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TERMINAL SLIDING MODE CONTROL OF SECOND-ORDER NONLINEAR UNCERTAIN SYSTEMS

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SUMMARY

In this paper, a terminal sliding mode control scheme is proposed for second-order nonlinear uncertain systems. By using a function augmented sliding hyperplane, it is guaranteed that the output tracking error converges to zero in *finite* time which can be set arbitrarily. In addition, the proposed scheme eliminates the reaching phase problem so that the closed-loop system always shows the invariance property to parameter uncertainties.

Key words: terminal sliding mode control; variable structure systems; uncertain systems; nonlinear systems

1. INTRODUCTION

It has been known that the control systems with sliding mode have robust and invariant property to parameter uncertainties and external disturbances.[1–3] The sliding mode control system is designed for the system state to be forced to stay on the predetermined sliding surface. When the system is in the sliding mode, the dynamics of the closed-loop system is totally determined by the prescribed sliding surface only, and the overall system shows the invariance property to parameter variations and external disturbances. However, before the system gets in the sliding mode, the output performance can be degraded by parameter variations, i.e. it is not able to guarantee the invariance property during this period, reaching phase problem.[1–3]

In general, the sliding surface has been designed as a linear dynamic equation, e.g. $s = \dot{e} + ce$. But, the linear sliding surface can guarantee the *asymptotic* error convergence in the sliding mode, i.e. the output error can not converge to zero in *finite* time.

Recently, to get a better performance in the sliding mode, terminal sliding mode control methods have been studied.[4–6] They have used nonlinear functions, $s = \dot{e} + ce^{p/q}$, so that the error's converged to zero in *finite* time. However, the nonlinear functions are not easy to

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implement.[6] Furthermore, these methods have a singularity problem.[7] It is a critical one because singular points are located around the origin in the state space, that is, a set of singular points is $\{(e, \dot{e}) \mid e = 0 \text{ and } \dot{e} \neq 0\}$. Especially, in the steady state, the conventional terminal sliding mode control methods may cause a problem since the saturation function is generally used instead of the switching function in order to avoid the chattering phenomena. In this case, the system state may drift around the origin ($e = \dot{e} = 0$) being bounded by the boundary layer thickness, and it may get across the axis, $e = 0$, in the phase space. On the axis, however, there is only one point, origin, that is not a singular point. Thus, conventional schemes may generate a very large control signal in the steady state.

Although a modified terminal sliding mode control method was proposed,[7] it also has the same problem because it does always generate a bounded range space for any bounded domain.

Su and Stepanenko proposed a sliding manifold which guarantees that the tracking error approaches zero in finite time.[8] For a scalar system, it gives a sliding manifold as $s = \dot{x} - k\,\mathrm{sgn}(x)$. It means that $|x|$ decreases with a constant speed of k in the sliding mode. For the practical mechanical system, however, it is so hard to stop a system moving with a constant speed abruptly since it may have a limitation for an acceleration.

Thus, a novel terminal sliding mode control scheme is proposed in this paper. By using a function augmented sliding hyperplane, it is guaranteed that the tracking error converges to zero in *finite* time. In addition, the reaching phase is totally eliminated wherever an initial state is located in the phase space. Therefore, the overall system is always in the sliding mode and shows the invariance property to parameter variations all the time.

2. PROBLEM FORMULATION

Consider a second-order nonlinear uncertain system described by

$$\ddot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t) + \mathbf{G}(\mathbf{x}, \dot{\mathbf{x}}, t)\mathbf{u}(t) \tag{1}$$

where $\mathbf{x} \in \Re^m$ is the state vector, $\mathbf{u} \in \Re^m$ is the control input vector, $\mathbf{f} \in \Re^m$ is the vector of nonlinear dynamics composed of f_i, and the matrix $\mathbf{G} \in \Re^{m \times m}$ is the control gain. It is assumed that the following equations are satisfied:

$$|\hat{f}_i - f_i| \leqslant F_i \quad \text{and} \quad \mathbf{G} = (\mathbf{I} + \boldsymbol{\Delta})\hat{\mathbf{G}}$$

where $i = 1, 2, \ldots, m$, $\mathbf{I}$ is the $m \times m$ identity matrix, and $\boldsymbol{\Delta} \in \Re^{m \times m}$ composed of Δ_{ij} satisfies the following inequality:

$$|\Delta_{ij}| \leqslant D_{ij}$$

where $D_{ij} > 0$, and $\|\mathbf{D}\| < 1$, and $(\hat{\cdot})$ represents a nominal value of $(\cdot)$

