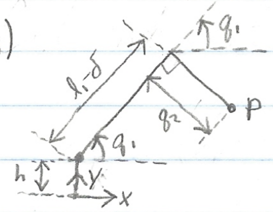
2. (a)

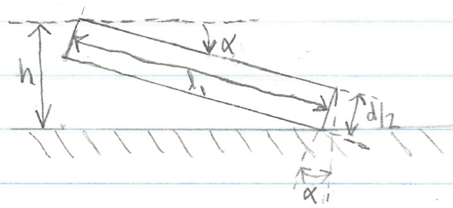


Locate endpoint P in given coordinate system.

$$\begin{aligned} x_p &= (l_1-\delta)\cos(q_1) + q_2\cos(-(\pi-\pi/2-q_1)) \\ &= (l_1-\delta)\cos(q_1) + q_2\cos(q_1-\pi/2) \\ &= (l_1-\delta)\cos(q_1) + q_2\sin(q_1) \end{aligned}$$

$$\begin{aligned} y_p &= h+(l_1-\delta)\sin(q_1) + q_2\sin(-(\pi-\pi/2-q_1)) \\ &= h+(l_1-\delta)\sin(q_1) + q_2\sin(q_1-\pi/2) \\ &= h+(l_1-\delta)\sin(q_1) - q_2\cos(q_1) \end{aligned}$$

Determine q_1 useable range.



$l_1\sin(\alpha) + d/2\cos(\alpha) = h$ at the limiting position

The left position is symmetric to this one.

$q_1 \in [-\alpha, \alpha+\pi]$, solve α from above equation numerically

q_2 useable range: $q_2 \in [d/2, D]$

Algorithm:

Select parameters: h, d, σ, D, and l_1

Solve nonlinear equation $l_1 \sin(\alpha) + d/2 \cos(\alpha) - h = 0$

Range of q_1: $[-\alpha, \alpha+\pi]$

Divide q_1 range into many points q_{1i}, $i = 1, 2, \ldots, N$

For each q_{1i}

    Range of q_2: $[d/2, D]$

    Divide q_2 range into many points q_{2j}, $j = 1, 2, \ldots, M$

    For each q_{2j}

        Calculate x_p and y_p

        If $y_p \geq 0$

            (x_p, y_p) belongs to reachable workspace, plot point

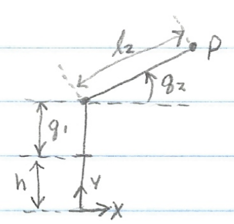
        (end if)

    (end for)

(end for)

See MATLAB m-file for implementation

(b)



Locate endpoint P in given coordinate system

$$x_p = l_2 \cos(q_2)$$
$$y_p = h + q_1 + l_2 \sin(q_2)$$

q_1 useable range: $q_1 \in [0, D]$

q_2 useable range: $[-\pi/2, 3\pi/2]$

Algorithm:

Select parameters: h, l_2, and D

Range of q_1: $[0, D]$

Divide q_1 range into many points q_{1i}, $i = 1, 2, \ldots, N$

For each q_{1i}

  Range of q_2: $[-\pi/2, 3\pi/2]$

  Divide q_2 range into many points q_{2j}, $j = 1, 2, \ldots, M$

  For each q_{2j}

    Calculate x_p and y_p

    If $y_p \geq 0$

      (x_p, y_p) belongs to reachable workspace, plot point

    (end if)

  (end for)

(end for)

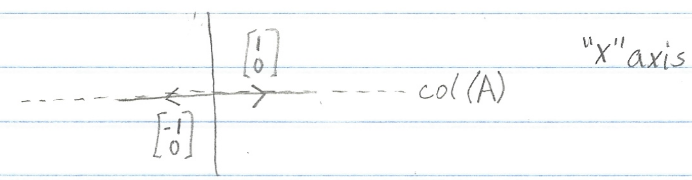
See MATLAB m-file for implementation

3. Set 2.1, ex. 3

Describe the column space and nullspace of:

i) $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

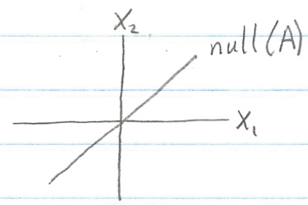
Column space $= \text{span}\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right\}$, dimension 1



Nullspace = set of vectors $v \in \mathbb{R}^2$ such that $Av = 0$

Let $v = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $Av = x_1 - x_2 = 0$

