Belief-Rule-Base Inference Method Based on Gradient Descent with Momentum

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ABSTRACT The belief-rule-base (BRB) inference methodology using evidential reasoning (ER) approach is widely used in different fields, such as fault diagnosis, system identification and decision analysis. Since the calculation of the individual matching degree in the inference process needs to set the reference value of the antecedent attribute parameter, it is easy to cause the rule zero activation problem, and the intelligent algorithm for parameter training is not as efficient and accurate as the gradient method, but the partial derivative of the gradient method is difficult to construct. In this paper, we propose a new belief rule structure and its gradient training method, aiming to solve zero activation during the inference process and improve inference accuray. We first used the Gaussian function to calculate the similarity of each attribute instead of the original method to avoid the zero activation problem caused by the non-adjacent reference values of the antecedent attributes. Then, according to the new activation weight calculation method, the corresponding distance-sensitive parameter is set for each attribute, and the weight parameter of each rule is cancelled. Is simplifies the calculation of rule weights in the inference process and enables the partial derivatives of the parameters of the inference system to be easily constructed. Finally, we use the momentum optimization gradient stochastic descent method to train the new BRB system, which improves the training speed and accuracy compared to the ordinary gradient method. Experiments on function fitting, oil pipeline leak detection and several public classification datasets are conducted to validate the new BRB system trained by stochastic gradient descent with momentum (SGDM-BRB) has better performance compared with others conventional methods. The experimental results show that the SGDM-BRB has higher accuray and faster training speed than conventional method.

INDEX TERMS belief rule base, structure optimization, stochastic gradient descent, momentum optimization.

I. INTRODUCTION

It is well known that rule-based intelligent systems are one of the most common frameworks for expressing various types of knowledge. The rule-based system has certain expression and processing capabilities through the use of existing human knowledge, and at the same time has the flexibility to deal with ambiguity, incompleteness, uncertainty and to combine different types of input data formats. In various rule-based systems for solving complex problems, belief rule-based inference methodology using evidential reasonging approach (ER) proposed by Yang *et al*. [1] baesd on conventional IF-THEN rules [2], Dempster-Shafer theory of evidence [3], [4], decision theory [5] and fuzzy set theory [6] shows its powerful function of representing and processing uncertain information. By introducing a belief distribution structure in the rules, this methodology can effectively handle incomplete and uncertain information involved in the datasets and widely used in various problem in different fields such as oil

pipeline leak detection [7], military capability estimation [8], consumer behavior prediction [9] and so on.

In the inference process of the belief rule base (BRB) system, the attribute weight, rule weight, belief distribution and other parameters directly affect the final accurary. Yang *et al.* [10] proposed optimization models for training BRB system using fmincon solver in Matlab, Chang *et al.* [11], [12] proposed an algorithm for training parameters in BRB system based on gradient and dichotomy methods, Wu *et al.* [13] used the accelerating of gradient algorithm to improve the convergence accuray and convergence speed. There are also a series of intelligent algorithms such as the particle swarm algorithm proposed by Su *et al.* [14] and the differential evolution algorithm proposed by Wang *et al.* [15] have excellent training effects on the BRB system. Liu *et al.* [16] introduces the belief distribution structure into the antecedent attributes and uses training data to build an extended belief rule base (EBRB) system, which simplifies the construction of the rule base and improves the inference speed.

At present, the parameter optimization model of the BRB system is mostly based on various intelligent algorithms. However, the process of those intelligent algorithms is complicated and there are many intermediate training parameters. When the conventional gradient method trains the parameters of the BRB system, the partial derivative of each parameter is difficult to construct, and the limit method is needed to solve the approximate value of the partial derivative [13]. The step size is limited by the corresponding parameter constraints and a search is required to find the best effective step size [11]–[13]. The EBRB system does not introduce a parameter training process, which makes the system have higher requirements for the representativeness of the training data used to construct the rule base. In the case of a large number of rules, it is necessary to perform rule reduction or use the data structure to optimize the storage and activation process of the rules. Because the conventional BRB system includes the rule attribute reference level setting, its potential zero activation problem may lead to the failure of the inference system.

In response to the above problems, a series of optimization modifications are proposed for the system structure and reasoning process, including:

1) We propose a new antecedent structure that does not need to set the attribute reference level, and propose a Gaussian function-based rule weight activation method for the new rule antecedent structure, which can effectively avoid the zero activation problem and has the feature of generating rules from the training data like EBRB.

2) We change the method of setting the weight of the global same antecedent attribute in the conventional BRB system, and set the corresponding rule attribute weight parameter for each rule, so that each rule has a finer activation granularity. On this basis, the rule weight and its related normalization process are cancelled, which simplifies the evidential reasoning process.

3) We further present a normalized exponential function

to preprocess the restricted parameters to avoid the problem of parameter failure during the training process.

The remainder of this paper is organized as follows: Section II introduces the conventional BRB system and our further improvements for common problems in the system. In Section III, we give the preprocessing method of the training model and prove that the gradient descent method can be effectively applied to the newly proposed BRB system. In Section IV, we compare the effects of different gradient descent parameters on training speed and inference accuracy. Experiments on a series of public data sets prove that the newly proposed BRB model and its training method have the best performance. Section V concludes this paper.

II. BRB SYSTEM WITH NEW ATTRIBUTE STRUCTURE AND RULE ACTIVATION WEIGHT CALCULATION METHOD

The BRB system proposed by Yang *et al.* [1] mainly refers to the rule activation and evidence reasoning method on the belief rule base. This section will briefly introduce the related concepts of the BRB system and propose the solutions for the common defects of the conventional BRB system.

A. REPRESENTATION OF BELIEF RULE BASE

On the basis of the conventional production rules, Yang *et al.* [1] proposed the expression form of the belief rules by introducing the belief distribution structure, the rule antecedent attribute parameter and the rule weight parameter. The specific expression is as follows:

$$R_k : if\{X_1 is A_1^k \wedge \cdots \wedge X_{T_k} is A_{T_k}^k\}$$
$$then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^{N} \beta_i^k \leq 1 \quad (1)$$

The equal sign is obtained when the rule information is complete. Each rule has its rule weight θ_k, antecedent attribute weight $\delta_1, \delta_2, \cdots, \delta_{T_k}$. A_i^k represents the reference value selected by the rule on the i-th attribute and β_i^k represents the belief degree of the rule in the i-th consequent attribute. On this basis, the extended belief rule base system introduces a belief distribution structure to the antecedent attributes, and its rule form is expressed as follows:

$$R_k : if\{[(A_{11}^k, \alpha_{11}^k), \cdots, (A_{1J_1}^k, \alpha_{1J_1}^k)] \wedge$$
$$\cdots \wedge [(A_{T_k 1}^k, \alpha_{T_k 1}^k), \cdots, (A_{T_k J_{T_k}}^k, \alpha_{T_k J_{T_k}}^k)]\}$$
$$then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^{N} \beta_i^k \leq 1 \quad (2)$$