3. DESIGN OF CONTROL SYSTEM

Let us define a novel function augmented sliding hyperplane $\mathbf{s}(t)$ as

$$\mathbf{s}(t) = \dot{\mathbf{e}}(t) + \mathbf{C}\mathbf{e}(t) - \mathbf{w}(t) \tag{2}$$

where $\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x}_d(t)$, $\mathbf{x}_d(t)$ is a given twice continuously differentiable reference trajectory, $\mathbf{C} = \mathrm{diag}(c_1, c_2, \ldots, c_m)$, $c_i > 0$, $\mathbf{w}(t) = \dot{\mathbf{v}}(t) + \mathbf{C}\mathbf{v}(t)$, and $v_i(t)$ is designed such that the following assumption holds.

Assumption 1

$v_i : \Re_+ \rightarrow \Re$, $v_i \in C^2[0, \infty)$, $\dot{v}_i, \ddot{v}_i \in L^\infty$, the support of v_i is a bounded interval $[0, T_f]$ for some $T_f > 0$, $v_i(0) = e_i(0)$, and $\dot{v}_i(0) = \dot{e}_i(0)$, where $C^2[0, \infty)$ represents the set of all second differentiable continuous functions defined on $[0, \infty)$, and $i = 1, 2, \ldots, m$.

Remark 1

From Assumption 1 and the definition of the proposed sliding hyperplane (2), it is clear that the system state is on the sliding hyperplane at an initial instant i.e.

$$\mathbf{s}(0) = \mathbf{0} \tag{3}$$

The assumption that an initial condition is available is not restrictive because the measured and/or estimated data of the system state can be obtained at each sampling time. For example, in case of robot manipulators, one can get the position and the velocity data at each sampling time. Thus, the initial condition can be regarded as the data obtained when the control system starts to operate. In fact, a lot of previous works proposed to overcome the reaching phase problem have used an initial condition of the system state.[9,10]

The Frobenium–Perron theorem is given to use in the proof of the stability of the closed-loop system.

Theorem 1 (Frobenium–Perron)

Consider a square matrix $\mathbf{A}$ with non-negative elements. Then, the largest real eigenvalue of $\mathbf{A}$, $\lambda_{\max}$, is non-negative. Furthermore, consider the following equation

$$(\mathbf{I} - \lambda^{-1}\mathbf{A})\mathbf{y} = \mathbf{z} \tag{4}$$

where all components of the vector $\mathbf{z}$ are non-negative. If $\lambda > \lambda_{\max}$, then the equation admits a unique solution $\mathbf{y}$, whose components y_i are all non-negative.

By using the proposed sliding surface (2) and the above theorem, the following theorem can be derived for the existence of the sliding mode.

Theorem 2

The overall system is in the sliding mode all the time ($\mathbf{s}(t) = \mathbf{0}\ \forall t \geqslant 0$) if the following controller is applied to the plant (1):

$$\mathbf{u} = \hat{\mathbf{G}}^{-1}[\ddot{\mathbf{x}}_d - \hat{\mathbf{f}} - \mathbf{C}\dot{\mathbf{e}} + \ddot{\mathbf{v}} - \mathbf{k} \bullet \mathbf{sgn}(\mathbf{s})] \tag{5}$$

where

$$\mathbf{k} = (\mathbf{I} - \mathbf{D})^{-1}(\mathbf{F} + \mathbf{D}|\ddot{\mathbf{x}}_d - \hat{\mathbf{f}} - \mathbf{C}\dot{\mathbf{e}} + \ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}}| + \boldsymbol{\Gamma})$$

$$\boldsymbol{\Gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_m]^{\mathrm{T}}, \quad \gamma_i > 0$$

$$\mathbf{sgn}(\mathbf{s}) = [\operatorname{sgn}(s_1), \operatorname{sgn}(s_2), \ldots, \operatorname{sgn}(s_m)]^{\mathrm{T}}$$

$$\operatorname{sgn}(s_i) = \begin{cases} 1 & \text{if} \quad s_i > 0, \\ 0 & \text{if} \quad s_i = 0, \quad i = 1, 2, \ldots, m \\ -1 & \text{if} \quad s_i < 0, \end{cases}$$

where $\mathbf{k} \bullet \mathbf{sgn}(\mathbf{s})$ is the vector of components $k_i \operatorname{sgn}(s_i)$, and $|\cdot|$ is the vector of components $|\cdot_i|$.

