Erivelton Gualter dos Santos

TERM PAPER PRESENTATION

Nonlinear System

Ri, S., Huang, J., Wang, Y., Kim, M., & An, S. (2014). **Terminal sliding mode control of mobile wheeled inverted pendulum system with nonlinear disturbance observer**. *Mathematical Problems in Engineering, 2014*.

INTRODUCTION

Terminal Sliding Mode Control with Disturbance Observer

INTRODUCTION

- MWIP stands for Mobile Wheeled Inverted Pendulum;
- MWIP can be used for delivery and touring;
- Examples of MWIP:
  - Segway (www.segway.com/);
  - JOE (Swiss Federal Institute of Technology Lausanne);
  - UW-Car
  - PMP (Faculty of Engineering, Tamagawa University).

MWIP SYSTEM

- Dynamics of MWIP is underactuated.

$$N_u < N_{DOF}$$

  - N_u: Number of Control Inputs.
  - N_{DOF}: Number of the Degrees of Freedom.
- Control this system is a challenge.
- Previous control approaches:
  - Feedback linearization;
  - Fuzzy control methods;
  - Neural Network-based;
  - Adaptive control;

SYSTEM FORMULATION

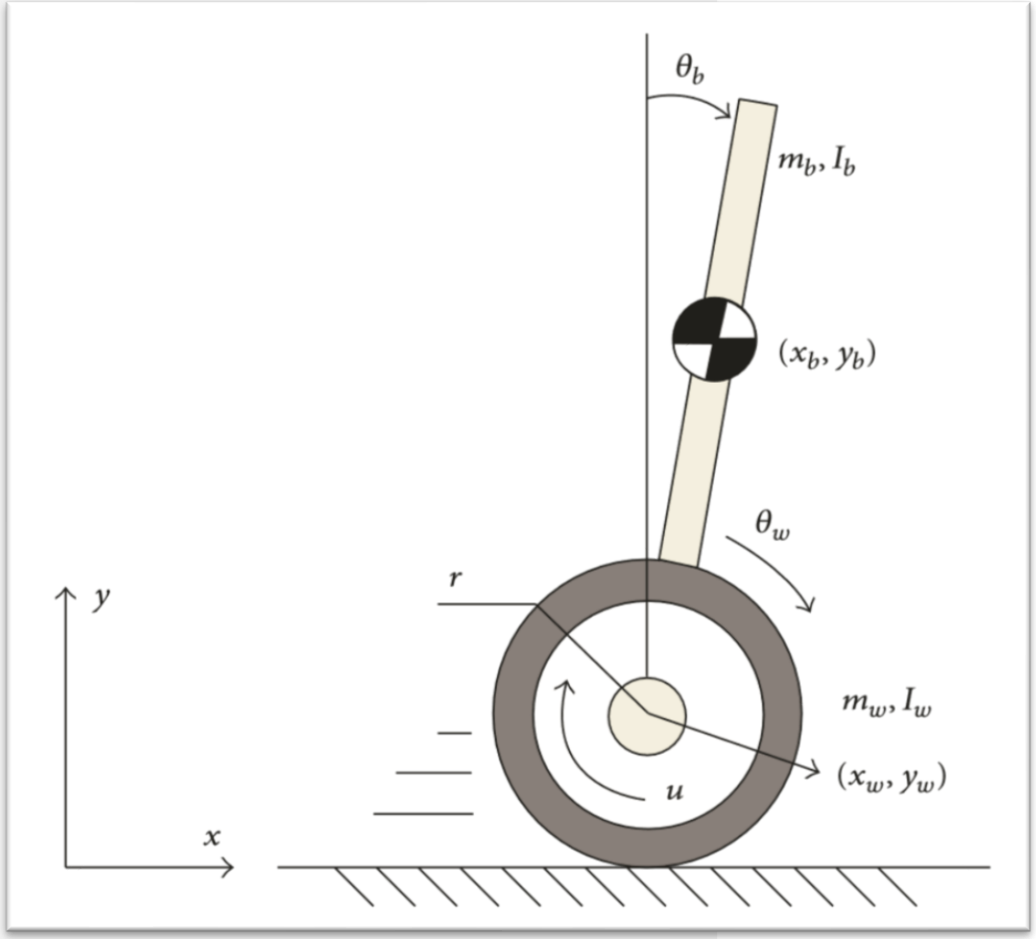
Terminal Sliding Mode Control with Disturbance Observer

SYSTEM FORMULATION

- One-dimensional inverted pendulum;
- Rotation and Translation motion of the body occurs on a place.
- States Variables:
  - $[\theta_b \quad \theta_w]$: Inclination angle of the body and the wheel’s rotation angle, respectively.

TABLE 1: Notations for MWIP parameters.

Parameter	Description
m_b, m_w	Masses of the body and the wheel
I_b, I_w	Moments of inertia of the body and the wheel
l	Length between the wheel axle and the center of gravity of the body
r	Radius of the wheel
D_b	Viscous resistance in the driving system
D_w	Viscous resistance of the ground
u	Rotation torque generated by the wheel motor



Mobile wheeled inverted pendulum (MWIP) system model.