The extended belief rule base converts the original training data into antecedent attributes with a belief distribution form. For the input data $X^k = (x_1^k, \cdots, x_T^k)$, convert the i-th

attribute parameter to construct the i-th antecedent attribute of the corresponding rule with a belief distribution form:

$$
\begin{aligned}
\alpha_{ij}^k &= \frac{\gamma_{i(j+1)} - x_i^k}{\gamma_{i(j+1)} - \gamma_{ij}}, \gamma_{ij} \leq x_i^k \leq \gamma_{i(j+1)} \\
\alpha_{i(j+1)}^k &= 1 - \alpha_{ij}^k, \gamma_{ij} \leq x_i^k \leq \gamma_{i(j+1)} \\
\alpha_{it}^k &= 0, t = 1, \cdots, (j-1), (j+2), \cdots, J_i
\end{aligned}
\tag{3}
$$

According to the same conversion method, the values of original data on other attributes can be converted into the corresponding berlief distribution form. We can also obtain the belief distribution form of the rule consequent attribute according to this method.

B. EVIDENCE REASONING APPROACH BASED ON BELIEF RULE BASE

The calculation and synthesis of activation weights for each rule in the rule base is the core part of the inference system of the belief rule base. The whole process mainly includes two steps: calculate the activation weight, and synthesize the rules according to the activation weight. The calculation of the activation weight of each rule in the belief rule base can be regarded as calculating the belief distribution similarity on each attribute and combining their results. Euclidean distance is used to calculate the individual matching degree of the i-th attribute. After converting the input data to have the same belief distribution form as the corresponding attribute, the individual matching degree of the attribute is calculated as:

$$
S_i^k = 1 - d_i^k = 1 - \sqrt{\frac{\sum_{j=1}^{J_i}(\alpha_{i,j} - \alpha_{i,j}^k)^2}{2}}
\tag{4}
$$

After the individual matching degree of each attribute is calculated, the individual matching degrees of all attributes are aggregated. The aggregation function in the form of conjunctive rules is:

$$
\alpha_k = \prod_{i=1}^{T_k}(S_i^k)^{\overline{\delta_i}}, \overline{\delta_i} = \frac{\delta_i}{\max_{j=1,\cdots,T_k}\delta_j}
\tag{5}
$$

The activation weight of this rule is calculated by the following formula:

$$
w_k = \frac{\theta_k \alpha_k}{\sum_{l=1}^L \theta_l \alpha_l}
\tag{6}
$$

Rule weight normalization operation makes all weights satisfy $0 \leq w_k \leq 1, \sum w_k = 1$.

After the rule weight calculation is completed, all the rules are synthesized and the inference result is obtained. First,

the belief distribution of the rule is transformed into the corresponding probability quality information:

$$
m_{j,k} = w_k \beta_j^k, j = 1, \cdots, N
\tag{7}
$$

$$
m_{D,k} = 1 - \sum_{j=1}^N m_{j,k} = 1 - w_k \sum_{j=1}^N \beta_j^k
\tag{8}
$$

$$
\overline{m}_{D,k} = 1 - w_k
\tag{9}
$$

$$
\widetilde{m}_{D,k} = w_k(1 - \sum_{j=1}^N \beta_j^k)
\tag{10}
$$

$m_{j,k}$ represents the credibility of the k rule on the j consequent attribute, where $\overline{m}_{D,k}$ represents the credibility that the k-th rule is not assigned to any consequent attribute, and $\widetilde{m}_{D,k}$ represents the credibility of the missing reference attribute of the k-th rule. The total uncertainty credibility is given by $m_{D,k} = \overline{m}_{D,k} + \widetilde{m}_{D,k}$. Synthesize the credibility information of all rules and obtain the final belief result of each consequent attribute:

$$
m_j = k[\prod_{i=1}^L(m_{j,i} + m_{D,i}) - \prod_{i=1}^L m_{D,i}], j = 1, \cdots, N
\tag{11}
$$

$$
\overline{m}_D = n[\prod_{i=1}^L \overline{m}_{D,i}]
\tag{12}
$$

$$
\widetilde{m}_D = k[\prod_{i=1}^L m_{D,i} - \prod_{i=1}^L \overline{m}_{H,i}]
\tag{13}
$$

$$
k = [\sum_{j=1}^N \prod_{i=1}^L(m_{j,i} + m_{D,i}) - (N-1)\prod_{i=1}^L m_{D,i}]^{-1}
\tag{14}
$$

$$
\beta_j = \frac{m_j}{1 - \overline{m}_D}, j = 1, \cdots, N
\tag{15}
$$

$$
\beta_D = \frac{\widetilde{m}_D}{1 - \overline{m}_D}
\tag{16}
$$

C. NEW ATTRIBUTE STRUCTURE AND RULE ACTIVATION WEIGHT CALCULATION METHOD

The reference value of antecedent attribute information needs to be set before generating rules. If the input is not in the neighborhood of the reference value of the antecedent attribute of a rule, the rule cannot be activated. If all the rules in the library are not activated, the reasoning system will fail. In order to solve the above problems, we propose an improved form of belief rules and corresponding activation weight calculation method as follows:

$$
\begin{aligned}
R_k &: if(x_1^k, \cdots, x_{T_k}^k) \\
&then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^N \beta_i^k \leq 1
\end{aligned}
\tag{17}
$$

The simplified belief rule structure can directly use the training data to generate the rule antecedent attribute information without manually setting the reference values of the antecedent attributes.
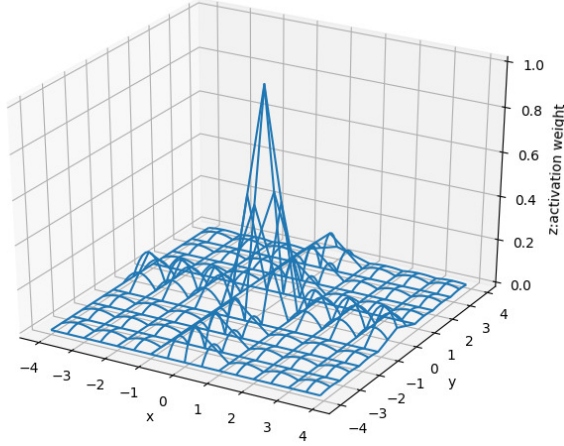
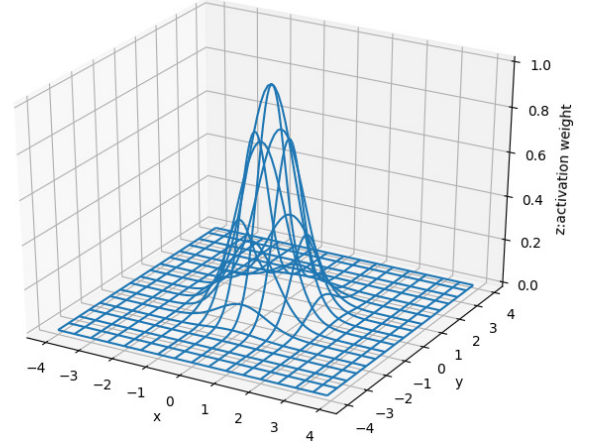**FIGURE 1.** Activation weight calculated by conventional methods

