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Inference and learning methodology of belief-rule-based expert system for pipeline leak detection

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Abstract

Belief rule based expert systems are an extension of traditional rule based systems and are capable of representing more complicated causal relationships using different types of information with uncertainties. This paper describes how the belief rule based expert systems can be trained and used for pipeline leak detection. Pipeline operations under different conditions are modelled by a belief rule base using expert knowledge, which is then trained and fine tuned using pipeline operating data, and validated by testing data. All training and testing data are collected and scaled from a real pipeline. The study demonstrates that the belief rule based system is flexible, can be adapted to represent complicated expert systems, and is a valid novel approach for pipeline leak detection.

Keywords: Belief rule base; Expert system; The evidential reasoning approach; Leak detection; Optimisation

1. Introduction

The belief rule base concept and its inference methodology were proposed by Yang, Liu, Wang, Sii, and Wang (2005a) based on the Evidential Reasoning approach (Yang, 2001; Yang & Xu, 2002a, 2002b). In the belief rule base, each possible consequent of a rule is associated with a belief degree. Such a rule base is capable of capturing more complicated and continuous causal relationships between different factors while the traditional *IF-THEN* rules are its special cases (Hodges et al., 1999; Parson, 1996; Sun, 1995).

When applying a belief rule base, the input of an antecedent is transformed into a belief distribution over the referential values of an antecedent. The distribution is then used to calculate the activation weights of the rules in the rule base. Subsequently, inference in the belief rule base is through the aggregation of all the activated rules using the Evidential Reasoning approach.

The rules in the belief rule base may come in different ways, such as extracted from experts, by examining historical data, or through self-learning from training data. The process of manually extracting rules may be time consuming and the rules may be approximate. As such, optimal learning methods for training the belief rule base have been proposed and investigated by Yang, Liu, Xu, Wang, and Wang (2005b).

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Though the new scheme was demonstrated using simple numerical examples, its validity and capability in dealing with more practical and complicated problems need to be tested. In this paper, we will apply the new scheme to build an expert system for pipeline leak detection.

Leaks from pipelines may cause immeasurable damage to the environment and losses to the pipeline operating companies. To minimise the damage and losses, many methods and types of systems for pipeline leak detection are developed, such as those based on mass balance (Rougier, forthcoming) and real time transit models (Abhulimen & Susu, 2004; Carpenter, Nicholas, & Henrie, 2005), statistical analysis (Buchberger & Nadimpalli, 2004) and acoustic emission detection (Gao, Brennan, Joseph, Muggleton, & Hunaidi, 2005; Lee, Lee, & Park, 2004). Real time systems based on mass balance corrected with pressure are among the very popular ones. The belief rule based expert system for leak detection investigated below will also be based on the mass balance principle.

When a leak develops in a pipeline, flow and pressure in the pipeline will change following certain patterns. Experts are able to provide a set of rules to distinguish patterns between operations under normal and leak situations. Traditional rule based expert systems may be capable of capturing those rules for leak detection purpose. However, such systems may not be capable of detecting small leaks and provide accurate information on leak sizes.

In this paper, the process and the outcomes of applying the belief rule based system for pipeline leak detection are described. In the following sections, the concept and methods for building a belief rule based expert system will be outlined, followed by a detailed description of how a pipeline leak detection expert system can be developed using the belief rule base. The system is trained and tested using real pipeline data, scaled to protect the interests of the pipeline owner. The paper is concluded by a discussion on the advantages of the new system and the needs for further development.

2. Outline of belief rule based expert systems

Based on Dempster-Shafer theory of evidence (Shafer, 1976), decision theory and fuzzy set theory, Yang et al. proposed a new methodology for building a hybrid rule-base using a belief structure and for inference in the rule-based system using the evidential reasoning approach (Yang, 2001; Yang & Sen, 1994; Yang & Singh, 1994; Yang & Xu, 2002a, 2002b). The methodology is referred to as a generic rule-base inference methodology using the evidential reasoning approach – RIMER (Yang et al., 2005a).

2.1. Belief rule-base

In the RIMER approach (Yang et al., 2005a), a belief *IF-THEN* rule, for example the kth rule R_k, is expressed as follows:

$$\begin{aligned}
&\text{IF } (X_1 \text{ is } A_1^k) \wedge (X_2 \text{ is } A_2^k) \wedge \cdots \wedge (X_{T_k} \text{ is } A_{T_k}^k)\\
&\text{THEN } \{(D_1, \beta_{1k}), (D_2, \beta_{2k}), \ldots, (D_N, \beta_{Nk})\}, \left(\sum_{i=1}^{N} \beta_{ik} \leqslant 1\right),\\
&\text{with a rule weight } \theta_k (k = 1, \ldots, L)\\
&\text{and attribute weights } \delta_1, \delta_2, \ldots, \delta_{T_k}. \qquad (1)
\end{aligned}$$

where $A_i^k (i = 1, \ldots, T_k)$ is the referential value of the ith antecedent attribute and T_k the number of antecedent attributes used in the kth rule. $\beta_{ik}(i = 1, \ldots, N)$ is the belief degree to which D_i is believed to be the consequent if $(X_1, X_2, \ldots, X_{T_k}) = (A_1^k, A_2^k, \ldots, A_{T_k}^k)$. L is the number of all rules in the rule-base. If $\sum_{i=1}^{N} \beta_{ik} = 1$, the kth rule is said to be complete; otherwise, it is incomplete. Note that $\sum_{i=1}^{N} \beta_{ik} = 0$ denotes total ignorance about the output given the input in the kth rule. Rule (1) is also referred to as a belief rule. It is further supposed that T is the total number of antecedent attributes used in the rule base.

Let

$$X = (X_1, X_2, \ldots, X_{T_k}), \quad A^k = (A_1^k, A_2^k, \ldots, A_{T_k}^k), \quad D = (D_1, D_2, \ldots, D_N)$$
$$\beta^k = (\beta_{1k}, \beta_{2k}, \ldots, \beta_{Nk}), \quad \text{and } \delta = (\delta_1, \delta_2, \ldots, \delta_T)$$

X is referred to as an input vector to the kth rule, A^k a packet antecedent, $A_i^k (i = 1, 2, \ldots, T_k)$ the ith referential values of the packet antecedent A^k, D the consequent vector, β^k the vector of the belief degrees, and δ the attribute weights of all the T antecedent attributes in the rule base.

