Intelligent Control – Interactive Session on Gaussian Process Regression

February 28, 2018

Problem 1:

Consider a pendulum with the equation of motion:

$$\frac{ml^2\ddot{\theta}}{3} + \frac{mgl\cos\theta}{2} = \tau$$

where m is the mass of the pendulum, l is the length, θ is the angle from horizontal, and τ is the torque applied at the base of the pendulum. We are going to do Gaussian process regression with the input $\mathbf{x} = [q \;\; \dot{q} \;\; \ddot{q}]^\top$ and output $y = \tau$.

(a) Get data. Load the file "pendulumdata.mat" into your Matlab workspace. It contains vectors of joint positions, velocities, and accelerations along with torques sampled from the motion of a single pendulum.

(b) Covariance function. Write a Matlab function to compute the covariance of the outputs for two different inputs $\mathbf{x}$ and $\mathbf{x}'$. Use a squared exponential covariance function with the form

$$k(\mathbf{x}, \mathbf{x}') = p_1 e^{\frac{(\mathbf{x}-\mathbf{x}')^\top(\mathbf{x}-\mathbf{x}')}{2p_2^2}}$$

where p_1 and p_2 are hyperparameters to choose.

(c) initial try. Choose $p_1 = 1$, pick a value of p_2 and do GPR using all the data for training. Use each training input as a query and predict the output for each training input. Compute the expected variance of the output at the query point.

(d) cross-validation. Write code to do leave-one-out cross validataion. Play with the value of p_2 to get a smaller mean squared leave-one-out cross validation error.