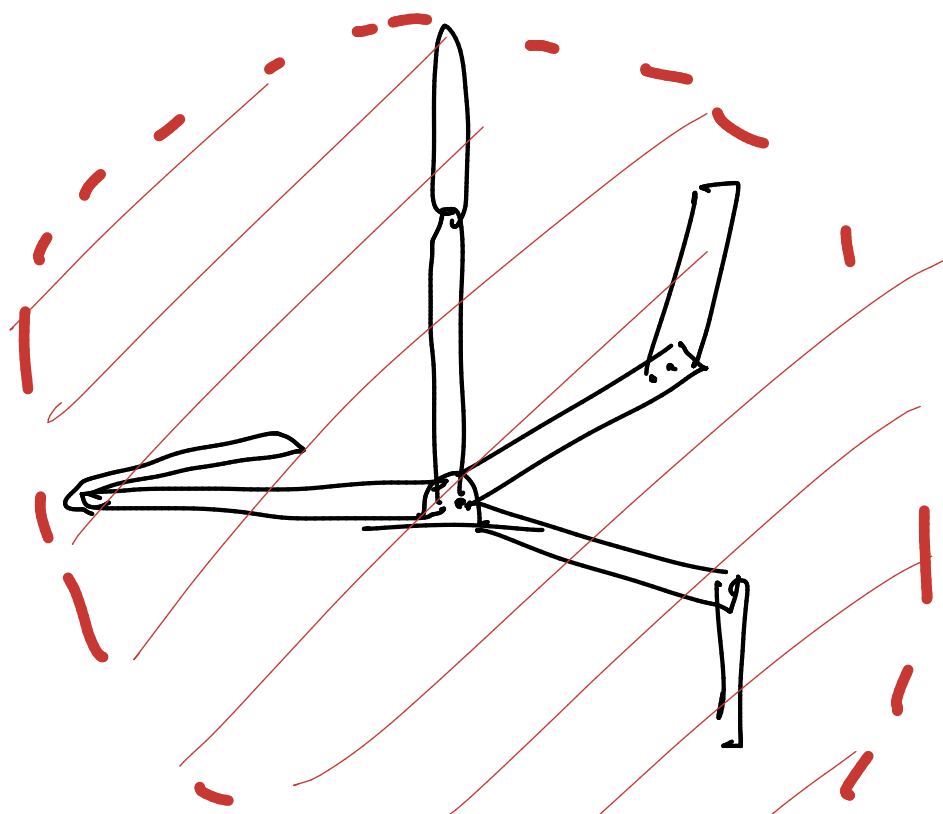
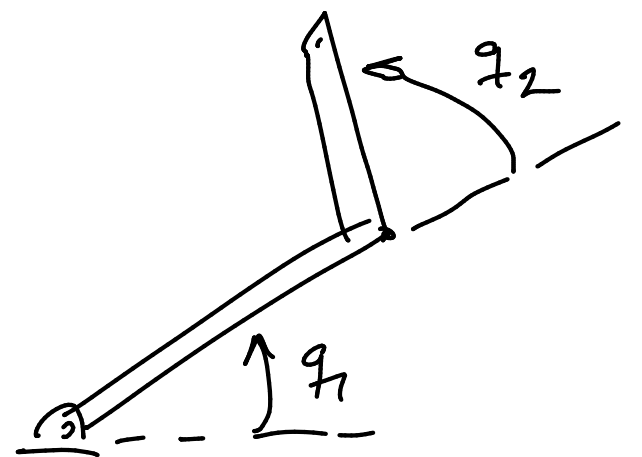


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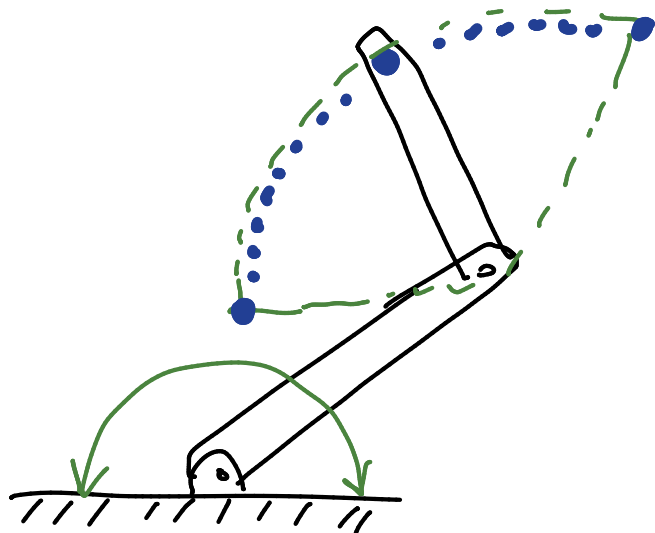


configuration space  
(mathematical concept)

