The 6th IASTED International Conference on

Modelling, Simulation and Identification

~MSI 2016~

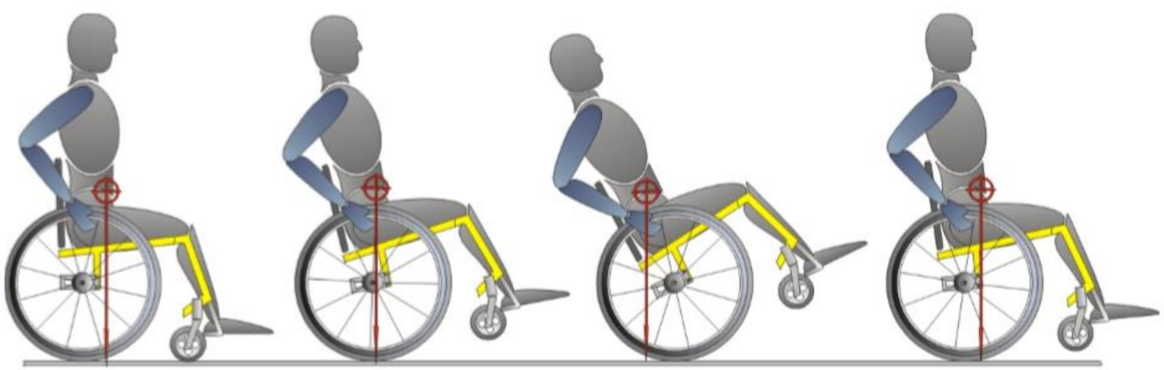
Optimal Control of the Weelchair Wheelie

Erivelton G. Dos Santos, Fabrizio Leonardi, Marko Ackermann

centro universitário FEI

Introduction

Wheelie in wheelchairs



(Denison, 2013)

Introduction

Manual

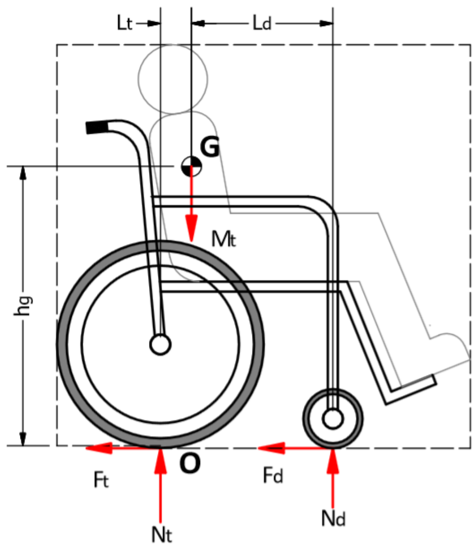
Motorized

Power-Assisted

Objective

Propose a control strategy to initiate and sustain the wheelie in power-assisted wheelchairs using a model of the user and wheelchair system and an optimal control formulation.

Model of Phase 1



- Sagittal plane (2D);
- 2 rigid bodies;
- Front wheels on the ground (stable);
- 1 DoF;
- Control: wheel torque
- Equation of motion:

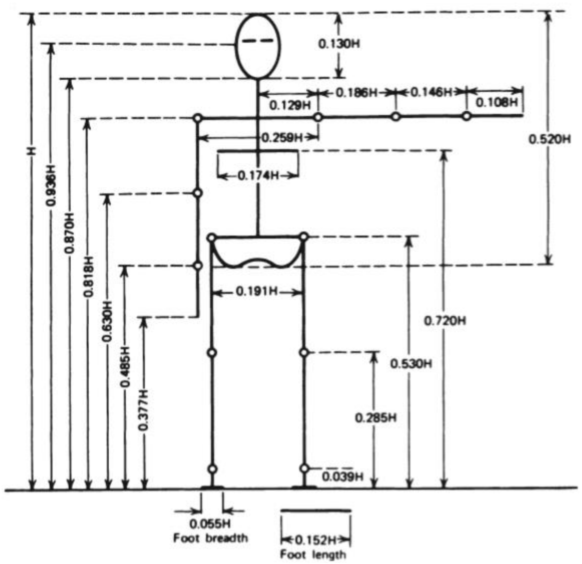
$$\frac{\tau}{R} = \left(M + \frac{J_R}{R^2}\right) \cdot \ddot{x} + F_R$$

- Transition acceleration:

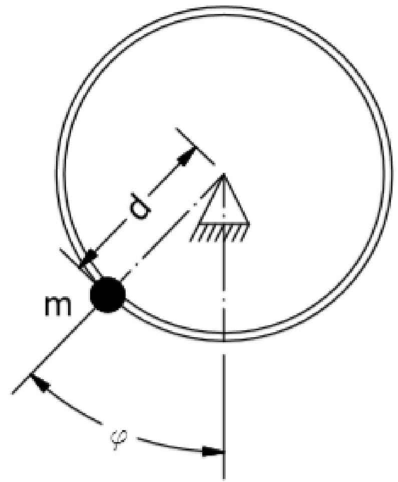
$$\ddot{x}_{nf} = \frac{d_{xcg}}{h_{cg}} \cdot g$$

Estimation of Parameters

Center of mass locations



(Winter, 2009)



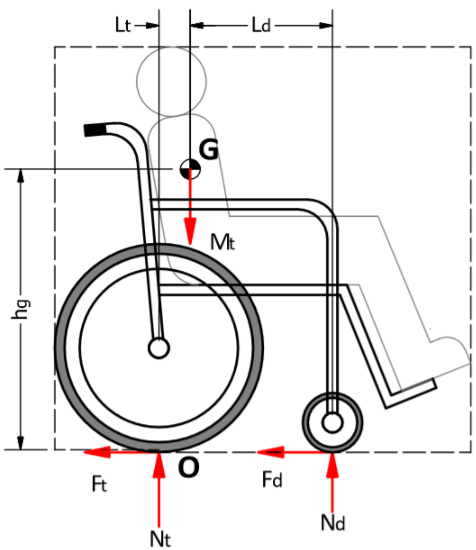
Moments of inertia

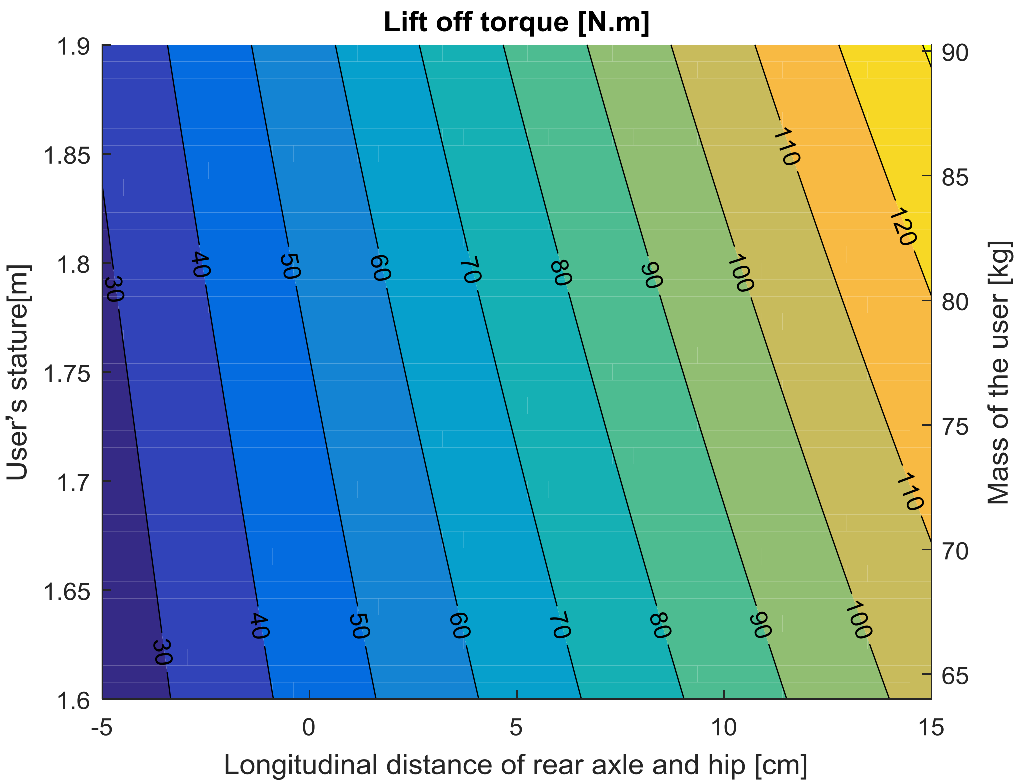
Lift-Off Torque

$$\frac{\tau}{R} = \left(M + \frac{J_R}{R^2}\right) \cdot \ddot{x} + F_R$$

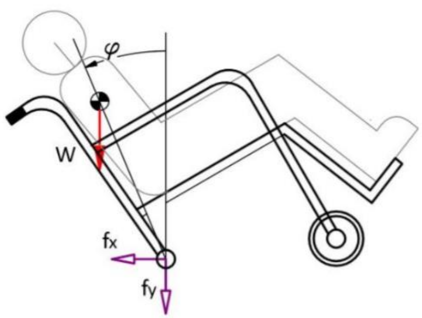
$$\ddot{x}_{nf} = \frac{d_{xcg}}{h_{cg}} \cdot g$$

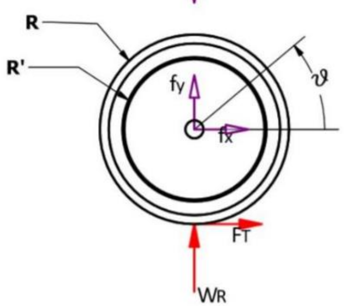
$$\tau_{nf} = \left[\left(M + \frac{J_R}{R^2}\right) \cdot \frac{d_{xcg}}{h_{cg}} \cdot g + F_R\right] \cdot R$$





Model of Phase 2





- Sagittal plane (2D);
- 2 rigid bodies;
- 2 DoF’s;
- Front wheels off the ground (unstable);
- Control: wheel torque;
- Equations of motion:

$$\tau - F_R \cdot R = \left[J_R + (M_r + M_c) \cdot R^2\right] \cdot \ddot{\theta} + (M_c \cdot R \cdot l \cdot \cos\varphi) \cdot \ddot{\varphi}$$
$$-M_c \cdot R \cdot l \cdot \dot{\varphi}^2 \cdot \sin\varphi$$
$$-\tau = (M_c \cdot l \cdot R \cdot \cos\varphi) \cdot \ddot{\theta} + (J_c + M_c \cdot l^2) \cdot \ddot{\varphi} - M_c \cdot g \cdot l \cdot \sin\varphi$$