MODELING

- The modeling method of this system is based on **Euler-Lagrange equations**.
- Lagrangian: $\mathcal{L} = \mathcal{K} - \mathcal{P}$
  - $\mathcal{K}$: Kinetic Energy
  - $\mathcal{P}$: Potential Energy.
- Then, the equation of motion can be found by:

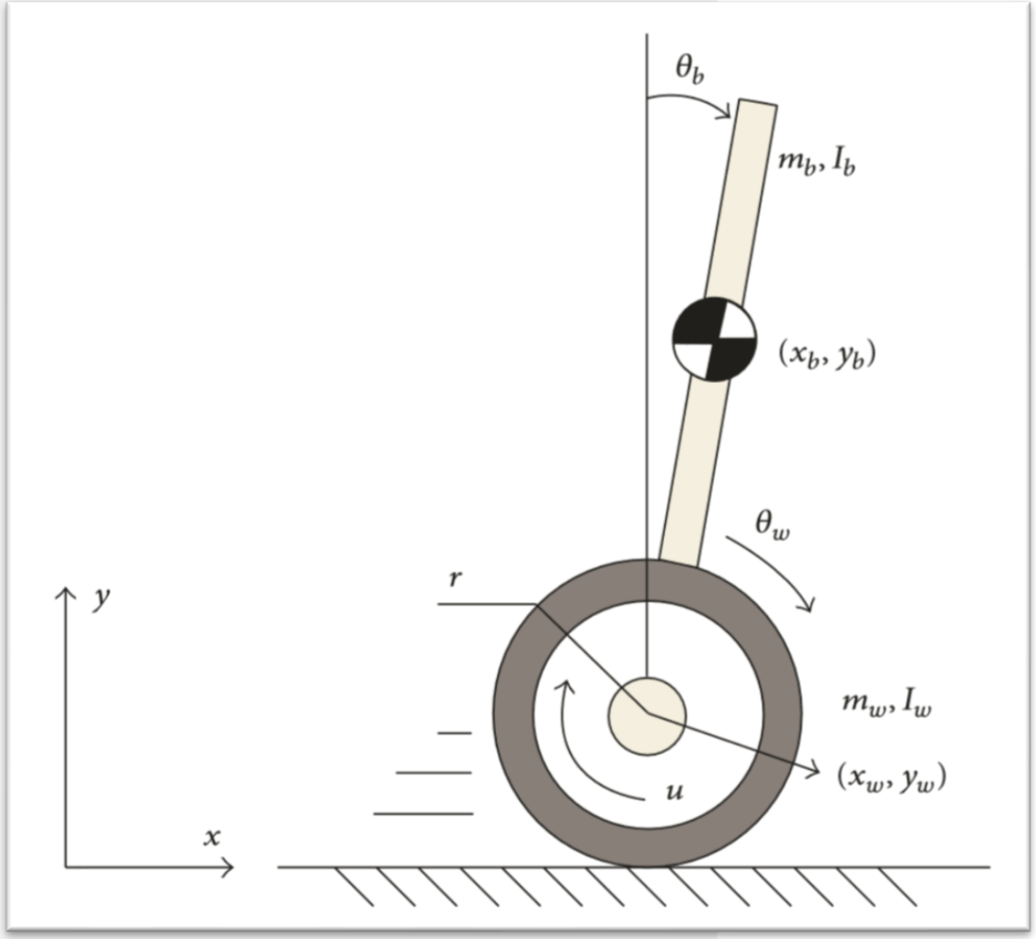
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_k} - \frac{\partial \mathcal{L}}{\partial \dot{q}_k} = \tau_k$$

$k = 1, \dots, n$

- Nonlinear underactuated system with disturbances:

$$M(q)\ddot{q} + N(q,\dot{q}) + F(\dot{q}) = \tau + \tau_{ext}$$

Where q $= [\theta_b \quad \theta_w]^T = [\theta_b \quad \theta_w]^T$



Mobile wheeled inverted pendulum (MWIP) system model.

TERMINAL SLIDING MODEL CONTROLLER DESIGN

INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust Nonlinear Control **9**, 769–780 (1999)

TERMINAL SLIDING MODE CONTROL OF SECOND-ORDER NONLINEAR UNCERTAIN SYSTEMS

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SUMMARY

In this paper, a terminal sliding mode control scheme is proposed for second-order nonlinear uncertain systems. By using a function augmented sliding hyperplane, it is guaranteed that the output tracking error converges to zero in *finite* time which can be set arbitrarily. In addition, the proposed scheme eliminates the reaching phase problem so that the closed-loop system always shows the invariance property to parameter uncertainties.

Key words: terminal sliding mode control; variable structure systems; uncertain systems; nonlinear systems

1. INTRODUCTION

It has been known that the control systems with sliding mode have robust and invariant property

Park, K. B., & Tsuji, T. (1999). **Terminal sliding mode control of second-order nonlinear uncertain systems**. International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal, 9(11), 769-780.