Proof. The main procedure follows up that of Slotine and Li.[13] Let us define V as a positive-definite function of s:

$$V = \tfrac{1}{2}\mathbf{s}^{\mathrm{T}}\mathbf{s} \tag{6}$$

Differentiating (6) along the controlled system (1) and (5) yields

$$\begin{aligned}
\dot{V} &= \mathbf{s}^{\mathrm{T}}\dot{\mathbf{s}} \\
&= \mathbf{s}^{\mathrm{T}}[\mathbf{f} + \mathbf{G}\mathbf{u} - \ddot{\mathbf{x}}_d + \mathbf{C}\dot{\mathbf{e}} - \ddot{\mathbf{v}} - \mathbf{C}\dot{\mathbf{v}}] \\
&= \mathbf{s}^{\mathrm{T}}[\mathbf{f} - \hat{\mathbf{f}} + \boldsymbol{\Delta}(-\hat{\mathbf{f}} + \ddot{\mathbf{x}}_d - \mathbf{C}\dot{\mathbf{e}} + \ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}}) - (\mathbf{I} + \boldsymbol{\Delta})\mathbf{k} \bullet \mathbf{sgn}(\mathbf{s})] \\
&= \sum_{i=1}^{m} s_i \left[f_i - \hat{f}_i + \sum_{j=1}^{m} \Delta_{ij}(\ddot{x}_{d_j} - \hat{f}_j - c_j\dot{e}_j + \ddot{v}_j + c_j\dot{v}_j) - \sum_{j \neq i}^{m} \Delta_{ij}k_j \operatorname{sgn}(s_j) - (1 + \Delta_{ii})k_i \operatorname{sgn}(s_i) \right]
\end{aligned} \tag{7}$$

It is clear that the above $\dot{V}$ is negative-definite provided that

$$(1 - D_{ii})k_i \geqslant F_i + \sum_{j=1}^{m} D_{ij}|\ddot{x}_{d_j} - \hat{f}_j - c_j\dot{e}_j + \ddot{v}_j + c_j\dot{v}_j| + \sum_{j \neq i} D_{ij}k_j \tag{8}$$

where $i = 1, 2, \ldots, m$. The above inequality can be satisfied if $\mathbf{k}$ is designed such that

$$(1 - D_{ii})k_i - \sum_{j \neq i} D_{ij}k_j = F_i + \sum_{j=1}^{m} D_{ij}|\ddot{x}_{d_j} - \hat{f}_j - c_j\dot{e}_j + \ddot{v}_j + c_j\dot{v}_j| + \gamma_i \tag{9}$$

where $i = 1, 2, \ldots, m$. It can be rewritten as

$$(\mathbf{I} - \mathbf{D})\mathbf{k} = \mathbf{F} + \mathbf{D}|\ddot{\mathbf{x}}_d - \hat{\mathbf{f}} - \mathbf{C}\dot{\mathbf{e}} + \ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}}| + \Gamma \tag{10}$$

It is shown that the vector $\mathbf{k}$ is uniquely determined with non-negative elements by applying the Frobenius–Perron theorem and the assumption $\|\mathbf{D}\| < 1$. Thus, the following inequality can be derived:

$$\dot{V} \leqslant -\sum_{i=1}^{m} \gamma_i |s_i| \tag{11}$$

Therefore, V is a positive-definite function and $\dot{V}$ is a negative-definite function. From Remark 1, it is easily known that $\mathbf{s}(0)=\mathbf{0}$, and it implies that $V(0)=0$. Thus, it implies that

$$V(t)\equiv 0$$

Clearly, it is equivalent to

$$\mathbf{s}(t)\equiv\mathbf{0} \tag{12}$$

□

Since the overall system was shown to be always in the sliding mode and the augmenting function $\mathbf{v}(t)$ can be designed arbitrarily provided that Assumption 1 holds, the following theorem can be derived.

Remark 2

Since $\mathbf{s}(t)\equiv\mathbf{0}$ from (12), there is no reaching phase and the overall system is in the sliding mode all the time. It also implies that the closed-loop system always shows the invariance property to parameter uncertainties and variations.