It is not difficult to see the difference between a traditional *IF-THEN* rule and a belief *IF-THEN* rule. In a traditional rule, the consequent is either 100% true or 100% false. Such a rule base has limited capacity in representing knowledge in a real world. For example, it is incapable of capturing continuous causal relationship between antecedents and consequents. The belief structure in the belief rule base provides better flexibility in representing knowledge of different structures and complexity, such as continuous and uncertain relationships between antecedents and consequents.

2.2. Rule inference using the evidential reasoning (ER) approach

Given an input to the system, $U = (U_i, i = 1, \ldots, T)$, how can the rule base be used to inference and generate an output? As mentioned earlier, T is the total number of antecedents in the rule base, $U_i (i = 1, \ldots, T)$ is the ith attribute, which can be one of the following types (Yang et al., 2005a): continuous, discrete, symbolic and ordered symbolic.

Before the start of an inference process, the matching degree of an input to each referential value in the antecedents of a rule needs to be determined so that an activation weight for each rule can be generated. This is equivalent to transforming an input into a distribution on referential values using belief degrees and can be accomplished using different techniques such as the rule or utility-based equivalence transformation techniques (Yang, 2001).

Using the notations provided above, the activation weight of the kth rule, w_k, is calculated as (Yang et al., 2005a):

$$w_k = \frac{\theta_k \times \prod_{i=1}^{T_k} (\alpha_{ik})^{\bar{\delta}_i}}{\sum_{j=1}^{L} [\theta_j \times \prod_{l=1}^{T_k} (\alpha_{lj})^{\bar{\delta}_l}]}, \tag{2}$$

where $\bar{\delta}_i = \frac{\delta_i}{\max_{i=1,\ldots,T_k} \{\delta_i\}}$, $\alpha_{ik}(i = 1, \ldots, T_k)$, is the *individual matching degree* to which the input $X_i \in U$ matches the ith referential value A_i^k of the packet antecedent A^k in the kth rule, and $\alpha_{ik} \geqslant 0$ and $\sum_{i=1}^{T_k} \alpha_{ik} \leqslant 1 . \alpha_k = \prod_{i=1}^{T_k} (\alpha_{ik})^{\bar{\delta}_i}$ is called *the combined matching degree*.

Having determined the activation weight of each rule in the rule base, the ER approach (Yang, 2001; Yang & Xu, 2002a) can be directly applied to combine the rules and generate final conclusions. Suppose the outcome of the combination yields the following:

$$O(U) = \{(D_j, \beta_j), \ j = 1, \ \ldots, \ N\} \tag{3}$$

The outcome expressed by Eq. (3) reads that if the input is given by $U = (U_i, i = 1, \ldots, T)$ then the consequent is D_1 to a degree of β_1, D_2 to a degree of $\beta_2, \ldots$, and D_N to a degree of β_N. Using the analytical format of the ER algorithm (Wang, Yang, & Xu, in press) the combined belief degree β_j in D_j can be generated as follows:

$$\beta_j = \frac{\mu \times \left[\prod_{k=1}^{L} (w_k \beta_{jk} + 1 - w_k \sum_{j=1}^{N} \beta_{jk}) - \prod_{k=1}^{L} (1 - w_k \sum_{j=1}^{N} \beta_{jk})\right]}{1 - \mu \times \left[\prod_{k=1}^{L} (1 - w_k)\right]},$$
$$j = 1, \ldots, N \tag{4}$$

where $\mu = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} (w_k \beta_{jk} + 1 - w_k \sum_{j=1}^{N} \beta_{jk}) - (N-1) \times \prod_{k=1}^{L} (1 - w_k \sum_{j=1}^{N} \beta_{jk})\right]^{-1}$, and w_k is as given in Eq. (2).

2.3. Optimal learning methods for training belief rule bases in RIMER

Although it is possible to establish a belief rule base by extracting knowledge from experts, the performance of the system can be improved if the rules are fine tuned through learning from available historical data. The adjustable parameters of a rule base are belief degrees ($\beta_{1k}, \beta_{2k}, \ldots, \beta_{Nk}$), rule weights θ_k for $k = 1, 2, \ldots, L$ and attribute weights ($\delta_1, \delta_2, \ldots, \delta_T$) (Yang et al., 2005b).

Fig. 1 sketches the process of training a belief rule base, where U is a given input, $\hat{O}$ the corresponding observed output either measured using instruments or assessed by experts, O the simulated output generated by the belief rule based system, $\xi(P)$ the difference between $\hat{O}$ and O, and

$$P = (\beta_{ik}, \ \theta_k, \ \delta_j; \quad i = 1, \ \ldots, \ N; \quad k = 1, \ \ldots, \ L; \quad j = 1, \ \ldots, \ T) \tag{5}$$

are the adjustable parameters. The objective of the training is to minimise the difference $\xi(P)$ by adjusting the parameters P. This objective is difficult to achieve manually even by experts, however there are computer algorithms available to solve the problem. Yang et al. (2005b) discussed in more details on how the problems can be constructed for different types of output and algorithms applied to solve them.

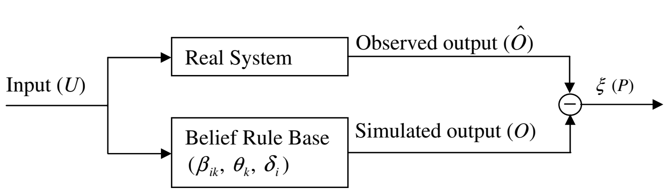

Fig. 1. Illustration of optimal learning process.

3. Belief-rule-based expert system for pipeline leak detection

3.1. Problem description

In this section, we will consider a pipeline more than 100 km in length with the mass flow meters at the inlet and outlet and the pressure meters at the inlet, outlet and 8 middle points along the line. Data are collected from those meters in every 10 s. The pipeline is mostly operated in leak free (normal) condition. However, during a leak trial period, a series of leaks were created in the pipeline. Each leak lasted up to a few hours and the size of the leaks was controlled through a valve. Fig. 2 shows the inlet and outlet flow and pressure readings ($f0$, $f1$, $p0$ and $p9$, respectively) collected in about 5 and half hours during a leak trail, with the leak period clearly marked by the large discrepancy between the inlet and outlet flow readings.

