Homework 1 - Robot Dynamics and Control

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1 Article summary

Supernumerary Robotic Limbs for Human Body Support [1]
F. Parietti and H.H. Asada

The journal paper introduces a new wearable device to support a person in dangerous and repetitive tasks, such as, carrying heavy loads in a working environment. Motivation boils down to the fact that there are several activities which is still not easily to be replaced by robots due to the slow learning rate of complex tasks. Therefore, a new device, called *Supernumerary Robotic Limbs* SRL, is proposed and it is characterized as a wearable robot to maximize the human performance. Additionally, a studying of body support stability is presented by take in consideration the stiffness matrix evaluation.

The SRL system illustrated in Fig. 1 consist on: two robotics limbs; a harness to protect the hip bone; and a control unit. The device provides support to the user without constraining its motion with the help of the three links: one prismatic joint and two revolute joints, consequently, three degrees of freedom (DOFs). According to kinematic arrangements of manipulators in [2], the SRL system is observed to be a system with two RRP robot.

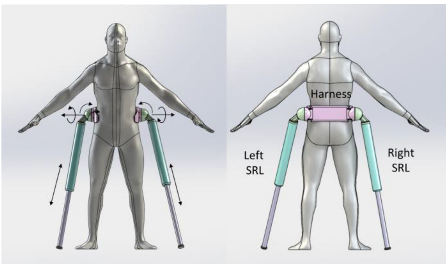

Figure 1: Design concept of SRL.

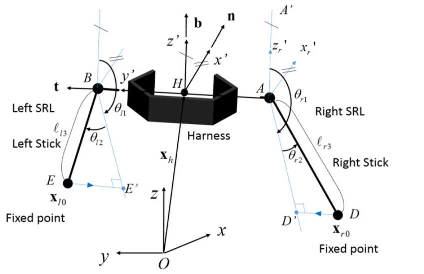

Figure 2: Schematic of the SRL system.

In order to study the support stability, the forward kinematic is presented, in additional, the differentiation of its equation results in Jacobian Matrix. Some assumptions are made by the authors, such as, massless of the SRL links and center of the mass equivalent to the center of the human body in combination with the harness.

References

[1] F. Parietti and H. H. Asada, "Supernumerary robotic limbs for human body support.," *IEEE Trans. Robotics*, vol. 32, no. 2, pp. 301–311, 2016.

[2] M. W. Spong, S. Hutchinson, M. Vidyasagar, *et al.*, *Robot modeling and control*, vol. 3. Wiley New York, 2006.

2 Question 2

The figure shows two planar manipulators of the RP and PR types. For each, write Matlab code that displays the reachable workspace for a given set of parameters. Show the shape of the reachable workspaces for the following parameter values: $l_1 = l_2 = 1$, $\delta = 0.2$, $d = 0.2$, $h = 0.5$ and $D = 0.75$.

RP robot: The range of motion of the prismatic link is $0 \leq q_2 \leq D$. The range of motion of the revolute joint is limited by interference between the first link and the ground and between the end effector and the ground.

PR robot: The range of motion of the prismatic link is $0 \leq q_1 \leq D$. The range of motion of the revolute joint is limited only by intereference between the end effector and the ground

3 Question 3

3. **Set 2.1:** Describe the column space and the nullspace of the matrices.

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \text{and } B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix} \text{and } C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. **Set 2.1:** Which of the following descriptions are correct? The solutions x of form

$$Ax = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

17. **Set 2.1:** The four types of subspaces of $\mathbf{R}^3$ are planes, lines, $\mathbf{R}^3$ itself, or $\mathbf{Z}$ containing only $(0, 0, 0)$.
    (a) Describe the three types of subspaces of $\mathbf{R}^2$.
    (b) Describe the five types of subspaces of $\mathbf{R}^4$.

2. **Set 2.3:** Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

20. **Set 2.3:** Find a basis for each of these subspaces of $\mathbf{R}^4$:

    (a) All vectors whose components are equal.

    (b) All vectors whose components add to zero.

    (c) All vectors that are perpendicular to $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.

    (d) The column space (in $\mathbf{R}^2$) and nullspace (in $\mathbf{R}^5$) of $U = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

31. **Set 2.3:** Find a counterexample to the following statement: If v_1, v_2, v_3, v_4 is a basis for the vector space $\mathbf{R}^4$, and if $\mathbf{W}$ is a subspace, then some subset of the v's is a basis for $\mathbf{W}$.

5. **Set 2.6:** The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ yields a shearing transformation, which leaves the y-axis unchanged. Sketch its effect on the x-axis, by indicating what happens to $(1, 0)$ and $(2, 0)$ and $(1, 0)$ — and how the whole axis is transformed

6. **Set 2.6:** What 3 by 3 matrices represent the transformations that

    - project every vector onto the x-y plane?
    - reflect every vector through the x-y plane?
    - rotate the x-y plane through 90° , leaving the z-axis alone?
    - rotate the x-y plane, then x-z, then y-z, through 90°?
    - carry out the same three rotations, but each one through 180° ?

7. **Set 2.6:** On the space $\mathbf{P}_3$ of cubic polynomials, what matrix represents d_2/d_{t^2}? Construct the 4 by 4 matrix from the standard basis 1, t, t^2 , t^3 . Find its nullspace and column space. What do they mean in terms of polynomials?