Remark 3

From (2) and (12), it can be easily known that the system output is totally governed by the following equation all the time

$$\dot{\mathbf{e}}(t)+\mathbf{C}\mathbf{e}(t)=\mathbf{w}(t),\quad \forall t\geqslant 0 \tag{13}$$

that is, the tracking error $\mathbf{e}(t)$ can be predetermined and it is not affected by parameter variations at all.

Theorem 3

If the control system (5) is applied to plant (1), the tracking error $\mathbf{e}(t)$ converges to zero in finite time T_{f}.

Proof. From the definition of $\mathbf{s}(t)$, it is easy to know that

$$\mathbf{s}=\dot{\mathbf{e}}+\mathbf{C}\mathbf{e}-\mathbf{w}=\dot{\mathbf{e}}+\mathbf{C}\mathbf{e}-(\dot{\mathbf{v}}+\mathbf{C}\mathbf{v})=\dot{\varepsilon}+\mathbf{C}\varepsilon \tag{14}$$

where $\varepsilon(t)=\mathbf{e}(t)-\mathbf{v}(t)$. From Assumption 1, it is obvious that $\varepsilon(0)=\dot{\varepsilon}(0)=\mathbf{0}$. Since $\mathbf{s}(t)=\mathbf{0}$ $\forall t\geqslant 0$, it can be known that $\varepsilon(t)=\mathbf{0}$ $\forall t\geqslant 0$, which is equivalent to

$$\mathbf{e}(t)\equiv\mathbf{v}(t) \tag{15}$$

The function $\mathbf{v}(t)$ can be designed *arbitrarily* if Assumption 1 holds, and Assumption 1 says that $\mathbf{v}(t)$ is designed so that $\mathbf{v}(t)=\mathbf{0}$ $\forall t\geqslant T_{\mathrm{f}}$. Thus, the tracking error $\mathbf{e}(t)=\mathbf{x}(t)-\mathbf{x}_d(t)$ converges to zero in *finite* time T_{f}, i.e.

$$\mathbf{e}(t)=\mathbf{0},\quad \forall t\geqslant T_{\mathrm{f}}$$

Remark 4

While the conventional terminal sliding mode control methods have a complex nonlinear sliding surface ($s = \dot{e} + c \cdot e^{p/q}$) and have a singularity problem,[7] the proposed scheme is easy to design and has no such a problem. By using the function augmented sliding hyperplane, it is guaranteed that the tracking error goes to zero in finite time. In addition, the relaxation time,[12] T_{f}, can be set arbitrarily.

Corollary 1

If the switching function $\mathbf{sgn}(\mathbf{s})$ in the proposed control system (5) is replaced by the saturation function $\mathbf{sat}(\mathbf{s})$ defined by

$$\mathrm{sat}(s_i) = \begin{cases} \mathrm{sgn}(s_i) & \text{if } |s_i| > \phi_i, \\ \dfrac{s_i}{\phi_i} & \text{if } |s_i| \leqslant \phi_i, \end{cases} \qquad \phi_i > 0 \quad \text{and} \quad i = 1, 2, \ldots, m$$

then it can be guaranteed that each tracking error $|e_i(t)|$ goes to be bounded by ϕ_i / c_i in finite time T_{f}, i.e.

$$|e_i(t)| \leqslant \frac{\phi_i}{c_i}, \quad \forall t \geqslant T_{\mathrm{f}},$$

where $i = 1, 2, \ldots, m$.

Proof. From (7)–(9) in Theorem 2, it is clear that the control gain $\mathbf{k}$ was designed such that the following inequality holds for all $i \in \{i \,|\, |s_i| > \phi_i\}$.

$$\begin{aligned} s_i \dot{s}_i &= s_i \left[f_i - \hat{f}_i + \sum_{j=1}^{m} \Delta_{ij} (\ddot{x}_{d_j} - \hat{f}_j - c_j \dot{e}_j + \ddot{v}_j + c_j \dot{v}_j) - \sum_{j \neq i}^{m} \Delta_{ij} k_j \, \mathrm{sat}(s_j) - (1 + \Delta_{ii}) k_i \, \mathrm{sgn}(s_i) \right] \\ &\leqslant 0 \end{aligned} \tag{16}$$

where $i \in \{i \,|\, |s_i| > \phi_i\}$. Thus, from (3) and (16), it is obvious that

$$|s_i(t)| \leqslant \phi_i \quad \forall t \geqslant 0 \tag{17}$$

where $i = 1, 2, \ldots, m$. It also implies the following inequality because $s_i = \dot{\varepsilon}_i + c_i \varepsilon_i$, $s_i(0) = 0$, and $\varepsilon_i(0) = 0$ from Assumption 1.