We will use the data to train and validate a belief rule based expert system for detecting those leaks and estimate the leak sizes without generating false alarms.

3.2. Antecedent and consequent attributes of the rule base

Under normal operations, when inlet flow is larger (or less) than outlet flow, the pressure in the pipeline will build up (or decrease) because the total content in the pipeline is increasing (or decreasing, respectively). However, if the pattern is violated, for example, if the inlet flow is larger than the outlet flow, yet the pressure in the line still decreases, then it is highly likely that there is a leak in the pipeline. Therefore, the difference between inlet flow and outlet flow, denoted by *FlowDiff*, and the average pipeline pressure change over time, denoted by *PressureDiff*, are the two very important factors in detecting whether there is a leak in the pipeline. They are the two antecedent attributes of the rule base and are calculated as follows:

$$U_1 = FlowDiff\ (t) = f1(t) - f0(t) \tag{6}$$

$$U_2 = PressureDiff\ (t) = [p0(t) + \cdots + p9(t)]/10 - [p0(t-1) + \cdots + p9(t-1)]/10 \tag{7}$$

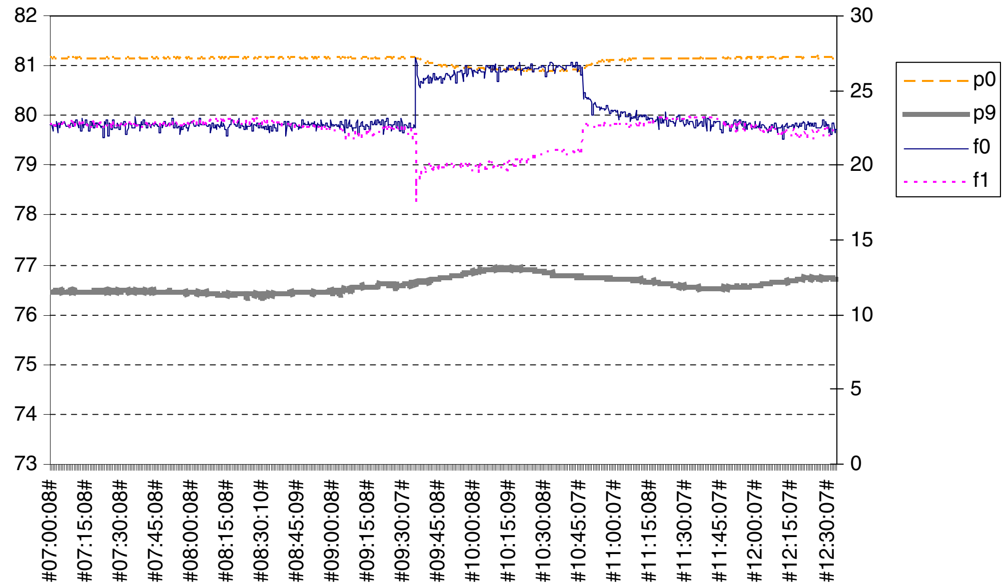

Fig. 2. Inlet and outlet flow and pressure readings.

Table 1
Calculated flow and pressure differences

	FlowDiff ($f1 - f0$)	*PressureDiff* ($p_{\text{average}}(t) - p_{\text{average}}(t-1)$)	*LeakSize t* (h)
#07:00:18#	0.05	0	0
...	...	...	...
#10:04:08#	−6.15	−0.007	6.35
#10:04:18#	−6.15	0	6.38
#10:04:28#	−6.15	0	6.40
#10:04:39#	−6.1	0.0085	6.39
#10:04:49#	−6.1	0	6.35
...	...	...	...
#12:34:27#	−0.1	0	0

where $f1(t)$, $f0(t)$, $p0(t)$, ..., and $p9(t)$ are collected instrument readings at time t. The consequent attribute is the leak rate, denoted by *LeakSize*. *LeakSize* values are controlled during the leak trial and therefore are more or less (though not exactly) known. Table 1 lists a few antecedent attribute values and the corresponding consequent attribute values.

3.3. Referential points of the antecedents and consequent

The number of referential points used for each antecedent decides the size of the rule base. If the number is too large, there will be too many rules in the rule base, and the subsequent training and inference process will be more demanding. If it is too small, the points may not be able to cover the value range of an antecedent attribute. This is especially true for a conventional rule base. Normally 5–9 referential points are used. The number of referential points for a consequent attribute is also comparable to those of the antecedent attributes. In this paper we use 8 points for *FlowDiff* and they are negative large (NL), negative medium (NM), negative small (NS), negative very small (NVS), zero (Z), positive small (PS), positive medium (PM), and positive large (PL). That is

$$A_1^k \in \{\text{NL}, \text{NM}, \text{NS}, \text{NVS}, \text{Z}, \text{PS}, \text{PM}, \text{PL}\} \tag{8}$$

Similarly we use 7 points for *PressureDiff* and they are NL, NM, NS, Z, PS, PM, and PL, i.e.,

$$A_2^k \in \{\text{NL}, \text{NM}, \text{NS}, \text{Z}, \text{PS}, \text{PM}, \text{PL}\} \tag{9}$$

For the consequent attribute, *LeakSize*, 5 referential points are used: zero (Z), very small (VS), medium (M), high (H) and very high (VH), i.e.

$$\boldsymbol{D} = (D_1, D_2, D_3, D_4, D_5) = (\text{Z}, \text{VS}, \text{M}, \text{H}, \text{VH}) \tag{10}$$

In order to use the data as shown in Table 1, the referential points defined above for the antecedent and consequent attributes are in linguistic terms and need to be quantified. By examining the calculated *FlowDiff* and *PressureDiff*, and the recorded *LeakSize* value, the following equivalent relationships between the linguistic terms and numerical values are assumed so that the values roughly cover the corresponding attribute value range.

