Homework 1 - Robot Dynamics and Control

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1 Article summary

Supernumerary Robotic Limbs for Human Body Support [1]
F. Parietti and H.H. Asada

The journal paper introduces a new wearable device to support a person in dangerous and repetitive tasks, such as, carrying heavy loads in a working environment. Motivation boils down to the fact that there are several activities which is still not easily to be replaced by robots due to the slow learning rate of complex tasks. Therefore, a new device, called *Supernumerary Robotic Limbs* SRL, is proposed and it is characterized as a wearable robot to maximize the human performance. Additionally, a studying of body support stability is presented by take in consideration the stiffness matrix evaluation.

The SRL system illustrated in Fig. 1 consist on: two robotics limbs; a harness to protect the hip bone; and a control unit. The device provides support to the user without constraining its motion with the help of the three links: one prismatic joint and two revolute joints, consequently, three degrees of freedom (DOFs). According to kinematic arrangements of manipulators in [2], the SRL system is observed to be a system with two RRP robot.

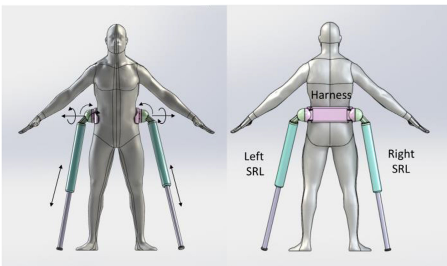


Figure 1: Design concept of SRL.

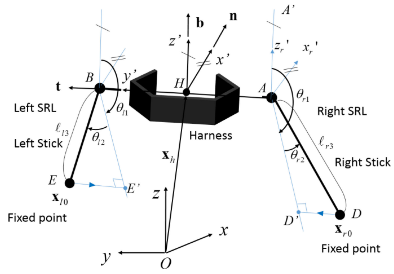


Figure 2: Schematic of the SRL system.

In order to study the support stability, the forward kinematic is presented, in additional, the differentiation of its equation results in Jacobian Matrix. Some assumptions are made by the authors, such as, massless of the SRL links and center of the mass equivalent to the center of the human body in combination with the harness.

The authors conclude the work by provind the SRL design works weel. A prototype was built and tested in diferent working environments. Additionally two control techniques presented and demostrated sucess to stabilize the body support:

- Null-space stabilization using Hessian;
- Joint servo stiffness control based on the Jacobian.

References

[1] F. Parietti and H. H. Asada, "Supernumerary robotic limbs for human body support.," *IEEE Trans. Robotics*, vol. 32, no. 2, pp. 301–311, 2016.

[2] M. W. Spong, S. Hutchinson, M. Vidyasagar, *et al.*, *Robot modeling and control*, vol. 3. Wiley New York, 2006.

2 Question 2

The figure shows two planar manipulators of the RP and PR types. For each, write Matlab code that displays the reachable workspace for a given set of parameters. Show the shape of the reachable workspaces for the following parameter values: $l_1 = l_2 = 1$, $\delta = 0.2$, $d = 0.2$, $h = 0.5$ and $D = 0.75$.

RP robot: The range of motion of the prismatic link is $0 \leq q_2 \leq D$. The range of motion of the revolute joint is limited by interference between the first link and the ground and between the end effector and the ground.

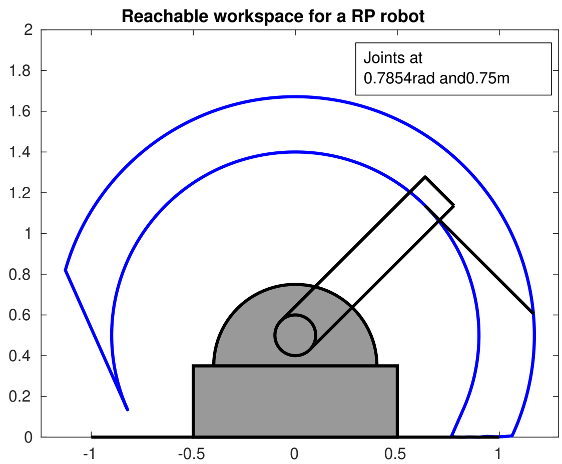


Figure 3: Reachable Workspace.

