Homework 1

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Abstract

1 Question

The circuit shown in the figure below contains a nonlinear inductor and is driven by a time-dependent current source. Suppose that the nonlinear inductor is described by $i_L = I_0 \sin(k\phi_L)$, where ϕ_L is the magnet flux of the inductor and I_0 and k are constants. Using ϕ_L and v_C as state variables, find the state equations.

2 Question

Use Matlab/Simulink to simulate the stable electronic oscillator in Example 8 in Lecture 1. Choose two sets of initial conditions that are different from the ones on pages 28-30 in this lecture, and produce the phase plane (or XY plane) plots and plot output responses with the various initial conditions. In your simulation, please choose $A = 1.5$, $V_1 = V_2 = 1$, $L = 1$, $C = 1F$, and $R = 0.1\Omega$.

3 Question

For the following system, find the equilibrium points and determine the type of each isolated equilibrium point:

$$\begin{aligned} \dot{x}_1 &= 2x_1 - x_1x_2 \\ \dot{x}_2 &= 2x_1^2 - x_2 \end{aligned}$$

By definition, the following equation must hold:

$$\begin{aligned} 0 &= 2x_1 - x_1x_2 \quad (1) \\ 0 &= 2x_1^2 - x_2 \quad (2) \end{aligned}$$

Replacing Equation 2 in Equation 1, we have:

$$\begin{aligned} 0 &= 2x_1 - 2x_1^3 \\ 0 &= 2x_1(1 - x_1^2) \end{aligned}$$

Then, there are three solutions for x_1 and x_2 where $x_1 = [0\ -1\ 1]$.

$$\begin{pmatrix} x_{1e} \\ x_{2e} \end{pmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \tag{3}$$

4 Question

By plotting trajectories starting at different initial conditions, draw the phase portrait of the following LTI systems:

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = -10x_1 - 10x_2 \tag{5}$$

5 Question

The phase portrait (or phase-plane plot) of the following system is shown below. Mark the arrowheads and discuss the stability of each isolated equilibrium point

$$\dot{x}_1 = x_2 \tag{6}$$

$$\dot{x}_2 = x_1 - 2\tan^{-1}(x_1 + x_2) \tag{7}$$