For *FlowDiff*, it is assumed that

$$\left\{ \begin{array}{lcl} \text{NL} & = & -10 \\ \text{NM} & = & -5 \\ \text{NS} & = & -3 \\ \text{NVS} & = & -1 \\ \text{Z} & = & 0 \\ \text{PS} & = & 1 \\ \text{PM} & = & 2 \\ \text{PL} & = & 3 \end{array} \right\} \tag{11}$$

For *PressureDiff*, it is assumed that

$$\left\{\begin{array}{lcl} NL & = & -0.01 \\ NM & = & -0.005 \\ NS & = & -0.002 \\ Z & = & 0 \\ PS & = & 0.002 \\ PM & = & 0.005 \\ PL & = & 0.01 \end{array}\right\} \tag{12}$$

For *LeakSize*, it is assumed that

$$\left\{\begin{array}{lcl} Z & = & 0 \\ VS & = & 2 \\ M & = & 4 \\ H & = & 6 \\ VH & = & 8 \end{array}\right\} \tag{13}$$

Although the referential values cover the attribute value ranges in historical data and can detect leaks with sizes of up to 30% of the pipeline throughput, it is possible that future leaks may be above such a figure. When this happens, the leak size will be estimated as VH with 100% belief degrees.

3.4. Rules

Using the linguistic terms or their equivalent referential numerical values, one of the conventional rules for leak detection and leak size estimate may look like this:

IF *FlowDiff* is NL AND *PressureDiff* is NL
THEN *LeakSize* is VH.

However, such conventional *IF-THEN* rules cannot capture the continuous relationships between the antecedents and the consequents. Therefore, the expert system based on those rules may not be able to achieve high detection performances.

Use the belief rule concept, the conventional rules can be extended as follows:

$$R_k: \quad \text{IF } \mathit{FlowDiff} \text{ is } A_1^k \text{AND } \mathit{PressureDiff} \text{ is } A_2^k$$
$$\text{THEN } \mathit{LeakSize} \text{ is } \{(Z, \beta_{1k}), (\text{VS}, \beta_{2k}), (\text{M}, \beta_{3k}), (\text{H}, \beta_{4k}), (\text{VH}, \beta_{5k})\}, \left(\sum_{i=1}^{N} \beta_{ik} \leqslant 1\right) \quad k \in \{1, \ldots, 56\} \tag{14}$$

Here A_1^k, A_2^k are the referential values as defined in Eqs. (8) and (9), respectively. Because *FlowDiff* is divided into 8 terms and *PressureDiff* 7 terms, there are 56 combinations of the 2 antecedents leading to 56 rules in total in the rule-base.

The initial belief rules can be established in the following four ways:

(1) Extracting rules from expert knowledge, either in forms of conventional rules or belief rules.
(2) Extracting rules by examining historical data.
(3) Using previous rule bases used for leak detection of similar pipelines if available. The rule bases can be conventional ones.
(4) Random rules without any pre-knowledge.

Table 2
Summary of effect of increasing leak on system variables

FlowDiff	*PressureDiff*	*LeakSize*
Negative and constant	More negative	↑
Zero	More negative	↑
More negative	Negative and constant	↑
More negative	More negative	↑

In our case, there is no previous rule base to start with. Rules are extracted by examining the data and using expert experiences, and are used as the starting point for training. Table A1 in Appendix lists the initial 56 belief rules provided by an expert experienced in pipeline leak detection. For example, Rule 8 in the table is stated as follows:

R_8: IF *FlowDiff* is NM AND *PressureDiff* is NL
THEN *LeakSize* is $\{(\text{H}, 0.2), (\text{VH}, 0.8)\}$

At the time when the expert was asked to provide the rules, the equivalent referential values for the linguistic terms shown in Eqs. (11) and (12) were not finalised. Therefore those rules may be qualitatively correct, i.e., the leak sizes vary with the *FlowDiff* and *PressureDiff* in the right trend (as shown in Table 2). However, the belief degrees for *LeakSize* distribution (Table A1) may not be accurate. Further training is therefore necessary to fine tune the belief degrees so that the performance of the expert system can be improved or optimised in a sense.

3.5. Training and testing of the rule base

3.5.1. Training data

During the leak trial, 2008 samples were collected at the rate of 10 s per sample, as shown in Fig. 2. In order to train the rule-base, 500 data samples are selected and about half of them are collected during the leak period. They are the data collected in the three periods of 7 a.m. to 7:33 a.m., 9.46 a.m. to 10:20 a.m. and 10:50 a.m. to 11:08 a.m. (Fig. 2).

3.5.2. Training

Using the notations defined in Section 2.3, the parameters that need to be trained are given in Eq. (5). More specifically, they are

$$P = (\beta_{ik}, \theta_k, \delta_j;\ i = 1, \ldots, N;\ k = 1, \ldots, L;\ j = 1, \ldots, T) \tag{15}$$

where $N = 5$, $L = 56$ and $T = 2$.

The input to both the real system and the rule based system are *FlowDiff*(t) and *PressureDiff*(t) as defined by Eqs. (6) and (7), that is

$U(t) = (FlowDiff(t),\ PressureDiff(t))$

The observed output $\widehat{O}(t)$ is the controlled (or observed) leak size recorded in a data file. The simulated output $O(t)$ is calculated using the inference method described in *Step 5* below. The learning process is outlined in the following seven steps. The process is implemented using MATLAB.

Step 1: Set initial parameters

The initial belief degrees are given by an expert and listed in Table A1. θ_k, and δ_j ($k = 1, \ldots, 56$; $j = 1, 2$) are all assumed to be 1.

Step 2: Transform the input

For each of the 500 training samples, the input values [*FlowDiff*(t), *PressureDiff*(t)] need to be transformed and represented in terms of the referential values as defined in Eqs. (11) and (12) using belief degrees to which the input values match the referential values. The transformation is based on the rule-based transformation technique (Yang, 2001) for the quantitative data transformation. For example, if *FlowDiff*(t) = −6.25, then using the referential values of this attribute, it is equivalently transformed to *FlowDiff*(t) = {(NL,0.25),(NM,0.75)} because $-6.25 = \mathrm{NL} \times 0.25 + \mathrm{NM} \times 0.75 = -10 \times 0.25 + (-5) \times 0.75$. The belief degree 0.25 is the matching degree of the input *FlowDiff* = −6.25 to the referential value NL = −10. Because the referential value NL is used in Rule 1 to Rule 7 (refer to Table A1 in Appendix), the α_{1k} in Eq. (2) is 0.25 for $k = 1, 2, \ldots, 7$. Similarly, $\alpha_{1k} = 0.75$ for $k = 8, 9, \ldots, 14$.

