Homework 10 - Optimal Control Systems

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1 Stochastic Problem - RC circuit

The following tables contain the parameters used for this work. The step size is 1ms for 10 s of simulation. The white noise is specified for zero mean and covariance 1e-4. In order to verify the E(J) for numerical and analytical expression, the simulation was executed 1000 times. It results in a numerical and analytical cost is 0.4884 and 0.4019.

Table 1: Controller parameters

Parameter	Value
H	1
Q	1
R	1

Table 2: Circuit parameters

Parameter	Value
R	1
C	1

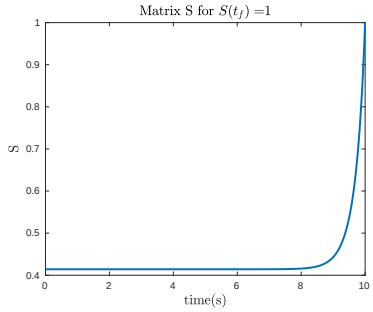

Figure 1: Random part.

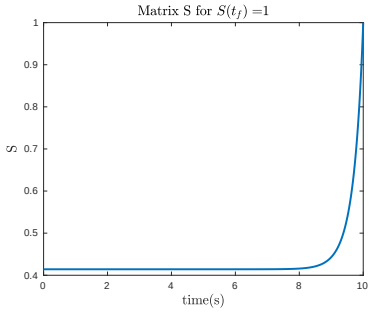

Figure 2: Deterministic part.

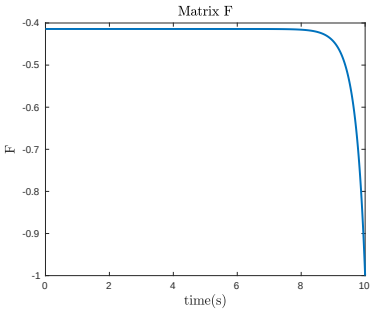

Figure 3: Random part.

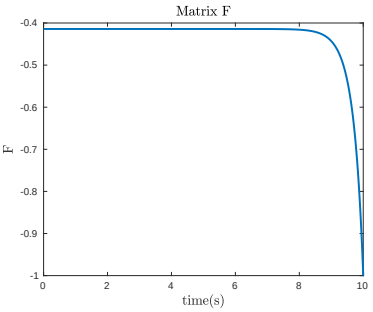

Figure 4: Deterministic part.

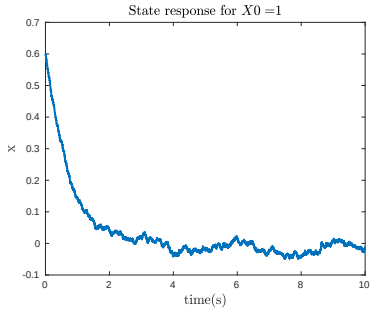

Figure 5: Random part.

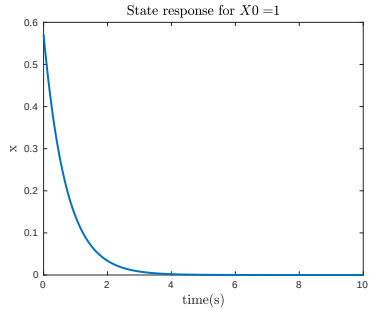

Figure 6: Deterministic part.

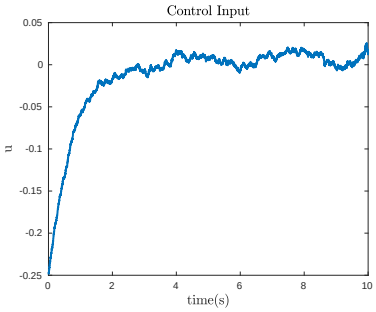

Figure 7: Random part.

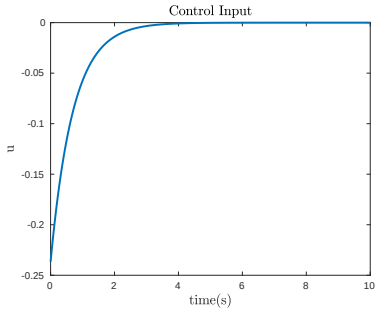

Figure 8: Deterministic part.

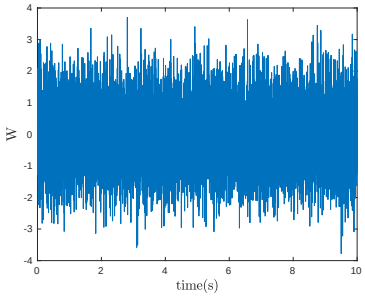

Figure 9: Random part.