$$|\varepsilon_i(t)| \leqslant \frac{\phi_i}{c_i} \quad \forall t \geqslant 0 \tag{18}$$

From Assumption 1, it can be also said that

$$v_i(t) = 0 \quad \forall t \geqslant T_{\mathrm{f}} \tag{19}$$

Thus, from (18) and (19), and the definition of $\varepsilon = \mathbf{e} - \mathbf{v}$, the following inequality can be derived:

$$|e_i(t)| \leqslant \frac{\phi_i}{c_i} \quad \forall t \geqslant T_{\mathrm{f}}$$

where $i = 1, 2, \ldots, m$. □

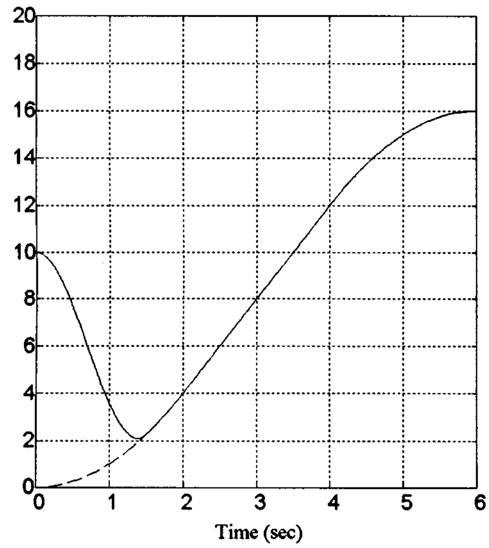

Figure 1. Reference and actual trajectories

4. ILLUSTRATIVE EXAMPLE

The simulation has been carried out for an underwater vehicle. A simplified model of the motion of the vehicle can be written as[13]

$$m\ddot{x} + c\dot{x}|\dot{x}| = u$$

where the mass m and the drag coefficient c are assumed to be bounded as follows: $1 \leqslant m \leqslant 5$, and $0{\cdot}5 \leqslant c \leqslant 1{\cdot}5$. The estimated and actual values for m and c are as follows: $\hat{m} = \sqrt{5}$, $\hat{c} = 1$, $m = 3 + 1{\cdot}5\sin(|\dot{x}|t)$, and $c = 1{\cdot}2 + 0{\cdot}2\sin(|\dot{x}|t)$ as the same as that of[13]. The augmenting function $\mathbf{v}(t)$ was designed as a cubic polynomial:

$$v_i(t) = \begin{cases} a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3 & \text{if } 0 \leqslant t \leqslant T_f \\ 0 & \text{if } t > T_f \end{cases}$$

where $a_{i0} = e_i(0)$. $a_{i1} = \dot{e}_i(0)$, $a_{i2} = -3(e_i(0)/T_f^2) - 2(\dot{e}_i(0)/T_f)$, $a_{i3} = 2(e_i(0)/T_f^3) + (\dot{e}_i(0)/T_f^2)$ and $i = 1, 2, \ldots, m$. For the sliding hyperplane, $c_i = 10$ and $T_f = 1{\cdot}5$ s were used. The desired trajectory consisted of a constant-acceleration phase at 2 m/s^2 for 2 s, a constant-velocity phase (at 4 m/s) for two seconds, and a constant-acceleration phase at -2 m/s^2 for 2 s.[11]

The actual and desired trajectories are shown in Figure 1. The solid line represents the actual trajectory, and a dashed line shows the desired one. From this figure, one can easily know that the actual output converges to the desired trajectory within $T_f = 1{\cdot}5$ s.

Figure 2 shows the output tracking error. As can be seen in this figure, the output tracking error converges to zero in $T_f = 1{\cdot}5$ s. In other words, after the relaxation time T_f that can be set arbitrarily, the system output tracks the reference trajectory perfectly.

