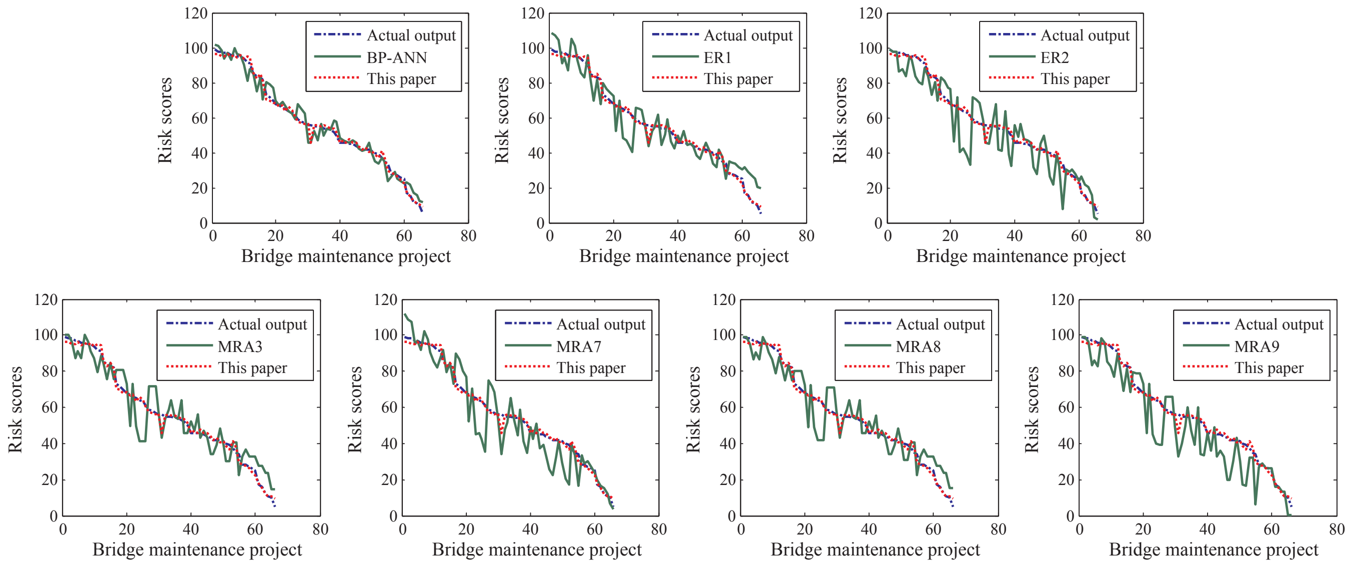

Fig. 9. Comparison of DBRB expert system and other approaches.

Table 9
Comparisons of DBRB expert system and ANFIS.

Performance criterion	ANFIS1			ANFIS2			This paper
	Trapezoidal	Triangular	Gauss	Trapezoidal	Triangular	Gauss	
MAPE (%)	7.0300	15.4865	14.3908	0.6446	0.5909	0.5932	4.9645
RMSE	3.6302	7.2022	5.8063	1.0698	1.0679	1.0678	2.5111
R	0.9898	0.9593	0.9737	0.9991	0.9991	0.9991	0.9952
Number of rules	16	16	16	81	81	81	5

ANFIS1 and ANFIS2, respectively.

Table 9 shows that the MAPE, RMSE, and R of the DBRB expert system are better than those of ANFIS1 but worse than those of ANFIS2. The main reason is that ANFIS2 includes 81 rules for 66 bridge maintenance projects and it is far more than 16 rules in ANFIS1. However, there are only 5 rules in the DBRB expert system. In other words, the DBRB expert system is not only more concise than the ANFIS to model bridge risks, but also more accurate than the ANFIS while