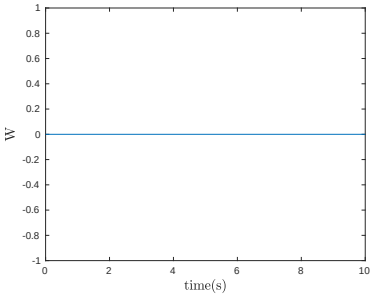

Figure 10: Deterministic part.

```
% Erivelton Gualter, 04/23/2018

clear all
close all
clc

% Paramters
R = 1;
C = 1;

%state equations
A = -1/(R*C);
B = +1/(R*C);

%Cost function
Q = 1;
R = 1;
H = 1;

dt = 0.001;      % Integration step size
tf = 10;         % Simulation length
t = 0:dt:tf;     % time array
```

```
23 X0 = 1;          % Initial state
24 N = round(tf/dt) + 1; %Number of steps
25
26 S = zeros(1,N);
27 S(:,N) = H;
28 F(N,:)=-inv(R)*B'*S(:,N);
29
30 %Backwards integration in time
31 for i = 1 : N-1
32     SDot = - A'*S(:,N-i+1) - S(:,N-i+1)*A + ...
           S(:,N-i+1)*B*inv(R)*B'*S(:,N-i+1) - Q;
33     S(:,N-i) = S(:,N-i+1)-SDot*dt;
34     F(N-i,:) = -inv(R)*B'*S(:,N-i);
35 end
36
37 mtimes = 100;        % Number of runs
38 for k=1:mtimes
39     v = 1e-4;        % Covariance
40     p0 = 0.1;        % Covariance for Initial Conditions
41     X = X0+(p0).^0.5*randn(1,1);
42
43     % System simulation
44     for i=1:N
45         w(i) = (v/dt).^0.5*randn(1,1);
46         u(i) = F(i,:)*X(i);
47         XDot = A*X(i) + B*u(i) + w(i);
48
49         if i < N
50             X(i+1) = X(i) + XDot*dt;
51         end
52     end
53
54     % compute cost
55     for j=1:N-1
56         xQxuRu(j) = X(:,j)'*Q*X(:,j) + u(j)*R*u(j);
57     end
58
59     Jn(k) = X(:,end)'*H*X(:,end) + trapz(t(1:end-1),xQxuRu);
60     disp([num2str(k)])
61 end
62
63
64 mean(Jn) % Numerical cost
65
66 % Analytical cost
67 for j=1:N
68     vS(j) = trace(v*S(:,j));
69 end
70
71 disp('Analytical cost')
72 Ja = trace(S(1)*p0) + trapz(t(1:end),vS(1:end))
73
74 %% Plots
75 close all
76 f1 = figure;
77 plot(t, S, 'LineWidth',2);
78 title(strcat('Matrix S for $S(t_f) =  $', ...
```

```
        num2str(H)),'Fontsize',14,'interpreter','latex');
xlabel('time(s)', 'Interpreter','Latex', 'FontSize',14);
ylabel('S', 'Interpreter','Latex', 'FontSize',14);

f2 = figure;
plot(t, F, 'LineWidth',2);
title(strcat('Matrix F'), 'Fontsize',14,'interpreter','latex');
xlabel('time(s)', 'Interpreter','Latex', 'FontSize',14);
ylabel('F', 'Interpreter','Latex', 'FontSize',14);

f3 = figure;
plot(t, X, 'LineWidth', 2);
title(strcat('State response for $X0 =  $', ...
        num2str(X0)),'Fontsize',14,'interpreter','latex');
xlabel('time(s)', 'Interpreter','Latex', 'FontSize',14);
ylabel('x', 'Interpreter','Latex', 'FontSize',14);

f4 = figure;
plot(t, u, 'LineWidth', 2);
title('Control Input','Fontsize',14,'interpreter','latex');
xlabel('time(s)', 'Interpreter','Latex', 'FontSize',14);
ylabel('u', 'Interpreter','Latex', 'FontSize',14);

f5 = figure;
plot(t, w);
xlabel('time(s)', 'Interpreter','Latex', 'FontSize',14);
ylabel('W', 'Interpreter','Latex', 'FontSize',14);

if v == 0
    saveFigureToPdf('fig6',f1);
    saveFigureToPdf('fig7',f2);
    saveFigureToPdf('fig8',f3);
    saveFigureToPdf('fig9',f4);
    saveFigureToPdf('fig10',f5);
else
    saveFigureToPdf('fig1',f1);
    saveFigureToPdf('fig2',f2);
    saveFigureToPdf('fig3',f3);
    saveFigureToPdf('fig4',f4);
    saveFigureToPdf('fig5',f5);
end
```

You can access the code at: https://github.com/EriveltonGualter/EEC-744-Optimal-Control-Systems