Step 3: Calculate rule activation weight

The matching degrees, α_{ik}, of input values to their attribute referential values are used in Eq. (2) to calculate the activation weight of the kth rule, w_k ($k = 1, \ldots, 56$).

Step 4: Combine activated rules

The ER approach (Yang & Xu, 2002a) is employed to combine the activated rules. The ER approach is implemented in the IDS software (Yang & Xu, 2005). Using the IDS software, the activated rules can be combined to yield the following outcome:

$$O(U(t)) = \{(D_j, \beta_j)\ j = 1, \ldots, 5\} \tag{16}$$

where β_j is given by Eq. (4), and $[D_1, D_2, D_3, D_4, D_5] = $ [Z, VS, M, H, VH] = [0, 2, 4, 6, 8] as defined in Eq. (13).

Step 5: Estimate leak sizes

Having obtained the outcome shown in Eq. (16), the simulated outcome, the *LeakSize* is calculated as follows:

$$EstimatedLeakSize(t) = D_1\beta_1 + D_2\beta_2 + D_3\beta_3 + D_4\beta_4 + D_5\beta_5 \tag{17}$$

The estimated *LeakSize* calculated using Eq. (17) on the basis of the initial belief rule base is plotted in Fig. 3, together with the observed *LeakSize* in the training data. It is obvious that the estimated values do not match the observed values well. This means the initial rule base provided by an expert is not good enough.

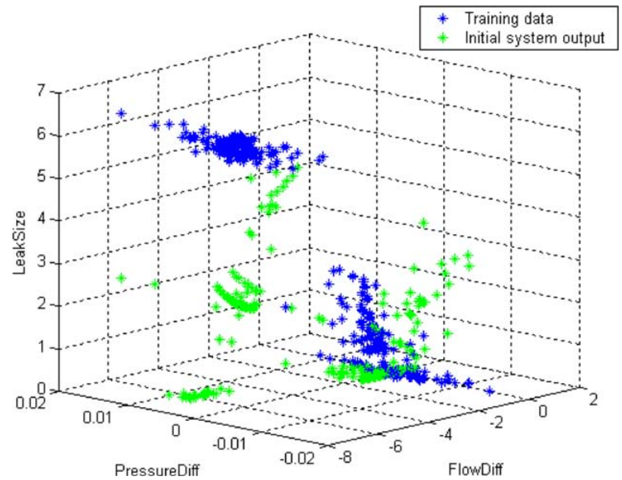

Fig. 3. Training data and the output by the initial belief rule based system.

Step 6: Calculate the difference between the observed leak size and estimated leak size

The difference between the observed and estimated leak sizes, $\xi(P)$, is calculated as follows:

$$\xi(P) = \frac{1}{500} \times \sum_{t=1}^{500} (ObservedLeakSize(t) - EstimatedLeakSize(t))^2 \tag{18}$$

Step 7: Find a new set of parameters P so that the difference defined in Eq. (18) is minimised

The objective of the learning process is to find a set of parameters P so that the difference between the observed and the estimated leak sizes is minimised, i.e.,

$$\min_{P} \quad \{\xi(P)\} \tag{19}$$

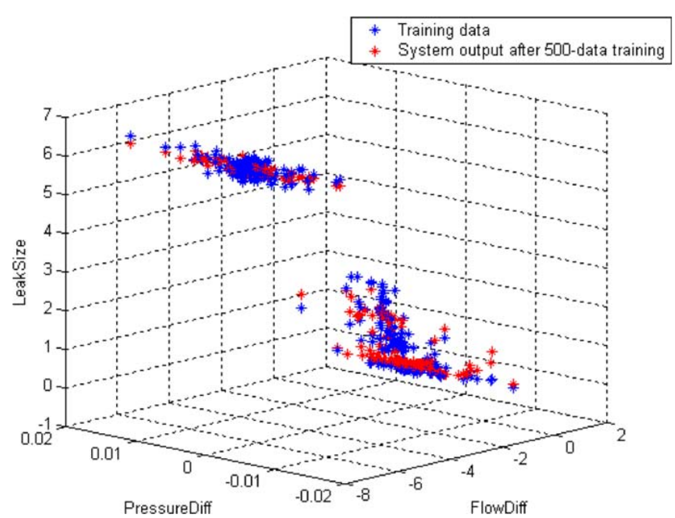

Fig. 4. Training data and the output by the trained belief rule based system.

$$\text{s.t.} \qquad 0 \leqslant \beta_{ik} \leqslant 1 \tag{19a}$$

$$\sum_{i=1}^{5} \beta_{ik} = 1 \tag{19b}$$

$$0 \leqslant \theta_k \leqslant 1 \tag{19c}$$

$$0 \leqslant \delta_j \leqslant 1 \tag{19d}$$

with $P = (\beta_{ik}, \theta_k, \delta_j;\ i = 1, \ldots, 5;\ k = 1, \ldots, 56;\ j = 1,2)$.

The above problem was solved using the MATLAB optimisation toolbox (Coleman, Branch, & Grace, 1999). The trained belief rules are listed in Table A2 of Appendix. The difference between the observed and the estimated leak sizes as defined in Eq. (18) is less than 0.001.

Fig. 4 shows that the trained belief rule base can closely replicate the relationship among *FlowDiff*, *PressureDiff* and *LeakSize* in the training data.

3.5.3. Testing

For testing the trained belief rules, all the 2008 samples shown in Fig. 2 are used. Fig. 5 shows the observed *LeakSize* and the estimated *LeakSize* for the same antecedent values (*FlowDiff* and *PressureDiff*). It demonstrates that the estimated outcomes match the observed ones very closely.

Fig. 6 displays the observed *LeakSize* and the estimated *LeakSize* on the time scale. It shows that the rule base can clearly detect the leak which happened at around 9:36 a.m. and ended at around 10:47 a.m.

