Exercises for Lecture Course on Numerical Optimization (NUMOPT)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015-2016

Exercise 10: Inequality Constrained Optimization

(to be completed during exercise session on Jan 20, 2016 or sent by email to dimitris.kouzoupis@imtek.uni-freiburg.de before Jan 22, 2016)

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Aim of this last (and optional) exercise sheet is to teach you the concepts of inequality constrained optimization using CasADi, a powerful optimization tool that might be useful for your projects.

Exercise Tasks

1. **Hanging chain, the last episode:** Recall the optimization problem of the hanging chain with non-convex inequality constraints and without considering a rest length:

$$\begin{aligned}
\underset{y,z}{\text{minimize}} \quad & \frac{1}{2}\sum_{i=1}^{N-1} D\left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2\right) + g_0 \sum_{i=1}^{N} m\, z_i && \text{(1a)}\\
\text{subject to} \quad & (y_1, z_1) = (-2, 1) && \text{(1b)}\\
& (y_N, z_N) = (2, 1) && \text{(1c)}\\
& z_i \geq -y_i^2, \qquad \text{for } i = 1, \ldots, N. && \text{(1d)}
\end{aligned}$$

   (a) Download the latest CasADi binaries (currently v2.4) for your operating system from:

   `http://install.casadi.org`

   Unzip the folder and add it to your MATLAB path.

   (0 bonus points)

   (b) Solve the optimization problem (1) with CasADi and the NLP solver IPOPT (included already in the CasADi binaries). For that purpose, complete the template file `main.m` to build the objective and constraints as CasADi *expressions*. Familiarize with the output of the NLP solver, did the algorithm converge? How many iterations were required?

   (2 bonus points)

   (c) Identify the active set at the solution. Does LICQ hold? Are there any weakly active constraints? *Hint: For the latter you will need to check both constraint values and multipliers.*

   (2 bonus points)

   (d) Evaluate the Hessian of the Lagrangian at the solution. You may first build a CasADi expression for the Lagrangian that you then differentiate twice with respect to the primal variables. *Hint: The linear constraints can be neglected as they do not contribute to the Hessian matrix.*

   (2 bonus points)

   (e) Form the Jacobian of the active constraints at the solution and calculate its null-space using a QR decomposition. Use the result to calculate the reduced Hessian. Is the reduced Hessian positive definite? What does this mean?

   (2 bonus points)

This sheet gives in total 8 bonus points.