Example $\quad T(e_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $\quad T(e_2) = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$$\rightarrow \quad A = \left[\begin{array}{c|c} 1 & 1 \\ 1 & -3 \end{array}\right]$$

Transform $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2e_1 + e_2$

Using the definition of T: $\quad T(v) = T(2e_1 + e_2)$

$$= 2T(e_1) + T(e_2) = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

or: $\quad Av = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

New basis: $\quad u_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $\quad u_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$u_1 = -e_1 + e_2$; $\quad u_2 = 2e_1 + 2e_2$

$T(u_1) = T(-e_1 + e_2) = -T(e_1) + T(e_2) = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

$T(u_2) = \dots \quad \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

expressed in e_1, e_2 basis

→ change them to new basis:

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \alpha u_1 + \beta u_2 = \alpha\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \beta\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

→ solving: $\quad \alpha = -2$, $\quad \beta = -1$

$$\rightarrow \begin{bmatrix} 0 \\ 4 \end{bmatrix} = -2u_1 - u_2$$

Similarly: $\quad \begin{bmatrix} 4 \\ -4 \end{bmatrix} = -4u_1 \qquad \rightarrow B = \begin{bmatrix} -2 & -4 \\ -1 & 0 \end{bmatrix}$

Transform v: $\quad \underset{\text{new basis}}{v} = \alpha u_1 + \beta u_2$

→ solving: $\quad \alpha = -1/2$, $\quad \beta = 3/4$

$$\underset{\text{new basis}}{v} = -\tfrac{1}{2} u_1 + \tfrac{3}{4} u_2$$

$$B \underset{\text{new basis}}{v} = \begin{bmatrix} -2 & -4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1/2 \\ 3/4 \end{bmatrix} = \begin{bmatrix} -2 \\ 1/2 \end{bmatrix}$$

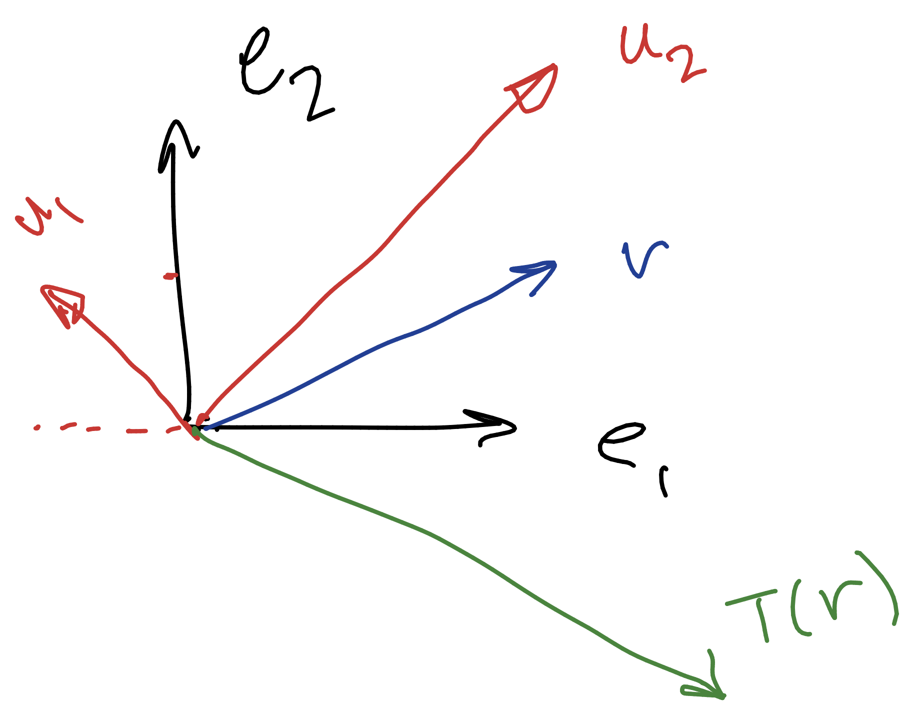
(in the new basis)

Check

$$T(v) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}_{\text{(original basis)}} \longrightarrow \text{new basis:}$$

$$\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \alpha u_1 + \beta u_2 \longrightarrow \text{solving:} \quad \alpha = -2, \quad \beta = 1/2$$

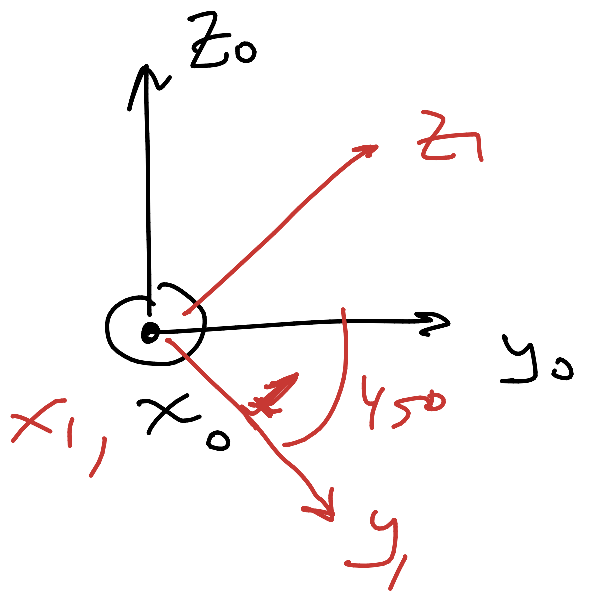
$$\longrightarrow \underset{\text{(new basis)}}{T(v)} = \begin{bmatrix} -2 \\ 1/2 \end{bmatrix}$$