FIGURE 2. Activation weight calculated by Gaussian function methods with distance-sensitive parameter a set to 1.0
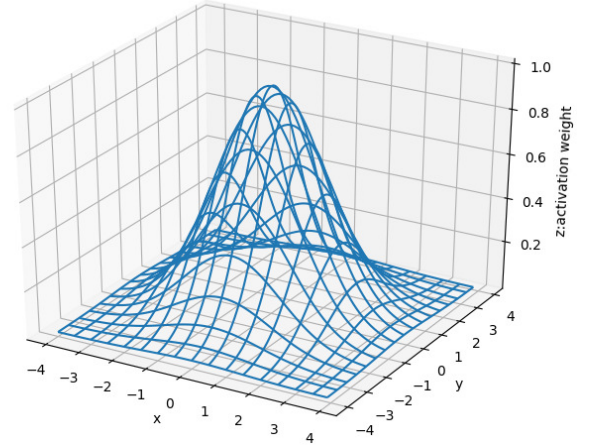
The conventional activation weight calculation method is no longer suitable for the simplified form of belief rules. In order to perform the weight activation, we use Gaussian function to calculate the individual matching degree for activation weight calculation. The degree of individual matching of input $X(x_1, \cdots, x_{T_k})$ and rule R_k : $if(x_1^k, \cdots, x_{T_k}^k)then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}$ on i-th attribute is calculated using the Gaussian function as:

$$S_i^k = e^{-[a_i^k \times (x_i - x_i^k)]^2} \quad (18)$$

The parameter a_i^k represents the sensitivity of the i-th attribute to the distance at the position x_i^k. When the distance between the rule antecedent attribute and the input data remains unchanged, the value of parameter a inversely proportional to the matching degree. The activation weight of a single rule under conjunctive conditions is calculated by the following formula:

$$w_k = \prod_{i=1}^{T_k} S_i^k = e^{-\sum_{i=1}^{T_k}[a_i^k(x_i - x_i^k)]^2} \quad (19)$$

Assuming a rule with two attributes x and y located at the $(0,0)$, the conventional method and Gaussian function method are used to calculate the activation weights. Set the reference reference values on the x-attributes and y-attributes to be $[-4, -3, -2, -1, 0, 1, 2, 3, 4]$. We ignore setting rule weights for both methods. The activation weight distributions shown in Figure 1, Figure 2 with distance-sensitive parameter a set to 1.0 and Figure 3 with distance-sensitive parameter a set to 0.5 can be obtained.

According to Figure 1 we can know that if the input on a certain attribute is not in the neighborhood of the rule attribute, the activation weight of this rule fluctuates around zero and gets zero when the attribute gets a reference value. If the activation weight of all rules is zero, the inference cannot be performed. However, the weight in Figure 2 and Figure 3 smoothly drops close to zero according to the distance from

FIGURE 3. Activation weight calculated by Gaussian function methods with distance-sensitive parameter a set to 0.5

the rule and will not take a value of zero. Moreover, different distance-sensitive parameters enable the rule to adapt to the calculation of activation weights at different scales. It eliminates the impact of rule zero activation on system inference performance, improves the fitting performance of the inference system by adding the distance-sensitive parameters.

Another benefit brought by the new rule structure and activation weight calculation method is that there is no need to adjust the activation weight of the rule by the attribute weight and rule weight. By adjusting the distance-sensitive parameters on each attribute of each rule, a good activation effect and activation granularity can be obtained. Due to the characteristics of the Gaussian function, the activation weight of each rule belongs to $(0, 1]$ without normalization operations. It greatly simplifies the redundant weight adjustment and calculation in the inference process.
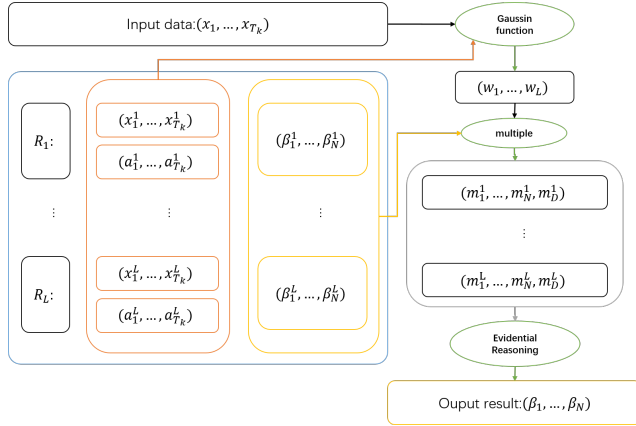
FIGURE 4. Inference system operation process

III. MOMENTUM OPTIMIZED GRADIENT DESCENT TRAINING PARAMETER

When the conventional gradient method is applied to the parameter training process of the inference system of the belief rule base, it is difficult to construct the partial derivative formula of the rule attribute and the training step is restricted by the parameter constraints. Compared with the conventional BRB system, the newly proposed rule structure with its activation weight calculation methid make it easier to construct the partial derivative of each part of the parameter.

In this section, we will:

1) Calculate the partial derivatives of the parameters of the new BRB system.

2) Prove that the new BRB system is differentiate.

3) Propose exponential normalization function for pre-processing to avoid specific constraints during parameter training.

4) Propose a parameter training method using stochastic gradient descent with momentum optimization.

A. PARTIAL DERIVATIVE OF THE PARAMETERS OF THE BRB SYSTEM

The reasoning process of the improved BRB system is shown in Figure 4. According to the inference process and compound function chain derivation rules, we can obtain the partial derivatives of the output to the different parameters of the system.