TERMINAL SLIDING MODEL CONTROLLER DESIGN

- Classic sliding mode control (SMC) technique may not converge the tracking errors on sliding surface $s(t)$ to zero in finite time.
- Terminal Sliding Model Controller (**TSMC**) is a variation of SMC, where the sliding surface is defined with additional nonlinear terms.
- It leads tracking errors on sliding surface to converge to zero in finite time.
- Also, the TSMC nonlinear function can be designed to reduce chattering and speed up convergence.
- General Sliding Surface of TSMC:

$$S(t) = \dot{e}(t) + Ce(t) - w(t)$$

$C = diag(c_1, c_2, \dots, c_m),\ \ c_i > 0,$

$e(t) = x(t) - x_d(t)$,

$w = \dot{v}(t) - cv(t)$: Nonlinear function as cubic polynomial.

ASSUMPTIONS AND THEOREMS

- Sliding surface for MWIP system:

$$S(t) = \dot{\theta}_b(t) + c\theta_b(t) - \dot{v}(t) - cv(t)$$

- *Assumption 1*. The tracking errors of the nonlinear disturbance observer are bounded and satisfy the following, where d_1 and d_2 are bounded:

$$|\tilde{\tau}_{d1}| \leq |d_1|, \qquad |\tilde{\tau}_{d2}| \leq |d_2|$$

- **Theorem 2**. The overall system is in sliding mode all the time if the following controller is applied to the plant:

$$\ddot{x}(t) = f(x, \dot{x}, t) + G(x, \dot{x}, t)u(t)$$

$$u = \hat{G}^{-1}\left[\ddot{x}_d - \hat{f} - C\dot{e} + \ddot{v} - k \cdot \mathrm{sgn}(s)\right]$$

- **Theorem 3.** For the internal dynamic model of the MWIP system, the proposed controller guarantees that the angular velocity $\dot{\theta}_w$ can converge to zero.

