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Belief-Rule-Base Inference Method Based on Gradient Descent with Momentum

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ABSTRACT

The belief-rule-base (BRB) inference methodology using the evidential reasoning (ER) approach is widely used in different fields, such as fault diagnosis, system identification, and decision analysis. The calculation characteristic of the conventional rule activation weight makes the inference system have the rule zero activation problem. The difficulty of constructing partial derivatives restricts the optimization of parameters using the gradient method. This paper proposes a new belief rule structure and its gradient training method to solve the rule zero activation problem during the inference process and improve inference accuracy. The Gaussian function is applied to calculate the activation weight of the rule with the new structure. Its characteristics avoid the zero activation problem caused by the attribute reference value set in the original method. Based on the newly proposed method, the corresponding distance-sensitive parameter is set for each attribute, and the weight parameter of each rule is discarded. It simplifies the calculation of rule activation weights in the inference process and enables the partial derivatives of the parameters of the inference system to be easily constructed. In the parameter optimization, the momentum optimization gradient stochastic descent method is used to train the new BRB system, which improves the training speed and accuracy compared with the conventional method. Experiments with nonlinear function fitting, oil pipeline leak detection, and classification of several public datasets are carried out to verify whether the new BRB system trained with momentum stochastic gradient descent (SGDM-BRB) has better performance than other conventional methods. The experimental results show that the SGDM-BRB has higher accuracy and faster training speed than the conventional methods.

INDEX TERMS belief rule base, structure optimization, stochastic gradient descent, momentum optimization.

I. INTRODUCTION

It is well known that rule-based intelligent systems are one of the most common frameworks for expressing various types of knowledge. The rule-based system has certain expression and processing capabilities through the use of existing human knowledge, and at the same time has the flexibility to deal with ambiguity, incompleteness, uncertainty, and to combine different types of input data formats. Yang *et al*. [1] proposed belief rule-based inference methodology using the evidential reasoning approach (ER) based on conventional IF-THEN rules [2], Dempster-Shafer theory of evidence [3], [4], decision theory [5] and fuzzy set theory [6]. It shows a powerful function of representing and processing uncertain information. By introducing a belief distribution structure in the rules, this methodology can effectively handle incomplete and uncertain information involved in the datasets and widely used in various problems in different fields such as oil pipeline leak detection [7], military capability estimation [8], consumer behavior prediction [9] and so on.

In the inference process of the belief rule base (BRB) system, the attribute weight, rule weight, belief distribution, and other parameters directly affect the final accuracy. Yang

et al. [10] proposed optimization models for training BRB system using fmincon solver in Matlab. Chang *et al*. [11], [12] proposed an algorithm for training parameters in the BRB system based on the gradient and dichotomy methods. Wu *et al*. [13] used the accelerating of the gradient algorithm to improve the convergence accuracy and convergence speed. There are also a series of intelligent algorithms such as the particle swarm optimization algorithm proposed by Su *et al*. [14] and the differential evolution algorithm proposed by Wang *et al*. [15] which have excellent training performance on the BRB system. Liu *et al*. [16] introduced the belief distribution structure into the antecedent attributes and used training data to build an extended belief rule base (EBRB) system, which simplifies the construction of the rule base and improves the inference speed.

At present, the parameter optimization of the BRB system is mostly based on various intelligent algorithms. However, the process of those intelligent algorithms is complicated and there are many intermediate training parameters. When the conventional gradient method trains the parameters of the BRB system, the partial derivative of each parameter is difficult to construct, and the limit method is needed to solve the approximate value of the partial derivative [13]. Additional algorithms are needed to search the efficient update step size for the restricted parameters [11]–[13]. The EBRB system does not introduce a parameter training process, which makes the system have higher requirements for the representativeness of the training data used to construct the rule base. For a large number of rules, it is necessary to perform rule reduction or use the data structure to optimize the storage and activation process of the rules. Due to the conventional BRB system rule attribute reference level setting, its potential zero activation problem may cause the inference system to malfunction.

In response to the above problems, a series of optimization modifications are proposed for the system structure and reasoning process, including:

1) A new antecedent structure that does not need to set the attribute reference level is proposed, and a Gaussian function-based rule weight activation method is proposed for the new rule antecedent structure. It effectively avoids the rule zero activation problem and has the feature of generating rules from the training data like EBRB.

2) Setting the global weight parameters for antecedent attributes are canceled, and each rule is set with its own antecedent attribute weight parameters so that each rule has a better activation granularity. On this basis, the setting of rule weight parameters and the related normalization process are canceled, which simplify the evidential reasoning process.

3) A normalized exponential function is proposed to preprocess the restricted parameters to avoid the problem of parameter failure during the training process.

The remainder of this paper is organized as follows: Section II introduces the conventional BRB system and our further improvements for common problems in the system. Section III gives the preprocessing method of the training model and proves that the gradient descent method can be effectively applied to the newly proposed BRB system. Section IV compares the performance of different gradient descent parameters on training speed and inference accuracy. Experiments on a series of public datasets prove that the newly proposed BRB model and its training method have a better performance than other conventional methods. Section V concludes this paper.

II. BRB SYSTEM WITH NEW ATTRIBUTE STRUCTURE AND RULE ACTIVATION WEIGHT CALCULATION METHOD

The BRB system proposed by Yang *et al*. [1] mainly refers to the rule activation and evidence reasoning method on the belief rule base. This section will briefly introduce the related concepts of the BRB system and propose solutions for the common defects of the conventional BRB system.

A. REPRESENTATION OF BELIEF RULE BASE

Based on the conventional production rules, Yang *et al*. [1] proposed the expression form of the belief rules by introducing the belief distribution structure, the rule antecedent attribute parameter, and the rule weight parameter. The specific expression is as follows:

$$\begin{aligned} &R_k : if\{X_1 is A_1^k \wedge \cdots \wedge X_{T_k} is A_{T_k}^k\} \\ &then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^{N} \beta_i^k \leq 1 \end{aligned} \tag{1}$$

The equal sign is obtained when the rule information is complete. Each rule has its rule weight θ_k and each antecedent attribute has its weight $\delta_1, \delta_2, \cdots, \delta_{T_k}$. A_i^k represents the reference value selected by the rule on the ith attribute and β_i^k represents the belief degree of the rule in the ith consequent attribute. On this basis, the extended belief rule base system introduces a belief distribution structure to the antecedent attributes, and its rule form is expressed as follows:

$$\begin{aligned} &R_k : if\{[(A_{11}^k, \alpha_{11}^k), \cdots, (A_{1J_1}^k, \alpha_{1J_1}^k)] \wedge \\ &\cdots \wedge [(A_{T_k 1}^k, \alpha_{T_k 1}^k), \cdots, (A_{T_k J_{T_k}}^k, \alpha_{T_k J_{T_k}}^k)]\} \\ &then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^{N} \beta_i^k \leq 1 \end{aligned} \tag{2}$$