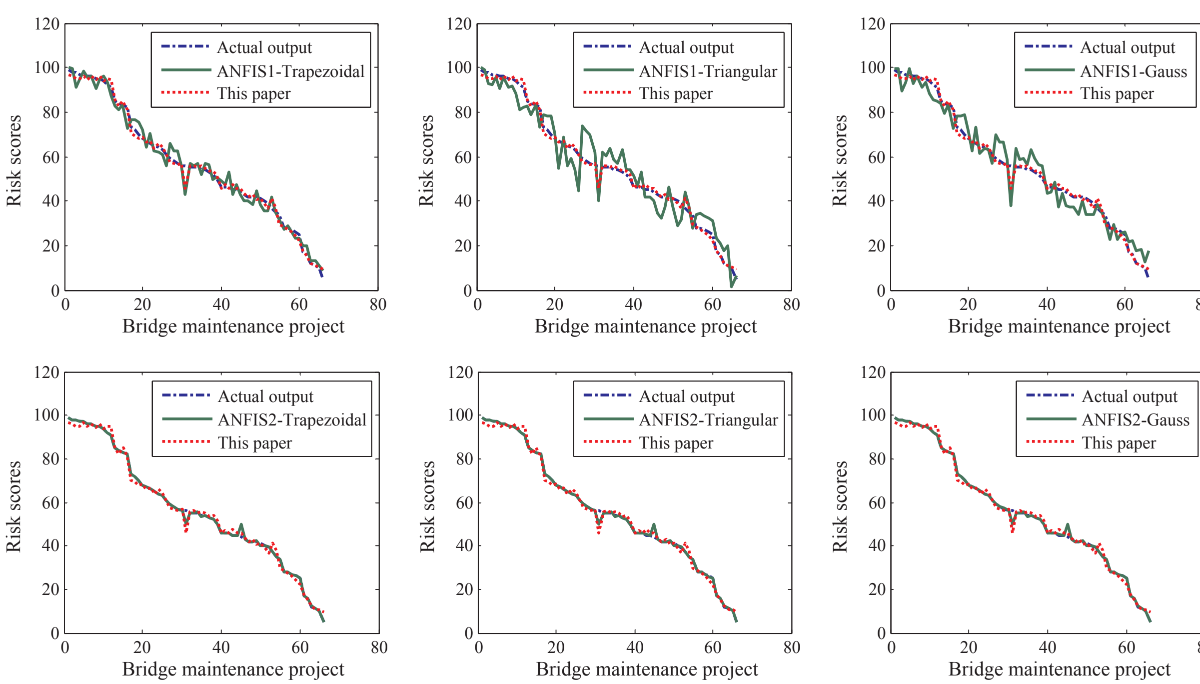

Fig. 10. Comparison of DBRB expert system and ANFIS.

considering the same number of rules. In addition, the DBRB expert system can be more efficient than the ANFIS, because the number of fuzzy rules needed to be trained is exponential based on the number of fuzzy numbers and/or antecedent attributes. Fig. 10 illustrates that ANFIS1 with three kinds of membership functions fail to generate accurate risk scores for 66 bridge maintenance projects. However, the predicted risk scores of ANFIS2 and the DBRB expert system almost match all actual risk scores. Therefore, for bridge risk assessment, the DBRB expert system is believed to be a good choice and powerful tool.

5. Conclusion

In this study, the novel DBRB expert system is developed for bridge risks assessment, where the DBRB is one type of BRBs by considering DBRs, so that it can overcome the combinatorial explosion problem of the most common BRB expert system which consists of CBRs. With the proposed dynamic parameter optimization model and IDE algorithm, the DBRB expert system has been proved to be complete and obtain the global optimal parameter values in modeling bridge risks, compared with other existing parameter optimization models related with CBRs and DBRs.

To overcome the limitations of conventional approaches such as the ANN, ERL, MRA, and ANFIS in modeling bridge risks, this study analyzes the advantages of the DBRB expert system and compares their performance. The comparison results show the superiority and validity of the DBRB expert system, so that it can determine the priority of bridge structures for maintenance and avoid the safety accidents of bridges. In summary, the DBRB expert system with the proposed dynamic parameter optimization model is a good choice and powerful tool for modeling bridge risks

In this paper, the connectivity between bridge risks and bridge criteria is based on the previous researches and may not be always simple, i.e. more bridge criteria and more hierarchical structure should be used to determine bridge risks. Hence, the accuracy of the proposed DBRB expert system would be affected. Also, the number of DBRs in the DBRB expert system may not be reasonable. In these regards, other enhancements of the BRB expert system must be proposed to model bridge risks.

Acknowledgements

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Appendix A. Derivation of correction formula for IDE algorithm

For the parameter vector $\boldsymbol{P_c}$ obtained from C parameter vectors, suppose that the upper and lower bound of the kth ($k=1, \ldots, K$) parameter $p_{c,k}$ are ub_k and lb_k, respectively. For convenience, the value range of the parameter $p_{c,k}$ can be marked as

$$VR(p_{c,k}) = [lb_k,ub_k]; \quad k = 1,\ldots,K. \tag{A1}$$

Correspondingly, three different parameter vectors $\boldsymbol{P_{c_1}}$, $\boldsymbol{P_{c_2}}$, and $\boldsymbol{P_{c_3}}$, which are randomly selected from C parameter vectors, have the same value range in the kth parameter and it can be expressed as

$$VR(p_{c_1,k}) = VR(p_{c_2,k}) = VR(p_{c_3,k}) = [lb_k,ub_k]; \quad k = 1,\ldots,K \tag{A2}$$