$x_2 = x_1$, dimension 1

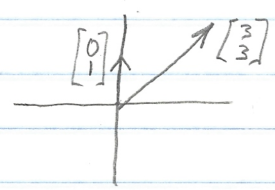


ii) $B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

one pair is independent

Column space $= \text{span}\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix}\right\}$, dimension 2

dependent

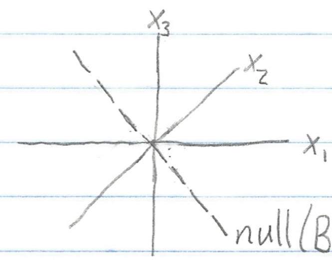


$col(B) = \mathbb{R}^2$

Nullspace:

Let $v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $Bv = \begin{bmatrix} 3x_3 \\ x_1 + 2x_2 + 3x_3 \end{bmatrix} = 0$

$x_3 = 0$, $x_1 + 2x_2 = 0 \rightarrow x_2 = -\frac{1}{2}x_1$



iii) $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Column space $= \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Nullspace $= \mathbb{R}^3$, for every vector $v \in \mathbb{R}^3$; $Cv = 0$

Set 2.1, ex. 8

The solutions of $Ax = 0$ are the nullspace of A by definition. The nullspace is a specific subspace.

Evaluate $Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 + x_2 + x_3 = 0$, $x_1 + 2x_3 = 0 \rightarrow x_1 = -2x_3$

$\rightarrow -2x_3 + x_2 + x_3 = 0$
$x_2 = x_3$

Nullspace is a line defined by $\{x_1 = -2x_3, x_2 = x_3\}$

(b), (d), and (e) are correct

Set 2.1, ex. 17

(a) The three types of subspaces of $\mathbb{R}^2$ are lines through $(0,0)$ $\mathbb{R}^2$ itself and $\mathbb{Z}$, which contains $(0,0)$ alone.

(b) The five types of subspaces of $\mathbb{R}^4$ are lines through $(0,0,0,0)$, $\mathbb{R}^4$ itself, planes and hyperplanes through the origin, and $\mathbb{Z}$, which contains $(0,0,0,0)$ alone.

Set 2.3, ex. 2

Use echelon form to identify the number of independent vectors.

$[v_3\ v_1\ v_2\ v_6\ v_4\ v_5]$, ordered for convenience

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \xrightarrow{r_4+1r_1} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & -1 \end{bmatrix} \xrightarrow{r_4+1r_2}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix} \xrightarrow{r_4+1r_3} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The first three vectors (v_1, v_2, v_3) span the space.

The largest possible number of independent vectors is three.

This number is the <u>dimension</u> of the space spanned by the v's.

Set 2.3, ex. 20 - No basis chosen is unique

(a) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ Independence automatic, vector spans the space

(b) $\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$ $\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$ $\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}$ These vectors are independent and span the space.

(c) Perpendicular requires $[x_1\ x_2\ x_3\ x_4]\begin{bmatrix}1\\1\\0\\0\end{bmatrix}=0$ and $[x_1\, x_2\, x_3\, x_4]\begin{bmatrix}1\\0\\1\\1\end{bmatrix}=0$

$$x_1+x_2=0\ ;\quad x_1+x_3+x_4=0$$

$\begin{bmatrix}1\\-1\\0\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\-1\\-1\\0\end{bmatrix}$ work. They are independent and span the space.

(d) column space (U) = span $\left\{\underbrace{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}}_{\text{independent}},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix}\right\}$

basis can be chosen as $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$

nullspace $(U) = Ux = \begin{bmatrix}1&0&1&0&1\\0&1&0&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=0$

$$x_1+x_3+x_5=0 \qquad x_2+x_4=0 \rightarrow x_2=-x_4$$

$\begin{bmatrix}1\\1\\0\\-1\\-1\end{bmatrix}$, $\begin{bmatrix}-1\\0\\1\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\-1\\0\\1\\0\end{bmatrix}$ work. They are independent and span the space.

Set 2.3, ex. 31

Let the v's form the identity matrix:

$$V = [v_1 \ v_2 \ v_3 \ v_4] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find basis for $\underbrace{\text{nullspace}}_{W}$ $\quad Vx = V \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$

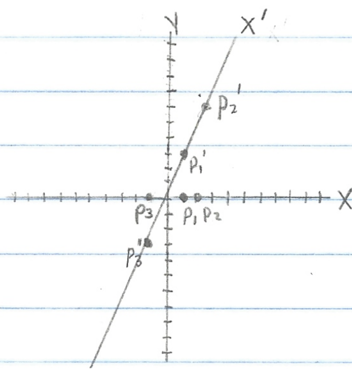
$x_1 = x_2 = x_3 = x_4 = 0 \rightarrow$ basis is zero vector

No subset of the v's can form the zero vector.