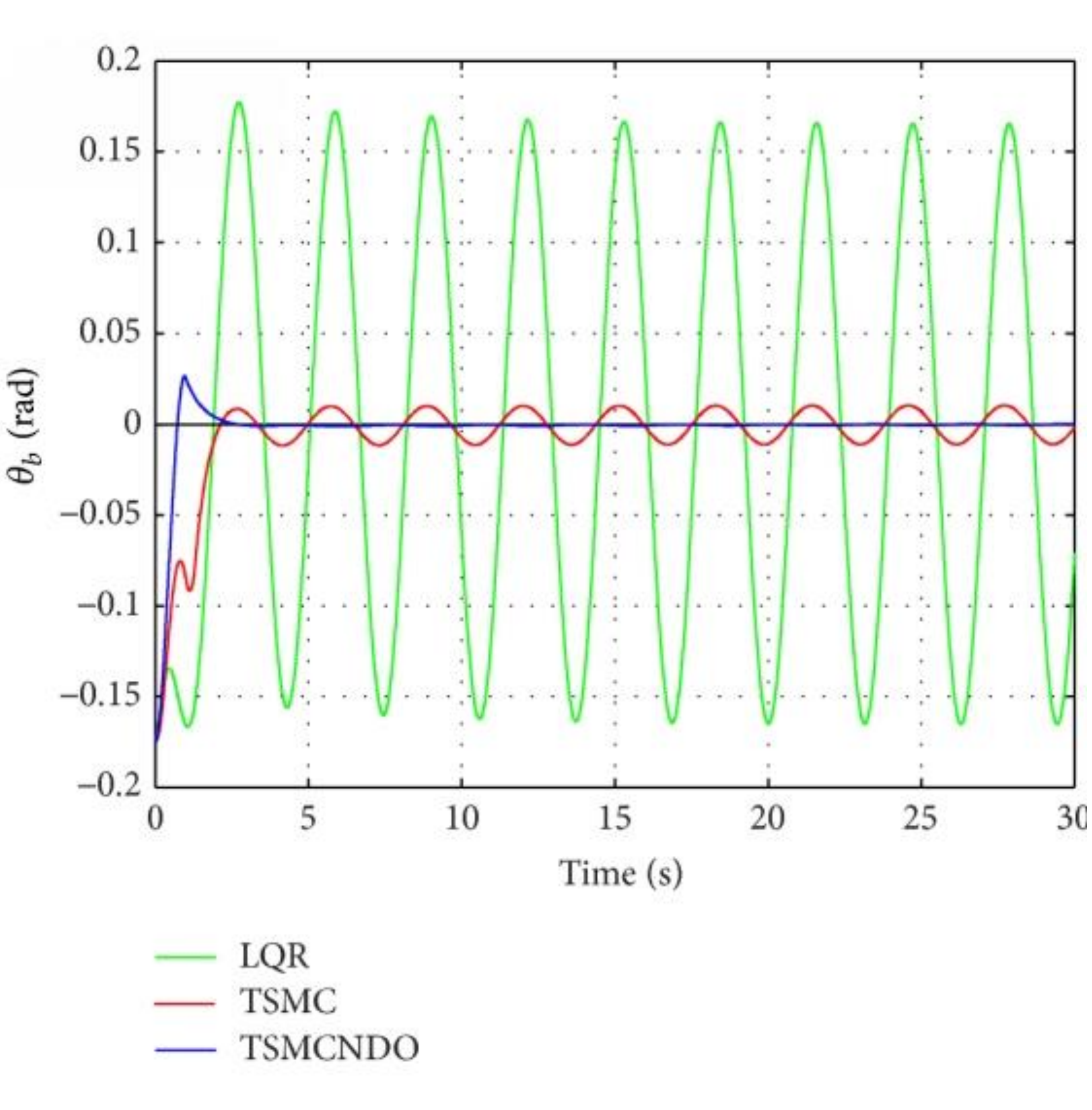
RESULTS

Terminal Sliding Mode Control with Disturbance Observer

OUTPUT: ANGLE POSITION

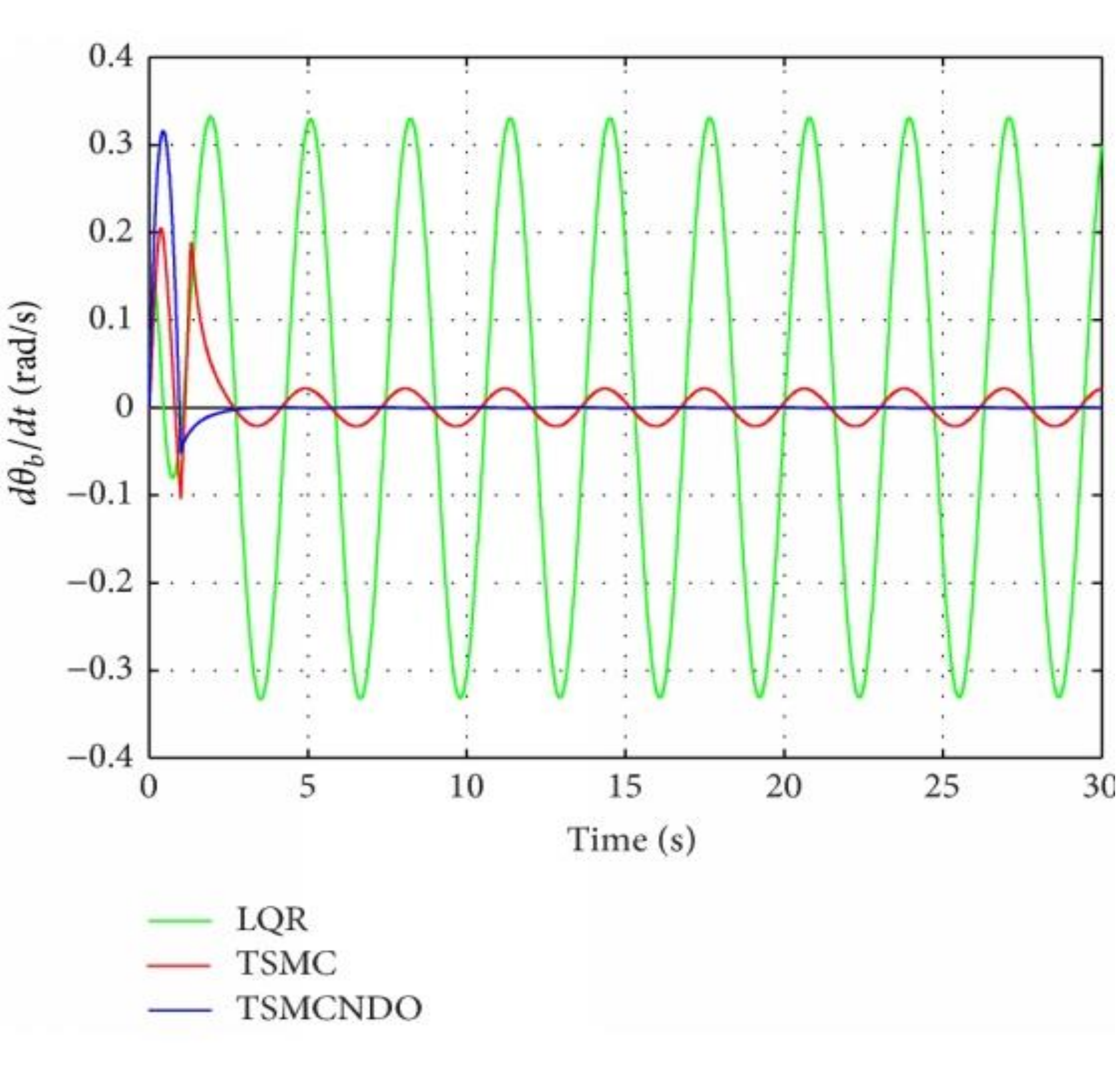
- Inclination angles of the MWIP system by employing LQR, TSMC, and TSMCNDO control strategies.
- Initial Condition: $\theta_b(t) = -10°$
- Parameters of SMC: $\varphi_{TSMC} = \varphi_{TSMCNDO} = 0.05$

- Different control strategies are compared:
  - LQR;
  - TSMC;
  - TSMCNDO.