According to Eq. (18), the value range of the parameter $p_{c_0,k}$ in parameter vector $\boldsymbol{P_{c_0}}$ is represented as follow:

$$\begin{aligned} VR(p_{c_0,k}) &= VR(p_{c_1,k}) + F*(VR(p_{c_2,k}) - VR(p_{c_3,k})) \\ &= [lb_k,ub_k] + F*([lb_k,ub_k]-[lb_k,ub_k]) \\ &= [lb_k,ub_k] + F*[lb_k-ub_k,ub_k-lb_k] \\ &= [lb_k + F*(lb_k-ub_k),ub_k + F*(ub_k-lb_k)] \end{aligned} \tag{A3}$$

Obviously, the lower and upper bound of $p_{c_0,k}$ are smaller and bigger than the predetermined lower and upper bound, namely $p_{c_0,k} < lb_k$ or $p_{c_0,k} > ub_k$. Hence, a correction formula must be provided for the parameter $p_{c_0,k}$ and the derivation process is shown as follows:

$$\begin{aligned} & VR(p_{c_0,k}) = [lb_k + F*(lb_k-ub_k),ub_k + F*(ub_k-lb_k)] \\ \Leftrightarrow\ & VR(p_{c_0,k} - lb_k + F*(ub_k-lb_k)) = [0,\ (2F + 1)*(ub_k-lb_k)] \\ \Leftrightarrow\ & VR\left(\frac{p_{c_0,k} - lb_k + F*(ub_k - lb_k)}{2F+1}\right) = [0,\ ub_k-lb_k] \\ \Leftrightarrow\ & VR\left(\frac{p_{c_0,k} + F*(ub_k + lb_k)}{2F+1}\right) = [lb_k,\ ub_k] \end{aligned} \tag{A4}$$

As a consequence of the derivation process, the correction formula is represented as follows:

$$p_{c_0,k} = \frac{p_{c_0,k} + F*(ub_k + lb_k)}{2F + 1}; \quad k = 1,\ldots,K \tag{A5}$$

Appendix B. Trained DBRB for bridge risk assessment

See Tables B1–B9.

Table B1
Trained DBRB obtained from the 2nd run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.5250)	*Functionality* (0.7916)	*Sustainability* (0.9666)	*Environment* (0.3338)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.0815	−1.0000	2.8666	2.8919	3.9175	0.4054	0.0203	0.0800	0.0572	0.4371
2	0.0104	0.1006	1.1056	0.8699	3.4412	0.4210	0.1096	0.0882	0.1439	0.2373
3	0.9483	4.0000	3.5376	4.0000	4.0000	0.0025	0.0030	0.0021	0.0050	0.9874
4	0.1419	1.9912	4.0000	−1.0000	−1.0000	0.0913	0.0679	0.4719	0.2679	0.1010
5	0.0303	−0.4252	−1.0000	−0.8301	−0.3764	0.9139	0.0521	0.0036	0.0184	0.0120

Table B2
Trained DBRB obtained from the 3rd run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1404)	*Functionality* (0.1585)	*Sustainability* (0.8171)	*Environment* (0.4059)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.0844	4.0000	2.3379	1.8844	3.8755	0.0137	0.0022	0.3524	0.0398	0.5918
2	0.1181	−1.0000	0.9863	−0.5580	1.2784	0.6335	0.1283	0.0731	0.1098	0.0553
3	0.8133	2.9573	4.0000	4.0000	4.0000	0.0009	0.0021	0.0029	0.0106	0.9834
4	0.8890	0.6739	−1.0000	−1.0000	−1.0000	0.5003	0.3517	0.0303	0.0698	0.0479
5	0.0949	0.6239	−0.1362	−0.1082	−0.4234	0.9353	0.0039	0.0179	0.0185	0.0244

Table B3
trained DBRB obtained from the 4th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1889)	*Functionality* (0.1679)	*Sustainability* (0.8741)	*Environment* (0.3148)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.8487	2.9957	4.0000	4.0000	4.0000	0.0011	0.0027	0.0060	0.0014	0.9887
2	0.0956	4.0000	2.1862	1.7454	2.3380	0.1255	0.1654	0.2218	0.3572	0.1301
3	0.1319	0.1495	−0.5696	−0.8196	−0.9093	0.9524	0.0109	0.0037	0.0187	0.0142
4	0.9411	0.3973	−1.0000	−1.0000	−1.0000	0.5505	0.2666	0.0581	0.1242	0.0007
5	0.0573	−1.0000	1.0000	−0.7250	3.6694	0.6049	0.1357	0.1281	0.0930	0.0383

Table B4
Trained DBRB obtained from the 5th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1245)	*Functionality* (0.1115)	*Sustainability* (0.8271)	*Environment* (0.8018)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.8670	0.4507	−1.0000	−1.0000	−1.0000	0.9010	0.0386	0.0087	0.0276	0.0241
2	0.0220	−0.0002	0.9986	0.5873	1.0957	0.1515	0.2496	0.2343	0.1406	0.2240
3	0.8419	2.8843	4.0000	4.0000	4.0000	0.0047	0.0167	0.0004	0.0120	0.9662
4	0.1184	4.0000	2.1283	1.9318	3.6009	0.1545	0.0737	0.0678	0.0517	0.6523
5	0.1059	−1.0000	−0.0593	−0.9652	−0.9680	0.7238	0.1918	0.0504	0.0196	0.0143

