1. **The risk modeling**
   Given
$$y_n = \beta_n^T \mathcal{Z} + \sqrt{1-\beta_n^T\beta_n}\epsilon_n$$
   where $\mathcal{Z} \sim \mathcal{N}(0, I_S)$ and $\epsilon \sim \mathcal{N}(0, I_N)$, the resulting $y_n \sim \mathcal{N}(0,1)$ as shown below
$$\mathbb{E}\{y_n\} = \sum_i \beta_{n,i}\mathbb{E}\{\mathcal{Z}_i\} + \sqrt{1-\beta_n^T\beta_n}\mathbb{E}\{\epsilon_n\} = 0$$
$$\begin{aligned} var\{y_n\} &= \mathbb{E}\left\{(y_n - \mathbb{E}\{y_n\})^2\right\} \\ &= \mathbb{E}\left\{(\beta_n^T\mathcal{Z} + \sqrt{1-\beta_n^T\beta_n}\epsilon_n)^2\right\} \\ &= \beta_n^T\beta_n\mathbb{E}\left\{(\mathcal{Z}-0)^2\right\} + (1-\beta_n^T\beta_n)\mathbb{E}\left\{(\epsilon_n-0)^2\right\} \\ &= \beta_n^T\beta_n var\{\mathcal{Z}\} + (1-\beta_n^T\beta_n) var\{\epsilon_n\} \\ &= 1 \end{aligned}$$

2. **Motivation for threshold between different states $H^c_{c(n)}$**
   The motivation is to model discrete probability in the credit state matrix with a continuous distribution such as the gaussian. In this cawe, we want to set $H^c_{c(n)}$ such that
$$p(H^{c-1}_{c(n)} \leq y_n \leq H^c_{c(n)}) = p^c_{c(n)}$$
   therefore we can write
$$p(y_n \leq H^c_{c(n)}) = \sum_{\gamma=1}^{c} p^{\gamma}_{c(n)} \qquad \xrightarrow{y_n\sim\mathcal{N}(0,1)} \qquad \Phi(H^c_{c(n)}) = \sum_{\gamma=1}^{c} p^{\gamma}_{c(n)}$$

3. **Confidence interval for monte carlo estimation**
$$p(L_N(\mathcal{Z},\epsilon) \geq l) \in p(L_N(\mathcal{Z},\epsilon) \geq l) \pm CI$$
   Idea is that for the two naive algorithms that are purported to be equivalent, the CI should be approximately the same

1. **Likelihood Function** Note
$$Z \sim \mathcal{N}(0, I) \qquad \leftarrow \qquad Z \sim \mathcal{N}(\mu, I)$$
   The quotient is then
$$\frac{\sqrt{2\pi}^S e^{-Z^TZ}2}{\sqrt{2\pi}^S e^{-(Z-\mu)^T(Z-\mu)}2}$$