Since the model construction and experiment in this paper are carried out with complete data and the rule consequent attribute does not include uncertain information, for any k-th rule:

$$\sum_{i=1}^{N} \beta_{ik} = 1 (k=1,\cdots,L) \quad (20)$$

$$m_{D,k} = \overline{m}_{D,k}, \widetilde{m}_{D,k} = 0 \quad (21)$$

In the case of completeness, the j-th consequent attribute is expressed as:

$$\beta_j = \frac{\prod_{i=1}^{L}(m_{j,i}+m_{D,i}) - \prod_{i=1}^{L}m_{D,i}}{\sum_{t=1}^{N}\prod_{i=1}^{L}(m_{t,i}+m_{D,i}) - N \times \prod_{i=1}^{L}m_{D,i}} \quad (22)$$

Then the result of the j-th consequent attribute before normalization is expressed as:

$$\overline{\beta}_j = \prod_{i=1}^{L}(m_{j,i}+m_{D,i}) - \prod_{i=1}^{L}m_{D,i}, \beta_j = \frac{\overline{\beta}_j}{\sum_{k=1}^{N}\overline{\beta}_k} \quad (23)$$

Substitute $m_{j,i}=w_i\beta_{j,i}$ and $m_{D,i}=1-w_i$ into the j-th consequent attribute before normalization expression:

$$\overline{\beta}_j = \prod_{i=1}^{L}(w_i\beta_j^i+1-w_i) - \prod_{i=1}^{L}(1-w_i) \quad (24)$$

We can get the partial derivative of the i-th consequent attribute β_i to the j-th unnormalized consequent attribute $\overline{\beta}_j$ as:

$$\frac{d\beta_i}{d\overline{\beta}_j} = \begin{cases} \frac{\sum_{k\neq j}^{N}\overline{\beta}_k}{(\sum_{k=1}^{N}\overline{\beta}_k)^2}, j=i \\ -\frac{\beta_i}{(\sum_{k=1}^{N}\overline{\beta}_k)^2}, j\neq i \end{cases} \quad (25)$$

Similarly, the partial derivative of the j-th unnormalized consequent attribute $\overline{\beta}_j$ to the activation weight of the k-th rule and the j-th consequent attribute of k-th rule can be obtained as:

$$\frac{d\overline{\beta}_j}{dw_k} = (\beta_j^k-1)\prod_{i=1,i\neq k}^{L}(w_i\beta_j^i+1-w_i) + \prod_{i=1,i\neq k}^{L}(1-w_i) \quad (26)$$

$$\frac{d\overline{\beta}_j}{d\beta_j^k} = w_k\prod_{i=1,i\neq k}^{L}(w_i\beta_j^i+1-w_i) \quad (27)$$

According to the activation weight expression of the k-th rule, we can obtain its partial derivatives of the rule antecedent attribute parameter and the corresponding attribute distance-sensitive parameter respectively:

$$\frac{dw_k}{dx_l^k} = 2(a_l^k)^2(x_l-x_l^k)e^{-\sum_{i=1}^{T_k}[a_i^k(x_i-x_i^k)]^2} \quad (28)$$

$$\frac{dw_k}{da_l^k} = 2a_l^k x_l^k(x_l-x_l^k)e^{-\sum_{i=1}^{T_k}[a_i^k(x_i-x_i^k)]^2} \quad (29)$$

Set the loss function expression of the final output to be $loss = Loss(\beta_1,\cdots,\beta_N)$. According to the compound

function chain derivation rule, we can obtain the partial derivative of the final loss on each parameter.

$$\frac{dloss}{d\beta_j^k} = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{dloss}{d\beta_i} \frac{d\beta_i}{d\overline{\beta}_j} \frac{d\overline{\beta}_j}{d\beta_j^k} \qquad (30)$$

$$\frac{dloss}{dx_l^k} = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{dloss}{d\beta_i} \frac{d\beta_i}{d\overline{\beta}_j} \frac{d\overline{\beta}_j}{dw_k} \frac{dw_k}{dx_l^k} \qquad (31)$$

$$\frac{dloss}{da_l^k} = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{dloss}{d\beta_i} \frac{d\beta_i}{d\overline{\beta}_j} \frac{d\overline{\beta}_j}{dw_k} \frac{dw_k}{da_l^k} \qquad (32)$$

B. DIFFERENTIABLE PROOF OF BRB SYSTEM

The evidential reasoning process of the belief rule base is a multivariate compound function process. According to the differentiable condition of the multivariate compound function, each intermediate function must satisfy the differential condition, and the partial derivative of each variable must exist and be continuous. Since any of the partial derivatives above are only obtained by elementary functions through four arithmetic operations and compound, the partial derivatives of any parameter are continuous in its domain. The existence and continuous partial derivative of any parameter can prove that the whole inference system is differentiable.

When the appropriate loss function is selected, the final output loss is differentiable to all the parameters of the model, which provides conditions for using the gradient method to optimize the model parameters. The gradient of the loss result on the parameters of each part of the model can be obtained. The gradient of the loss function on all the rule antecedent attribute parameters is:

$$\nabla_x loss = \begin{bmatrix} \frac{dloss}{dx_1^1} & \cdots & \frac{dloss}{dx_{T_k}^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{dx_1^L} & \cdots & \frac{dloss}{dx_{T_k}^L} \end{bmatrix} \qquad (33)$$

The gradient of the loss function on the distance-sensitive parameters of all rules is:

$$\nabla_a loss = \begin{bmatrix} \frac{dloss}{da_1^1} & \cdots & \frac{dloss}{da_{T_k}^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{da_1^L} & \cdots & \frac{dloss}{da_{T_k}^L} \end{bmatrix} \qquad (34)$$

The gradient of the loss function on all rule consequent attribute parameters is:

$$\nabla_\beta loss = \begin{bmatrix} \frac{dloss}{d\beta_{1,1}} & \cdots & \frac{dloss}{d\beta_{N,1}} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{d\beta_{1,L}} & \cdots & \frac{dloss}{d\beta_{N,L}} \end{bmatrix} \qquad (35)$$

According to the belief distribution output by the inference system and the loss function of the result, the gradient of each part of the parameters can be optimized by updating the parameters along the negative gradient direction.

C. EXPONENTIAL NORMALIZATION FUNCTION PREPROCESSING

In the training process, in order to meet the restriction that the sum of the consequent attributes of each rule is one and each result attribute is non-negative, we use the exponential normalization function to preprocess the consequent attribute parameters during the training process:

$$\beta_j^k = \frac{e^{\overline{\beta_j^k}}}{\sum_{i=1}^{N} e^{\overline{\beta_i^k}}} (\sum_{i=1}^{N} \beta_j^k = 1, \beta_j^k > 0) \qquad (36)$$

The derivative of the corresponding parameter can be obtained through the formula:

$$\frac{d\beta_j^k}{d\overline{\beta}_t^k} = \begin{cases} \beta_t^k \times (1 - \beta_j^k), t = j \\ -\beta_t^k \times \beta_j^k, t \neq j \end{cases} \qquad (37)$$

