- **Lyapunov's indirect method for Linear Time-Invariant (LTI) Systems**

Theorem1: The equilibrium of the LTI system $\dot{X} = AX + BU$ is Lyapunov stable if and only if all the eigenvalues of A have non-positive real parts, and those that have zero real parts are non-repeated.

Theorem2: The equilibrium point of the LTI system $\dot{X} = AX + BU$ is asymptotically Lyapunov stable if and only if all the eigenvalues of A have negative real parts.

Example 1: $\dot{X} = \begin{bmatrix} -11 & 0 \\ 0 & -2 \end{bmatrix} X$ is asymptotically stable because the eigenvalues of matrix A are "-11" and " -2".

Example 2: $\dot{X} = \begin{bmatrix} 11 & 0 \\ 0 & -2 \end{bmatrix} X$ is unstable because the eigenvalues of A are "11" and "-2". The positive value of "11" makes the system unstable.

Example 3: $\dot{X} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} X$ is unstable due to the repeated eigenvalues of "0" on imaginary axis.

Example 4: $\dot{X} = \begin{bmatrix} 0 & 0 \\ 0 & -10 \end{bmatrix} X$ is stable in the sense of Lyapunov but not asymptotically stable because one of the eigenvalues of A is "0" which is not negative.

- **Lyapunov's direct method for both linear and nonlinear time-invariant systems**

Lyapunov's direct method is based on the concept of energy and dissipative systems. Physically it means that any isolated system has a certain amount of nonnegative energy. If the system has not input and its energy is always decreasing, we think of it as being stable or asymptotically stable. If the energy increases, we think of it as being unstable. If the energy is constant, the system is Lyapunov stable.

Theorem:

For $\dot{x} = f(x)$ *or* $\dot{x} = Ax$, and f(0)=0, we have the following theorem.

If there is a continuously differentiable, positive definite function V(x) such that $\dot{V}(x) = \frac{d}{dt}V(x) = \frac{\partial V^T}{\partial x}\dot{x} = \frac{\partial V^T}{\partial x} f(x) = -\omega(x)$ is negative semi-definite, then $x_e=0$ is stable in the sense of Lyapunov.

If $\dot{V}(x)$ is negative definite, then $x_e=0$ is asymptotically stable.

If $\dot{V}(x) < 0$ for all $x \neq 0$, and $V(x) \to \infty$ as $\|x\| \to \infty$, then x=0 is globally asymptotically stable.