ESC794: Selected Topics in Engineering Science
Model Predictive Control

Homework 4. Due: 11/01/18 before class. Matlab code must be received by email by 5 PM.

1 Consider a discrete-time linear system with two states and one control:

$$x^+ = Ax + Bu$$

Take $\mathbb{X}$ to be the unit box centered at the origin and $\mathbb{U} = [-1, 1]$.

a. Describe (in words) the geometry of the viable subset of $\mathbb{X}$.

b. Write a Matlab function that receives A and B and plots the boundaries of the viable set.

c. Write a Matlab function that receives A and B and a horizon N and plots the boundaries of the feasible set $\mathbb{X}_N$ corresponding to the terminal set $\mathbb{X}_0$ given by a circle centered at the origin with radius 0.5.

2
Consider the system

$$x^+ = Ax + Bu$$

with `A=[1 0 1;0 0 -1;1 2 1];B=[2 0;-1 0;0 1]`. For $Q = I_3$, $R = I_2$ and $Q_f = 10I_3$, solve the finite-horizon LQR problem using the Riccati backward recursion. Write code to simulate the control system for any desired horizon. Choose a convenient horizon and plot the resulting trajectories as a function of time and in a 3D phase plot.

3
Work out every step of the proof of Theorem 4.3 in the textbook by Grüne and Pannek, finding a justification for each step taken for the case $\lambda = 0$. Then repeat the proof for arbitrary $\lambda \geq 0$ assuming that asymptotic controllability holds with the small control property. Be prepared to discuss your reasoning during class on 11/01.

4 Solve Prob. 3 of Chapter 3 in Grüne and Pannek.

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