PR robot: The range of motion of the prismatic link is $0 \leq q_1 \leq D$. The range of motion of the revolute joint is limited only by intereference between the end effector and the ground

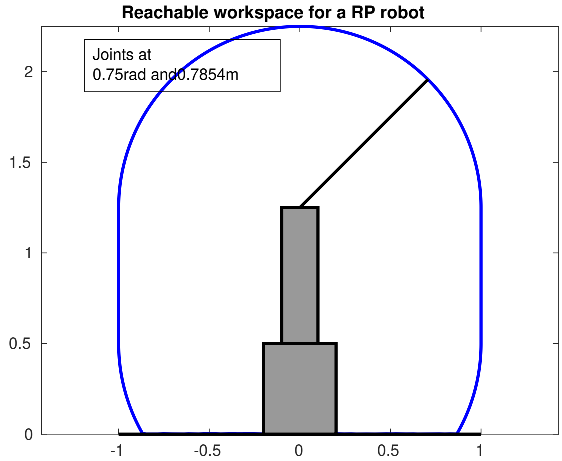


Figure 4: Reachable Workspace

3 Code Instruction

- Download the code (**MAINroboticsHW1.m**) attached in the email.
- Alternatively, the code is avaliable at `https://github.com/EriveltonGualter/MCE747-Robot-Dynamics-and-Control` after submission deadline.
- Run MAINroboticsHW1.m

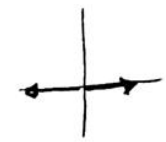
Problems from Section 2.1

(2.1-3) Describe the column space and the nullspace of the matrices:

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Column space of A : $Col(A)$, then

$$Col(A) = span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}$$



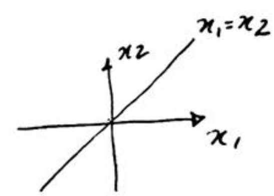
From the linear combination $c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 * \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 - c_2 \\ 0 \end{bmatrix}$

Therefore there is only a dimension which correspond to the abscissa.

In order to find the nullspace of matrix A, it is necessary to find all the vectors x such that $Ax = 0$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{then} \quad x_1 - x_2 = 0$$

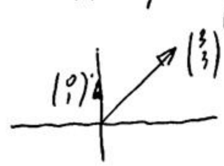
The representation of Nullspace of A is a line



$$B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$C(B) = span \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right\}$$

Since only two vectors out of three are independent, there are two dimensio $C(B) = \mathbb{R}^2$

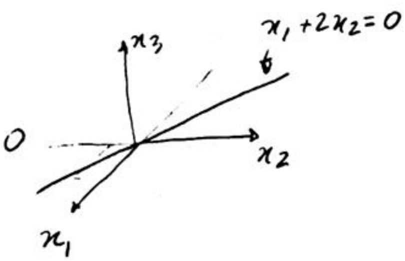


$Bx = 0$

$$\begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_3 \\ x_1 + 2x_2 + 3x_3 \end{bmatrix}$$

Then $x_3 = 0$

$x_1 + 2x_2 = 0$



$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $C(c) = span \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$N(C)$: $C.x = 0$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Then nullspace of C correspond to any combination of x $\mathbb{R}^3$

(2.1-17) The four types of subspaces of $\mathbb{R}^3$ are planes, lines, $\mathbb{R}^3$ itself, or Z containing only (0,0,0)

(a) Describe the three types of subspace of $\mathbb{R}^2$

The first two subspace are stronthforword defined by $\underline{\underline{\mathbb{R}^2}}$ itself and the smallest subspace of $\mathbb{R}^2$: $\underline{\underline{(0,0)}}$. (which is trivial).

The last one is a $\underline{\underline{\text{line which passes through the Origin in } \mathbb{R}^2}}$.

It boilus down to the fact that any other line which does not passes through the origin, does not satisfy the "scaling" property.

(b) Describe the five types of subspaces of $\mathbb{R}^4$.