The extended belief rule base converts the original training data into antecedent attributes with a belief distribution form. For the input data $X^k = (x_1^k, \cdots, x_T^k)$, convert the ith attribute parameter to construct the ith antecedent attribute of the corresponding rule in the form of belief distribution:

$$\begin{aligned} &\alpha_{ij}^k = \frac{\gamma_{i(j+1)} - x_i^k}{\gamma_{i(j+1)} - \gamma_{ij}}, \gamma_{ij} \leq x_i^k \leq \gamma_{i(j+1)} \\ &\alpha_{i(j+1)}^k = 1 - \alpha_{ij}^k, \gamma_{ij} \leq x_i^k \leq \gamma_{i(j+1)} \\ &\alpha_{it}^k = 0, t = 1, \cdots, (j-1), (j+2), \cdots, J_i \end{aligned} \tag{3}$$

Using the same conversion method, the values of original data on other attributes can be converted into the corre-

sponding belief distribution form. It is also possible to obtain the belief distribution form of the rule consequent attribute according to this method.

B. EVIDENCE REASONING APPROACH BASED ON BELIEF RULE BASE

The calculation and synthesis of activation weights for each rule in the rule base is the core part of the inference system of the belief rule base. The whole process mainly includes two steps: calculate the activation weight, and synthesize the rules activation weight to get the result. The calculation of the activation weight of each rule in the belief rule base can be regarded as calculating the belief distribution similarity on each attribute and combining the results. Euclidean distance is used to calculate the individual matching degree of the ith attribute. After converting the input data to have the same belief distribution form as the corresponding attribute, the individual matching degree of the attribute is calculated as:

$$S_i^k = 1 - d_i^k = 1 - \sqrt{\frac{\sum_{j=1}^{J_i}(\alpha_{i,j} - \alpha_{i,j}^k)^2}{2}} \tag{4}$$

After the individual matching degree of each attribute is calculated, the individual matching degrees of all attributes are aggregated. The aggregation function in the form of conjunctive rules is:

$$\alpha_k = \prod_{i=1}^{T_k}(S_i^k)^{\overline{\delta_i}}, \overline{\delta_i} = \frac{\delta_i}{\max_{j=1,\cdots,T_k}\delta_j} \tag{5}$$

The activation weight of this rule is calculated by the following formula:

$$w_k = \frac{\theta_k \alpha_k}{\sum_{l=1}^{L}\theta_l \alpha_l} \tag{6}$$

Rule weight normalization operation makes all weights satisfy $0 \le w_k \le 1, \sum w_k = 1$.

After the rule weight calculation is completed, all the rules are synthesized and the inference result is obtained. First, the belief distribution of the rule is transformed into the corresponding probability quality information:

$$m_{j,k} = w_k \beta_j^k, j = 1, \cdots, N \tag{7}$$

$$m_{D,k} = 1 - \sum_{j=1}^{N} m_{j,k} = 1 - w_k \sum_{j=1}^{N}\beta_j^k \tag{8}$$

$$\overline{m}_{D,k} = 1 - w_k \tag{9}$$

$$\widetilde{m}_{D,k} = w_k(1 - \sum_{j=1}^{N}\beta_j^k) \tag{10}$$

where $m_{j,k}$ represents the credibility of the k rule on the j consequent attribute, $\overline{m}_{D,k}$ represents the credibility that the kth rule is not assigned to any consequent attribute, and $\widetilde{m}_{D,k}$ represents the credibility of the missing reference attribute of the kth rule. The total uncertainty credibility is given by $m_{D,k} = \overline{m}_{D,k} + \widetilde{m}_{D,k}$. Synthesize the credibility information of all rules and obtain the final belief result of each consequent attribute:

$$m_j = k[\prod_{i=1}^{L}(m_{j,i} + m_{D,i}) - \prod_{i=1}^{L} m_{D,i}], j = 1, \cdots, N \tag{11}$$

$$\overline{m}_D = n[\prod_{i=1}^{L}\overline{m}_{D,i}] \tag{12}$$

$$\widetilde{m}_D = k[\prod_{i=1}^{L} m_{D,i} - \prod_{i=1}^{L}\overline{m}_{H,i}] \tag{13}$$

$$k = [\sum_{j=1}^{N}\prod_{i=1}^{L}(m_{j,i} + m_{D,i}) - (N-1)\prod_{i=1}^{L} m_{D,i}]^{-1} \tag{14}$$

$$\beta_j = \frac{m_j}{1 - \overline{m}_D}, j = 1, \cdots, N \tag{15}$$

$$\beta_D = \frac{\widetilde{m}_D}{1 - \overline{m}_D} \tag{16}$$

C. NEW ATTRIBUTE STRUCTURE AND RULE ACTIVATION WEIGHT CALCULATION METHOD

The reference value of antecedent attribute information needs to be set before generating rules. If the input is not in the neighborhood of the reference value of the antecedent attribute of a rule, the rule cannot be activated. If all the rules in the rule base are not activated, the inference system will fail. A simplified belief rules structure and corresponding activation weight calculation method are proposed to solve the above problems. Its structure is as follows:

$$\begin{aligned} &R_k : if(x_1^k, \cdots, x_{T_k}^k) \\ &then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}, \sum_{i=1}^{N}\beta_i^k \le 1 \end{aligned} \tag{17}$$

The simplified belief rule structure can directly use the training data to generate the rule antecedent attribute without manually setting the reference values of the antecedent attributes.

The conventional activation weight calculation method is no longer suitable for the simplified form of belief rules. The use of the Gaussian function to calculate individual matching degree for activation weight calculation is proposed to perform weight activation. The degree of individual matching of input $X(x_1, \cdots, x_{T_k})$ and rule R_k : $if(x_1^k, \cdots, x_{T_k}^k) then\{(D_1, \beta_1^k), \cdots, (D_N, \beta_N^k)\}$ on ith attribute is calculated using the Gaussian function as:

$$S_i^k = e^{-[a_i^k \times (x_i - x_i^k)]^2} \tag{18}$$

The parameter a_i^k represents the sensitivity of the ith attribute to the distance at the position x_i^k. When the distance between the rule antecedent attribute and the input data remains unchanged, the value of parameter a inversely proportional to the matching degree. The activation weight of a single rule

