Homework 1

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1 Question

The circuit shown in the figure below contains a nonlinear inductor and is driven by a time-dependent current source. Suppose that the nonlinear inductor is described by $i_L = I_0 \sin(k\phi_L)$, where ϕ_L is the magnet flux of the inductor and I_0 and k are constants. Using ϕ_L and v_C as state variables, find the state equations.

Knowing the capacitor current is related to the capacitor voltage by

$$i_c(t) = C\frac{dv_c(t)}{dt} \tag{1}$$

and the inductor voltage is related to the inductor current by

$$v_l(t) = L\frac{di_l(t)}{dt} \tag{2}$$

and after choosing the state variables as v_c and ϕ_l, we can find the state space representation.

Replacing the nonlinear inductor current in equation 2, we have:

$$\begin{aligned} v_l(t) &= L\tfrac{d}{dt}(I_0 \sin(k\phi_l)) \\ &= LkI_0 \cos(k\phi_l)\dot{\phi}_l \end{aligned}$$

as $v_l = v_c = v_R$, we can write the following equation in terms of state variables:

$$v_c(t) = LkI_0 \cos(k\phi_l)\dot{\phi}_l \tag{3}$$

Applying Kirchhoff's current law (KCL) in the circuit, we hav e $i_c = i_s - i_l - i_R$. By replacing the respective quantities:

$$i_c = i_s - I_0 \sin(k\phi_l) - \frac{v_c}{R} \tag{4}$$

Therefore, replacing equation 4 in equation 1:

$$i_s - I_0 \sin(k\phi_l) - \frac{v_c}{R} = C\frac{dv_c(t)}{dt} \tag{5}$$

Finally, after reoordering the equations 3 and 5:

$$\begin{cases} \dot{v}_c = \frac{1}{C}\left(-\frac{v_c}{R} - I_0\sin(k\phi_l) + i_s\right) \\ \dot{\phi}_l = \frac{v_c}{LkI_0\cos(k\phi_l)} \end{cases} \tag{6}$$

2 Question

Use Matlab/Simulink to simulate the stable electronic oscillator in Example 8 in Lecture 1. Choose two sets of initial conditions that are different from the ones on pages 28-30 in this lecture, and produce the phase plane (or XY plane) plots and plot output responses with the various initial conditions. In your simulation, please choose $A = 1.5$, $V_1 = V_2 = 1$, $L = 1$, $C = 1F$, and $R = 0.1\Omega$.

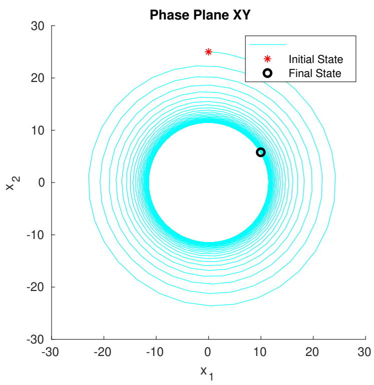

Figure 1: Phase plane for $(0, 30)$.

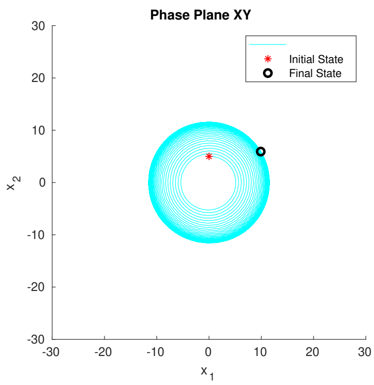

Figure 2: Phase plane for $(0, 5)$.

3 Question

For the following system, find the equilibrium points and determine the type of each isolated equilibrium point:

$$\begin{aligned} \dot{x}_1 &= 2x_1 - x_1x_2 \\ \dot{x}_2 &= 2x_1^2 - x_2 \end{aligned}$$

By definition, the following equation must hold:

$$0 = 2x_1 - x_1x_2 \tag{7}$$
$$0 = 2x_1^2 - x_2 \tag{8}$$

Replacing Equation 8 in Equation 7, we have:

$$\begin{aligned} 0 &= 2x_1 - 2x_1^3 \\ 0 &= 2x_1(1 - x_1^2) \end{aligned}$$

Then, there are three solutions for x_1 and x_2

$$\begin{pmatrix} x_{1e} \\ x_{2e} \end{pmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \quad (9)$$

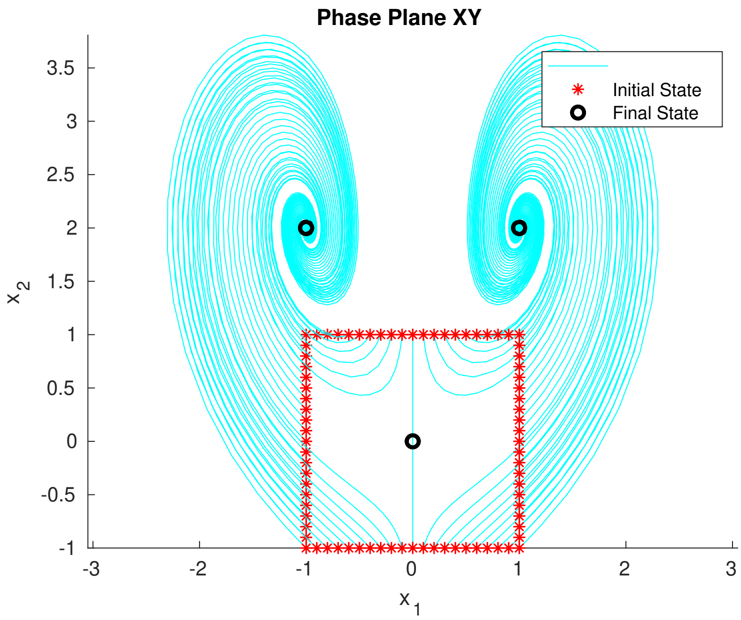

Figure 3: Phase plane for XY.

4 Question

By plotting trajectories starting at different initial conditions, draw the phase portrait of the following LTI systems:

$$\dot{x}_1 = x_2 \quad (10)$$

$$\dot{x}_2 = -10x_1 - 10x_2 \quad (11)$$

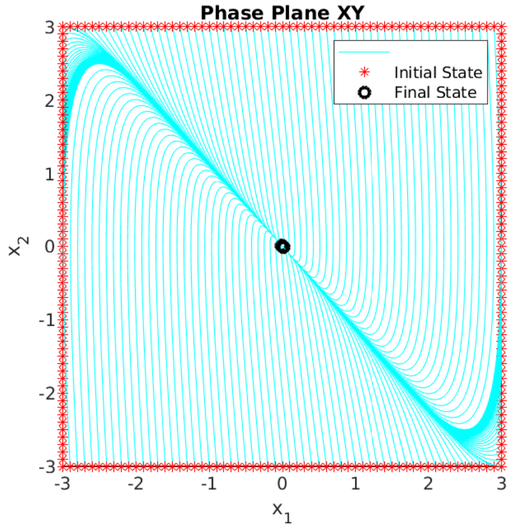

Figure 4: Phase plane for XY.

5 Question

The phase portrait (or phase-plane plot) of the following system is shown below. Mark the arrowheads and discuss the stability of each isolated equilibrium point

$$\dot{x}_1 = x_2 \tag{12}$$

$$\dot{x}_2 = x_1 - 2\tan^{-1}(x_1 + x_2) \tag{13}$$