The partial derivative of the unpreprocessed consequent attribute can be expressed as:

$$\frac{dloss}{d\overline{\beta}_j^k} = \sum_{i=1}^{N}\sum_{t=1}^{N} \frac{dloss}{d\beta_i} \frac{d\beta_i}{d\overline{\beta}_t} \frac{d\overline{\beta}_t}{d\beta_t^k} \frac{d\beta_t^k}{d\overline{\beta}_j^k} \qquad (38)$$

Also according to the elementary function chain derivation rule, the derivative of this parameter is continuous in its domain, and the parameter is differentiable. Its gradient is expressed as:

$$\nabla_{\overline{\beta}} loss = \begin{bmatrix} \frac{dloss}{d\beta_1^1} & \cdots & \frac{dloss}{d\beta_N^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{d\beta_1^L} & \cdots & \frac{dloss}{d\beta_N^L} \end{bmatrix} \qquad (39)$$

D. STOCHASTIC GRADIENT DESCENT WITH MOMENTUM OPTIMIZATION

The optimization process of using the gradient descent method to update the parameters of the inference model is given by the following equation:

$$M_{new}(x, a, \beta) = M_{old} - \mu \nabla_{M_{old}} loss \qquad (40)$$

The gradient information is given according to the loss function of the final output, and the learning rate μ is the updated step length information that needs to be set. Chang and Zhang [11] and Wu *et al.* [13] used dichotomy in the gradient training process to iteratively find the optimal step size in the constraint space and added perturbation parameters when the gradient is zero to avoid the training process stagnation. For the application of new rule structures, activation methods and preprocessing steps, the gradient training update step size is no longer limited. We propose the momentum-optimized stochastic gradient descent method for faster training.

The stochastic gradient descent method is an iterative optimization method when the objective function is differentiable. It used a random subset of the training data to calculate the gradient value as the estimated value of the entire training data gradient, which reduces the computational burden in high-dimensional optimization problems.

The output of the loss function in the conventional gradient descent method is determined by all samples, and the model parameters are updated according to its gradient:

$$loss = \frac{1}{n}\sum_{i}^{n} Loss(\beta_1^i, ..., \beta_N^i) \qquad (41)$$

We randomly select a single sample as the estimated value of the average value of the loss function output on all samples to update the model parameters:

$$loss = Loss(\beta_1^r, ..., \beta_N^r) \qquad (42)$$

Since the update direction completely depends on the gradient of the current sample, the stochastic gradient descent method is not stable. The momentum method improves the stability and speed by retaining a certain degree of historical gradient information and combining it with the current sample gradient. It also enhances the ability to get rid of local optimal solutions.

The momentum method uses a weighted fusion method to synthesize the historical gradient information and the current sample gradient, and use it as an update parameter for each round of training:

$$v_t = \nu v_{t-1} + \mu \nabla_{loss} M, M = M - v_t \qquad (43)$$

The initial value of v_0 is set to zero and ν represent the ratio of retaining historical gradient information. The same direction of the current gradient and the historical gradient will increase the speed of parameter training in this gradient direction. The different directions of the current gradient and the historical gradient will inhibit the current gradient from causing parameter training oscillations.

IV. EXPERIMENTAL RESULTS

In this section, we first compare the training performance of the gradient method under different momentum parameters through a nonlinear function fitting experiment, and then we compare the performance of the SGDM-BRB system under different parameter settings through the oil leak detection experiment. Finally we compared the SGDM-BRB system with conventional machine learning algorithms on several public datasets and summarize all the experimental results.

A. EXPERIMENTAL ENVIROMENT

The experiment runs on a Ubuntu 20.04 system equipped with Intel© Core™ i5 8500@3.0GHz CPU, 16GB RAM and GeForce GTX 1060 Graphics. Use TensorFlow 2.0 to build the evidential reasoning framework of the BRB system and use Scikit-learn machine learning library to collect and clean datasets.

B. PERFORMANCE OF GRADIENT DESCENT METHOD WITH DIFFERENT MOMENTUM PARAMETERS

Liu *et al.* [16] proved that the belief rule base system can approximate any function. In this section, a nonlinear
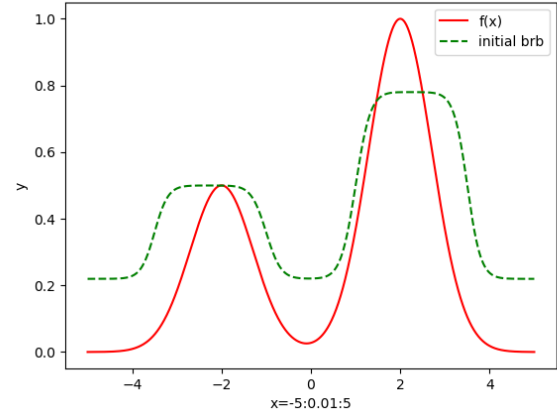

FIGURE 5. Untrained BRB system output

multi-extreme function is introduced to compare the training performance of the stochastic gradient descent method under different momentum parameters. The nonlinear multi-extreme function is as follows:

$$f(x) = e^{-(x-2)^2} + 0.5e^{-(x+2)^2}, x \in [-5, 5] \qquad (44)$$

1) Basic Settings And Optimization

In the domain defined by function (44), 1000 sample data are uniformly selected as the fitting dataset and the mean square error is used as the loss function. According to each extreme point on the function curve, the rule base consequent attribute evaluation level and corresponding utility value can be set:

$$\{D_1, D_2, D_3, D_4, D_5\} = \{-0.5, 0, 0.5, 1, 1.5\}$$

Select the five extreme points on the function curve to convert to the corresponding rule structure to build a belief rule base. Set the default distance-sensitive parameter of each rule to 1.0. The initial rule information is shown in Table 1. Figure 5 shows the untrained brb system output. Set the number of training samples in each batch to be 128 and the learning rate μ to be 0.001 for 1000 rounds of training. Set the momentum optimization parameters ν to be 0.0 (non momentum optimization), 0.5, 0.9 and 0.99 to compare their fitting performance.

2) Results And Analysis

Figure 6 shows the mean square error loss of each batch under different momentum parameters. Through the decreasing curve of the mean square error loss function, we can find that smaller momentum optimization parameters can significantly improve the model performance. The loss of the model with the momentum parameter set to 0.5 has never been lower than 0.001 during the training process. The loss of the model with the momentum parameter set to 0.9 and 0.99 dropped rapidly to 1×10^{-4}, and reached the level of 1×10^{-5} when the training is over.

