1. **The risk modeling**
Given
$$y_n = \beta_n^T \mathcal{Z} + \sqrt{1-\beta_n^T\beta_n}\epsilon_n$$
where $\mathcal{Z} \sim \mathcal{N}\left(0, I_S\right)$ and $\epsilon \sim \mathcal{N}\left(0, I_N\right)$, the resulting $y_n \sim \mathcal{N}\left(0, 1\right)$ as shown below
$$\mathbb{E}\left\{y_n\right\} = \sum_i \beta_{n,i}\mathbb{E}\left\{\mathcal{Z}_i\right\} + \sqrt{1-\beta_n^T\beta_n}\mathbb{E}\left\{\epsilon_n\right\} = 0$$
$$\begin{aligned}
var\left\{y_n\right\} &= \mathbb{E}\left\{(y_n - \mathbb{E}\left\{y_n\right\})^2\right\} \\
&= \mathbb{E}\left\{(\beta_n^T\mathcal{Z} + \sqrt{1-\beta_n^T\beta_n}\epsilon_n)^2\right\} \\
&= \beta_n^T\beta_n\mathbb{E}\left\{(\mathcal{Z}-0)^2\right\} + (1-\beta_n^T\beta_n)\mathbb{E}\left\{(\epsilon_n-0)^2\right\} \\
&= \beta_n^T\beta_n var\left\{\mathcal{Z}\right\} + (1-\beta_n^T\beta_n)var\left\{\epsilon_n\right\} \\
&= 1
\end{aligned}$$

2. **Motivation for threshold between different states $H_{c(n)}^c$**
The motivation is to model discrete probability in the credit state matrix with a continuous distribution such as the gaussian. In this cawe, we want to set $H_{c(n)}^c$ such that
$$p(H_{c(n)}^{c-1} \leq y_n \leq H_{c(n)}^c) = p_{c(n)}^c$$
therefore we can write
$$p(y_n \leq H_{c(n)}^c) = \sum_{\gamma=1}^{c} p_{c(n)}^{\gamma} \qquad \xrightarrow{y_n\sim\mathcal{N}(0,1)} \qquad \Phi(H_{c(n)}^c) = \sum_{\gamma=1}^{c} p_{c(n)}^{\gamma}$$

3. **Confidence interval for monte carlo estimation**
$$p(L_N(\mathcal{Z},\epsilon) \geq l) \in p(L_N(\mathcal{Z},\epsilon) \geq l) \pm CI$$
Idea is that for the two naive algorithms that are purported to be equivalent, the CI should be approximately the same