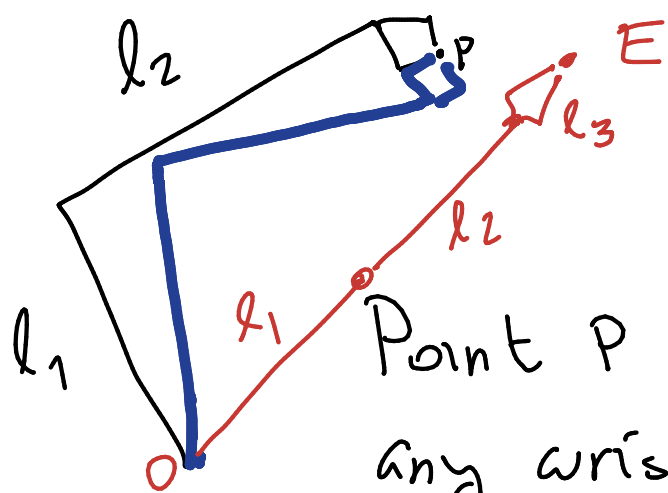
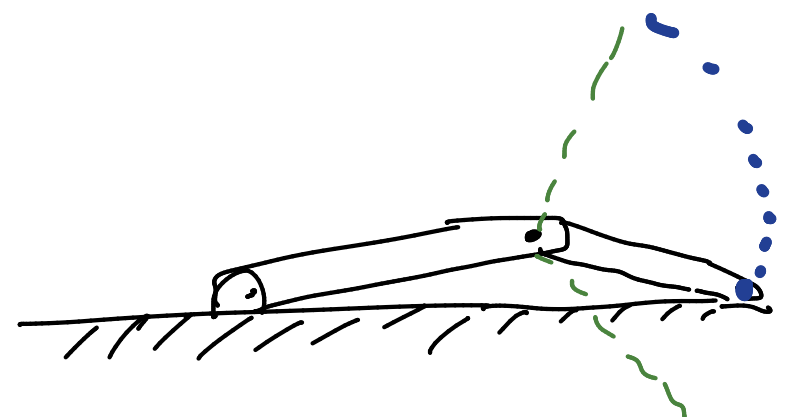


state:  $\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$  .... (where)  
.. -- (where next)

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$



(Reachable)  
Workspace



Point P can be reached using  
any wrist angle (within a range) : point P  
belongs to the dexterous workspace

Point E is  $l_1 + l_2 + l_3$  away from O. It can only be reached with zero  
wrist angle (an isolated value of wrist angle) : not a dexterous point

Linearity  $f$  is linear if  $\begin{cases} f(x+y) = f(x) + f(y) \\ f(\alpha x) = \alpha f(x) \end{cases}$

(superposition & scaling)

$f(x) = 3 + 2x$  (scaling doesn't hold —  
 $f$  is not linear)

- Vector space (or linear space): Set of vectors  
and 2 operations:  $\begin{cases} + & \text{(between vectors)} \\ \cdot & \text{multiplication btw. scalar and a vector} \end{cases}$

Example: functions on  $[a, b]$  : vectors

$$\text{Operations: } \begin{cases} (f_1 + f_2)(x) = f_1(x) + f_2(x) \\ (\alpha f)(x) = \alpha f(x) \end{cases}$$

Also: all  $n \times n$  matrices ✓

- Subspace: A linear space  $V$  is a subspace if

$$x \in V, y \in V \implies x + y \in V$$

$$x \in V \implies \alpha x \in V$$

Example: a plane in  $\mathbb{R}^3$

- Span of a set of vectors: set of all linear combinations of the vectors:

$$V = \{v_1, v_2, v_3, \dots, v_n\}$$

$$\text{span}(V) = \{ \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n, v_i \in V, \alpha_i \text{ scalars} \}$$

Example:  $\text{span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\} = \text{a plane in } \mathbb{R}^3$