TABLE 1. Initial rule information

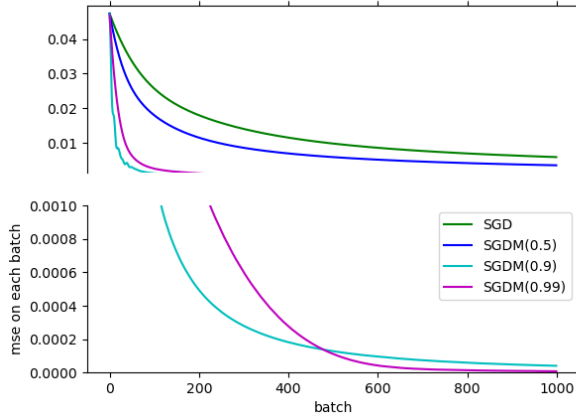| No.	Antecedent attribute x	Distance-sensitive a	Consequent attributes $\overline{\overline{\beta}}$				
1	-5	1	(-1,	1,	-1,	-1,	-1)
2	-2	1	(-1,	-1,	1,	-1,	-1)
3	0	1	(-1,	1,	-1,	-1,	-1)
4	2	1	(-1,	-1,	-1,	1,	-1)
5	5	1	(-1,	1,	-1,	-1,	-1)

FIGURE 6. Mean square error loss under different momentum parameters
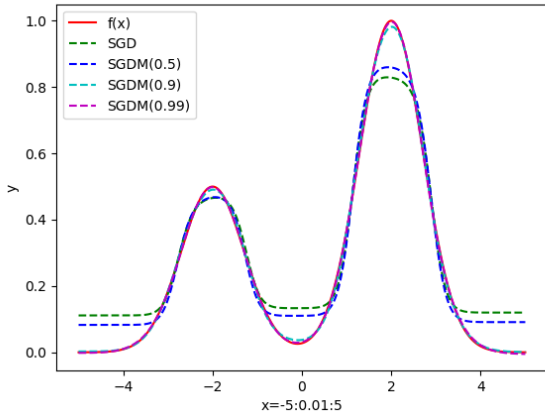

FIGURE 7. Fitting performance of different momentum parameter models

Figure 7 shows that higher momentum parameter values can obtain better fitting performance, greatly reducing the distance between the fitted curve and the original curve.

Table 2 records the information of each rule in the model with the momentum parameter set to 0.99 when the training is completed. The distance-sensitive parameters have become smaller after training, that is, the activation area of each rule has increased. The consequent attribute in their respective utility levels is significantly increased , while other levels have decreased. The antecedent attribute information of each rule is only slightly adjusted after training.

We compare the performance of the SGDM-BRB with

several conventional BRB system in Table 3, include the final mean square error results and training time of these models. It is obvious that the gradient method after the momentum optimization has excellent performance in improving the inference accuracy of the BRB system, but the training model without the effective momentum parameter optimization cannot reach the loss level of the conventional method. And the running speed of gradient method is much lower than the conventional BRB system optimized by intelligent algorithm and other gradient descent algorithm.

C. OIL PIPELINE LEAK DETECTION OF BRB SYSTEM WITH DIFFERENT SETTINGS

In this section, oil pipeline leak detection are applied to validate the SGDM-BRB and reveal its practicability. We also compare the performance of the model under different types of distance-sensitive parameters.

The oil pipeline leak detection dataset is a common benchmark used for testing BRB system and its improvements. There are 2008 sets of sample data collect every 10 seconds. Each sample includes three continuous variables: flow difference, pressure difference and leak size. The flow difference and pressure difference are used as independent variables, and the leak size is used as the dependent variable. In order to verify that the model with distance-sensitive parameters for each rule have better performance than the model with uniform distance-sensitive parameters for each attribute, we compare the mean square errors of these two parameter settings and other conventional BRB system optimization methods.

1) Basic Settings And Optimization

We randomly select 500 samples as the training set. The distance-sensitive of flow difference and pressure difference are set to be 1.0 and 0.01. There are 5 reference levels for leak size, including zero (Z), very small (VS), medium (M), high (H) and very high (VH). The reference values are:

$$D_{LS} \in \{(Z, 0), (VS, 2), (M, 4), (H, 6), (Vh, 8)\} \quad (45)$$

Randomly select 16 samples from the training set and convert them into the belief rule form and build the corresponding rule base. For flow difference and pressure difference information, convert them into antecedent attribute. For the leak size, set the consequent attribute to be the negative value of the distance from leak size to corresponding reference value.

TABLE 2. Trained rule information

No.	Antecedent attribute x	Distance-sensitive a	Consequent attributes $\overline{\overline{\beta}}$				
1	−4.55	0.207	(0.101,	1.58,	-1.83,	-1.73,	-1.74)
2	−2.01	0.446	(-1.35,	-1.54,	0.562,	-0.461,	-0.478)
3	0.0805	0.509	(-0.154,	1.48,	-1.55,	-1.52,	-1.61)
4	2.06	0.498	(-1.81,	-2.06,	-1.97,	1.54,	0.597)
5	4.75	0.214	(0.123,	1.55,	-1.83,	-1.74,	-1.78)

TABLE 3. Mean square error and training time using different optimization algorithms

Year	Method	Mean square error	Running time
2007	fmincon [10]	5.16×10^{-5}	496.17
2007	Chen-BRB [13]	6.3228×10^{-5}	-
2015	Wang-BRB [15]	3.9284×10^{-5}	386.63
2018	Li-BRB [21]	3.3322×10^{-5}	357
This study	SGD-BRB	5.90×10^{-3}	11.60
This study	SGDM-BRB(0.5)	3.50×10^{-3}	10.82
This study	SGDM-BRB(0.9)	4.01×10^{-5}	11.09
This study	SGDM-BRB(0.99)	7.40×10^{-6}	13.42

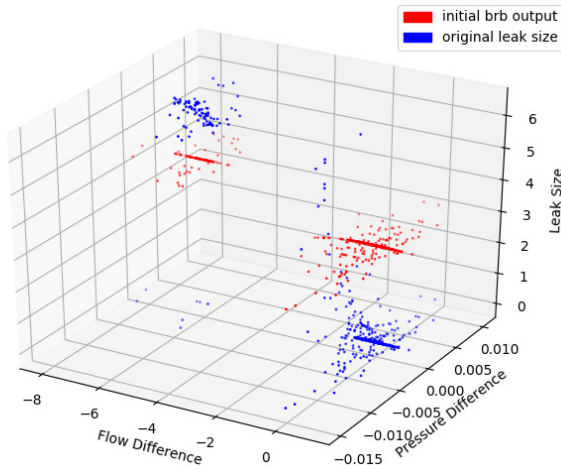**FIGURE 8.** Comparison of the initial model output and the leak size of the training set

FIGURE 9. Comparison of the trained model output and the leak size on all samples

The corresponding rule for a sample with flow difference fd, pressure difference pd and leak size ls is expressed as:

$$R : If(fd, pd) Then\{(\overline{\beta_Z}, -|ls - 0|), (\overline{\beta_{VS}}, -|ls - 2|),$$
$$(\overline{\beta_M}, -|ls - 4|), (\overline{\beta_H}, -|ls - 6|), (\overline{\beta_Z}, -|ls - 8|)\} \quad (46)$$

The initial belief rule base inference results for training set are shown in Figure 8.