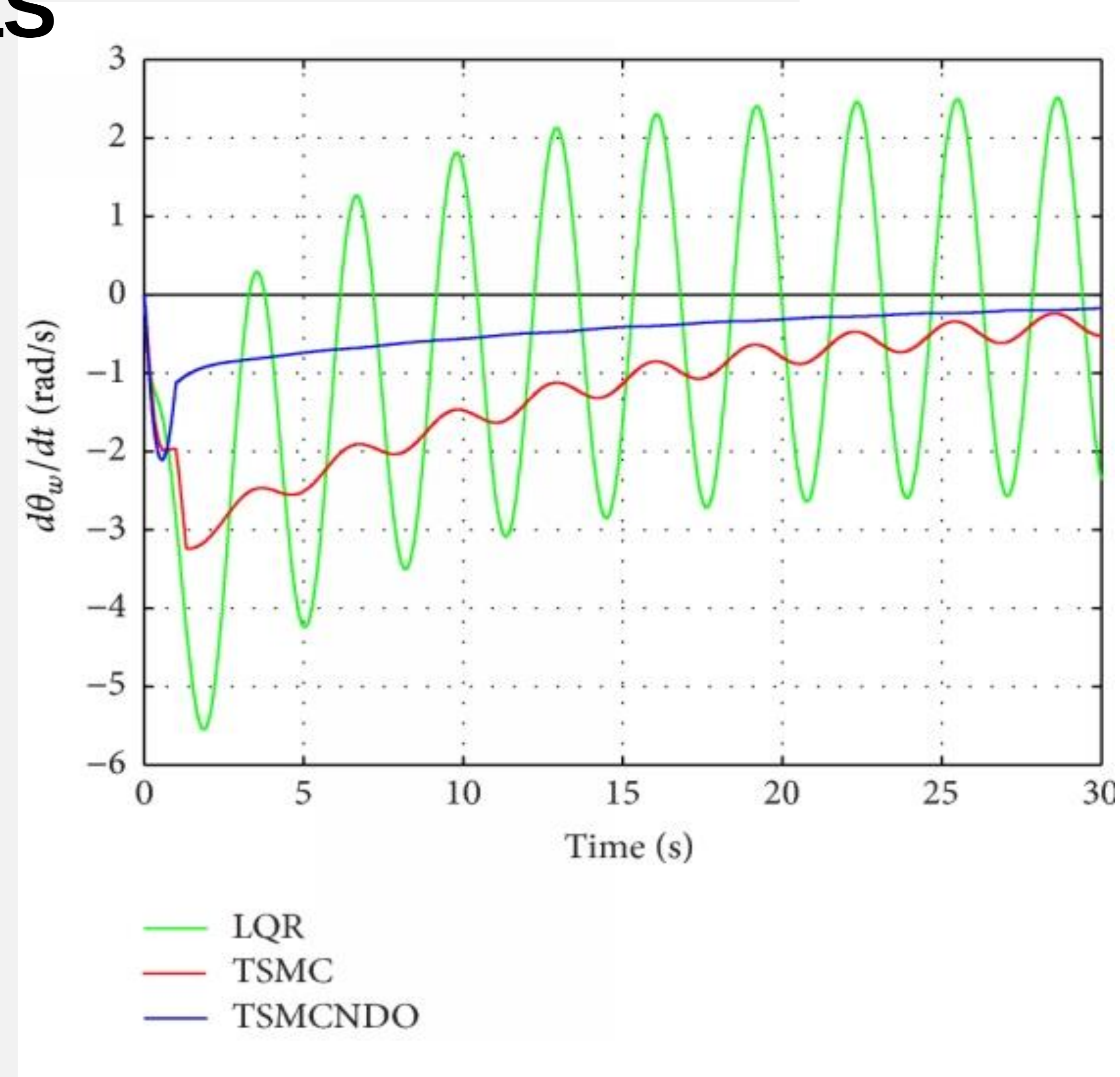
OUTPUT: ANGULAR VELOCITY

- Inclination Velocity of the MWIP system by employing LQR, TSMC, and TSMCNDO control strategies.
- Initial Condition: $\theta_b(t) = -10°$
- Parameters of SMC: $\varphi_{TSMC} = \varphi_{TSMCNDO} = 0.05$



