

Problem formulation

Let $\mathcal{M} = (\mathbf{V}, \mathbf{F})$ be mesh living in dimension $d \in \{2, 3\}$. Let $\mathbf{V} = \{\mathbf{v}_1^T, \dots, \mathbf{v}_n^T\}^T \in \mathbb{R}^{n \times d}$ be rest-pose vertex positions. Let $\mathcal{H} = \{\mathbf{h}_1, \dots, \mathbf{h}_m\}$ be a set of control handles. Let $\mathbf{T}_j \in \mathbb{R}^{d \times (d+1)}$ be affine transformation for each handle \mathbf{h}_j . Let $\mathbf{T} = \{\mathbf{T}_1^T, \dots, \mathbf{T}_m^T\}^T \in \mathbb{R}^{(d+1)m \times d}$. Linear blend skinning is a deformation method whereby deformed vertices $\mathbf{V}' = \{\mathbf{v}'_1^T, \dots, \mathbf{v}'_n^T\}^T \in \mathbb{R}^{n \times d}$ is a weighted linear combination of handles' transformations,

$$\mathbf{v}'_i = \sum_{j=1}^m w_j(\mathbf{v}_i) \mathbf{T}_j \begin{pmatrix} \mathbf{v}_i \\ 1 \end{pmatrix}$$

where $w_j : \mathcal{M} \rightarrow \mathbb{R}$ is computed using bounded biharmonic weights. Equivalently

$$\mathbf{V}' = \mathbf{M}\mathbf{T}$$

where, $\mathbf{M} \in \mathbb{R}^{n \times (d+1)m}$ is a matrix combining \mathbf{V} and \mathbf{W} .

arap energy

We can define *as-rigid-as-possible* deformation energy that measures local distortion,

$$E_{arap}(\mathbf{V}', \mathbb{R}) = \frac{1}{2} \sum_{f \in \mathbf{F}} \sum_{(i,j) \in f} c_{ij} \|(\mathbf{v}'_i - \mathbf{v}'_j) - (\mathbf{v}_i - \mathbf{v}_j)\|^2$$

and in matrix form

$$E_{arap}(\mathbf{V}', \mathbb{R}) = tr(\mathbf{V}'^T \mathbf{L} \mathbf{V}' + \mathbf{V}'^T \mathbf{K} \mathbf{R}) = tr(\mathbf{T}^T \tilde{\mathbf{L}} \mathbf{T} + \mathbf{T}^T \tilde{\mathbf{K}} \mathbf{R})$$

where $\tilde{\mathbf{L}} = \mathbf{M}^T \mathbf{L} \mathbf{M} \in \mathbb{R}^{(d+1)m \times (d+1)m}$, $\tilde{\mathbf{K}} = \mathbf{M}^T \mathbf{K} \in \mathbb{R}^{(d+1)m \times dn}$ and \mathbf{K} as defined in the deformation assignment

overhang energy

An overhanging region that can be 3D printed without support is called *self-supported*. We call the angle between region's tangent plane and printing direction be *self-supported angle* α . Let α_{max} be maximum supporting angle. Let $\tau = \sin(\alpha_{max})$ be *maximal supporting coefficient*. Let $\partial \mathcal{M} \subset \mathbf{F}$ be boundary of mesh \mathcal{M} , i.e. surface faces. A surface face f is *risky* and thus requires support if $\mathbf{n}(f) \cdot \mathbf{d}_p < -\tau$, where $\mathbf{n}(f)$ is unit normal of f and \mathbf{d}_p is the printing direction. We can define an overhang energy that measures support required for 3D printing,

$$E_{support}(\mathbf{V}') = \sum_{f \in \partial \mathcal{M}} \min(\mathbf{n}(f) \cdot \mathbf{d}_p - \tau, 0)$$