Set 2.6, ex. 5

Evaluate the effect of the transformation on $p_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $p_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, and $p_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$:

$p_1' = Ap_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad p_2' = Ap_2 = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \quad p_3' = Ap_3 = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$



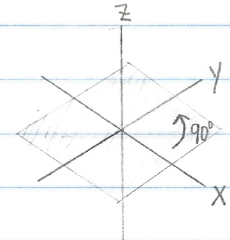
The x-axis is rotated. There is no effect on the y-axis.

Set 2.6, ex. 6

(a) $\begin{bmatrix} & & \\ & A? & \\ & & \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix} \rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c)



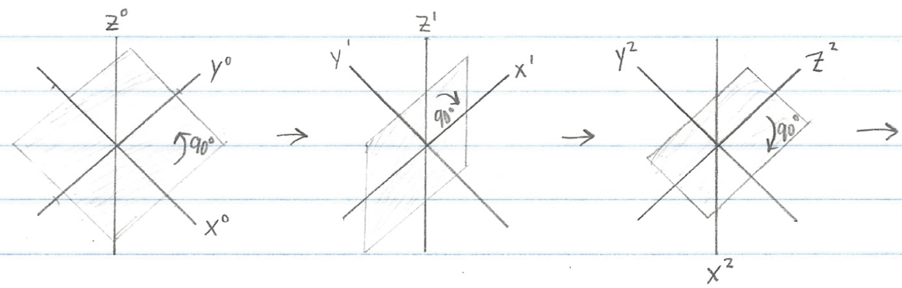
$x^{90°} = -y^{0°}$

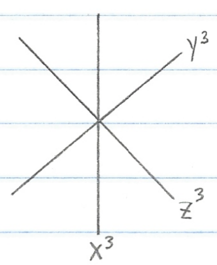
$y^{90°} = x^{0°}$

$z^{90°} = z^{0°}$

$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -y \\ x \\ z \end{bmatrix} \rightarrow A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d)





$\Rightarrow$
$x^3 = x^2 = -z' = -z^0$

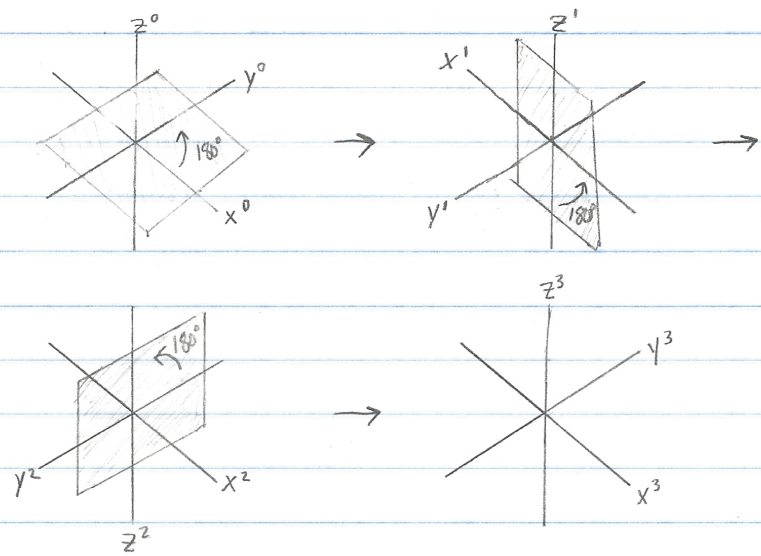
$y^3 = z^2 = x' = y^0$

$z^3 = -y^2 = -y' = x^0$

$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ y \\ x \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(e)



$$x^3 = x^0$$
$$y^3 = y^0$$
$$z^3 = z^0$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Set 2.6, ex. 7

$$p = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \rightarrow [a_0\ a_1\ a_2\ a_3]^T$$
$$p' = a_1 + 2a_2 t + 3a_3 t^2 \rightarrow [a_1\ 2a_2\ 3a_3\ 0]^T$$
$$p'' = 2a_2 + 6a_3 t \rightarrow [2a_2\ 6a_3\ 0\ 0]^T$$

$$A_p = p''$$

$$A \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 2a_2 \\ 6a_3 \\ 0 \\ 0 \end{bmatrix} \rightarrow A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Nullspace: $Aa = 0$

a_0, a_1 arbitrary, $a_2 = a_3 = 0$

Nullspace indicates a linear polynomial. Then the second derivative is zero.

Column space = span $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 0 \\ 0 \end{bmatrix} \right\}$

$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

The second derivative in P_3 contains only a_0 and a_1 terms. It is a line.