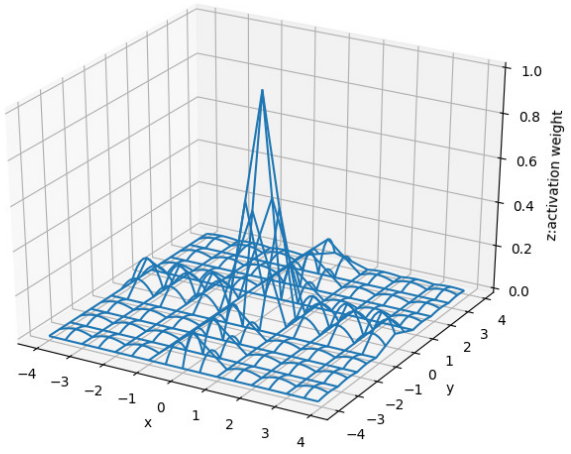

FIGURE 1. Activation weight calculated by conventional methods

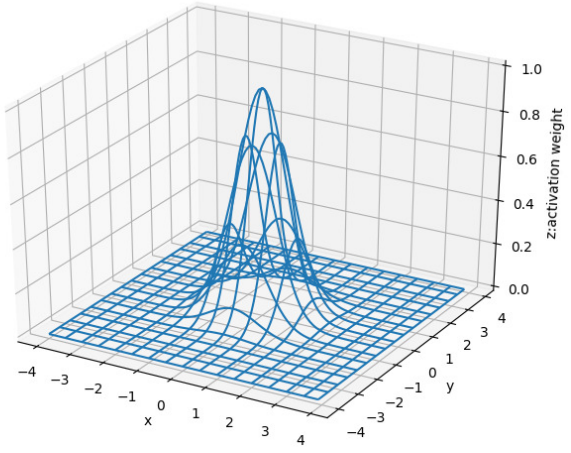

FIGURE 2. Activation weight calculated by Gaussian function methods with distance-sensitive parameter a set to 1.0

under conjunctive conditions is calculated by the following formula:

$$w_k = \prod_{i=1}^{T_k} S_i^k = e^{-\sum_{i=1}^{T_k}[a_i^k(x_i - x_i^k)]^2} \tag{19}$$

Assuming a rule with two attributes x and y located at the $(0, 0)$, the conventional method and Gaussian function method are used to calculate the activation weights. Set the reference values on the x-attributes and y-attributes to be $[-4, -3, -2, -1, 0, 1, 2, 3, 4]$. Ignore the rule weight setting of the two methods. The activation weight distributions shown in Figure 1, Figure 2 with distance-sensitive parameter a set to 1.0 and Figure 3 with distance-sensitive parameter a set to 0.5 can be obtained.

Figure 1 shows that if the input on a certain attribute is not in the neighborhood of the rule attribute, the activation weight of this rule fluctuates around zero and gets zero when the attribute gets a reference value. If the activation weight of

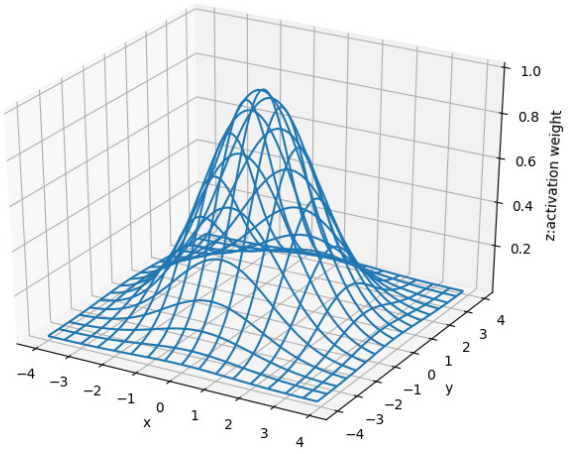

FIGURE 3. Activation weight calculated by Gaussian function methods with distance-sensitive parameter a set to 0.5

all rules is zero, the inference cannot be performed. However, the weight in Figure 2 and Figure 3 smoothly drops close to zero according to the distance from the rule and will not take a value of zero. Moreover, different distance-sensitive parameters enable the rule to adapt to the calculation of activation weights at different scales. It eliminates the impact of rule zero activation on system inference performance and improves the fitting performance of the inference system by adding the distance-sensitive parameters.

Another benefit brought by the new rule structure and activation weight calculation method is that there is no need to adjust the activation weight of the rule by the attribute weight and rule weight. By adjusting the distance-sensitive parameters on each attribute of each rule, a good activation effect and activation granularity can be obtained. Due to the characteristics of the Gaussian function, the activation weight of each rule belongs to $(0, 1]$ without normalization operations. It greatly simplifies the redundant weight adjustment and calculation in the inference process.

III. MOMENTUM OPTIMIZED GRADIENT DESCENT TRAINING PARAMETER

When the conventional gradient method is applied to the parameter training process of the inference system of the belief rule base, it is difficult to construct the partial derivative formula of the rule attribute and the training step is restricted by the parameter constraints. Compared with the conventional BRB system, the newly proposed rule structure with its activation weight calculation method make it easier to construct the partial derivative of each part of the parameter.

There are four subsections in this section, including:

1) Calculating the partial derivatives of the parameters of the new BRB system.

2) Proving the differentiability of the new BRB system.

3) Proposing exponential normalization function for preprocessing to avoid specific constraints during parameter training.

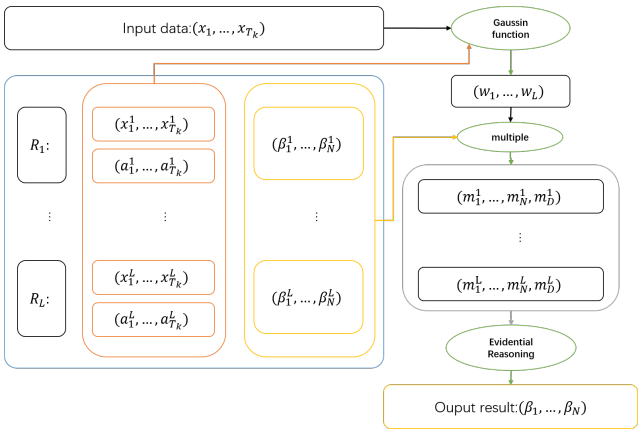

FIGURE 4. Inference system operation process

4) Proposing a parameter training method using stochastic gradient descent with momentum optimization.

A. THE PARTIAL DERIVATIVE OF THE PARAMETERS OF THE BRB SYSTEM

The inference process of the improved BRB system is shown in Figure 4. According to the inference process and compound function chain derivation rules, the partial derivatives of the output to the different parameters of the system can be obtained.

