EEC 643/743/ESC794: Homework 3

Due on Feb. 25, 2019

Please include all the equation development and Matlab codes, and simulation results in your homework. Please have a cover page with "Homework 1" title and your printed name. The problems should be in order and all the pages should be stapled together. Any deviation from the required format will result in a deduction from the homework grade. The homework has to be completed independently and individually. Identical submissions will result in grades of ZERO.

1. A phase locked loop can be described by the differential equation
$$\ddot{y} + \left(a + b\,cos(y)\right)\dot{y} + c\,sin(y) = 0$$
Assume c>0, and a and b are real numbers. Apply Lyapunov's indirect method to test the stability of the equilibrium point $y = \dot{y} = 0$. Are your results conclusive?

2. (For <u>EEC743 and ESC794</u> students only) Give an example of a system that is
a). stable in the sense of Lyapunov but not asymptotically stable
b). asymptotically stable but not globally asymptotically stable.
Please note that you are <u>only allowed to use first-order systems</u> as the examples.

3. Determine whether each of the functions below is positive definite or not.
a). $V(x_1, x_2) = x_1^2 + x_2^4$
b). $V(x_1, x_2) = (x_1 + x_2^2)^2$

4. A LTI system is given below. Please use Lyapunov's direct method to prove the stability of the system around equilibrium point. Find the Lyapunov function of the system with the help of Matlab command "lyap".
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -12 & -8 \end{bmatrix} X$$

5. Use La Salle Invariance Set Theorem to prove that the line x_1=0 is globally asymptotically stable.
$$\dot{x}_1 = -(x_2 - a)x_1$$
$$\dot{x}_2 = \gamma x_1^2$$
You may choose the Lyapunov function as $V(X) = \frac{1}{2}x_1^2 + \frac{1}{2\gamma}(x_2 - b)^2$, where b>a.

6. a). Show that the function
$$V(X) = \frac{1}{2}x_2^2 + \int_0^{x_1} g(x)dx$$
is a Lyapunov function for the system
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -f(x_2) - g(x_1)$$
where *xg(x), xf(x)>0* for $x \neq 0$, and *x* could be either x_1 or x_2, *f(0)=g(0)=0.*
Hint: please show that *V(X)* is positive definite, and $\dot{V}(X)$ is negative semi definite.
b). Is the equilibrium point *x=0* stable i. s. L.?
c). Use such a Lyapunov function to evaluate the stability of *x=0* for the system
$$\ddot{x} + \dot{x}^3 + x = 0$$