Optimal Control Formulation

Find (optimization variables):

- $\tau(t)$ – wheel torque
- $x(t) = [\varphi(t) \quad \dot{\varphi}(t) \quad \dot{\theta}(t)]$ - states

that minimize the cost function:

$$J = \int_0^{tf} \tau^2 \, dt$$

subject to (constraints):

- *Equations of motion:*

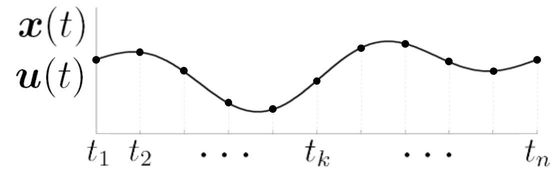
$$\tau - F_R \cdot R = [J_R + (M_r + M_c) \cdot R^2] \cdot \ddot{\theta} + (M_c \cdot R \cdot l \cdot \cos\varphi) \cdot \ddot{\varphi} - M_c \cdot R \cdot l \cdot \dot{\varphi}^2 \cdot \sin\varphi$$

$$-\tau = (M_c \cdot l \cdot R \cdot \cos\varphi) \cdot \ddot{\theta} + (J_c + M_c \cdot l^2) \cdot \ddot{\varphi} - M_c \cdot g \cdot l \cdot \sin\varphi$$

Solution of Optimal Control Problem (Direct Collocation)

Optimal Control Problem $\Longrightarrow$ Nonlinear Programming

Find $\boldsymbol{u}(t)$ $\boldsymbol{x}(t)$



$$\boldsymbol{x}_k = \boldsymbol{x}(t_k) \quad \boldsymbol{u}_k = \boldsymbol{u}(t_k)$$

system dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$$

$$\frac{\boldsymbol{x}_k - \boldsymbol{x}_{k-1}}{\Delta t} = \boldsymbol{f}_k \quad k = 1 \ldots n-1$$