Since the model construction and experiment in this paper are carried out with complete data and the rule consequent attribute does not include uncertain information, for any kth rule:

$$\sum_{i=1}^{N}\beta_{ik}=1(k=1,\cdots,L) \tag{20}$$

$$m_{D,k}=\overline{m}_{D,k},\widetilde{m}_{D,k}=0 \tag{21}$$

In the case of completeness, the jth consequent attribute is expressed as:

$$\beta_j=\frac{\prod_{i=1}^{L}(m_{j,i}+m_{D,i})-\prod_{i=1}^{L}m_{D,i}}{\sum_{t=1}^{N}\prod_{i=1}^{L}(m_{t,i}+m_{D,i})-N\times\prod_{i=1}^{L}m_{D,i}} \tag{22}$$

Then the result of the jth consequent attribute before normalization is expressed as:

$$\overline{\beta}_j=\prod_{i=1}^{L}(m_{j,i}+m_{D,i})-\prod_{i=1}^{L}m_{D,i},\beta_j=\frac{\overline{\beta}_j}{\sum_{k=1}^{N}\overline{\beta}_k} \tag{23}$$

Substitute $m_{j,i}=w_i\beta_{j,i}$ and $m_{D,i}=1-w_i$ into the jth consequent attribute before normalization expression:

$$\overline{\beta}_j=\prod_{i=1}^{L}(w_i\beta_j^i+1-w_i)-\prod_{i=1}^{L}(1-w_i) \tag{24}$$

Obtain the partial derivative of the ith consequent attribute β_i to the jth unnormalized consequent attribute $\overline{\beta}_j$:

$$\frac{d\beta_i}{d\overline{\beta}_j}=\begin{cases}\dfrac{\sum_{k\neq j}^{N}\overline{\beta}_k}{(\sum_{k=1}^{N}\overline{\beta}_k)^2},j=i\\ -\dfrac{\beta_i}{(\sum_{k=1}^{N}\overline{\beta}_k)^2},j\neq i\end{cases} \tag{25}$$

Similarly, the partial derivative of the jth unnormalized consequent attribute $\overline{\beta}_j$ to the activation weight of the kth rule and the jth consequent attribute of kth rule can be obtained as:

$$\frac{d\overline{\beta}_j}{dw_k}=(\beta_j^k-1)\prod_{i=1,i\neq k}^{L}(w_i\beta_j^i+1-w_i)+\prod_{i=1,i\neq k}^{L}(1-w_i) \tag{26}$$

$$\frac{d\overline{\beta}_j}{d\beta_j^k}=w_k\prod_{i=1,i\neq k}^{L}(w_i\beta_j^i+1-w_i) \tag{27}$$

According to the activation weight expression of the kth rule, the partial derivatives of the rule antecedent attribute parameter and the corresponding attribute distance-sensitive parameter can be obtained respectively:

$$\frac{dw_k}{dx_l^k}=2(a_l^k)^2(x_l-x_l^k)e^{-\sum_{i=1}^{T_k}[a_i^k(x_i-x_i^k)]^2} \tag{28}$$

$$\frac{dw_k}{da_l^k}=2a_l^kx_l^k(x_l-x_l^k)e^{-\sum_{i=1}^{T_k}[a_i^k(x_i-x_i^k)]^2} \tag{29}$$

Set the loss function expression of the final output to be $loss=Loss(\beta_1,\cdots,\beta_N)$. According to the compound function chain derivation rule, the partial derivative of the final loss on each parameter can be obtained.

$$\frac{dloss}{d\beta_j^k}=\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{dloss}{d\beta_i}\frac{d\beta_i}{d\overline{\beta}_j}\frac{d\overline{\beta}_j}{d\beta_j^k} \tag{30}$$

$$\frac{dloss}{dx_l^k}=\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{dloss}{d\beta_i}\frac{d\beta_i}{d\overline{\beta}_j}\frac{d\overline{\beta}_j}{dw_k}\frac{dw_k}{dx_l^k} \tag{31}$$

$$\frac{dloss}{da_l^k}=\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{dloss}{d\beta_i}\frac{d\beta_i}{d\overline{\beta}_j}\frac{d\overline{\beta}_j}{dw_k}\frac{dw_k}{da_l^k} \tag{32}$$

B. DIFFERENTIABLE PROOF OF BRB SYSTEM

The evidential reasoning process of the belief rule base system is a multivariate compound function process. According to the differentiable condition of the multivariate compound function, each intermediate function must satisfy the differential condition, and the partial derivative of each parameter must exist and be continuous. Since any of the partial derivatives above are only obtained by elementary functions through four arithmetic operations and compound, the partial derivatives of any parameter are continuous in its domain. The existence and continuous partial derivative of any parameter can prove that the whole inference system is differentiable.

When the appropriate loss function is selected, the final output loss is differentiable to all the parameters of the model, which provides conditions for using the gradient method to optimize the model parameters. The gradient of the loss result on the parameters of each part of the model can be obtained. The gradient of the loss function on all the rule antecedent attribute parameters is:

$$\nabla_x loss = \begin{bmatrix} \frac{dloss}{dx_1^1} & \cdots & \frac{dloss}{dx_{T_k}^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{dx_1^L} & \cdots & \frac{dloss}{dx_{T_k}^L} \end{bmatrix} \tag{33}$$

The gradient of the loss function on the distance-sensitive parameters of all rules is:

$$\nabla_a loss = \begin{bmatrix} \frac{dloss}{da_1^1} & \cdots & \frac{dloss}{da_{T_k}^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{da_1^L} & \cdots & \frac{dloss}{da_{T_k}^L} \end{bmatrix} \tag{34}$$

The gradient of the loss function on all rule consequent attribute parameters is:

$$\nabla_\beta loss = \begin{bmatrix} \frac{dloss}{d\beta_{1,1}} & \cdots & \frac{dloss}{d\beta_{N,1}} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{d\beta_{1,L}} & \cdots & \frac{dloss}{d\beta_{N,L}} \end{bmatrix} \tag{35}$$

According to the belief distribution output by the inference system and the loss function of the result, the gradient of each part of the parameters can be optimized by updating the parameters along the negative gradient direction.

C. EXPONENTIAL NORMALIZATION FUNCTION PREPROCESSING

In the training process, to satisfy the restriction that the sum of the consequent attributes of each rule is one and each consequent attribute is non-negative, the exponential normalization function is used to preprocess the consequent attributes:

$$\beta_j^k = \frac{e^{\overline{\beta_j^k}}}{\sum_{i=1}^N e^{\overline{\beta_i^k}}} (\sum_{i=1}^N \beta_j^k = 1, \beta_j^k > 0) \tag{36}$$

The derivative of the corresponding parameter can be obtained through the formula:

$$\frac{d\beta_j^k}{d\overline{\beta_t^k}} = \begin{cases} \beta_t^k \times (1 - \beta_j^k), t = j \\ -\beta_t^k \times \beta_j^k, t \neq j \end{cases} \tag{37}$$

The partial derivative of the consequent attribute without preprocessing can be expressed as:

$$\frac{dloss}{d\overline{\beta_j^k}} = \sum_{i=1}^N \sum_{t=1}^N \frac{dloss}{d\beta_i} \frac{d\beta_i}{d\overline{\beta}_t} \frac{d\overline{\beta}_t}{d\beta_t^k} \frac{d\beta_t^k}{d\overline{\beta_j^k}} \tag{38}$$