- $\mathbb{R}^4$ itself
- (0,0,0,0)
-

(2.1-18) For $Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ then $\begin{matrix} x_1 + x_2 + x_3 = 0 \\ x_2 - x_3 = 0 \end{matrix}$ Therefore it is a straight line in $\mathbb{R}^3$

Additionally, the nullspace of (A): $N(A)$ is $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \alpha$

Problem Set 2.3

(2.3-2) Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Note: If $c_1v_1 + c_2v_2 + \cdots + c_Kv_K = 0$ and the only solution is $c_1 = c_2 = \cdots = v_K = 0$, then $v_1, v_2, \cdots v_K$ are linearly independent, otherwise linear dependent.

one solution is to find the rank of $\left[v_1 | v_2 | v_3 | v_4 | v_5 | v_6 \right]$,

So For $V = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$ we have a echelon form: $\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Since rank(V) = 3 we have <u>3 independent vectors.</u> Note that the ~~rank~~ echelon form of V do not have leading 1's on columns 4, 5 and 6. It means the vectors v_4, v_5 and v_6 are dependent on the vectors v_1, v_2 and v_3.

(2.3-20) Find a basis for each of these subspace of $\mathbb{R}^4$

(a) All vectors whose components are equal.

Considering the following subspace $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$
So in order to these components became equal, we have:

$x_1 = x_2 = x_3 = x_4 = \alpha$ Ans: $(\alpha, \alpha, \alpha, \alpha)$ For $\forall \alpha$

example: $(2,2,2,2)$ or $(1,1,1,1)$

(b) All vectors whose components add to zero

Then $x_1 + x_2 + x_3 + x_4 = 0$ For a subspace (x_1, x_2, x_3, x_4)

Example: $(0,0,0,0)$
$(1,-1,1,-1)$ or any combination which respect
$x_1 + x_2 + x_3 + x_4 = 0$

(c) All vectors that are perpendicular to $(1,1,0,0)$ and $(1,0,1,1)$

For $A=(1,1,0,0)$ and $B=(1,0,1,1)$ Also Knowing the vector $X=(x_1, x_2, x_3, x_4)$. the following must be true $X^TA=0$ (to be perpendicular, also known orthogonal)

So:
$$(x_1 \ x_2 \ x_3 \ x_4)\begin{pmatrix}1\\1\\0\\0\end{pmatrix}=0 \qquad x_1+x_2=0$$

$$(x_1 \ x_2 \ x_3 \ x_4)\begin{pmatrix}1\\0\\1\\1\end{pmatrix}=0 \qquad x_1+x_3+x_4=0$$

Then $x_1=-x_2$ and $x_1=-x_3+x_4$ Therefore for $x_1=\alpha$

we know $(x_1, x_2, x_3, x_4) \Rightarrow (\alpha, -\alpha, x_3, x_4)$ where $x_3+x_4=-\alpha$

Example: $(1,-1,0,-1)$ $\begin{cases}1-1=0\\1+0-1=0\end{cases}$ ✓

or

$(1,-1,-1,0)$ $\begin{cases}1-1=0\\1-1+0=0\end{cases}$ ✓

(d) The column space (in $\mathbb{R}^2$) and nullspace (in $\mathbb{R}^5$) of $U=\begin{bmatrix}1&0&1&0&1\\0&1&0&1&0\end{bmatrix}$

$C(U)=\text{span}\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix}\right\}$ since we have only two vectors independent $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$

$N(U) \Rightarrow \begin{bmatrix}1&0&1&0&1\\0&1&0&1&0\end{bmatrix}$

(2.3-31) Find a counterexample to the following statement:
If v_1, v_2, v_3, v_4 is a basis for the vector space $\mathbb{R}^4$, and if W is a subspace, then some subset of the v's is a basis for W.

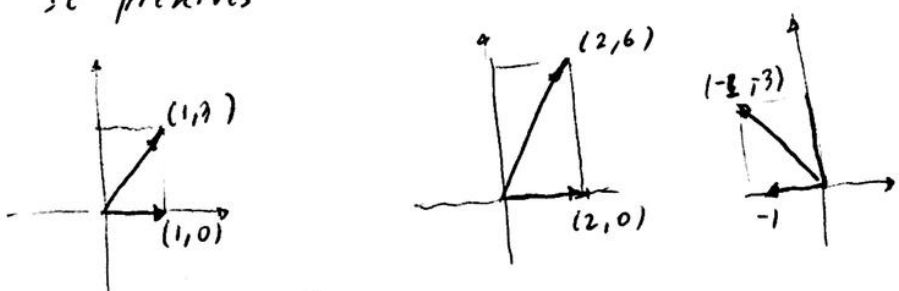
In order to v_1, v_2, v_3, v_4 to be a basis of $\mathbb{R}^4$, the rank of A must be 4, where, $A = [v_1\ v_2\ v_3\ v_4]$. So, for $\vec{v_1} = [1\ 0\ 0\ 0]^T$, $v_2 = [0\ 1\ 0\ 0]^T$, $v_3 = [0\ 0\ 1\ 0]^T$ and $v_4 = [0\ 0\ 0\ 1]^T$, we have $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ as basis for $\mathbb{R}^4$.
Since W is a subset of $\mathbb{R}^4$, it means superposition and scaling must hold.
Therefore, any combination of v_1, v_2, v_3, v_4 can be a basis for W, since they are independent linearly $\perp$