In the training process, the mean square error is used as the loss function, each batch uses 32 samples, and a total of 1000 rounds of training. A model with distance-sensitive parameter for each rule has a total of $144(16\times(2+2+5))$ parameters to be trained, and a model with uniform distance-sensitive parameters for each attribute has a total of $114(16\times(2+5)+2)$ parameters to be trained. The output of the model with distance-sensitive parameters for each rule after training is shown in Figure 9.
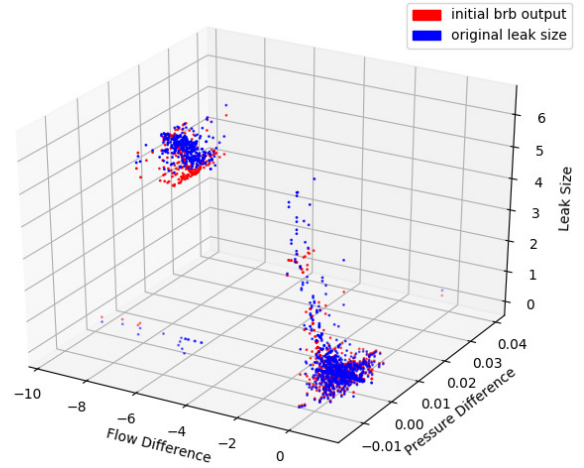
2) Results And Analysis

All of 2008 samples and estimated output are shown in Figure 10, it can be observed that the results of the two are roughly the same, especially during the time when the leak occurred. Note that there is a large leak estimation error around the 1000th sample, which may be caused by noisy data.

To further verify the validity of SGDM-BRB, we conducted 20 independent experiments on two different distance-sensitive parameters settings methods. Tabel 4 list the comparison of results of some existing works and two proposed methods, where the mean square error and mean absolute error are obtained from 2008 samples.

For the method of using uniform distance-sensitive parameters for each attribute, the mean absolute error and the mean square error are both ranks in the top four. For the method of setting distance-sensitive parameters for each rule respectively, the mean absolute error and the mean square error are both ranks in the top two. It only lags behind JOPS in the mean absolute error, and only lags behind Bi-level BRB model in the mean square error.

Since the new method simplifies the structure of using reference values to construct the belief distribution of the antecedent attributes, it reduces a large number of parameters compared with the conventional BRB model. If a uniform
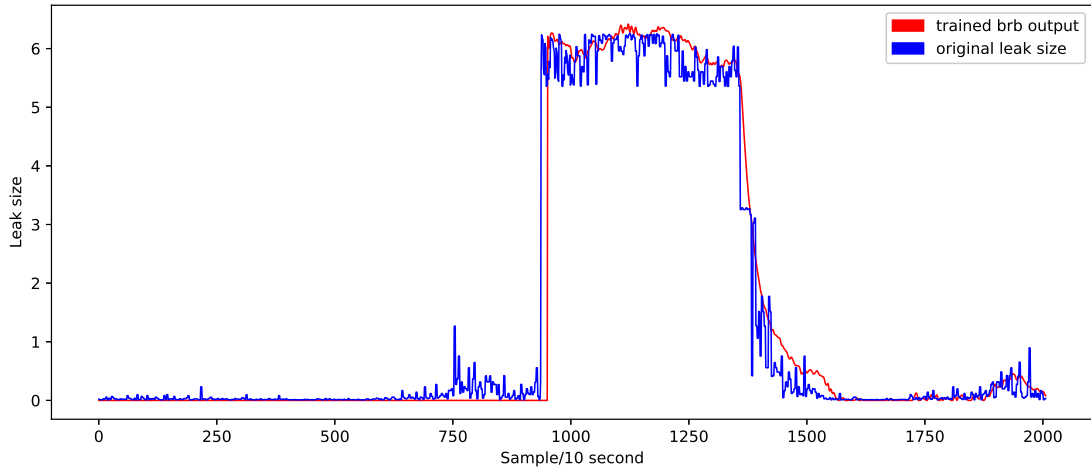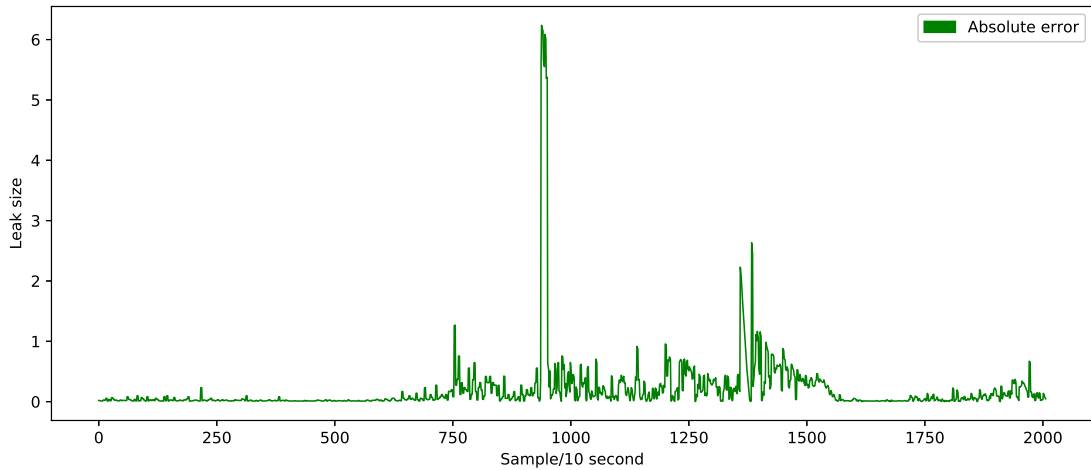
FIGURE 10. Estimated output and samples

FIGURE 11. Absolute error between estimated output and samples

distance-sensitive parameter is used for each attribute, the number of parameters can still be greatly reduced without significantly reducing the inference accuracy. However, compared with other newer methods, our method still needs to train a certain number of rule parameters to improve the inference accuracy and cannot achieve the ultimate parameter reduction.

In summary, the comparison of experimental results show that the proposed method achieves higher inference accuracy, and a large number of parameters are reduced through simplification based on the traditional rule structure.

D. EXPERIMENT ON PUBLIC CLASSIFICATION DATASETS

In this section, we select 7 UCI public classification datasets that are widely used to valid the inference performance of SGDM-BRB method. Table 6 shows the detailed information of these classification datasets. We repeat independent 10-

fold cross-validation experiments for 20 times to obtain the final results. The results for comparison are measured by average accuracy.