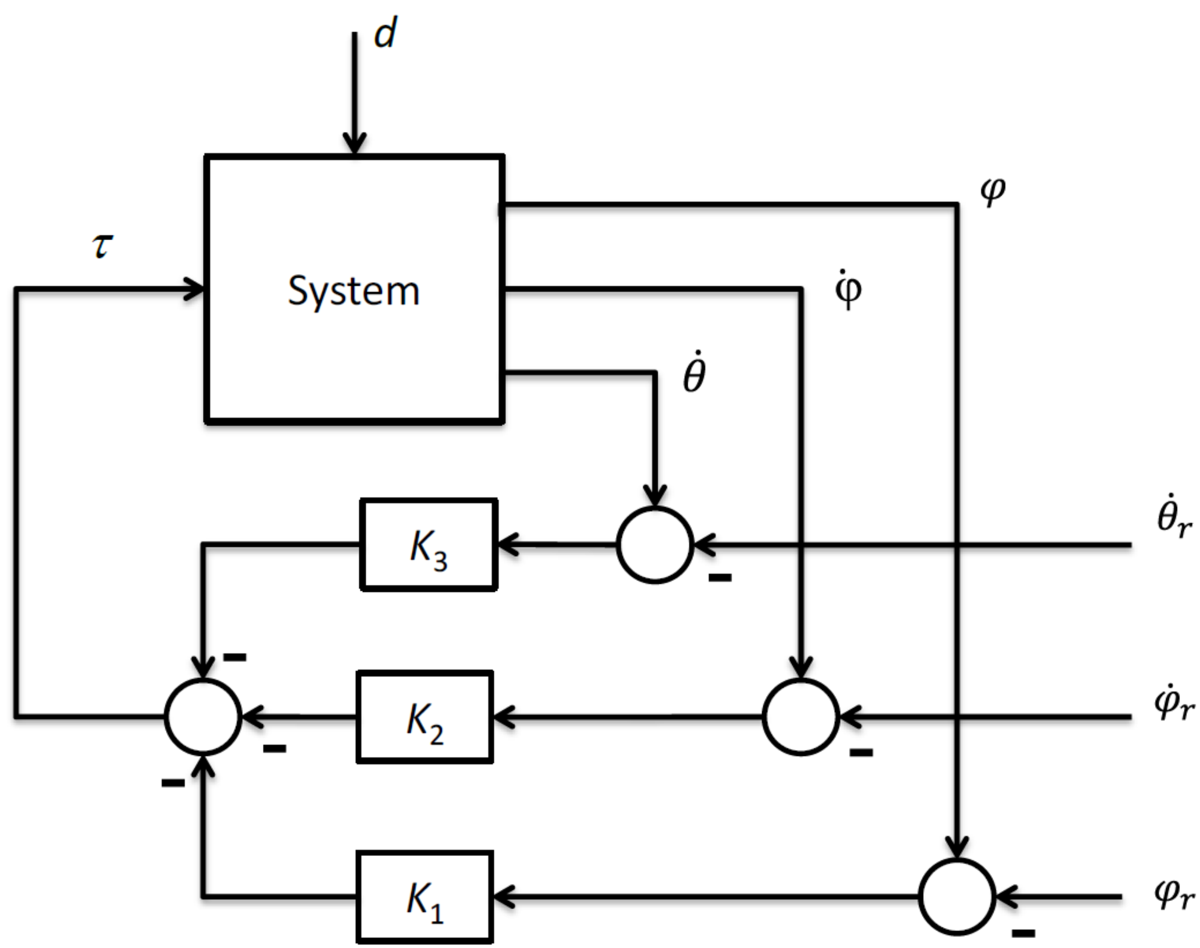
(Kaplan and Heegaard, J. Biomech., 2001); (Ackermann and van den Bogert, J. Biomech., 2010); (Lee and Umberger, PeerJ, 2016)

Solver: PROPT/SNOPT (Tomlab)