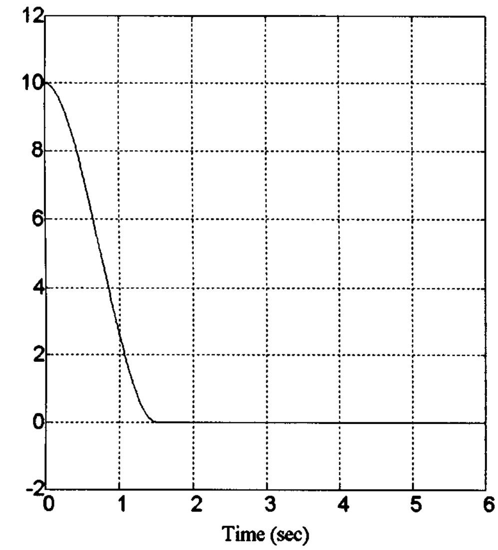

Figure 2. Trajectory tracking error

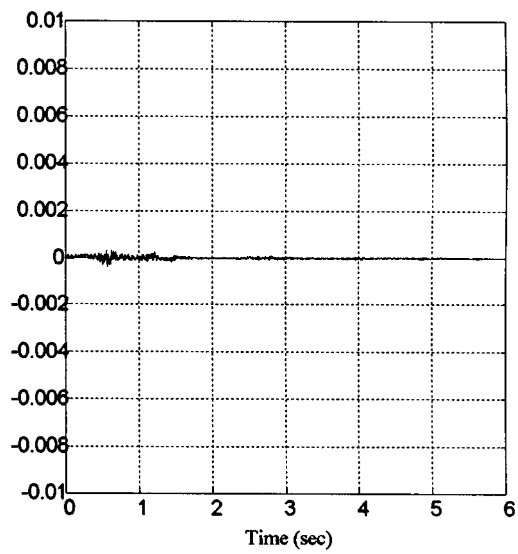

Figure 3. Sliding hyperplane variables (s)

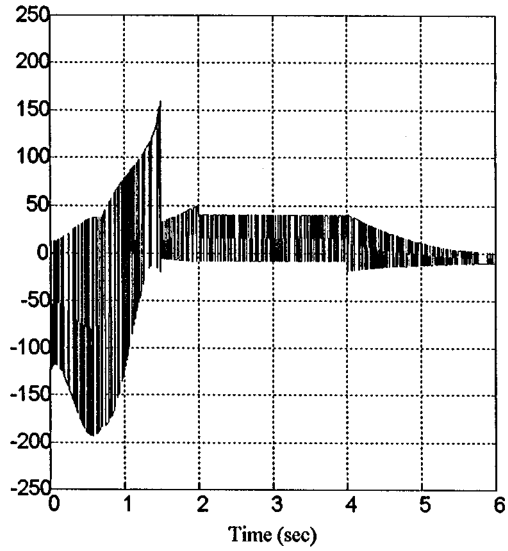

Figure 4. Control input (u)

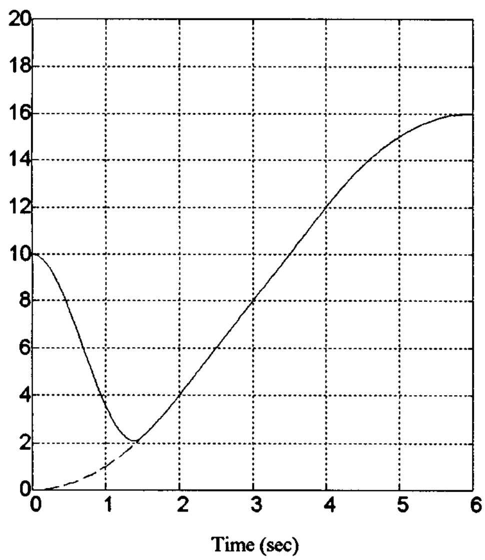

Figure 5. Reference and actual trajectories

The sliding hyperplane variables $s(t)$ is presented in Figure 3. From this figure, it can be known that the system state is always on the sliding hyperplane, i.e. the overall system is in the sliding mode all the time. Therefore, as can be seen in Figure 4, the control input signal always shows the chattering phenomena.