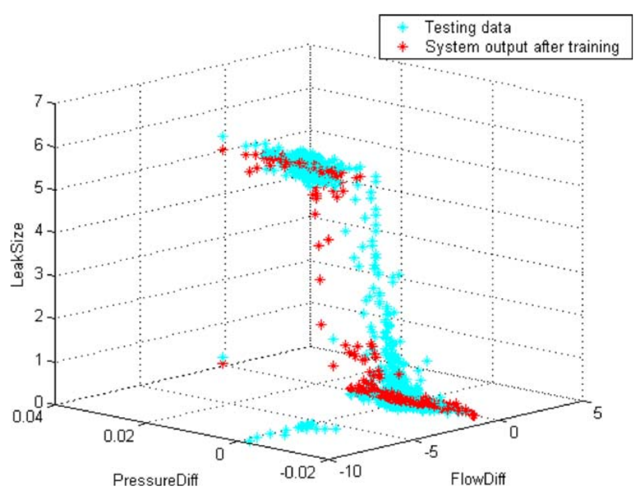

Fig. 5. Testing data and the output by the trained belief rule based system.

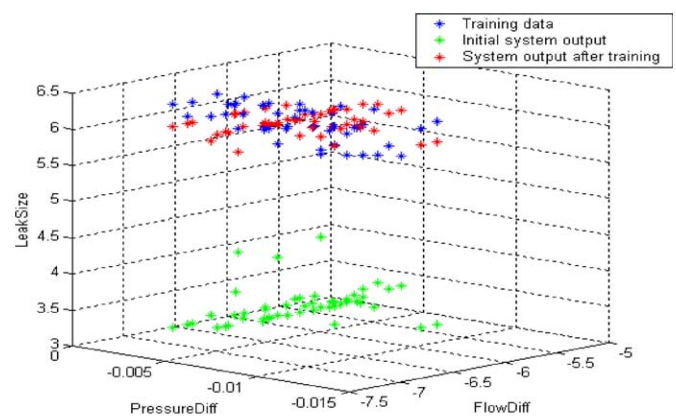

Fig. 7. Outcomes generated by random initial rule base and trained rule base.

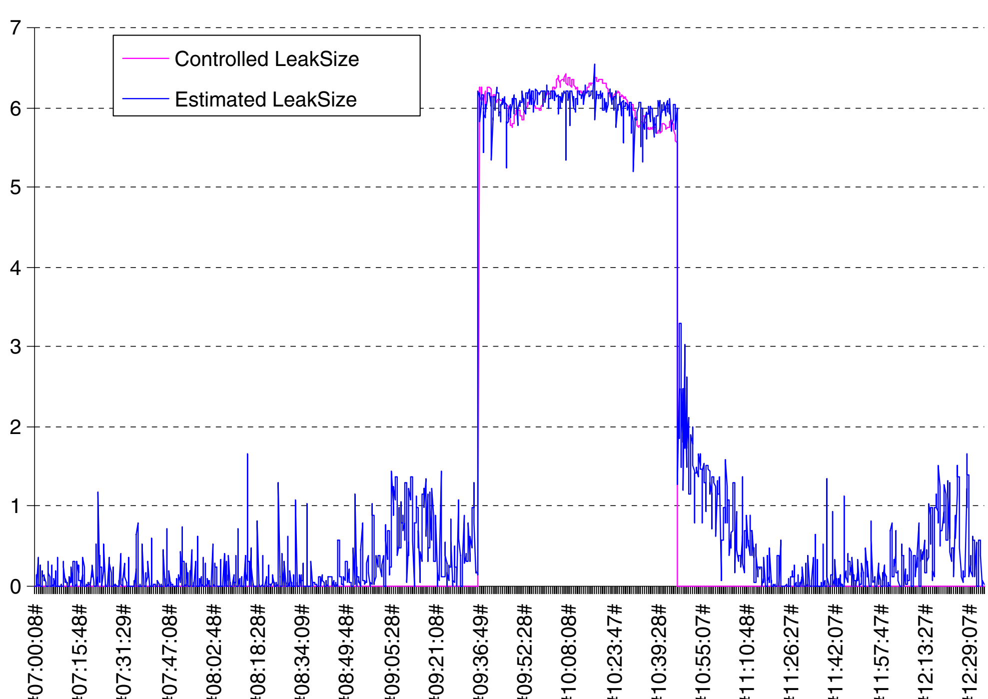

Fig. 6. Testing data and the output by the trained belief rule based system.

It is also noticed that the estimated *LeakSize* is somehow noisy and not constantly zero in the leak free periods (Fig. 6). The noise may be caused by turbulence and dynamic changes in the pipeline, and possible instrument and data communication errors. Such noise is intrinsic to almost all pipeline operation data and poses significant challenges to developing pipeline leak detection systems with high reliability and sensitivity (Bloom, 2004; Theakston, 2004; Theakston & Larnaes, 2002; Carpenter et al., 2005). The most common approach to preventing the noise