R_a^b : "the rotation matrix to convert points from frame a to frame b"

according to : $p^b = R_a^b p^a$

Example: $-45°$ 3D rotation around x_0 axis



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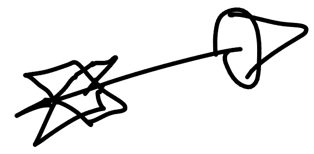

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{1}/2 \end{bmatrix}$$

$$y_1^0 = \begin{bmatrix} 0 \\ +\sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix}$$

$$z_1^0 = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

Example: find z_1^0

$$z_1^0 = R_1^0 z_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

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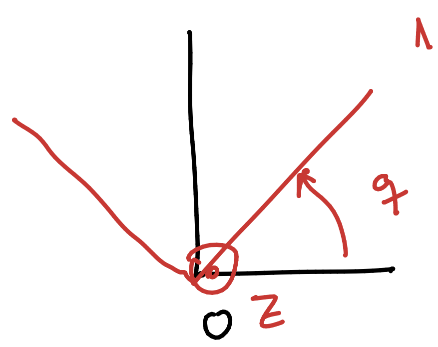


"frame 1 is obtained from frame 0 by rotating q rad about the z-axis"

$$R_1^0 = \begin{bmatrix} \text{fcn. of } q \end{bmatrix}$$

: $\rightarrow$ the matrix to bring points from frame 1 to frame 0

$$p^0 = R_1^0 p^1$$

Prob 2-14 – SHV

$0 \longrightarrow 1$: rot $\pi/2$, x_0

then rot $\pi/2$, y_0

shorthand notation

$\cos\theta_1$: c_1

$\sin\theta_1$: s_1

$\cos(\theta_1+\theta_2)$: c_{12}

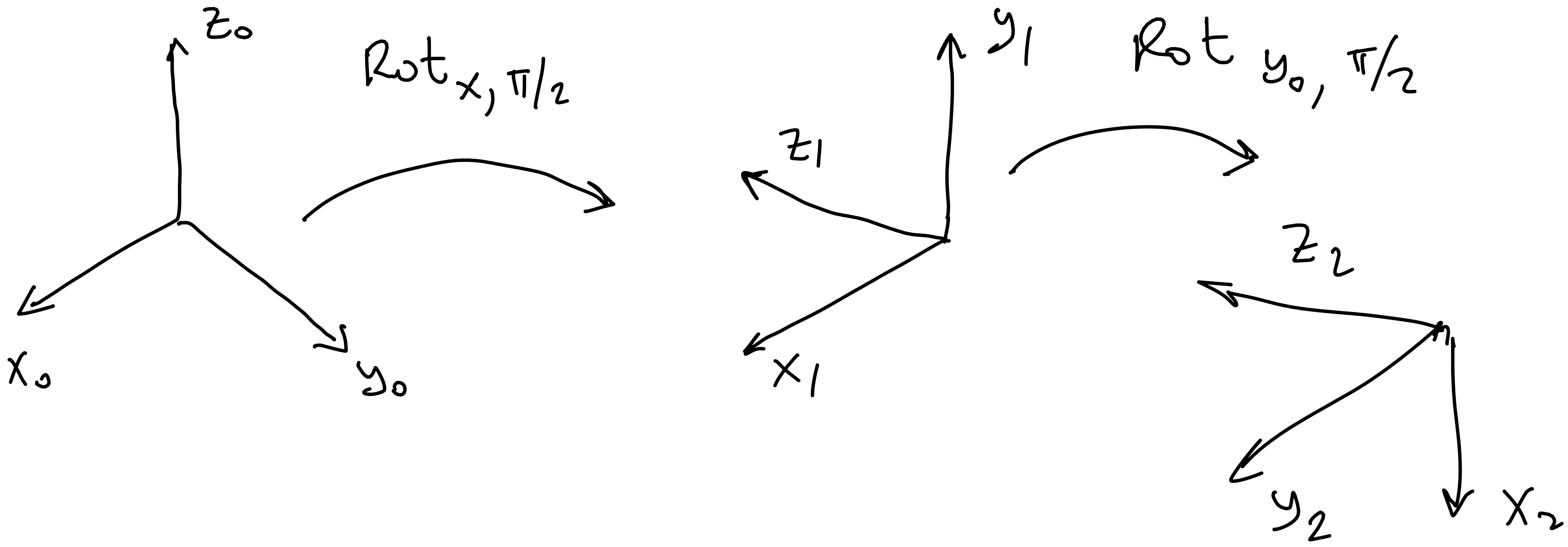
Find $R_1^0 = Rot_{y,\pi/2} \cdot Rot_{x,\pi/2}$

(see formulas 2.2, 2.6, 2.7)

$$Rot_{y,\theta} = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}, \quad Rot_{y,\pi/2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$Rot_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix}, \quad Rot_{x,\pi/2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



Test point : $p^2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$: By inspection

p^0 should be : $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

check :

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \quad \checkmark\checkmark$$

Example 2-8

$0 \to 1$	$Rot_{x_0,\theta}$
$1 \to 2$	$Rot_{z_1,\phi}$
$2 \to 3$	$Rot_{z_0,\alpha}$
$3 - 4$	$Rot_{y_3,\beta}$
$4 - 5$	$Rot_{x_0,\delta}$

B = first three rotations; C = first two rotations; A = first four rotations

Find

$$R_5^0 = Rot_{x_0,\delta} \cdot A$$

$$A = B \cdot Rot_{y_3,\beta}$$

$$B = Rot_{z_0,\alpha} \cdot C$$

$$C = Rot_{x_0,\theta} \cdot Rot_{z_1,\phi}$$