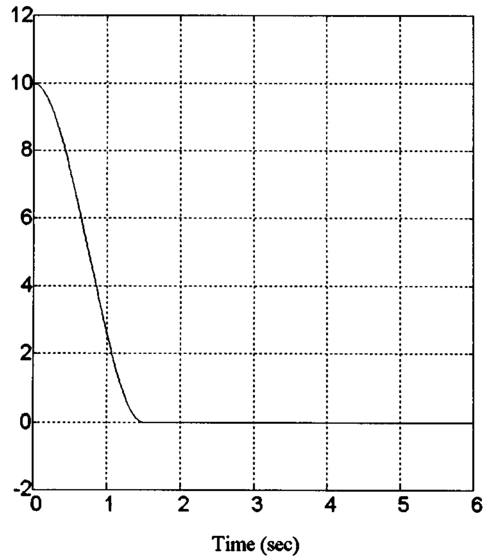

Figure 6. Trajectory tracking error

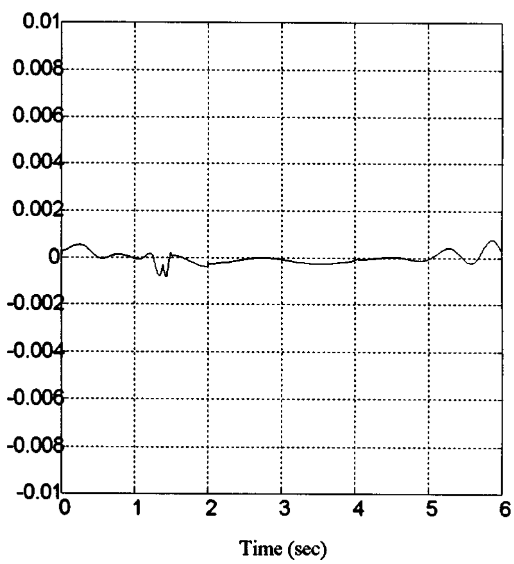

Figure 7. Sliding hyperplane variables (s)

A saturation function can be used to eliminate the chattering phenomena. The results when the switching function is substituted as a saturation function are given in Figures 5–8, where a saturation function with the boundary layer thickness of 0·001 was used.

The actual and desired trajectories are shown in Figure 5. The solid line represents the actual trajectory, and a dashed line shows the desired one. From this figure, one can easily know that the

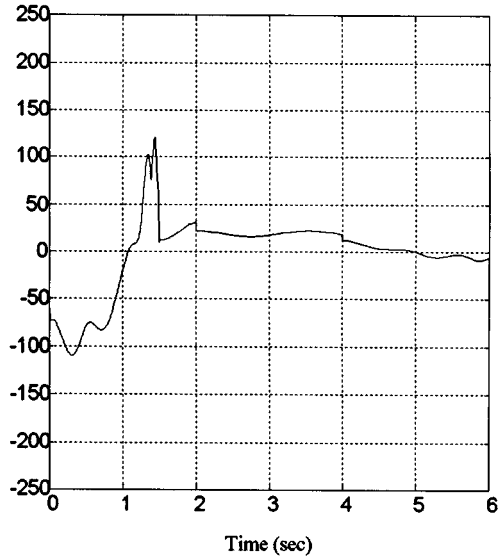

Figure 8. Control input (u)

difference between the actual output and the desired trajectory converges and is to be bounded by ϕ_i/c_i (= the boundary layer thickness/c_i = 0·0001) within $T_f = 1{\cdot}5$ s.

Figure 6 shows the output tracking error. As can be seen in this figure, the output tracking error converges to be bounded by ϕ_i/c_i in $T_f = 1{\cdot}5$ s. In other words, after the relaxation time T_f that can be set arbitrarily, the system output tracks the reference trajectory with the maximum error of 0·0001.

The sliding hyperplane variable $s(t)$ is presented in Figure 7. From this figure, it can be known that the system state is always bounded by the boundary layer thickness 0·001.

As can be seen in Figure 8, the control input signal shows a smooth curve because that the switching function was replaced by the saturation function.

5. CONCLUSIONS

The terminal sliding mode control scheme using the function augmented sliding hyperplane has been proposed for second-order nonlinear systems. By using the proposed sliding hyperplane, it was guaranteed that the reaching phase problem is overcome and the output tracking error converges to zero in *finite* time and the relaxation time T_f can be set *arbitrarily*. Since the reaching phase problem was also overcome, it has been shown that the overall system is always in the sliding mode and shows the invariance property to parameter variations all the time.

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