Table A1
Initial belief rules provided by an expert

Rule number	*FlowDiff* AND *PressureDiff*	*LeakSize* distribution $\{D_1, D_2, D_3, D_4, D_5\} = \{0, 2, 4, 6, 8\}$	*LeakSize* value
1	NL AND NL	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0), (D_5,1)\}$	8
2	NL AND NM	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0.3), (D_5,0.7)\}$	7.4
3	NL AND NS	$\{(D_1,0), (D_2,0), (D_3,0.2),(D_4,0.8), (D_5,0)\}$	5.6
4	NL AND Z	$\{(D_1,0), (D_2,0), (D_3,0.8), (D_4,0.2), (D_5,0)\}$	4.4
5	NL AND PS	$\{(D_1,0.65), (D_2,0.35), (D_3,0), (D_4,0), (D_5,0)\}$	0.7
6	NL AND PM	$\{(D_1,0.85), (D_2,0.15), (D_3,0), (D_4,0), (D_5,0)\}$	0.3
7	NL AND PL	$\{(D_1,0.95), (D_2,0.05), (D_3,0), (D_4,0), (D_5,0)\}$	0.1
8	NM AND NL	$\{(D_1,0), (D_2,0), (D_3,0.1), (D_4,0.9), (D_5,0)\}$	5.8
9	NM AND NM	$\{(D_1,0), (D_2,0), (D_3,0.7), (D_4,0.3), (D_5,0)\}$	4.6
10	NM AND NS	$\{(D_1,0), (D_2,0.7), (D_3,0.3), (D_4,0), (D_5,0)\}$	2.6
11	NM AND Z	$\{(D_1,0), (D_2,0.9), (D_3,0.1), (D_4,0), (D_5,0)\}$	2.2
12	NM AND PS	$\{(D_1,0.8), (D_2,0.2), (D_3,0), (D_4,0), (D_5,0)\}$	0.4
13	NM AND PM	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
14	NM AND PL	$\{(D_1,0.99), (D_2,0.01), (D_3,0), (D_4,0), (D_5,0)\}$	0.02
15	NS AND NL	$\{(D_1,0), (D_2,0), (D_3,0.4), (D_4,0.6), (D_5,0)\}$	5.2
16	NS AND NM	$\{(D_1,0), (D_2,0), (D_3,0.8), (D_4,0.2), (D_5,0)\}$	4.4
17	NS AND NS	$\{(D_1,0), (D_2,0.3), (D_3,0.6), (D_4,0.1), (D_5,0)\}$	3.6
18	NS AND Z	$\{(D_1,0.1), (D_2,0.7), (D_3,0.2), (D_4,0), (D_5,0)\}$	2.2
19	NS AND PS	$\{(D_1,0.7), (D_2,0.3), (D_3,0), (D_4,0), (D_5,0)\}$	0.6
20	NS AND PM	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
21	NS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
22	NVS AND NL	$\{(D_1,0), (D_2,0.1), (D_3,0.4), (D_4,0.5), (D_5,0)\}$	4.8
23	NVS AND NM	$\{(D_1,0), (D_2,0.8), (D_3,0.2), (D_4,0), (D_5,0)\}$	2.4
24	NVS AND NS	$\{(D_1,0.2), (D_2,0.7), (D_3,0.1), (D_4,0), (D_5,0)\}$	1.8
25	NVS AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
26	NVS AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
27	NVS AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
28	NVS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
29	Z AND NL	$\{(D_1,0), (D_2,0.4), (D_3,0.6), (D_4,0), (D_5,0)\}$	3.2
30	Z AND NM	$\{(D_1,0.2), (D_2,0.7), (D_3,0.1), (D_4,0), (D_5,0)\}$	1.8
31	Z AND NS	$\{(D_1,0.4), (D_2,0.6), (D_3,0), (D_4,0), (D_5,0)\}$	1.2
32	Z AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
33	Z AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
34	Z AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
35	Z AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
36	PS AND NL	$\{(D_1,0), (D_2,0.8), (D_3,0.2), (D_4,0), (D_5,0)\}$	2.4
37	PS AND NM	$\{(D_1,0.8), (D_2,0.2), (D_3,0), (D_4,0), (D_5,0)\}$	0.4
38	PS AND NS	$\{(D_1,0.95), (D_2,0.05), (D_3,0), (D_4,0), (D_5,0)\}$	0.1
39	PS AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
40	PS AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
41	PS AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
42	PS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
43	PM AND NL	$\{(D_1,0.1), (D_2,0.9), (D_3,0), (D_4,0), (D_5,0)\}$	1.8
44	PM AND NM	$\{(D_1,0.3), (D_2,0.7), (D_3,0), (D_4,0), (D_5,0)\}$	1.4
45	PM AND NS	$\{(D_1,0.85), (D_2,0.15), (D_3,0), (D_4,0), (D_5,0)\}$	0.3
46	PM AND Z	$\{(D_1,0.98), (D_2,0.02), (D_3,0), (D_4,0), (D_5,0)\}$	0.04
47	PM AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
48	PM AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
49	PM AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
50	PL AND NL	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
51	PL AND NM	$\{(D_1,0.99), (D_2,0.01), (D_3,0), (D_4,0), (D_5,0)\}$	0.02
52	PL AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
53	PL AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
54	PL AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
55	PL AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
56	PL AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0

from triggering false leak alarms is to use a confirmation period. That is, before a leak alarm can be confirmed, the estimated *LeakSize* needs to stay positive continuously for a specified period.

The lengths of the confirmation period vary with the quality of the instruments used, the estimated leak sizes, the transported material in the pipeline and whether pipeline is operated at steady, changing, starting up or shutting down status. Another belief rule based system can also be established, trained and used to detect the pipeline operating status and determine the lengths of the confirmation period, which is not discussed in this paper.

Table A2
Trained belief rules after 500-data training (the initial rule weights are assumed to be equal)