Also according to the elementary function chain derivation rule, the derivative of this parameter is continuous in its domain, and the parameter is differentiable. Its gradient is expressed as:

$$\nabla_{\overline{\beta}} loss = \begin{bmatrix} \frac{dloss}{d\overline{\beta}_1^1} & \cdots & \frac{dloss}{d\overline{\beta}_N^1} \\ \vdots & \ddots & \vdots \\ \frac{dloss}{d\overline{\beta}_1^L} & \cdots & \frac{dloss}{d\overline{\beta}_N^L} \end{bmatrix} \tag{39}$$

D. STOCHASTIC GRADIENT DESCENT WITH MOMENTUM OPTIMIZATION

The optimization process of using the gradient descent method to update the parameters of the inference model is given by the following equation:

$$M_{new}(x, a, \beta) = M_{old} - \mu \nabla_{M_{old}} loss \tag{40}$$

The gradient is given according to the loss function of the final output, and the learning rate μ is the updated step length that needs to be set. Chang and Zhang [11] and Wu *et al.* [13] used dichotomy in the gradient training process to iteratively find the optimal step size in the constraint space and added perturbation parameters when the gradient is zero to avoid the training process stagnation. For the application of new rule structures, activation methods, and preprocessing steps, the gradient training update step size is no longer limited. The momentum-optimized stochastic gradient descent method is proposed for faster training.

The stochastic gradient descent method is an iterative optimization method when the objective function is differentiable. It used a random subset of the training data to calculate the gradient value as the estimated value of the entire training data gradient, which reduces the computational burden in high-dimensional optimization problems.

The output of the loss function in the conventional gradient descent method is determined by all samples, and the model parameters are updated according to its gradient:

$$loss = \frac{1}{n} \sum_i^n Loss(\beta_1^i, ..., \beta_N^i) \tag{41}$$

Randomly select a single sample as the estimated value of the average value of the loss function output on all samples to update the model parameters:

$$loss = Loss(\beta_1^r, ..., \beta_N^r) \tag{42}$$

Since the update direction completely depends on the gradient of the current sample, the stochastic gradient descent method is not stable. The momentum method improves the stability and speed by retaining a certain degree of historical gradient information and combining it with the current sample gradient. It also enhances the ability to get rid of locally optimal solutions.

The momentum method uses a weighted fusion method to synthesize the historical gradient information and the current sample gradient and uses the result as an update parameter for each round of training:

$$v_t = \nu v_{t-1} + \mu \nabla_{loss} M, M = M - v_t \tag{43}$$

The initial value of v_0 is set to zero and ν represents the ratio of retaining historical gradient information. The same direction of the current gradient and the historical gradient will increase the speed of parameter training in this gradient direction. The different directions of the current gradient and the historical gradient will inhibit the current gradient from causing parameter training oscillations.

IV. EXPERIMENTAL RESULTS

This section first compares the training performance of the gradient method under different momentum parameters through a nonlinear function fitting experiment and then compares the performance of the SGDM-BRB system under different parameter settings through the oil leak detection experiment. Finally, the SGDM-BRB system is compared with conventional machine learning algorithms on several public classification datasets and all experimental results are summarized.

A. EXPERIMENTAL ENVIRONMENT

The experiment runs on a Ubuntu 20.04 system equipped with Intel© Core™ i5 8500@3.0GHz CPU, 16GB RAM and GeForce GTX 1060 Graphics. Use TensorFlow 2 to build the evidential reasoning framework of the BRB system and use the Scikit-learn machine learning library to collect and clean datasets.

B. PERFORMANCE OF THE GRADIENT DESCENT METHOD WITH DIFFERENT MOMENTUM PARAMETERS

Liu *et al.* [16] proved that the belief rule base system can approximate any function. In this section, a nonlinear multi-extreme function is introduced to compare the training performance of the stochastic gradient descent method under different momentum parameters. The nonlinear multi-extreme function is as follows:

$$f(x) = e^{-(x-2)^2} + 0.5e^{-(x+2)^2}, x \in [-5, 5] \tag{44}$$

1) Basic Settings And Optimization

In the domain defined by function (44), 1000 samples are uniformly selected as the fitting dataset, and the mean square error is used as the loss function. According to each extreme point on the function curve, the rule base consequent attribute evaluation level and corresponding utility value can be set:

$$\{D_1, D_2, D_3, D_4, D_5\} = \{-0.5, 0, 0.5, 1, 1.5\}$$

Select the five extreme points on the function curve and convert them into corresponding rules as the rules of the rule base. Set the default distance-sensitive parameter of each rule to be 1.0. The initial rule information is shown in Table 1. Figure 5 shows the untrained BRB system output. Set the number of training samples in each batch to be 128 and the learning rate μ to be 0.001 for 1000 rounds of training. Set the momentum optimization parameters ν to be 0.0 (non momentum optimization), 0.5, 0.9 and 0.99 to compare their fitting performance.

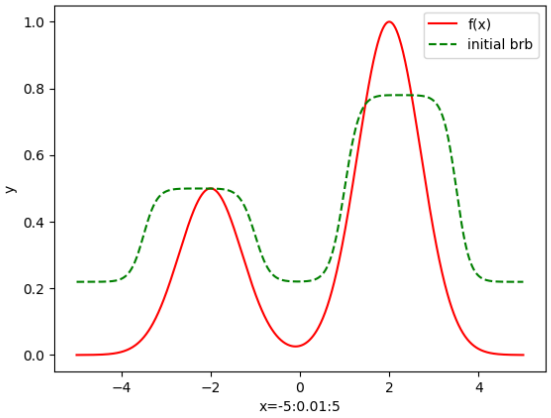

FIGURE 5. Untrained BRB system output

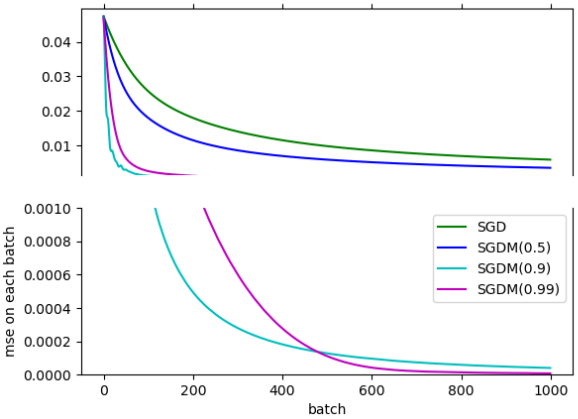

FIGURE 6. Mean square error loss under different momentum parameters

2) Results And Analysis

Figure 6 shows the mean square error loss of each batch under different momentum parameters. Through the decreasing curve of the mean square error loss function, it can be found that smaller momentum optimization parameters can significantly improve model performance. The loss value of the model with the momentum parameter set to 0.5 has never been lower than 0.001 during the training process. The loss of the model with the momentum parameter set to 0.9 and 0.99 rapidly decreased to about 1×10^{-4}, and decreased to about 1×10^{-5} after training.