Table B5
Trained DBRB obtained from the 6th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.7054)	*Functionality* (0.7748)	*Sustainability* (0.8975)	*Environment* (0.8698)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.0020	−0.4254	1.3571	1.0195	3.3229	0.3294	0.1983	0.1841	0.1676	0.1206
2	0.1597	1.9518	−1.0000	−1.0000	−1.0000	0.0172	0.2887	0.3203	0.2878	0.0861
3	0.0836	−1.0000	2.7069	2.8760	3.6437	0.3597	0.1360	0.0514	0.0623	0.3906
4	0.0245	−0.2331	−0.1027	−0.8059	−0.6632	0.9578	0.0077	0.0115	0.0206	0.0024
5	0.9092	4.0000	4.0000	4.0000	4.0000	0.0029	0.0007	0.0052	0.0083	0.9829

Table B6
trained DBRB obtained from the 7th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1196)	*Functionality* (0.0933)	*Sustainability* (0.7730)	*Environment* (0.6555)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.0609	−1.0000	0.9956	−0.5915	3.4257	0.2707	0.2231	0.1165	0.2866	0.1031
2	0.1299	0.2434	−0.1901	−0.9783	−0.3089	0.9190	0.0312	0.0182	0.0090	0.0226
3	0.7870	2.9525	4.0000	4.0000	4.0000	0.0023	0.0045	0.0068	0.0245	0.9619
4	0.8326	0.6480	−1.0000	−1.0000	−1.0000	0.7130	0.0815	0.1377	0.0008	0.0670
5	0.0587	4.0000	2.4234	1.7208	3.5080	0.1711	0.0985	0.1750	0.3277	0.2277

Table B7
trained DBRB obtained from the 8th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1173)	*Functionality* (0.0624)	*Sustainability* (0.7991)	*Environment* (0.7538)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.9123	0.4608	−1.0000	−1.0000	−1.0000	0.9758	0.0205	0.0018	0.0001	0.0018
2	0.1284	−1.0000	−0.8895	1.8190	−0.6967	0.2451	0.2686	0.2915	0.0090	0.1859
3	0.9184	2.8940	4.0000	3.9925	4.0000	0.0054	0.0154	0.0187	0.0129	0.9475
4	0.0020	−0.0005	0.9941	0.0177	0.5597	0.4183	0.1149	0.2466	0.1141	0.1061
5	0.1770	4.0000	2.2183	4.0000	3.8753	0.0224	0.0337	0.0584	0.1994	0.6860

Table B8
Trained DBRB obtained from the 9th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1430)	*Functionality* (0.0885)	*Sustainability* (0.8649)	*Environment* (0.7702)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.8594	0.3761	−1.0000	−1.0000	−1.0000	0.8001	0.0312	0.0487	0.0541	0.0659
2	0.7568	2.8336	4.0000	3.9956	4.0000	0.0054	0.0091	0.0185	0.0219	0.9452
3	0.1132	4.0000	2.1846	4.0000	3.8830	0.0138	0.0085	0.1750	0.1839	0.6189
4	0.0679	−1.0000	−0.5115	−0.4907	−0.5720	0.7937	0.0872	0.0438	0.0703	0.0050
5	0.0434	−0.0002	0.9890	1.7473	1.3098	0.6795	0.0528	0.1559	0.0579	0.0539

Table B9
Trained DBRB obtained from the 10th run.

Rule	Rule weight	Antecedent attributes (attribute weight)				Consequent (utility value)				
		Safety (0.1307)	*Functionality* (0.1336)	*Sustainability* (0.9654)	*Environment* (0.8904)	*Zero* (0)	*Small* (25)	*Medium* (50)	*High* (75)	*Very High* (100)
1	0.1284	−1.0000	2.0446	1.7880	3.6000	0.0913	0.3568	0.2728	0.2468	0.0322
2	0.8055	0.6492	−1.0000	−1.0000	−1.0000	0.7822	0.0736	0.0541	0.0583	0.0319
3	0.8170	2.8850	4.0000	4.0000	4.0000	0.0035	0.0025	0.0074	0.0029	0.9837
4	0.0059	4.0000	1.0000	−0.0675	3.3271	0.2041	0.0927	0.1818	0.2864	0.2350
5	0.1329	0.6105	−0.2137	−0.6862	−0.8995	0.9517	0.0159	0.0171	0.0104	0.0048

Appendix C. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cie.2017.09.027.

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