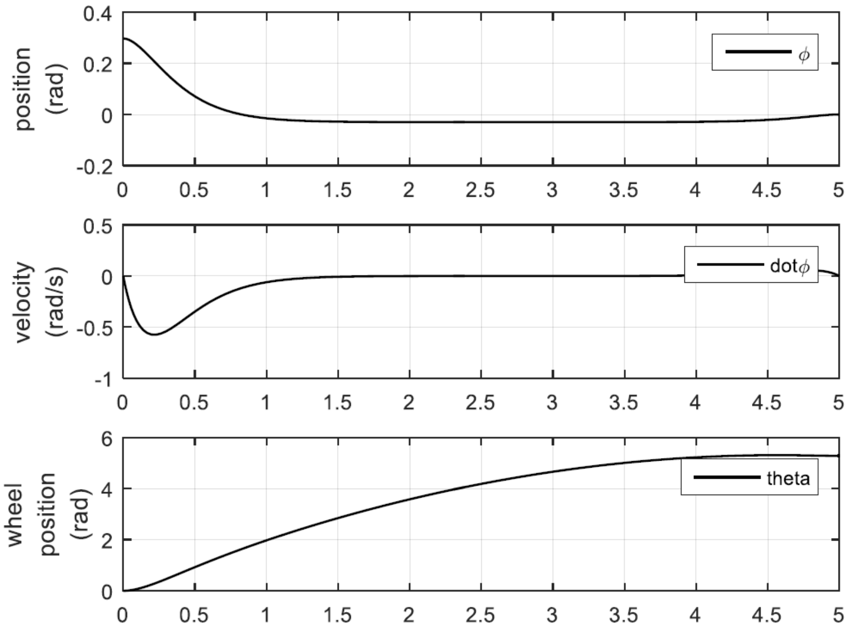
PROPT – transcription

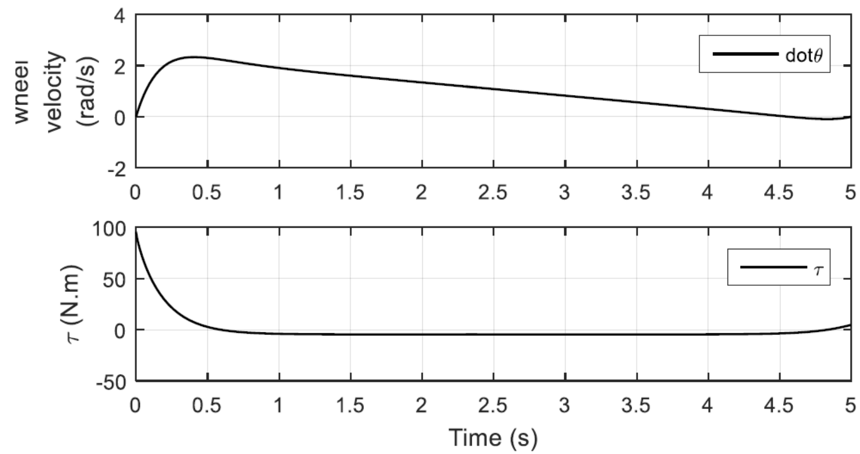
SNOPT – large-scale optimization

Closed-Loop Controller



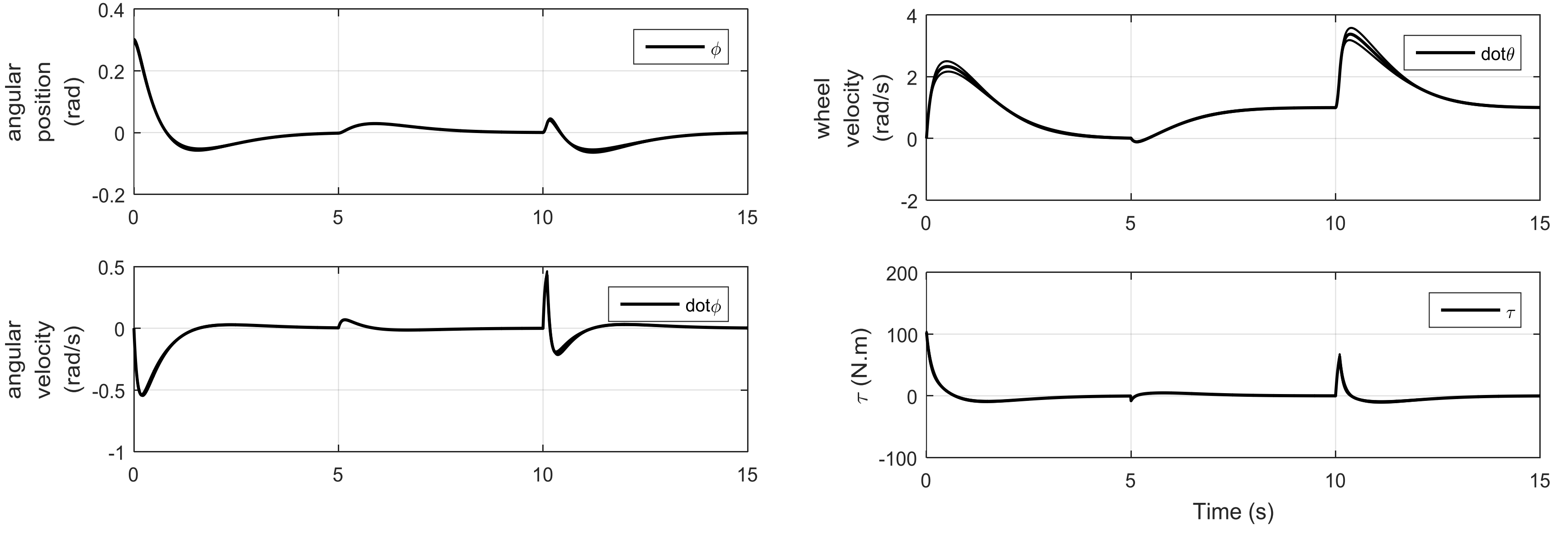
Results: Reference Patterns





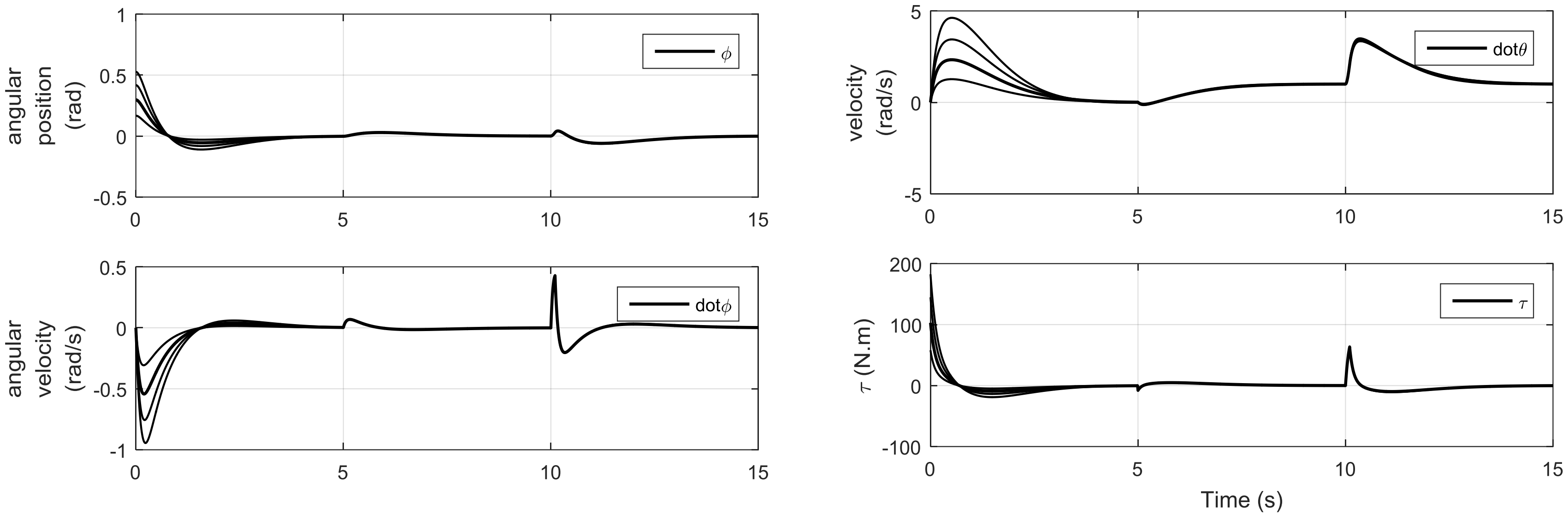
Closed-Loop Response

Different Statures: 1.6 m, 1.75 m and 1.90 m



Closed-Loop Response

Different fore-aft adjustments: -0.05 m, 0.00 m and 0.05 m



Discussion and Conclusions

- The modelling approach allowed for global changes in joint stiffness, independent from muscle properties;
- The optimal control framework was shown to have great potential in predicting gait pattern changes in musculoskeletal properties due to aging or desease;
- Observed changes in muscle activation patterns due to stiffening of ankle joint and decrease in muscle force capacity is consistent with findings in the literature;
- Future work: apply the presented approach to investigate typical changes in gait patterns of neuropathic diabetic patients.

Thank you!

Conselho Nacional de Desenvolvimento Científico e Tecnológico

centro universitário FEI