The following equation development should help you to understand how to acquire the output voltage on Slide 17 of Lecture 1. Please note that it could be either a minus or plus sign inside of the parenthesis in the final expression of $V_o(t)$ depending on if $\dot{V}_o(0)$ is positive or negative.

$$\frac{L}{R}\frac{d^2V_o}{dt^2}+\frac{1}{RC}V_o=0$$

Conducting Laplace Transform on both sides of equation, we have

$$\frac{L}{R}(s^2V_o(s)-sV_o(0)-\dot{V}_o(0))+\frac{1}{RC}V_o(s)=0$$

$$\frac{L}{R}s^2V_o(s)+\frac{1}{RC}V_o(s)=\frac{L}{R}(sV_o(0)+\dot{V}_o(0))$$

$$V_o(s)=\frac{\frac{L}{R}sV_o(0)}{\frac{L}{R}s^2+\frac{1}{RC}}+\frac{\frac{L}{R}\dot{V}_o(0)}{\frac{L}{R}s^2+\frac{1}{RC}}$$

$$V_o(s)=\frac{sV_o(0)}{s^2+\frac{1}{LC}}+\frac{\dot{V}_o(0)}{s^2+\frac{1}{LC}}$$

Conducting inversed Laplace Transform on both sides of the equation above

$$V_o(t)=\underbrace{V_o(0)}_{a}\cos\left(\frac{1}{\sqrt{LC}}t\right)+\underbrace{\dot{V}_o(0)}_{b}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

$$V_o(t)=a\cos\left(\frac{1}{\sqrt{LC}}t\right)+b\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

$$V_o(t)=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos\left(\frac{1}{\sqrt{LC}}t\right)+\frac{b}{\sqrt{a^2+b^2}}\sin\left(\frac{1}{\sqrt{LC}}t\right)\right)$$

Define $\frac{a}{\sqrt{a^2+b^2}}=\cos(\varphi)$, and $\frac{b}{\sqrt{a^2+b^2}}=\sin(\varphi)$

Then we have $V_o(t)=\sqrt{a^2+b^2}\left(\cos(\varphi)\cos\left(\frac{1}{\sqrt{LC}}t\right)+\sin(\varphi)\sin\left(\frac{1}{\sqrt{LC}}t\right)\right)$

$$V_o(t)=\underbrace{\sqrt{a^2+b^2}}_{A}\cos\left(\frac{1}{\sqrt{LC}}t-\varphi\right)$$

$$V_o(t)=A\cos\left(\frac{1}{\sqrt{LC}}t-\varphi\right)$$