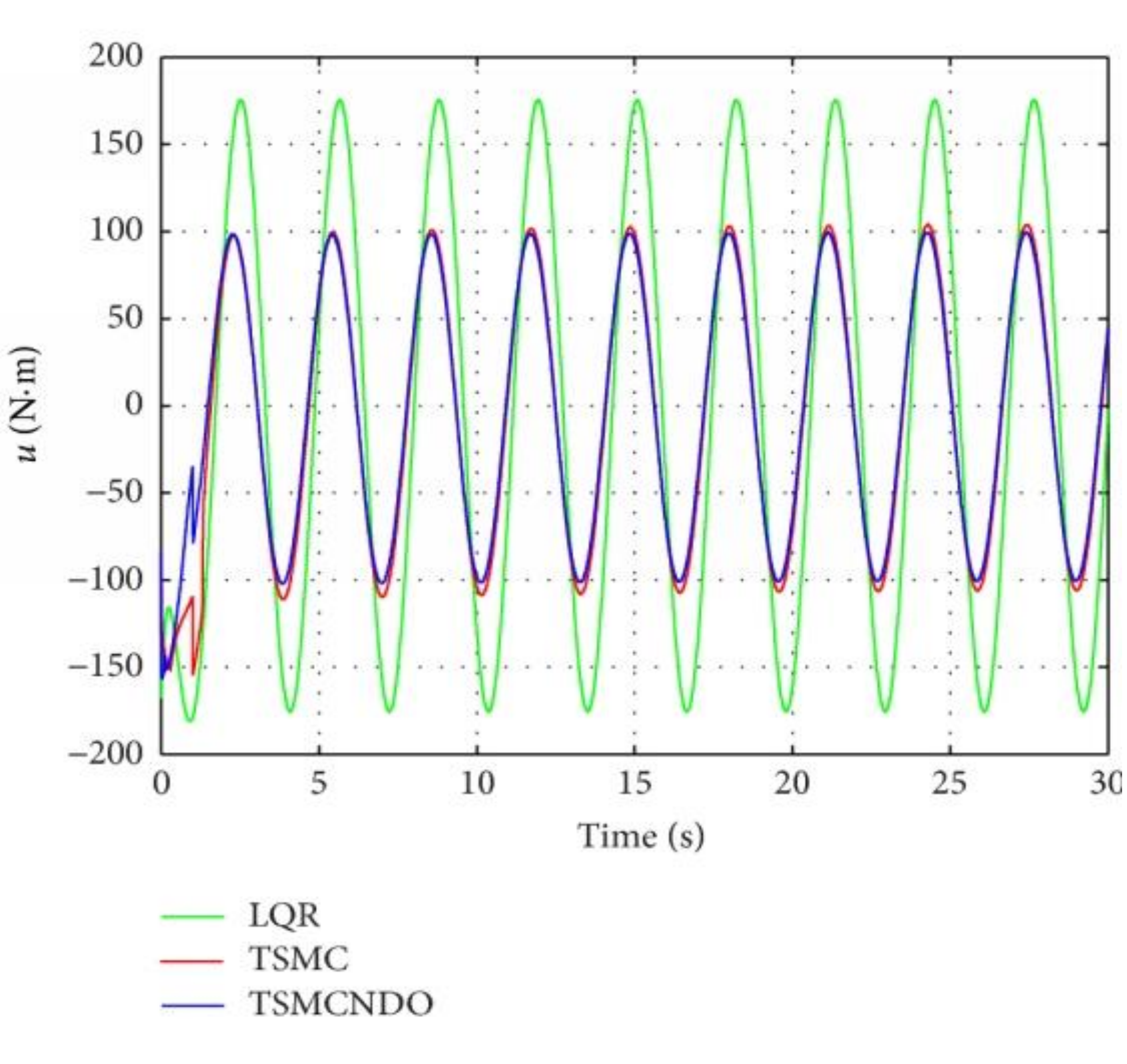
WHEEL ROTATION VELOCITIES

- Inclination Velocity of the MWIP system by employing LQR, TSMC, and TSMCNDO control strategies.
- Initial Condition: $\theta_b(t) = -10°$
- Parameters of SMC: $\varphi_{TSMC} = \varphi_{TSMCNDO} = 0.05$



CONTROL INPUTS

- Control inputs of the MWIP system by employing LQR, TSMC, and TSMCNDO control strategies.
- “The control performance of the MWIP system by using TSMCNDO control strategy is better than the one by using TSMC control strategy”.