Problem set 2.6

(2.6-5) The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ yields a shearing transformation, which leaves the y-axis unchanged. Sketch its effect on the x-axis, by indicating what happens to (1,0) and (2,0) and (-1,0) - and how the whole axis is transformed.

$$T: \mathbb{R}^2 \to \mathbb{R}^2 : \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 3x+y \end{bmatrix}$$

It preserves the x-axis unchanged.



ex.
$$\begin{bmatrix} x \\ 3x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

(2.6-6) What 3 by 3 matrices represent the transformations that

(a) project every vector onto the x-y plane

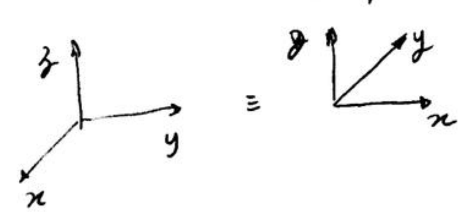
For a vector in the space (x, y, z) we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

(b) reflect every vector through the x-y plane

Transformation is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

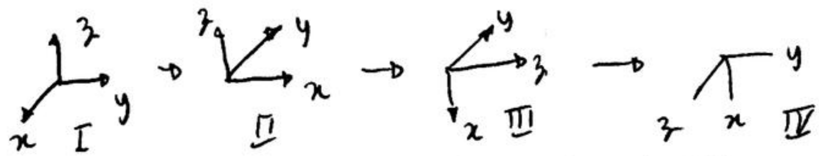
(c) rotate the x-y plane through $90°$, leaving the z-axis alone?



$y \Rightarrow x$
$x \Rightarrow -y$

Then: $T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) rotate the xy plane, then xz, then yz, through $90°$?

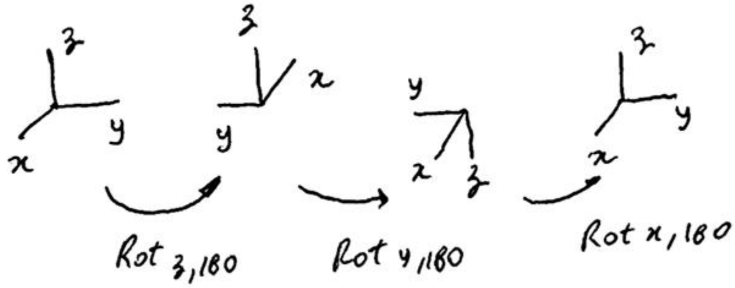


It can be done by relating I and IV, so $T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

or step by step

$$\underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathrm{I} \to \mathrm{II}} \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}}_{\mathrm{II} \to \mathrm{III}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}_{\mathrm{III} \to \mathrm{IV}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

(c) Carry out the same three rotations, but each one through 180°?



$T = Rot_{z,180}\, Rot_{y,180}\, Rot_{x,180}$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.6-7 On the space P_3 of cubic polynomials, what matrix represent d^2/dt^2? Construct the 4 by 4 matrix from the standard basis $1, t, t^2, t^3$. Find its nullspace and column space. What do they mean in terms of polynomials:

From $\{1, t, t^2, t^3\}$ we have $\{0, 0, 2, 6t\}$ after derivation.

So we can represent $\{0, 0, 2, 6t\}$ as $\underbrace{\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A}$

Then $N(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} = \left\{ x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

It means a linear function (or 1° degree polynomial)

$C(A)$: Column space

According to A matrix, the column space is equal to the nullspace.

$C(A) = N(A)$: Linear function