Figure 7 shows that higher momentum parameter values can obtain better fitting performance, greatly reducing the distance between the fitted curve and the original curve.

Table 2 records the information of each rule in the model with the momentum parameter set to 0.99 when the training is completed. The distance-sensitive parameters have become smaller after training, that is, the activation area of each rule has increased. The consequent attribute in their respective utility levels is significantly increased, while other levels have decreased. The antecedent attribute information of each rule

TABLE 1. Initial rule information

No.	Antecedent attribute x	Distance-sensitive a	Consequent attributes $\overline{\beta}$
1	-5	1	(-1, 1, -1, -1, -1)
2	-2	1	(-1, -1, 1, -1, -1)
3	0	1	(-1, 1, -1, -1, -1)
4	2	1	(-1, -1, -1, 1, -1)
5	5	1	(-1, 1, -1, -1, -1)

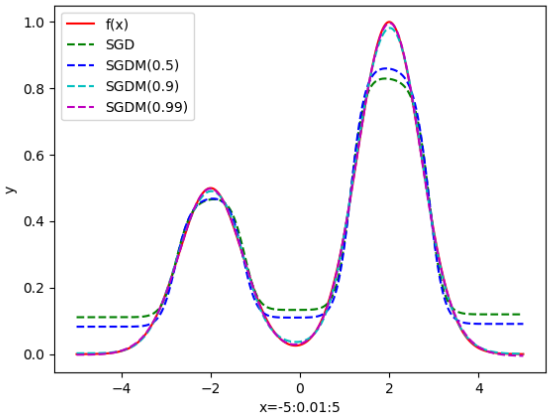

FIGURE 7. Fitting performance of different momentum parameter models

is only slightly adjusted after training.

Table 3 compares the performance of the SGDM-BRB with several conventional BRB systems, including the final mean square error results and training time of these models. It is obvious that the gradient method after the momentum optimization has an excellent performance in improving the inference accuracy of the BRB system, but the training model without the effective momentum parameter optimization cannot reach the loss level of the conventional method. And the running speed of the gradient method is much lower than the conventional BRB system optimized by other intelligent algorithms or gradient descent algorithms.

C. OIL PIPELINE LEAK DETECTION OF BRB SYSTEM WITH DIFFERENT SETTINGS

In this section, oil pipeline leak detection is applied to validate the SGDM-BRB and reveal its practicability, and compare the performance of models under different types of distance-sensitive parameters.

The oil pipeline leak detection dataset is a common benchmark used for testing the BRB system and its improvements. There are 2008 sets of sample data collect every 10 seconds. Each sample includes three continuous variables: flow difference, pressure difference, and leak size. The flow difference and pressure difference are used as independent variables, and the leak size is used as the dependent variable. By comparing the mean square errors of these two parameter settings and other conventional BRB system optimization methods, it is verified that the model with distance-sensitive parameters for each rule has better performance than the model with uniform distance-sensitive parameters for each attribute.

1) Basic Settings And Optimization

Randomly select 500 samples as the training set. The distance-sensitive parameters of flow difference and pressure difference are set to be 1.0 and 0.01. There are 5 reference levels for leak size, including zero (Z), very small (VS), medium (M), high (H), and very high (VH). The reference values are:

$$D_{LS} \in \{(Z,0),(VS,2),(M,4),(H,6),(VH,8)\} \quad (45)$$

Randomly select 16 samples from the training set and convert them into the belief rule form and build the corresponding rule base. For flow difference and pressure difference information, convert them into the antecedent attribute. For the leak size, set the consequent attribute to be the negative value of the distance from leak size to the corresponding reference value. The corresponding rule for a sample with flow difference fd, pressure difference pd ,and leak size ls is expressed as:

$$R: If(fd,pd)Then\{(\overline{\beta_Z},-|ls-0|),(\overline{\beta_{VS}},-|ls-2|),(\overline{\beta_M},-|ls-4|),(\overline{\beta_H},-|ls-6|),(\overline{\beta_Z},-|ls-8|)\} \quad (46)$$

The initial belief rule-base inference system results for the training set are shown in Figure 8.

In the training process, the mean square error is used as the loss function, each batch uses 32 samples, and a total of 1000 rounds of training. A model with distance-sensitive parameters for each rule has a total of 144(No. rule × (No. antecedent + No. distance-sensitive + No. consequent)=16×(2+2+5)) parameters to be trained, and a model with uniform distance-sensitive parameters for each attribute has a total of 114(No. rule × (No. antecedent + No. consequent) + No. distance-sensitive=16×(2+5)+2) parameters to be trained. The output of the model with distance-sensitive parameters for each rule after training is shown in Figure 9.

2) Results And Analysis

All of the 2008 samples and estimated output are shown in Figure 10, it can be observed that the results of the two are roughly the same, especially when the leak occurred. Note

TABLE 2. Trained rule information

No.	Antecedent attribute x	Distance-sensitive a	Consequent attributes $\overline{\beta}$				
1	-4.55	0.207	(0.101,	1.58,	-1.83,	-1.73,	-1.74)
2	-2.01	0.446	(-1.35,	-1.54,	0.562,	-0.461,	-0.478)
3	0.0805	0.509	(-0.154,	1.48,	-1.55,	-1.52,	-1.61)
4	2.06	0.498	(-1.81,	-2.06,	-1.97,	1.54,	0.597)
5	4.75	0.214	(0.123,	1.55,	-1.83,	-1.74,	-1.78)

TABLE 3. Mean square error and training time using different optimization algorithms

Year	Method	Mean square error	Running time
2007	fmincon [10]	5.16×10^{-5}	496.17
2007	Chen-BRB [13]	6.3228×10^{-5}	-
2015	Wang-BRB [15]	3.9284×10^{-5}	386.63
2018	Li-BRB [21]	3.3322×10^{-5}	357
This study	SGD-BRB	5.90×10^{-3}	11.60
This study	SGDM-BRB(0.5)	3.50×10^{-3}	10.82
This study	SGDM-BRB(0.9)	4.01×10^{-5}	11.09
This study	SGDM-BRB(0.99)	7.40×10^{-6}	13.42

