EEC 643/743/ESC794: Homework 6

Due on April 17, 2019

Please include all the equation development, Matlab codes, Simulink model, and simulation results in your homework. Please have a cover page with "Homework 1" title and your printed name. The problems should be in order and all the pages should be stapled together. Any deviation from the required format will result in a deduction from the homework grade. The homework has to be completed independently and individually. Identical submissions will result in grades of ZERO.

1. Show that the internal dynamics of the following system is unstable using the concept of zero dynamics. The output $y=x_1$.
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2^3 + u \\ -u \end{bmatrix}$$
2. Design a controller for the following system to track a constant reference signal $y_d=1$. Assume that the model is accurate, and the states $[x_1, x_2]^T$ are measurable. Write the full expression of the controller, as a function of the measured states $[x_1, x_2]^T$. Please simulate your control system in Matlab/Simulink and test the effectiveness of your controller design. Plot time response y. Is y driven to y_d in your simulation?
$$\dot{x}_1 = \sin(x_2)$$
$$\dot{x}_2 = x_1^4 \cos(x_2) + u$$
$$y = x_1$$
3. (For EEC743/ESC794 students only) Find the internal dynamics and zero dynamics of the following system where y is output and u is input. Is the zero dynamics stable?
$$\dot{y} + z^3 e^{yz^2} = u$$
$$\ddot{z} - (\dot{y} + y^3)(\dot{z}^4 + 1) + z^5 + yz = 0$$
4. Globally stabilize the following nonlinear system, where u is the control input.
$$\dot{y} + y^2 e^{y^4 z} = u$$
$$\ddot{z} + \dot{z}^2 - z^7 + yz^2 = 0$$
5. The following pendulum system has a control input u which is used to stabilize the angular position (θ) of pendulum at $\delta_1 = \frac{\pi}{2}$.
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\left(\frac{g_0}{l}\right)\sin(x_1 + \delta) - \frac{k_0}{m}x_2 + \frac{1}{ml^2}u$$
In the equations above, $x_1 = \theta - \delta_1$ and $x_2 = \dot{\theta}$. The parameter values are m=0.1, l=1, and k_0=0.02. The controller is designed as $u = -ksgn(a_1 x_1 + x_2)$, where a_1=1 and k=4. We suppose the output $y=\theta$. Simulate the control system and show that the output is driven to $\frac{\pi}{2}$.