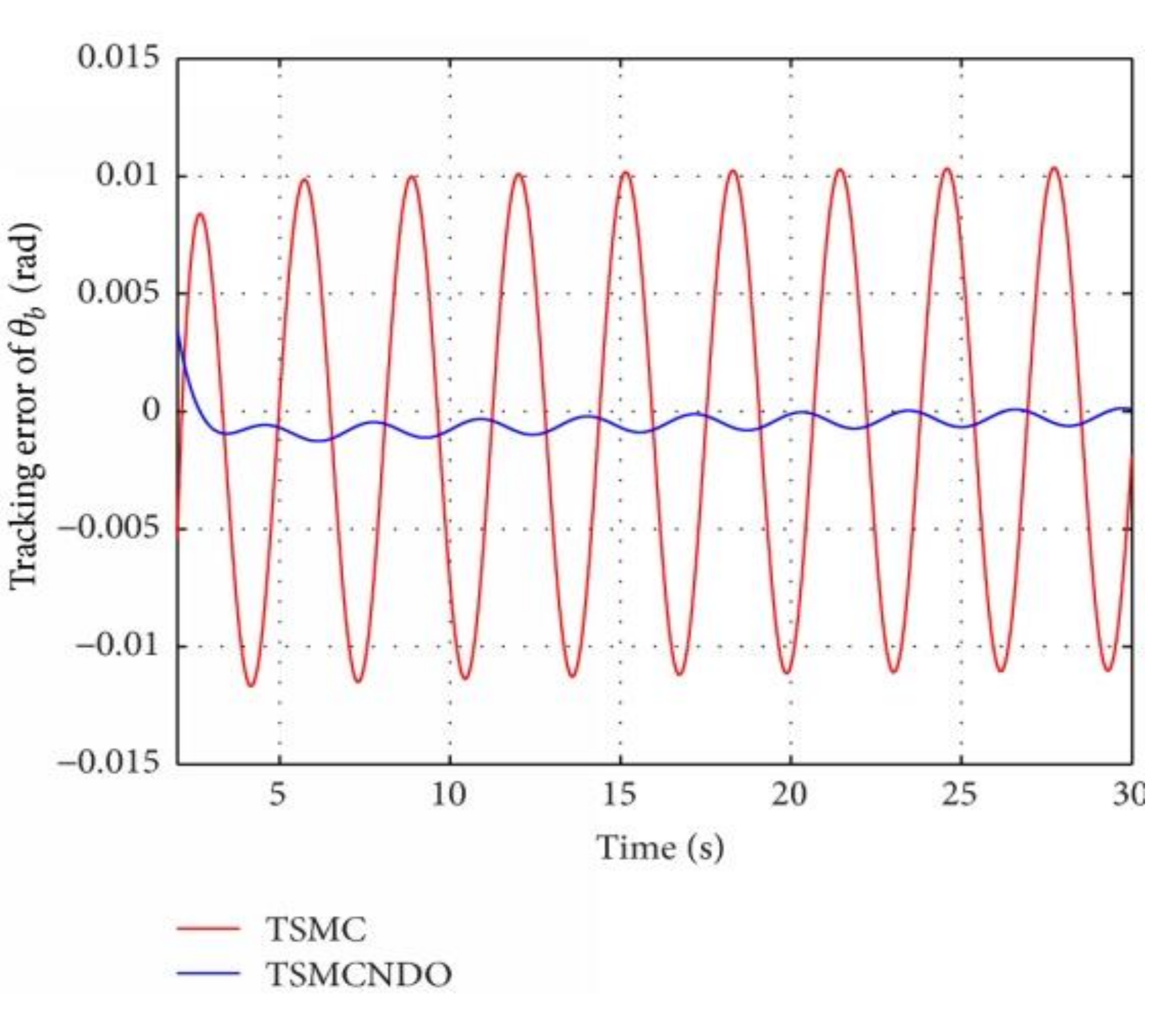


CONCLUSION

Terminal Sliding Mode Control with Disturbance Observer

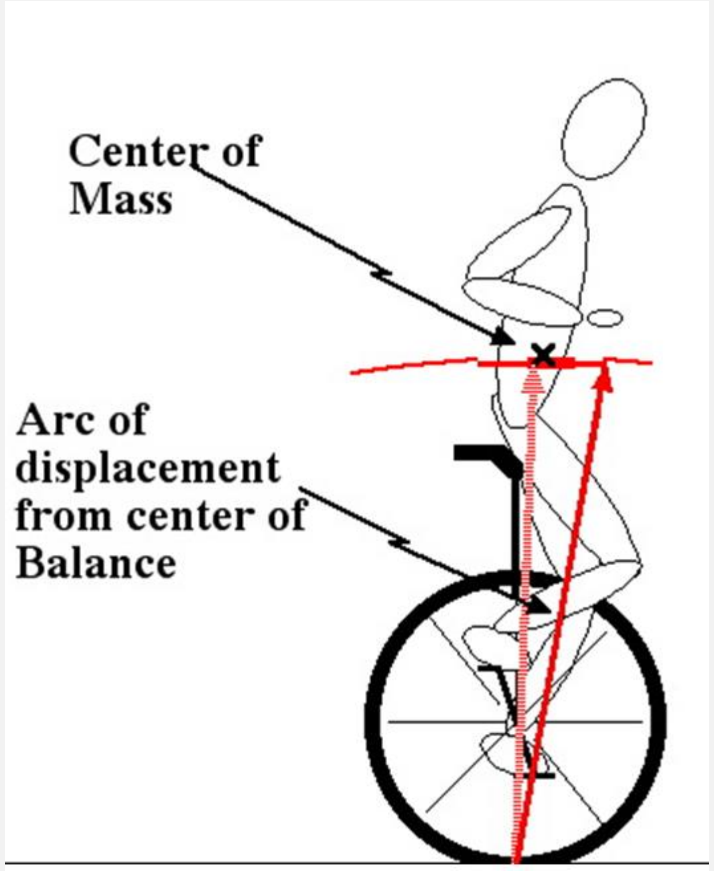
CONCLUSION

- TSMC can deal with modeling uncertainties and external disturbances.
- It guarantees the system trajectory converges in a finite time.
- Remove chattering caused by the SMC.
- Decreased Tracking error.