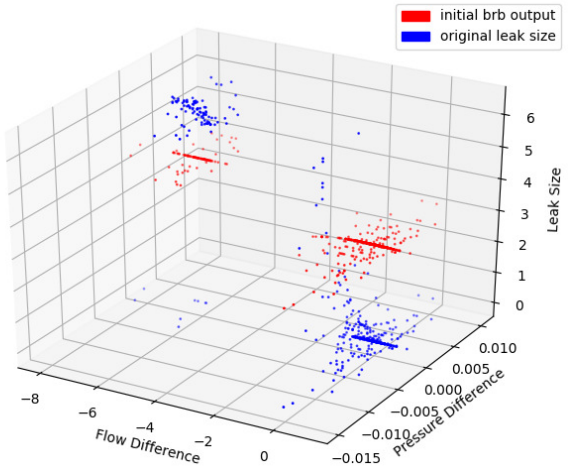

FIGURE 8. Comparison of the initial model output and the leak size of the training set

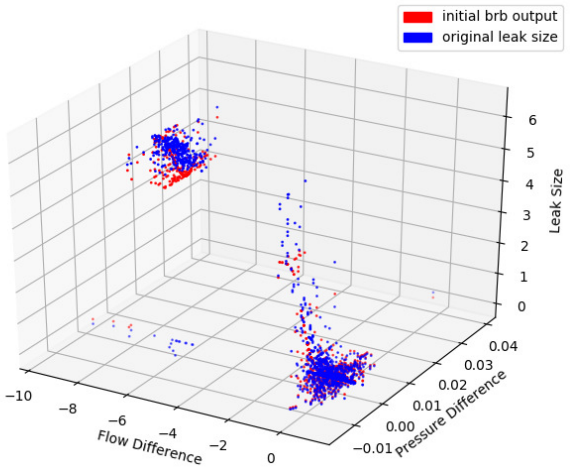

FIGURE 9. Comparison of the trained model output and the leak size on all samples

that there is a large leak estimation error around the 1000th sample, which may be caused by noisy data.

To further verify the validity of SGDM-BRB, 20 independent experiments on two different distance-sensitive parameter settings methods are conducted. Tabel 4 lists the comparison of results of some existing works and two proposed methods, where the mean square error and mean absolute error are obtained from 2008 samples.

For the method of using uniform distance-sensitive parameters for each attribute, the mean absolute error and the mean square error are both ranks in the top four. For the method of setting distance-sensitive parameters for each rule respectively, the mean absolute error and the mean square error are both ranks in the top two. It only lags behind the JOPS algorithm in the mean absolute error, and only lags behind the Bi-level BRB model in the mean square error.

Since the new method simplifies the structure of using reference values to construct the belief distribution of the antecedent attributes, it reduces a large number of parameters compared with the conventional BRB model. If a uniform distance-sensitive parameter is used for each attribute, the number of parameters can still be greatly reduced without significantly reducing the inference accuracy. However, compared with other newer methods, our method still needs to train a certain number of rule parameters to improve the inference accuracy and cannot achieve the ultimate parameter reduction.

In summary, the comparison of experimental results show that the proposed method achieves higher inference accuracy, and a large number of parameters are reduced through simplification based on the traditional rule structure.

D. EXPERIMENT ON PUBLIC CLASSIFICATION DATASETS

This section selects 7 UCI public classification datasets that are widely used to validate the inference performance of the SGDM-BRB method. Table 6 shows the detailed information of these classification datasets. Repeat independent 10-fold cross-validation experiments for 20 times to obtain the final results. The results for comparison are measured by average accuracy.

The results obtained by the SGDM-BRB method will be compared with the results obtained by other machine learning

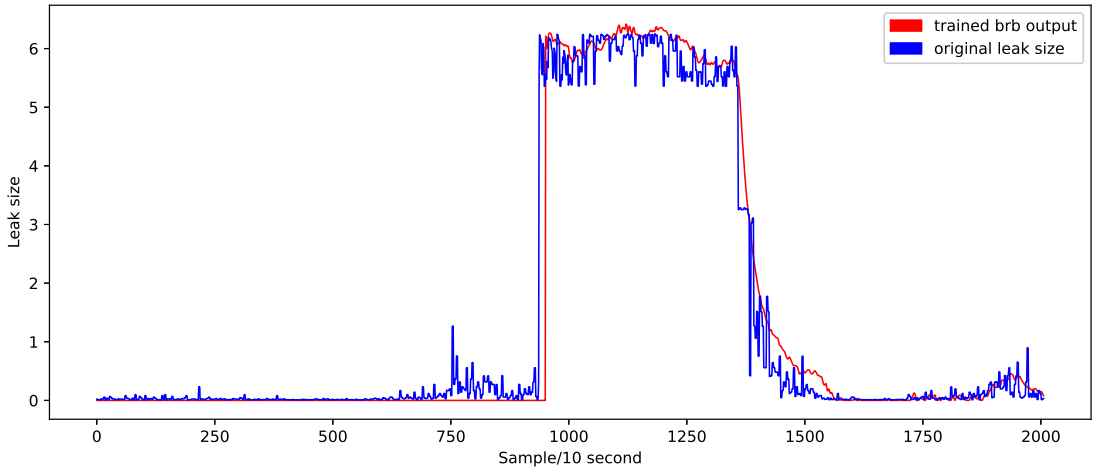

FIGURE 10. Estimated output and samples

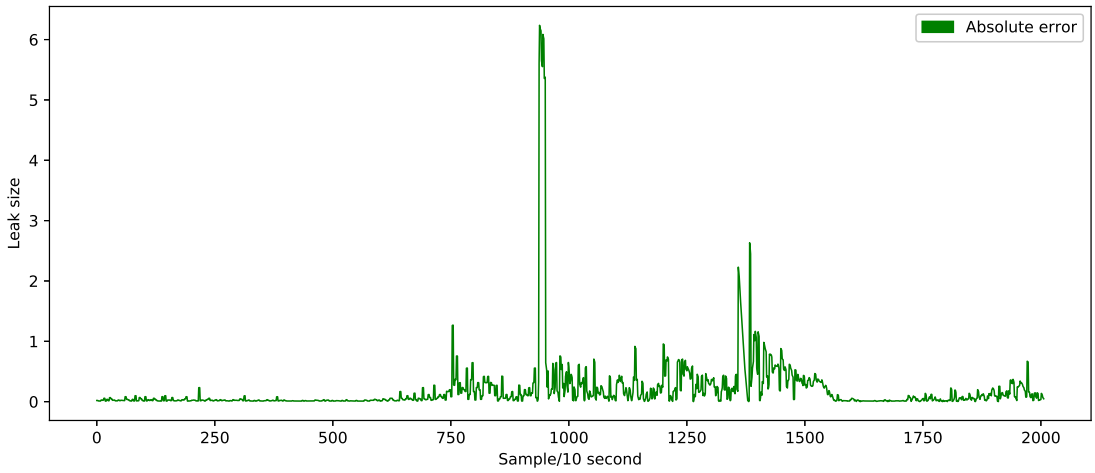

FIGURE 11. Absolute error between estimated output and samples

methods and belief rule-base inference systems. The machine methods for comparison include KNN, Naive Bayes (NB) and C4.5, support vector machine (SVM) and their results are cited from previous papers [24], [29]. The BRB systems used for comparison include SRA-EBRB [22], MVP-EBRB [23] and BA-EBRB [24].