Rule number	Trained rule weight	*FlowDiff* AND *PressureDiff*	*LeakSize* distribution $\{D_1, D_2, D_3, D_4, D_5\} = \{0, 2, 4, 6, 8\}$	*LeakSize* value
1	0.90	NL AND NL	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0), (D_5,1)\}$	8
2	0.90	NL AND NM	$\{(D_1,0), (D_2,0), (D_3,0), (D_4,0.3), (D_5,0.7)\}$	7.4
3	0.50	NL AND NS	$\{(D_1,0), (D_2,0), (D_3,0.2), (D_4,0.8), (D_5,0)\}$	5.6
4	0.50	NL AND Z	$\{(D_1,0), (D_2,0), (D_3,0.8), (D_4,0.2), (D_5,0)\}$	4.4
5	0.50	NL AND PS	$\{(D_1,0.65), (D_2,0.35), (D_3,0), (D_4,0), (D_5,0)\}$	0.7
6	0.80	NL AND PM	$\{(D_1,0.85), (D_2,0.15), (D_3,0), (D_4,0), (D_5,0)\}$	0.3
7	0.90	NL AND PL	$\{(D_1,0.95), (D_2,0.05), (D_3,0), (D_4,0), (D_5,0)\}$	0.1
8	0.90	NM AND NL	$\{(D_1,0), (D_2,0), (D_3,0.1), (D_4,0.9), (D_5,0)\}$	5.8
9	0.80	NM AND NM	$\{(D_1,0), (D_2,0), (D_3,0.7), (D_4,0.3), (D_5,0)\}$	4.6
10	0.50	NM AND NS	$\{(D_1,0), (D_2,0.7), (D_3,0.3), (D_4,0), (D_5,0)\}$	2.6
11	0.50	NM AND Z	$\{(D_1,0), (D_2,0.9), (D_3,0.1), (D_4,0), (D_5,0)\}$	2.2
12	0.50	NM AND PS	$\{(D_1,0.8), (D_2,0.2), (D_3,0), (D_4,0), (D_5,0)\}$	0.4
13	0.90	NM AND PM	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
14	0.90	NM AND PL	$\{(D_1,0.99), (D_2,0.01), (D_3,0), (D_4,0), (D_5,0)\}$	0.02
15	0.80	NS AND NL	$\{(D_1,0), (D_2,0), (D_3,0.4), (D_4,0.6), (D_5,0)\}$	5.2
16	0.80	NS AND NM	$\{(D_1,0), (D_2,0), (D_3,0.8), (D_4,0.2), (D_5,0)\}$	4.4
17	0.50	NS AND NS	$\{(D_1,0), (D_2,0.3), (D_3,0.6), (D_4,0.1), (D_5,0)\}$	3.6
18	0.50	NS AND Z	$\{(D_1,0.1), (D_2,0.7), (D_3,0.2), (D_4,0), (D_5,0)\}$	2.2
19	0.50	NS AND PS	$\{(D_1,0.7), (D_2,0.3), (D_3,0), (D_4,0), (D_5,0)\}$	0.6
20	0.50	NS AND PM	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
21	1.00	NS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
22	0.77	NVS AND NL	$\{(D_1,0.02), (D_2,0.11), (D_3,0.39), (D_4,0.48), (D_5,0)\}$	4.66
23	0.36	NVS AND NM	$\{(D_1,0.10), (D_2,0.78), (D_3,0.12), (D_4,0), (D_5,0)\}$	2.04
24	0.30	NVS AND NS	$\{(D_1,0.36), (D_2,0.65), (D_3,0.10), (D_4,0), (D_5,0)\}$	1.75
25	1.00	NVS AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
26	1.00	NVS AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
27	1.00	NVS AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
28	1.00	NVS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
29	0.64	Z AND NL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
30	0.88	Z AND NM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
31	0.55	Z AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
32	1.00	Z AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
33	1.00	Z AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
34	1.00	Z AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
35	1	Z AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5, 0)\}$	0
36	0.77	PS AND NL	$\{(D_1,0.39), (D_2,0.61), (D_3,0), (D_4,0), (D_5,0)\}$	1.22
37	0.64	PS AND NM	$\{(D_1,0.90), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
38	0.57	PS AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
39	1.00	PS AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
40	1.00	PS AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
41	1.00	PS AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
42	1.00	PS AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
43	0.90	PM AND NL	$\{(D_1,0.1), (D_2,0.9), (D_3,0), (D_4,0), (D_5,0)\}$	1.8
44	0.50	PM AND NM	$\{(D_1,0.3), (D_2,0.7), (D_3,0), (D_4,0), (D_5,0)\}$	1.4
45	0.50	PM AND NS	$\{(D_1,0.85), (D_2,0.15), (D_3,0), (D_4,0), (D_5,0)\}$	0.3
46	0.50	PM AND Z	$\{(D_1,0.98), (D_2,0.02), (D_3,0), (D_4,0), (D_5,0)\}$	0.04
47	1.00	PM AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
48	1.00	PM AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
49	1.00	PM AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
50	0.90	PL AND NL	$\{(D_1,0.9), (D_2,0.1), (D_3,0), (D_4,0), (D_5,0)\}$	0.2
51	0.70	PL AND NM	$\{(D_1,0.99), (D_2,0.01), (D_3,0), (D_4,0), (D_5,0)\}$	0.02
52	1.00	PL AND NS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
53	1.00	PL AND Z	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
54	1.00	PL AND PS	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
55	1.00	PL AND PM	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0
56	1.00	PL AND PL	$\{(D_1,1), (D_2,0), (D_3,0), (D_4,0), (D_5,0)\}$	0

In our case, if the confirmation period is set at 24 min, then all false alarms caused by data noises will be completely eliminated.

3.6. Training with random initial rules

In the previous section, the initial rules are given by an expert. To test the learning ability of the belief rule based system, a number of randomly generated rule bases are used as the starting points for learning. The study shows that the initial random rules can be trained to representing the relationship in the training data, as demonstrated in Fig. 7, where the initial system output is generated by the random belief rule base. After the training, the system output closely matches the observed output. Such learning capability of the belief rules base makes it applicable to a wide range of situations where experts' knowledge may or may not be available.

4. Concluding remarks

This paper describes a feasibility study of applying the belief rule based system for pipeline leak detection. The study demonstrates that by using the optimal learning methods (Yang et al., 2005b) for training the belief rules, the belief rule based system can learn from pipeline operating data the relationship between leak sizes and the pipeline flow and pressure readings. It has also demonstrated that learning could start with a random rule base and therefore pre-knowledge does not have to be provided.

For pipeline leak detection systems based on mass balance principles, identifying the relationship between flow difference and pressure changes is very important and involves intensive parameter tuning. It is a time-consuming process and pipeline specific. The optimal parameters may also change with time with the aging of the pipeline and instruments. The self-learning capability of the belief rule based system can significantly reduce the parameter tuning time and improve the performance of the system.

Conventional rule base is a special case of the belief rule base and may also be employed for the purpose. However, as it cannot model the continuous relationship between the leak sizes and the instrument readings of the pipeline, an accurate leak size estimate will be difficult.

Neural networks (Gallant, 1993) are also a popular tool for modelling causal relationship in different variables. In a belief rule based system, while human expert knowledge is used to construct a roughly correct belief rule base, the optimal learning mechanism can help to fine tune system performance if the system input–output data are available. As such, we believe that reasoning with fine-tuned logical rules is more acceptable to human users than the recommendations given by a black box system, because such reasoning is comprehensible, provides explanations, and can be validated by human inspection. It also increases confidence in the system, and may help to discover important relationships and combinations of features.

To apply the belief rule based system for real time online leak detection, auxiliary belief rule bases are needed so that intelligent judgments can be made online on whether any of the instruments is faulty, the pipeline is starting up, shutting down or running at steady states, and the instrument readings are drifting. Such judgments help to dynamically adjust the lengths of the confirmation periods, and correct the real flow and pressure differences fed to the belief rules. With the help of such auxiliary belief rule based systems, we expect that the sensitivity, reliability and response time of the new leak detection system would be improved. However further research and feasibility studies are needed before this can be confirmed.

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Appendix

See Tables A1 and A2.

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