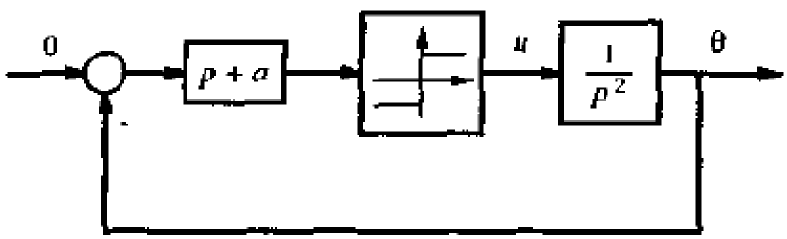
EEC 643/743/ESC794: Homework 2

Due on Feb. 11, 2019

Please include all the equation development, Matlab codes, Simulink model, and simulation results in your homework. Please have a cover page with "Homework 1" title and your printed name. The problems should be in order and all the pages should be stapled together. Any deviation from the required format will result in a deduction from the homework grade. The homework has to be completed independently and individually. Identical submissions will result in grades of ZERO.

1. The system shown in the figure below represents a satellite control system. The block diagram consists of a first-order system with a zero of $-a$, a sign function, and a double integrator. Please note that the parameter "p" is equivalent to the Laplace transform variable "s". The parameter "a" is a positive real number. You may choose an "a" at your convenience for a successful computer simulation. Draw the phase portrait of the system using Matlab/Simulink, and determine the system's stability through the phase portrait.



2. (For EEC743 and ESC794 students only) The following nonlinear system has limit cycles. Without solving the state equations explicitly, show that the number of limit cycles is infinity. (Hint: Use polar coordinates)

$$\dot{x} = y + x(x^2 + y^2 - 1)sin\frac{1}{x^2 + y^2 - 1}$$
$$\dot{y} = -x + y(x^2 + y^2 - 1)sin\frac{1}{x^2 + y^2 - 1}$$

3. Using Bendixson's theorem to show that the following system has no limit cycles.

$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_1 = g(x_1) + ax_2, \qquad a \neq 1$$

4. Using Poincare's theorem to show that the following system has no limit cycles.

$$\dot{x}_1 = 1 - x_1 x_2^2$$
$$\dot{x}_2 = x_1$$

5. Using Lyapunov's direct method to prove the stability of the following system.

$$\dot{x} = -y - x^3$$
$$\dot{y} = x - y^3$$

6. Consider the following autonomous system. Find the domain of attraction for the stability of the system.

$$\dot{x}_1 = (x_1 - x_2)(x_1^2 + x_2^2 - 1)$$
$$\dot{x}_2 = (x_1 + x_2)(x_1^2 + x_2^2 - 1)$$