1) Basic Settings And Optimization

To enable a unified classification process, standardized operation is performed on each dataset:

$$x' = \frac{x - \overline{x}}{\delta} \tag{47}$$

where $\overline{x}$ is the mean of x and δ is the standard deviation of x. After standardization, the data on each attribute can be approximated to the standard normal distribution, and the distance-sensitive parameter on all attributes can be conveniently set to 1.0. Then choose cross-entropy as the loss function to train the BRB system. For a multi-classification task with N categories, the loss function on each sample y, and the corresponding prediction result $\overline{y}$ is defined as follows:

$$Loss(y, \overline{y}) = -\sum_{i=1}^{N} y_i \log \overline{y}_i \tag{48}$$

For the experiment on each dataset, 32 samples are randomly selected from the training set as the initial rule and convert the classification result into the corresponding belief result distribution according to the above conversion method. The rule corresponding to the kth training sample

$$X_k : (x_1, ..., x_{T_k}), Y_k : c, 1 \leq c \leq N \tag{49}$$

TABLE 4. Comparison with present researches

Year	Method	Size(training/test)	Parameters	Mean absolute error	Mean square error
2007	Local training [31]	500/2008	353	0.2223	0.4049
2011	Adaptive learning [32]	500/2008	353	0.2064	0.3990
2013	Extended BRB [16]	-/2008	18902	0.2169	-
2016	Dynamic rule adjustment [33]	900/2008	43	0.2080	0.4450
2017	Bi-level BRB [34]	500/2008	40	0.1941	0.2917
2018	JOPS [35]	900/2008	43	0.1738	0.3998
This study	SGDM-BRB(uniform distance-sensitive parameters)	500/2008	114	0.2024	0.3496
This study	SGDM-BRB(respective distance-sensitive parameters)	500/2008	144	0.1998	0.3469

TABLE 5. Accuracy of the SGDM-BRB method compare with conventional machine learning methods

	KNN	NB	C4.5	SVM	SRA-EBRB	MVP-EBRB	BA-EBRB	SGDM-BRB
iris	96.67(**1**)	96.00(4)	96.00(4)	96.67(**1**)	94.80(8)	95.87(6)	95.26(7)	96.50(3)
wine	96.05(5)			96.40(4)	96.85(3)		97.02(2)	97.44(**1**)
diabetes	74.09(3)	76.30(**1**)	73.82(4)	65.10(8)	71.71(7)	72.59(5)	72.32(6)	75.29(2)
ecoli	85.71(**1**)	85.42(4)	84.23(5)	75.60(7)	84.85(5)	85.61(2)		85.43(3)
glass	66.63(7)	48.60(8)	66.82(6)	68.69(5)	73.08(2)	72.06(4)	72.32(3)	74.75(**1**)
seeds	92.38(3)	91.43(5)	91.90(6)	90.48(8)	91.24(7)	92.38(3)	93.95(2)	94.02(**1**)
yeast	58.22(3)	57.61(4)	55.39(7)	43.26(8)	56.85(6)	57.49(5)	58.63(2)	59.49(**1**)
Average rank	3.28(2)	4.33(5)	5.33(6)	6.44(8)	5.42(7)	4.16(4)	3.66(3)	1.71(**1**)

TABLE 6. Details of the classification datasets

Dataset name	#Instances	#Features	#Classes
iris	150	5	3
wine	178	14	3
diabetes	768	9	2
ecoli	336	8	8
glass	214	10	6
seeds	210	8	3
yeast	1484	9	10

is expressed as:

$$\begin{aligned} &R_k : if(x'_1, \cdots, x'_{T_k}) \\ &then\{(\overline{\beta_1^k}, -1), \cdots, (\overline{\beta_{c-1}^k}, -1), (\overline{\beta_c^k}, 1), (\overline{\beta_{c+1}^k}, -1), \\ &\cdots, (\overline{\beta_N^k}, -1)\} \\ &with\, distance - sensitive\, parameter(a_1^k, \cdots, a_{T_k}^k) \end{aligned} \tag{50}$$

Assign each rule its own distance-sensitive parameters on all attributes to obtain a finer inference granularity by increasing the number of parameters.

In the process of parameter optimization using momentum optimization stochastic gradient descent method, the learning rate is set to 0.001 and the momentum optimization parameter is set to 0.99. There are 128 training samples in each batch, and 2000 rounds training are performed on each dataset.

2) Results And Analysis

Table 5 lists the classification accuracy of the SGDM-BRB method and other machine learning methods include several conventional improved BRB methods. Although the SGDM-BRB method is not always the best, it's ranking never fell out of the top 3, and it is also the method with the highest average ranking. As shown in Table 5, our method achieves higher accuracy than any other method on the four datasets of wine, glass, seeds, and yeast. For the other results shown in Table 5, the SGDM-BRB method achieves an accuracy of 96.50% on the dataset iris which is worse than the 96.67% accuracies obtained from KNN and SVM, and accuracy of 85.43% on dataset ecoli which is worse than the 85.71% accuracy obtained from KNN and the 85.61% accuracy obtained from MVP-BRB, and accuracy of 75.29% on dataset pima is worse than the 76.30% accuracy obtained from NB.

Thanks to the fixed number of rules to be trained, the number of parameters for each data set experiment only increases linearly with the number of antecedent attributes and the number of consequent attributes. It effectively avoids the problem of parameter explosion that the conventional BRB method may encounter.

V. CONCLUSIONS

This paper proposes a new rule structure and its activation method with the corresponding momentum optimization gradient training method. The new rule structure uses original data to form rules directly, avoiding the parameter explosion problem caused by excessive reference value settings. The rule activation weight calculation method improved by using the Gaussian function has effectively improved the inference accuracy by introducing distance-sensitive parameters, the characteristic of the Gaussian function avoids the rule zero activation problem and simplifies the process of normalizing rule activation weights. The new structure and optimization method makes it easier to construct the partial derivatives of the inference system, providing conditions for high-performance gradient training methods. The further conclusions of this paper are summarized as follow:

1) The method of combining attribute distance-sensitive parameters and Gaussian function avoids the rule zero activation problem. It also makes the activation weight change more gentle and has a good fitting performance.

2) The stochastic gradient method combined with larger momentum optimization parameters greatly improves the accuracy and convergence speed of the model.

Due to its good fitting performance, future research will focus on using integrated methods to further improve inference performance and reduce potential over-fitting risks.

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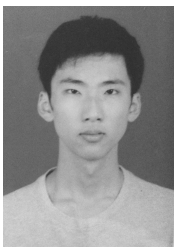

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