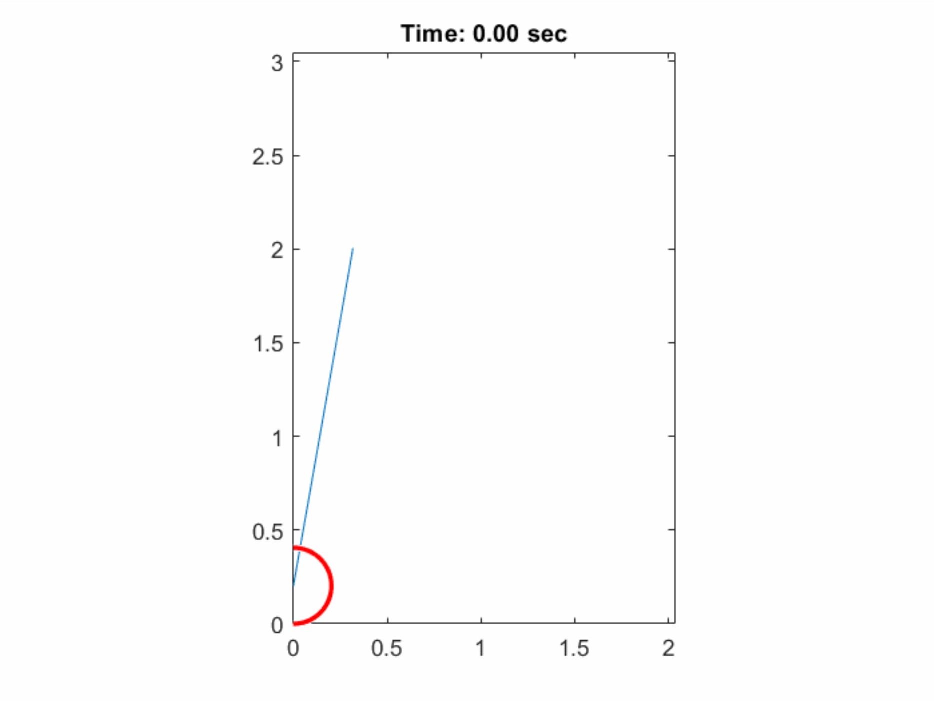


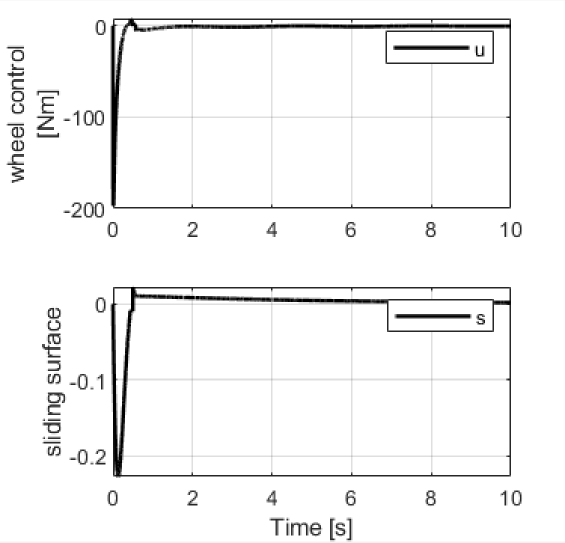
WORKING IN PROGRESS …

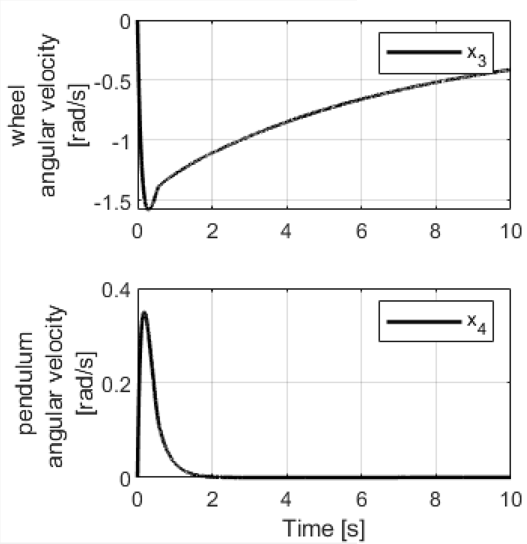
TSMC with Nonlinear Disturbance Observer applied to a Unicycle System



Apply TSMC to a Unicycle system taking in consideration anthropometric data.







WORKING IN PROGRESS…

TSMC with Nonlinear Disturbance Observer applied to a Unicycle System

THANK YOU

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EXTRA REFERENCES

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