The results obtained by SGDM-BRB method will be compared with the results obtained by other machine learning methods and belief rule base inference system. The machine methods for comparison include KNN, Naive Bayes (NB) and C4.5, support vector machine (SVM) and their results are cited from previous papers [24], [29]. The BRB systems used for comparison include SRA-EBRB [22], MVP-EBRB [23] and BA-EBRB [24].

1) Basic Settings And Optimization

To enable a unified classification process, we perform a standardized operation on each dataset:

$$x' = \frac{x - \overline{x}}{\delta} \qquad (47)$$

TABLE 4. Comparison wih present researches

Year	Method	Size(training/test)	Parameters	Mean absolute error	Mean square error
2007	Local training [31]	500/2008	353	0.2223	0.4049
2011	Adaptive learning [32]	500/2008	353	0.2064	0.3990
2013	Extended BRB [16]	-/2008	18902	0.2169	-
2016	Dynamic rule adjustment [33]	900/2008	43	0.2080	0.4450
2017	Bi-level BRB [34]	500/2008	40	0.1941	0.2917
2018	JOPS [35]	900/2008	43	0.1738	0.3998
This study	SGDM-BRB(uniform distance-sensitive parameters)	500/2008	114	0.2024	0.3496
This study	SGDM-BRB(respective distance-sensitive parameters)	500/2008	144	0.1998	0.3469

TABLE 5. Accuracy of SGDM-BRB compare with conventional classification approaches

	KNN	NB	C4.5	SVM	SRA-EBRB	MVP-EBRB	BA-EBRB	SGDM-BRB
iris	96.67(**1**)	96.00(4)	96.00(4)	96.67(**1**)	94.80(8)	95.87(6)	95.26(7)	96.50(3)
wine	96.05(5)			96.40(4)	96.85(3)		97.02(2)	97.44(**1**)
diabetes	74.09(3)	76.30(**1**)	73.82(4)	65.10(8)	71.71(7)	72.59(5)	72.32(6)	75.29(2)
ecoli	85.71(**1**)	85.42(4)	84.23(5)	75.60(7)	84.85(5)	85.61(2)		85.43(3)
glass	66.63(7)	48.60(8)	66.82(6)	68.69(5)	73.08(2)	72.06(4)	72.32(3)	74.75(**1**)
seeds	92.38(3)	91.43(5)	91.90(6)	90.48(8)	91.24(7)	92.38(3)	93.95(2)	94.02(**1**)
yeast	58.22(3)	57.61(4)	55.39(7)	43.26(8)	56.85(6)	57.49(5)	58.63(2)	59.49(**1**)
Average rank	3.28(2)	4.33(5)	5.33(6)	6.44(8)	5.42(7)	4.16(4)	3.66(3)	1.71(**1**)

TABLE 6. Details of the classification datasets

Dataset name	#Instances	#Features	#Classes
iris	150	5	3
wine	178	14	3
diabetes	768	9	2
ecoli	336	8	8
glass	214	10	6
seeds	210	8	3
yeast	1484	9	10

where $\overline{x}$ is the mean of x and δ is the standard deviation of x. After standardization, the data on each attribute can be approximated to the standard normal distribution. In this way, we can conveniently set the distance-sensitive parameter on all attributes to 1.0. Then we select cross entropy as the loss function to train the BRB system. For a multi-classification task with N categories, the loss function on each sample y and the corresponding prediction result $\overline{y}$ is defined as follows:

$$Loss(y,\overline{y}) = -\sum_{i=1}^{N} y_i \log \overline{y}_i \qquad (48)$$

For the experiment on each dataset, 32 samples are randomly selected from training set as the initial rule and convert the classification result into the corresponding belief result distribution according to the above conversion method. The rule corresponding to the k-th training sample

$$X_k : (x_1, ..., x_{T_k}), Y_k : c, 1 \le c \le N \qquad (49)$$

is expressed as:

$$R_k : if(x'_1, \cdots, x'_{T_k})$$
$$then\{(\overline{\beta_1^k}, -1), \cdots, (\overline{\beta_{c-1}^k}, -1), (\overline{\beta_c^k}, 1), (\overline{\beta_{c+1}^k}, -1),$$
$$\cdots, (\overline{\beta_N^k}, -1)\}$$
$$with\ distance-sensitive\ parameter(a_1^k, \cdots, a_{T_k}^k)$$
$$\qquad (50)$$

Assign each rule its own distance-sensitive parameters on all attributes to obtain a finer inference granularity by increasing the number of parameters.

In the process of parameter optimization using momentum optimization stochastic gradient descent method, the learning rate is set to 0.001 and the momentum optimization parameter is set to 0.99. There are 128 training samples in each batch, and 2000 rounds trainings are performed on each dataset.

2) Results And Analysis
Table 5 lists the classification accuracy of SGDM-BRB method and other machine learning methods include several conventional improved BRB methods. Although the SGDM-BRB method is not always the best method, its ranking never fell out of the top 3, and it is also the method with the highest average ranking. As shown in Table 5, our method achieves higher accuracy than any other methods on the four datasets of wine, glass, seeds, and yeast. For the other results shown in Table 5, SGDM-BRB method achieves an accuracy of 96.50% on tthe dataset iris which is worse than the 96.67% accuracies obtained from KNN and SVM, an accuracy of 85.43% on dataset ecoli which is worse than the 85.71% accuracy obtained from KNN and the 85.61% accuracy obtained from MVP-BRB, and an accuracy of 75.29% on

dataset pima is worse than the 76.30% accuracy obtained from NB.

Thanks to the fixed number of rules to be trained, the number of parameters for each data set experiment only increases linearly with the number of antecedent attributes and the number of consequent attributes, It effectively avoids the problem of parameter explosion that the conventional BRB method may encounter.

V. CONCLUSION

This paper proposes a new rule structure and its activation method with the corresponding momentum optimization gradient training method. The new rule structure uses original data to directly form rules, it the problem of parameter explosion caused by too many reference value settings. The rule activation weight calculation method improved by using Gaussian function has effectively improved the inference accuracy by introducing distance-sensitive parameters, the characteristic of Gaussian function avoids the rule zero activation problem and simplifies the process of normalizing rule activation weights. The new structure and optimization method make it easier to construct the partial derivatives of the inference system, providing conditions for high-performance gradient training methods. The further conclusions of this paper are summarized as follow:

1) Using the method of combining attribute distance-sensitive parameters and Gaussian function instead of the conventional belief distribution method to calculate the individual matching degree effectively avoids the defect that the conventional method may cause the individual matching degree to be 0. This makes the activation weight change more gentle and has good fitting performance.

2) The stochastic gradient method combined with larger momentum optimization parameters greatly improves the accuracy and convergence speed of the model.

Due to its good fitting performance, future research will focus on using integrated methods to further improve inference performance and reduce potential over-fitting risks.

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